# Flow induced vibrations in valves

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The control of vibrations induced in hydraulic systems is becoming one of the most important problems facing engineers and research workers. Significant vibration sources come from fluid valves present in these systems. Suitable mathematical models and advanced numerical techniques have provided good opportunity to examine the flow induced-vibration in fluid valves thoroughly and with good accuracy. The flow pulsation-induced vibrations in valves resulting from periodic variations of upstream fluid pressure are a particularly difficult problem because of the complexity of the geometry, the time dependence of the flow and the suitable turbulence model to be used. The study of fluid forces in valves at different values of Reynolds number and for large Strouhal number is required for controlling the vibrations in these valves. In this paper the vibration of a fluid disc valve is examined and factors controlling it are predicted. Some geometry factors and valve opening together with Reynolds and Strouhal numbers are found to have great influence on the amplitude of the exciting fluid forces and thereby on the vibration characteristics.

الصمامات المستخدمة في عدد من النظم الهيدروليكية تمثل مصدرا جوهريا للاهتزازات في تلك النظم ويعد التحكم فيها من اهم المشاكل التي تواجه العديد من الباحثين في هذا المجال. باستخدام النموذج الرياضي المناسب والتحليل العددي المتقدم فانه يمكن تقديم فحص مناسب للاهتزازات المتوادة في الصمامات المستخدمة للتحكم في سريان المائع. نظرا لعدم انتظام شكل الصمام وتغير خصائص المائع خلال سريانه مع الزمن واختيار نموذج السريان الاضطرابي يجعل دراسة الاهتزازات الناجمة عن سريان المائع خلال الصمام من المشاكل المعقدة. يقوم هذا البحث بدراسة القوى الناشئة عن سريان المائع خلال الصمام ذات القرص والتي تسبب تولد الاهتزازات، مع ايضاح العوامل التي تتحكم في هذه القوى. وقد وجد أن بعض معاملات تحديد الشكل وفتحة سريان المائع خلال الصمام مع كل من رقم رينولدز ورقم ستروهل لها تأثير جوهرى على ما القوى الديناميكية الناشئة وبالتالي خصائص الاهتزازات المتولدة.

**Keywords:** Flow-induced vibration, Vibration control, Models of valves, Modeling of fluid valves, Forces in valves

#### 1. Introduction

A great deal of research has been carried out on the flow behavior in fluid valves. This indicates that this type of flow is a highly complex process, which is strongly dependent on the details of the valve geometrical parameters and fluid operating conditions. Early works studied the flow in valves assuming it inviscid and irrotational [1-2]. The more recent development of computational fluid dynamics techniques has provided broad scope for study of valve flow. Hayashi et al. [3] studied the laminar flow through valves where they found the existence of jet separation and re-attachment. Pountney et al. [4] examined the turbulent case of flow in disc valves but made no experimental work to support their theoretical predictions. Fleming et al. [5],

Leschziner [6] and Vaughan et al [7] studied the turbulent flow through fluid valves and recirculating flows where they found considerable differences between the theoretical results and those obtained from experiments. They stated that the differences are due to the difficulty in selecting adequate turbulence model. They compared results for different models of turbulence and found to give considerable differences. They concluded that the k-E and most other practical formulations do not perform well in regions of recirculating flows and curved streamlines, which is always the case in the flow through fluid valves. In addition they noticed some computational inaccuracies appearing due to the improper of the governing equations formulation suitable for the numerical solution. An attempt to study the laminar case for valve

flow was made by Sharara, Saber and El-Gamal [8]. They considered the flow to be unsteady and studied the fluid forces on a disc valve. They assumed the Strouhal number and Reynolds number to be large enough to neglect the coupling between the induced pressure field and the flow pattern inside the valve. They concluded that some geometry factors as well as valve opening affect the fluid forces on the valve considerably. It is the aim of this paper to include the effect of the product of Reynolds number Re and the Strouhal number S on the unsteady flow inside the valve and thereby the hydrodynamic forces causing valve vibrations.

