A model for a labyrinth seal of arbitrary shape under stationary conditions

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Although many investigators have examined the leakage from labyrinth seals of turbo-machinery a reliable prediction of the effect of various labyrinth parameters on leakage rate has not been reported so far. The theoretical models for the seals proposed in most of the previous literature provided results with considerable differences and contradictions in some cases. Different geometrical configurations of the seal have been considered in the literature for example the up step, the down step, the grooved shaft, the grooved casing, the staggered ...etc. Despite the importance of the effect of the geometrical configuration on the seal performance it is far from being examined with the care it merits. The aim of this paper is to provide a theoretical model for the flow in a labyrinth seal of arbitrary geometrical configuration and to study the different shape parameters. For the present model it is assumed from the outset that the transverse static pressure gradient inside the seal is negligibly small and the seal size is small compared to shaft radius. Under stationary conditions a coordinate transformation is used for the mathematical model and solutions to the momentum and continuity equations are obtained using the finite difference technique for the leakage rate across the seal. Case studies for single seals and multiple seals having non-linear solid boundaries are given. The effect of the number of cavities on the performance of the seal is clearly demonstrated. The effects of surface shape, maximum height deviation to clearance ratio and the clearance to seal length ratio on the leakage rate are also discussed.

بالرغم من قيام العديد من الباحثين بفحص ودراسة التسرب خلال الموانع اللابيرنثية للآلات الدوارة إلا أن تأثير شكل مانع التسرب على معدل السريان لم يقدم بشكل كاف ، فالعديد من النماذج النظرية آلتي تمت دراستها في الأبحاث السابقة قدمت نتائج ذات فروق متباينة أحيانا ومتعارضة أحيانا أخرى. يهتم هذا البحث بتقديم نموذج رياضي لسريان المائع خلال مانع تسرب ذو شكل اختياري متغير في الاتجاه المحوري ودراسة تأثير معاملات تحديد الشكل على معدل السريان ، كما يقوم البحث بدراسة تأثير كل من مانع التسرب المنفرد ومانع التسرب المتعدد والذي يتغير شكله في الاتجاه المحوري على أداء مانع التسرب.

Keywords: Labyrinth seals, Contact-less seals, Fluid seals, Models for seals, Cavities

1. Introduction

The problem of leakage through labyrinth seals has been subjected to many investigations. Several theoretical models have been put forward and some experimental works have been performed in order to study the various parameters affecting the leakage rate. Many of the previous theoretical models of the problem were found to provide results with discrepancies and contradictions [1]. Some predictions for the turbulent flow through labyrinth seals were given by Stoff [2] and Rhode et al. [3-4]. For the laminar case the performance and dynamic characteristics of labyrinth seals of the grooved shaft type was analyzed by El-Gamal et al. [5]. Their analysis dealt with the effect of lateral misalignment, which was shown to produce significant lateral forces on the seal causing inevitable vibrations. Some other investigations on the effect of geometrical shape of a single seal, rotation of the shaft, and blowing through the seal gland were also reported by El-Gamal et al. [6-7]. More complex geometrical configurations are required to be analyzed for the sake of improving the performance and dynamic characteristics of labyrinth seals. It is the aim of this paper to introduce a theoretical model for labyrinth seals of arbitrary shapes provided that the size of the seal is small compared to the shaft radius.

2. Analysis

The arbitrary geometry of the seal and the coordinate system are shown in fig. 1. The equations of motion for a steady two-dimensional incompressible and isoviscous fluid flowing across a stationary labyrinth seal in the absence of swirl motion may be written in Cartesian coordinates as,

$$\left.\begin{aligned} & v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\frac{\partial^2 w}{\partial y^2} \\ & 0=-\frac{1}{\rho}\frac{\partial p}{\partial y} \\ & \frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \end{aligned}\right\} \quad (1)$$