### 2. Analysis

A disc valve geometry is selected for the analysis and is shown in fig. 1. The flow through the valve is assumed laminar, incompressible and axisymetric (no swirl motion). Using the cylindrical polar coordinates the momentum and continuity equations may be written as,

$$\rho \left( \frac{\partial \mathbf{v}}{\partial \mathbf{t}} + \mathbf{v} \frac{\partial \mathbf{v}}{\partial \mathbf{r}} + \mathbf{w} \frac{\partial \mathbf{v}}{\partial \mathbf{z}} \right) = -\frac{\partial \mathbf{p}}{\partial \mathbf{r}} + \mathbf{v} \left( \frac{\partial^2 \mathbf{v}}{\partial \mathbf{r}^2} + \frac{1}{\mathbf{r}} \frac{\partial \mathbf{v}}{\partial \mathbf{r}} - \frac{\mathbf{v}}{\mathbf{r}^2} + \frac{\partial^2 \mathbf{v}}{\partial \mathbf{z}^2} \right), \tag{1}$$

$$\rho \left( \frac{\partial \mathbf{w}}{\partial \mathbf{t}} + \mathbf{v} \frac{\partial \mathbf{w}}{\partial \mathbf{r}} + \mathbf{w} \frac{\partial \mathbf{w}}{\partial \mathbf{z}} \right)$$

$$= -\frac{\partial \mathbf{p}}{\partial \mathbf{z}} + \mu \left( \frac{\partial^2 \mathbf{w}}{\partial \mathbf{r}^2} + \frac{1}{\mathbf{r}} \frac{\partial \mathbf{w}}{\partial \mathbf{r}} + \frac{\partial^2 \mathbf{w}}{\partial \mathbf{z}^2} \right). \tag{2}$$

$$\frac{\partial \mathbf{v}}{\partial \mathbf{r}} + \frac{\mathbf{v}}{\mathbf{r}} + \frac{\partial \mathbf{w}}{\partial \mathbf{z}} = 0.$$
 (3)

The following dimensionless variables are introduced:

$$p^* = \frac{p}{\rho U^2}, v^* = \frac{v}{U}, w^* = \frac{w}{U}, t^* = \alpha t,$$
 $z^* = \frac{z}{\ell}, r^* = \frac{r}{R}$ 

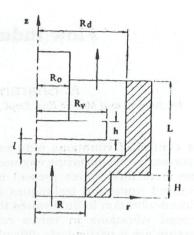


Fig. 1. The system of coordinates and valve geometry.

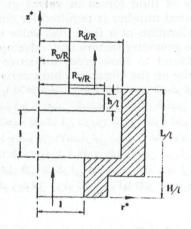


Fig. 2. Non-dimensional valve geometric parameters.

Momentum and continuity equations in nondimensional form become:

$$\frac{\partial \mathbf{v}^{*}}{\partial \mathbf{t}^{*}} + \frac{1}{S} \left( \mathbf{v}^{*} \frac{\partial \mathbf{v}^{*}}{\partial \mathbf{r}^{*}} + \left( \frac{R}{\ell} \right) \mathbf{w}^{*} \frac{\partial \mathbf{v}^{*}}{\partial \mathbf{z}^{*}} \right) = -\frac{1}{S} \frac{\partial \mathbf{p}^{*}}{\partial \mathbf{r}^{*}} + \frac{1}{R_{e} S}$$

$$\left( \frac{\partial^{2} \mathbf{v}^{*}}{\partial \mathbf{r}^{*2}} + \frac{1}{\mathbf{r}^{*}} \frac{\partial \mathbf{v}^{*}}{\partial \mathbf{r}^{*}} - \frac{\mathbf{v}^{*}}{\mathbf{r}^{*2}} + \left( \frac{R}{\ell} \right)^{2} \frac{\partial^{2} \mathbf{v}^{*}}{\partial \mathbf{z}^{*2}} \right). \tag{4}$$