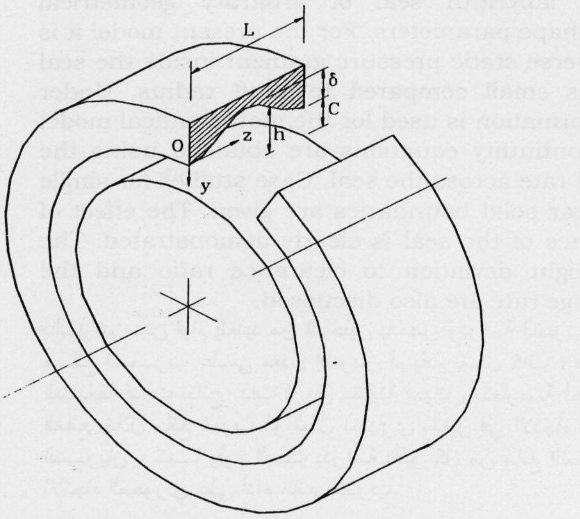

Fig. 1. The labyrinth seal geometry and the coordinate system (sectioned for clarity).

It is also assumed from the outset that the change of static transverse pressure gradient in the seal is negligibly small and that the seal size is small compared to the radius of the shaft. Introducing the following dimensionless variables:

$$\bar{v}=v\left(\frac{c}{\nu}\right),\ \bar{w}=w\left(\frac{c}{\nu}\right),\ \bar{p}=p\left(\frac{c^2}{\rho\,\nu^2}\right),\ \bar{y}=\frac{y}{c},\ \bar{z}=\frac{z}{L},\ h^*=\frac{h}{c}$$

Making the transformation of coordinates: $\eta=\dfrac{\bar{y}}{h^*}$, $\varsigma=\bar{z}$ we get,

$$\left.\begin{aligned} & \frac{\bar{v}}{h^*}\frac{\partial\bar{w}}{\partial\eta}+\bar{w}\left(\frac{c}{L}\right)\left[\frac{\partial\bar{w}}{\partial\varsigma}-\frac{\eta}{h^*}\frac{dh^*}{d\varsigma}\frac{\partial\bar{w}}{\partial\eta}\right] =-\left(\frac{c}{L}\right)\frac{\partial\bar{p}}{\partial\varsigma}+\frac{1}{h^{*2}}\frac{\partial^2\bar{w}}{\partial\eta^2} && \text{(a)} \\ & 0=\frac{\partial\bar{p}}{\partial\eta} && \text{(b)} \\ & \frac{1}{h^*}\frac{\partial\bar{v}}{\partial\eta}+\left(\frac{c}{L}\right)\left[\frac{\partial\bar{w}}{\partial\varsigma}-\frac{\eta}{h^*}\frac{dh^*}{d\varsigma}\frac{\partial\bar{w}}{\partial\eta}\right]=0 && \text{(c)} \end{aligned}\right\} \quad (2)$$

As a first approximation for the seal model it is assumed that inertia forces are small compared to viscous forces and consequently eq. (2-a) can be easily integrated twice with respect to η subject to the boundary conditions,

$$\bar{w}(0,\varsigma)=\bar{w}(1,\varsigma)=0\,.$$

The approximate velocity components become,

$$\left.\begin{aligned} & \bar{w}=\frac{1}{2}\left(\frac{c}{L}\right)h^{*2}\frac{d\bar{p}}{d\varsigma}\left(\eta^2-\eta\right) \\ & \bar{v}=-\frac{1}{2}\left(\frac{c}{L}\right)^2\left[-h^{*2}\frac{dh^*}{d\varsigma}\frac{d\bar{p}}{d\varsigma}\frac{\eta^2}{2}+h^{*3}\frac{d^2\bar{p}}{d\varsigma^2}\left(\frac{\eta^3}{3}-\frac{\eta^2}{2}\right)\right] \end{aligned}\right\}. \quad (3)$$

Back substitution into eq. (2), taking the spatial average in the transverse direction and integrating the pressure twice with respect to ς subject to the boundary conditions: $\bar{p}(0)=\Delta\bar{p}$, $\bar{p}(1)=0$, the following expression for the static pressure distribution across the seal is obtained,

$$\bar{p}=-\Delta\bar{p}^{*}\int_{1}^{\varsigma}\frac{d\varsigma}{h^{*3}}$$

with $\Delta p^* = \left(\Delta \bar{p} \Big/ \int_0^1 \frac{d\varsigma}{h^{*3}} \right)$.

The pressure gradient $\frac{d\bar{p}}{d\varsigma}$ will then be,

$$\frac{d\bar{p}}{d\varsigma} = -\frac{\Delta p^*}{h^{*3}}. \tag{5}$$

The approximate eq. (3) for the velocity components $\bar{v}$ and $\bar{w}$ may be then written as,

$$\left.\begin{aligned} \bar{w} &= -\frac{1}{2}\left(\frac{c}{L}\right)\frac{1}{h^*}\left(\eta^2 - \eta\right)\Delta p^* \\ \bar{v} &= -\frac{1}{2}\left(\frac{c}{L}\right)^2 \frac{1}{h^*}\frac{dh^*}{d\varsigma}\left(\eta^3 - \eta^2\right)\Delta p^* \end{aligned}\right\}. \tag{6}$$

A better improvement to the velocity components $\bar{v}$ and $\bar{w}$ may be obtained by substitution from eqs. (5) and (6) into the inertia terms on the left hand side of eq. (2-a), using eq. (2-c) and upon integration we have,

$$\left.\begin{aligned} \bar{w} &= -\frac{1}{2}\left(\frac{c}{L}\right)\frac{\Delta p^*}{h^*} \\ &\left[\left(\eta^2 - \eta\right) + \frac{1}{2}\left(\frac{c}{L}\right)^2 \frac{\Delta p^*}{h^{*2}}\frac{dh^*}{d\varsigma}\left(\frac{\eta^6}{30} - \frac{\eta^5}{10} + \frac{\eta^4}{12} - \frac{\eta}{60}\right)\right] \\ \bar{v} &= -\frac{1}{2}\left(\frac{c}{L}\right)^2 \frac{\Delta p^*}{h^*}\frac{dh^*}{d\varsigma}\left(\eta^3 - \eta^2\right) \\ &+ \frac{1}{4}\left(\frac{c}{L}\right)^4 \Delta p^{*2} \\ &\left[\left(\frac{1}{h^{*2}}\frac{d^2h^*}{d\varsigma^2} - \frac{3}{h^{*3}}\frac{dh^*}{d\varsigma}\right)\left(\frac{\eta^7}{210} - \frac{\eta^6}{60} + \frac{\eta^5}{60} - \frac{\eta^2}{120}\right)\right] \end{aligned}\right\}. \tag{7}$$

However, instead of repeating the procedure for obtaining improved eq. (7) and further expressions for the velocity components $\bar{v}$ and $\bar{w}$ in the seal it would be more convenient to confine the approximation to only a part of the inertia terms namely;

$$\frac{\bar{v}}{h^*}\frac{\partial \bar{w}}{\partial \eta} - \bar{w}\left(\frac{c}{L}\right)\frac{\eta}{h^*}\frac{dh^*}{d\varsigma}\frac{\partial \bar{w}}{\partial \eta},$$

and to solve the resulting equation numerically. This can be done by substituting from eq. (5) and (6) into eq. (2-a) to obtain a satisfactory approximation of eq. (2-a) as follows:

$$\left(\frac{c}{L}\right)\bar{w}\frac{\partial \bar{w}}{\partial \varsigma} = \left(\frac{c}{L}\right)\frac{\Delta p^*}{h^{*3}} + \frac{1}{h^{*2}}\frac{\partial^2 \bar{w}}{\partial \eta^2}. \tag{8}$$

Eq. (8) governs the flow inside the seal with the parametric dependence on the dimensionless pressure difference across the seal Δp^*. The boundary conditions for eq. (8) may be written as:

$$\bar{w}(0,\varsigma) = 0, \bar{w}(1,\varsigma) = 0, \left.\frac{\partial \bar{w}}{\partial \varsigma}\right|_{\varsigma=0} = 0.$$

With the additional condition from continuity consideration we therefore have,

$$\bar{W} = \int_0^1 \bar{w}(\eta, 0)\, d\eta = \int_0^1 \bar{w}(\eta, 1)\, d\eta$$

Where $\bar{W}$ is the mean leakage velocity.