$$\frac{\partial \mathbf{w}^{\star}}{\partial \mathbf{t}^{\star}} + \frac{1}{S} \left( \mathbf{v}^{\star} \frac{\partial \mathbf{w}^{\star}}{\partial \mathbf{r}^{\star}} + \left( \frac{R}{\ell} \right) \mathbf{w}^{\star} \frac{\partial \mathbf{w}^{\star}}{\partial \mathbf{z}^{\star}} \right) = -\frac{1}{S} \left( \frac{R}{\ell} \right) \frac{\partial \mathbf{p}^{\star}}{\partial \mathbf{z}^{\star}} + \frac{1}{R_{e} S}$$

$$\left( \frac{\partial^{2} \mathbf{w}^{\star}}{\partial \mathbf{r}^{\star^{2}}} + \frac{1}{\mathbf{r}^{\star}} \frac{\partial \mathbf{w}^{\star}}{\partial \mathbf{r}^{\star}} + \left( \frac{R}{\ell} \right)^{2} \frac{\partial^{2} \mathbf{w}^{\star}}{\partial \mathbf{z}^{\star^{2}}} \right). \tag{5}$$

$$\frac{\partial \mathbf{v}^{\star}}{\partial \mathbf{r}^{\star}} + \frac{\mathbf{v}^{\star}}{\mathbf{r}^{\star}} + \left(\frac{\mathbf{R}}{\ell}\right) \frac{\partial \mathbf{w}^{\star}}{\partial \mathbf{z}^{\star}} = 0. \tag{6}$$

Where,

$$S = \frac{\alpha R}{U}$$
 and  $R_e = \frac{\rho U R}{\mu}$  are the Strouhal and Reynolds numbers, respectively.

Assuming that the Strouhal number is very large S >> 1, eqs. (4) and (5) may be written as,

$$\frac{\partial \mathbf{v}^{\star}}{\partial t^{\star}} = -\frac{1}{S} \frac{\partial \mathbf{p}^{\star}}{\partial \mathbf{r}^{\star}} + \frac{1}{R_{e} S} \left( \frac{\partial^{2} \mathbf{v}^{\star}}{\partial \mathbf{r}^{\star 2}} + \frac{1}{\mathbf{r}^{\star}} \frac{\partial \mathbf{v}^{\star}}{\partial \mathbf{r}^{\star}} - \frac{\mathbf{v}^{\star}}{\mathbf{r}^{\star}} + \left( \frac{R}{\ell} \right)^{2} \frac{\partial^{2} \mathbf{v}^{\star}}{\partial \mathbf{z}^{\star 2}} \right), \tag{7}$$

$$\frac{\partial w^{*}}{\partial t^{*}} = -\frac{1}{S} \left( \frac{R}{\ell} \right) \frac{\partial p^{*}}{\partial z^{*}} + \frac{1}{R_{e} S} \left( \frac{\partial^{2} w^{*}}{\partial r^{*2}} + \frac{1}{r^{*}} \frac{\partial w^{*}}{\partial r^{*}} + \left( \frac{R}{\ell} \right)^{2} \frac{\partial^{2} w^{*}}{\partial z^{*2}} \right)$$
(8)

Using eqs. (6), (7) and (8), the full pressure equation can be obtained as,

$$\frac{1}{r^{\star}} \frac{\partial}{\partial r^{\star}} \left( r^{\star} \frac{\partial p^{\star}}{\partial r^{\star}} \right) + \left( \frac{R}{\ell} \right)^{2} \frac{\partial^{2} p^{\star}}{\partial z^{\star^{2}}} = 0.$$
 (9)

To solve eq. (9) we need complete information about the velocity field  $w^*$  and  $v^*$ .

The boundary conditions may also be given in the following form:

At 
$$r^* = 1$$
 and  $z^* \le \overline{H}$  OR  
 $r^* = \overline{R}_d$  and  $\overline{H} \le z^* \le (\overline{H} + \overline{L})$ :

$$v^* = w^* = 0$$
 with  $\frac{\partial p^*}{\partial r^*} = \frac{1}{R_e} \frac{\partial^2 v^*}{\partial r^{*2}}$ .

At 
$$1 \le r^* \le \overline{R}_d$$
 and  $z^* \le \overline{H}$  OR  $\overline{R}_i \le r^* \le \overline{R}_v$  and  $z^* = (\overline{H} + 1)$  OR  $\overline{R}_0 \le r^* \le \overline{R}_v$  and  $z^* = (\overline{H} + \overline{h})$ :

(6) 
$$v^* = w^* = 0 \text{ with } \frac{\partial p^*}{\partial z^*} = \frac{1}{R_e} \left(\frac{R}{\ell}\right) \frac{\partial^2 w^*}{\partial z^{*2}}$$

At 
$$\overline{R}_i \le r^* \le 1$$
 and  $z^* = 0$ :

$$v^* = 0$$
,  $\frac{\partial w^*}{\partial z^*} = 0$  and  $p^* = P^*(t^*)$ 

At 
$$\overline{R}_o \le r^* \le \overline{R}_d$$
 and  $z^* = (\overline{H} + \overline{L})$ :

$$v^* = 0, \frac{\partial w^*}{\partial z^*} = 0$$
 and  $p^* = 0$ 

It is expected from practice that  $\frac{\partial p^*}{\partial r^*} << \frac{\partial p^*}{\partial z^*}$  especially at small values of  $\frac{\ell}{R}$ . So as a first approximation the variation of  $p^*$  with respect to  $r^*$  is considered function of  $z^*$  and  $t^*$  only. Therefore, as a first approximation and using eq. (9),  $\frac{\partial^2 p^*}{\partial z^*} = 0$ . Integrating twice to get the

following approximate expression for the pressure,

$$p^* \approx P^* \left(t^*\right) \left[1 - z^* \left(\frac{\ell}{H + L}\right)\right] \tag{10}$$

Substituting from (10) into (8), we have

$$\frac{\partial w^{\star}}{\partial t^{\star}} = \frac{1}{S} \left( \frac{R}{\ell} \right) \left( \frac{\ell}{H + L} \right) P^{\star} \left( t^{\star} \right) + \frac{1}{R_{e} S} \left( \frac{\partial^{2} w^{\star}}{\partial r^{\star^{2}}} + \frac{1}{r^{\star}} \frac{\partial w^{\star}}{\partial r^{\star}} + \left( \frac{R}{\ell} \right)^{2} \frac{\partial^{2} w^{\star}}{\partial z^{\star^{2}}} \right) \tag{11}$$

$$\mathbf{v}^{*} = -\left(\frac{R}{\ell}\right) \frac{1}{r^{*}} \int_{\overline{R}_{i}}^{r^{*}} \operatorname{or} \overline{R}_{v} \operatorname{or} \overline{R}_{d} r^{*} \left(\frac{\partial \mathbf{w}^{*}}{\partial \mathbf{z}^{*}}\right) dr^{*}. \tag{12}$$

A further substitution can be made into eqs. (11) and (12) so that,

$$w^* = R_e w', v^* = R_e v'$$
 and  $t^* = R_e St'$ 

Thus eqs. (11) and (12) become,

$$\frac{\partial w'}{\partial t'} = \left(\frac{R}{\ell}\right) \left(\frac{\ell}{H+L}\right) P^{\star}(t') + \left(\frac{\partial^2 w'}{\partial r^{\star^2}} + \frac{1}{r^{\star}} \frac{\partial w'}{\partial r^{\star}} + \left(\frac{R}{\ell}\right)^2 \frac{\partial^2 w'}{\partial z^{\star^2}}\right) (13) \qquad \frac{1}{r^{\star}} \frac{\partial}{\partial r^{\star}} \left(r^{\star} \frac{\partial P_r}{\partial r^{\star}}\right) + \left(\frac{R}{\ell}\right)^2 \frac{\partial^2 P_r}{\partial z^{\star^2}} = 0.$$

$$\mathbf{v}' = -\left(\frac{R}{\ell}\right) \frac{1}{r^*} \int_{\overline{R}_i}^{r^*} \operatorname{or} \overline{R}_v \operatorname{or} \overline{R}_d r^* \left(\frac{\partial \mathbf{w}'}{\partial \mathbf{z}^*}\right) d\mathbf{r}^*. \tag{14} \qquad \frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* \frac{\partial P_i}{\partial r^*}\right) + \left(\frac{R}{\ell}\right)^2 \frac{\partial^2 P_i}{\partial z^{*2}} = 0.$$