Eq. (8) together with its boundary conditions are to be solved here by the aid of the finite difference approximation and using the successive over-relaxation iterative method. Solutions are obtained using a mesh size of 0.05 for both coordinates η and ς. A sinusoidal-shaped seal configuration (see fig. 1) is selected for the analysis having the form, $h^* = 1 + \left[\left(1 - \cos 2\pi n\varsigma\right)\delta / c\right]$, which includes single and multi-cavity seal depending on the value of n.

3. Results and discussion

Fig. 2 shows the relation between the dimensionless pressure drop across the seal

Δp^* and the dimensionless leakage velocity $\overline{W}$ for a single cavity seal and multi-cavity seals with $\delta/c = 10$ and $c/L = 0.05$. For small values of Δp^* the relation is approximately linear and the leakage decreases as the number of the cavities increased for the same pressure drop across the seal. The percentage deviation from linearity is pronounced for larger values of Δp^*.

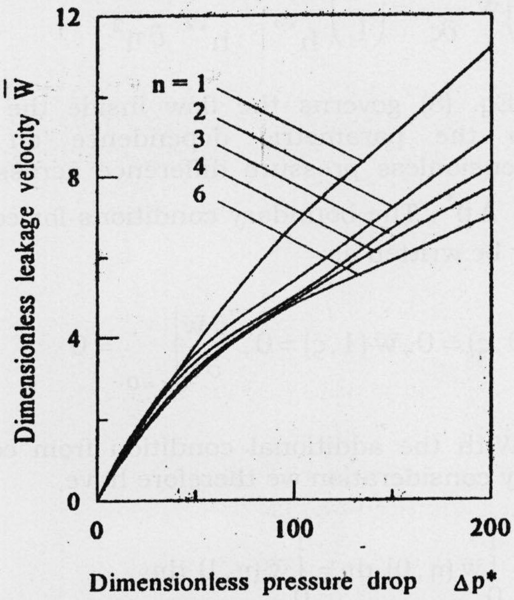

Fig. 2. The dimensionless pressure drop across the seal versus dimensionless leakage velocity for different number of cavities.

Increasing the number of cavities n still decreases the leakage velocity for the same pressure drop but the percentage reduction in leakage does decrease. Fig. 3 shows the effect of the maximum seal height deviation to clearance ratio δ/c on the performance characteristics of a single cavity seal with $c/L = 0.05$. A linear relationship between Δp^* and $\overline{W}$ is predicted here for the case of plain annular cavity ($\delta/c = 0$). As δ/c increases the leakage velocity decreases. Deviation from linearity takes place at relatively large values of δ/c. It is clear that the percentage reduction in leakage due to the increase in δ/c decreases as Δp^* decreases. In fig. 4, Δp^* is plotted versus $\overline{W}$ for different values of clearance to seal length ratio c/L. The smaller the value of c/L the lesser the leakage is from the seal. The decrease in leakage is more pronounced for small values of Δp^*

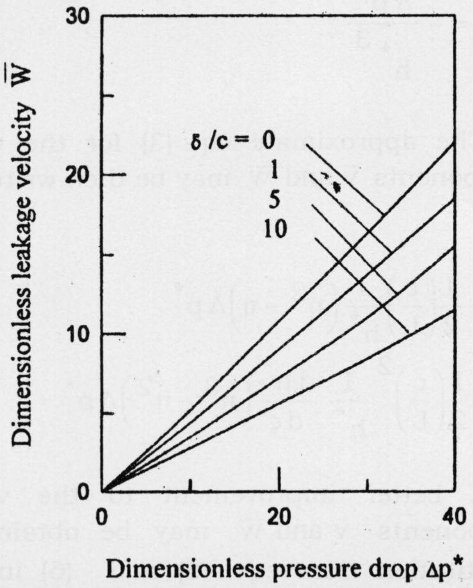

Fig. 3. The dimensionless pressure drop across the seal versus dimensionless leakage velocity for different values of δ/c.