It seems convenient to adopt the complex notation for eqs. (13) and (14) by writing,

$$P^* = \lambda e^{i \operatorname{Re S} t'}$$
,  $p^* = P(r^*, z^*) e^{i \operatorname{Re S} t'}$   
 $w' = W(r^*, z^*) e^{i \operatorname{Re S} t'}$ ,  $v' = V(r^*, z^*) e^{i \operatorname{Re S} t'}$ 

and substituting into eqs. (13) and (14) to get,

i Re SW = 
$$\left(\frac{R}{\ell}\right) \left(\frac{\ell}{H+L}\right) \lambda + \left(\frac{\partial^2 W}{\partial r^{\star 2}} + \frac{1}{r^{\star}} \frac{\partial W}{\partial r^{\star}} + \left(\frac{R}{\ell}\right)^2 \frac{\partial^2 W}{\partial z^{\star 2}}\right)$$
. (15)

$$V = -\left(\frac{R}{\ell}\right) \frac{1}{r^*} \int_{\overline{R}_i}^{r^*} \operatorname{or} \overline{R}_v \operatorname{or} \overline{R}_d r^* \left(\frac{\partial W}{\partial z^*}\right) dr^*.$$
 (16)

Let.

 $W = W_r + iW_i$  and  $V = V_r + iV_i$  we thus have,

$$-\text{ReSW}_{i} = \left(\frac{R}{\ell}\right) \left(\frac{\ell}{H+L}\right) \lambda + \left(\frac{\partial^{2} W_{r}}{\partial r^{*2}} + \frac{1}{r^{*2}} \frac{\partial W_{r}}{\partial r^{*2}} + \left(\frac{R}{\ell}\right)^{2} \frac{\partial^{2} W_{r}}{\partial z^{*2}}\right) \quad (17)$$

$$ReSW_{r} = \left(\frac{\partial^{2}W_{i}}{\partial r^{*2}} + \frac{1}{r^{*}}\frac{\partial W_{i}}{\partial r^{*}} + \left(\frac{R}{\ell}\right)^{2}\frac{\partial^{2}W_{i}}{\partial z^{*2}}\right)$$
(18)

and

$$V_{r} = -\left(\frac{R}{\ell}\right) \frac{1}{r^{*}} \int_{\bar{R}_{i}}^{r^{*}} \operatorname{or} \bar{R}_{v} \operatorname{or} \bar{R}_{d} r^{*} \left(\frac{\partial W_{r}}{\partial z^{*}}\right) dr^{*} . \tag{19}$$

$$V_{i} = -\left(\frac{R}{\ell}\right)\frac{1}{r^{*}}\int_{\overline{R}_{i}}^{r^{*}} \operatorname{or} \overline{R}_{v} \operatorname{or} \overline{R}_{d} r^{*}\left(\frac{\partial W_{i}}{\partial z^{*}}\right) dr^{*}. \tag{20}$$

Substituting into eq. (9) we have,

$$\frac{1}{r^{\star}} \frac{\partial}{\partial r^{\star}} \left( r^{\star} \frac{\partial P_{r}}{\partial r^{\star}} \right) + \left( \frac{R}{\ell} \right)^{2} \frac{\partial^{2} P_{r}}{\partial z^{\star^{2}}} = 0.$$
 (21)

$$\frac{1}{r^{\star}} \frac{\partial}{\partial r^{\star}} \left( r^{\star} \frac{\partial P_{1}}{\partial r^{\star}} \right) + \left( \frac{R}{\ell} \right)^{2} \frac{\partial^{2} P_{1}}{\partial z^{\star^{2}}} = 0.$$
 (22)

Eqs. (17) and (18) are to be solved prior to solving eqs. (21) and (22). The solution for the pressure distribution inside the valve will be in the form.

$$p^* = \text{Real}[P_r + iP_i]e^{i\text{ReSt'}}$$
 (23)

The velocity components may be obtained from.

$$w' = \text{Real}\left[ (W_r + i W_i) e^{i \text{ReSt'}} \right] , \qquad (24)$$

$$v' = \operatorname{Real}\left[\left(V_{r} + i V_{i}\right) e^{i \operatorname{Re} S t'}\right] \qquad (25)$$

Let,

$$\begin{split} \hat{W}_r &= \frac{W_r}{\lambda}, \hat{V}_r = \frac{V_r}{\lambda}, \ \hat{W}_i = \frac{W_i}{\lambda}, \\ \hat{V}_i &= \frac{V_i}{\lambda}, \quad \hat{P}_r = \frac{P_r}{\lambda}, \ \hat{P}_i = \frac{P_i}{\lambda} \end{split}$$

and substituting into eqs. (17) and (18), we get

$$-\text{ReS}\hat{W}_{i} = \left(\frac{R}{\ell}\right) \left(\frac{\ell}{H+L}\right) + \left(\frac{\partial^{2}\hat{W}_{r}}{\partial r^{*2}} + \frac{1}{r^{*}}\frac{\partial\hat{W}_{r}}{\partial r^{*}} + \left(\frac{R}{\ell}\right)^{2}\frac{\partial^{2}\hat{W}_{r}}{\partial z^{*2}}\right) (26)$$

$$\operatorname{ReS} \hat{W}_{r} = \left( \frac{\partial^{2} \hat{W}_{i}}{\partial r^{\star^{2}}} + \frac{1}{r^{\star}} \frac{\partial \hat{W}_{i}}{\partial r^{\star}} + \left( \frac{R}{\ell} \right)^{2} \frac{\partial^{2} \hat{W}_{i}}{\partial z^{\star^{2}}} \right), \tag{27}$$

$$\hat{V}_{r} = -\left(\frac{R}{\ell}\right) \frac{1}{r^{\star}} \int_{\overline{R}_{i}}^{r^{\star}} \operatorname{or} \overline{R}_{v} \operatorname{or} \overline{R}_{d} r^{\star} \left(\frac{\partial \hat{W}_{r}}{\partial z^{\star}}\right) dr^{\star}, \qquad (28)$$

$$\hat{V}_{i} = -\left(\frac{R}{\ell}\right) \frac{1}{r^{*}} \int_{\overline{R}_{i}}^{r^{*}} \operatorname{or} \overline{R}_{v} \operatorname{or} \overline{R}_{d} r^{*} \left(\frac{\partial \hat{W}_{i}}{\partial z^{*}}\right) dr^{*}.$$
 (29)

Substituting into eqs. (21) and (22) we have,

$$\frac{1}{r^{\star}} \frac{\partial}{\partial r^{\star}} \left( r^{\star} \frac{\partial \hat{P}_{r}}{\partial r^{\star}} \right) + \left( \frac{R}{\ell} \right)^{2} \frac{\partial^{2} \hat{P}_{r}}{\partial z^{\star^{2}}} = 0.$$
 (30)

$$\frac{1}{r^{*}} \frac{\partial}{\partial r^{*}} \left( r^{*} \frac{\partial \hat{P}_{i}}{\partial r^{*}} \right) + \left( \frac{R}{\ell} \right)^{2} \frac{\partial^{2} \hat{P}_{i}}{\partial z^{*2}} = 0.$$
 (31)

The boundary conditions for eqs. (26), (27), (30) and (31) become:

At 
$$r^* = 1$$
 and  $z^* \le \overline{H}$  OR  $r^* = \overline{R}_d$  and  $\overline{H} \le z^* \le (\overline{H} + \overline{L})$ 

$$\hat{W}_r = \hat{W}_i = 0 \text{ with } \frac{\partial \hat{P}_r}{\partial r^*} = \frac{1}{R_e} \frac{\partial^2 \hat{V}_r}{\partial r^{*2}}$$
and  $\frac{\partial \hat{P}_i}{\partial r^*} = \frac{1}{R_e} \frac{\partial^2 \hat{V}_i}{\partial r^{*2}}$ 

At  $1 \le r^* \le \overline{R}_d$  and  $z^* = \overline{H}$  OR  $\overline{R}_i \le r^* \le \overline{R}_v$  and  $z^* = (\overline{H} + 1)$ 