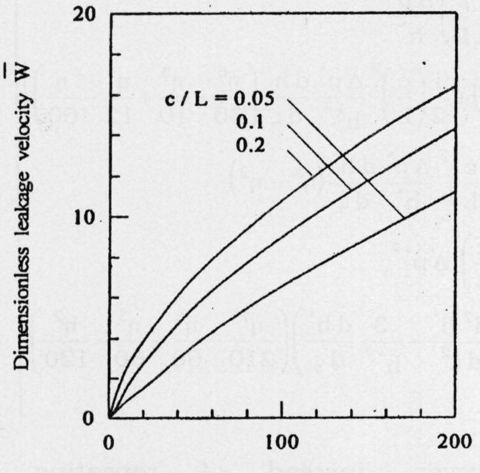

Fig. 4. The dimensionless pressure drop across the seal versus dimensionless leakage velocity for different values of c/L.

Fig. 5 is a reproduction of fig. 2 for the case of a single cavity ($n = 1$, $\delta/c = 10$) and

$c/L = 0.05$ with the addition of some of the results given in [7] for the sake of qualitative comparison. Although the case of a single rectangular grooved cavity examined in [7] is quite different geometry from the cavity considered here, there is a satisfactory qualitative agreement for the case when the aspect ratio of the rectangular cavity $(\delta + c)/L$ lies between 1 to 1.25. This is not surprising, however, as the rectangular cavity of order unity aspect ratio may be considered as geometrically equivalent to the wavy cavity presented here. Such interpretation is enough to support the results qualitatively based on physical ground but obviously not sufficient as far as the correctness of the numerical procedure is concerned.

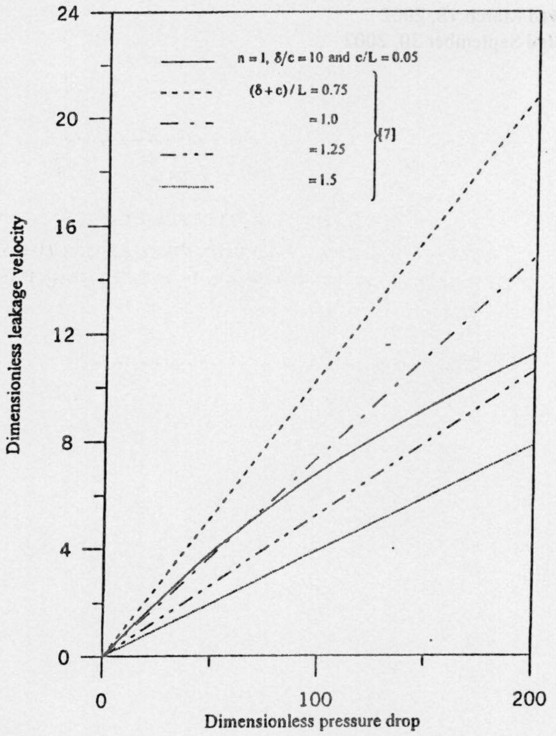

Fig. 5. Comparison between the present results and the results of El-Gamal et al. [7].

4. Conclusions

The mathematical model put forward in this work has provided theoretical predictions for the seal performance and the effect of various geometry parameters. Case studies for single and multi-cavity seals of sinusoidal-shaped geometry have shown that when the maximum seal height deviation to clearance ratio increases the average leakage velocity decreases especially at relatively large values of deviation and pressure drop across the seal. It has also been shown that as the seal clearance to length ratio decreases the leakage decreases especially for small values of pressure drop. Design curves are provided for single and multiple seals.

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Nomenclature

c	radial clearance, m.
h	seal shape function, m.
h^*	dimensionless shape function, $h^* = h/c$.
L	seal length, m.
N	waviness number.
p	fluid pressure, Pa.
$\overline{p}$	dimensionless fluid pressure, $\overline{p} = p\left(c^2/\rho \nu^2\right)$.
v, w	velocity components in y and z, m/s.
$\overline{v}, \overline{w}$	dimensionless velocity components, $\overline{v} = v(c/\nu), \overline{w} = w(c/\nu)$.
y, z	radial and axial coordinates, m.
$\overline{y}, \overline{z}$	dimensionless radial and axial. coordinates, $\overline{y} = y/c$, $\overline{z} = z/L$.

Greek symbols

δ	maximum seal height deviation, m.
ν	fluid kinematic viscosity, m^2/s.
ρ	fluid density, kg/m^3.
η, ζ	dimensionless coordinates, $\eta = y/h^*, \varsigma = \overline{z}$.

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