OR  $\overline{R}_o \le r^* \le \overline{R}_v$  and  $z^* = (\overline{H} + \overline{h})$ 

$$\hat{W}_r = \hat{W}_i = 0 \text{ with } \frac{\partial \hat{P}_r}{\partial r^*} = \left(\frac{R}{\ell}\right) \frac{\partial^2 \hat{W}_r}{\partial r^{*2}}$$

and

$$\frac{\partial \hat{\mathbf{P}}_{i}}{\partial \mathbf{z}^{\star}} = \left(\frac{\mathbf{R}}{\ell}\right) \frac{\partial^{2} \hat{\mathbf{W}}_{i}}{\partial \mathbf{z}^{\star^{2}}}$$

At 
$$\overline{R}_i \le r^* \le 1$$
 and  $z^* = 0$ 

$$\frac{\partial \hat{W}_{r}}{\partial z^{*}} = \frac{\partial \hat{W}_{i}}{\partial z^{*}} = 0 \text{ with } \hat{P}_{r} = 1 \text{ and } \hat{P}_{i} = 0$$

At 
$$\overline{R}_0 \le r^* \le \overline{R}_d$$
 and  $z^* = (\overline{H} + \overline{L})$ 

$$\frac{\partial W_r}{\partial z^*} = \frac{\partial W_i}{\partial z^*} = 0 \text{ with } \hat{P}_r = \hat{P}_i = 0$$

The solution for the pressure distribution inside the valve will be in the form,

$$p^* = \lambda \operatorname{Real}\left[\left(\hat{P}_r + i\,\hat{P}_i\right)e^{i\operatorname{ReSt'}}\right] \,. \tag{32}$$

A further substitution can be made into eqs.

(30) and (31) so that 
$$\hat{P}_r = \varphi_1(r^*)^{-\frac{1}{2}}$$
 and

 $\hat{P}_i = \phi_2 \left(r^*\right)^{-\frac{1}{2}}$ . The aim of this transformation is to eliminate first derivatives appearing in these governing equations and thereby render them more suitable to numerical solution by iterative methods.

Thus eqs. (30) and (31) become,

$$\frac{\partial^2 \varphi_1}{\partial r^{\star 2}} + \left(\frac{R}{\ell}\right)^2 \frac{\partial^2 \varphi_1}{\partial z^{\star 2}} + \frac{\varphi_1}{4r^{\star 2}} = 0, \text{ and}$$
 (33)

$$\frac{\partial^2 \varphi_2}{\partial r^{\star 2}} + \left(\frac{R}{\ell}\right)^2 \frac{\partial^2 \varphi_2}{\partial z^{\star 2}} + \frac{\varphi_2}{4r^{\star 2}} = 0. \tag{34}$$

The boundary conditions for eqs. (33) and (34) become,

At 
$$r^* = 1$$
 and  $z^* \le \overline{H}$ , OR  $r^* = \overline{R}_d$  and  $\overline{H} \le z^* \le (\overline{H} + \overline{L})$   $r^* = \overline{R}_i$  and  $z^* \le (\overline{H} + 1)$ , OR  $r^* = \overline{R}_v$  and  $(\overline{H} + 1) \le z^* \le (\overline{H} + 1 + \overline{h})$ , OR  $r^* = \overline{R}_o$  and  $(\overline{H} + 1 + \overline{h}) \le z^* \le (\overline{H} + \overline{L})$ 

$$\frac{\partial \varphi_1}{\partial r^*} - \frac{1}{2\sqrt{r^*}} \varphi_1 = \frac{1}{\sqrt{r^*}} \frac{\partial^2 \hat{V}_r}{\partial r^{*2}},$$
and
$$\frac{\partial \varphi_2}{\partial r^*} - \frac{1}{2\sqrt{r^*}} \varphi_2 = \frac{1}{\sqrt{r^*}} \frac{\partial^2 \hat{V}_i}{\partial r^{*2}}.$$

At 
$$1 \le r^* \le \overline{R}_d$$
 and  $z^* = \overline{H}$  OR
$$\overline{R}_i \le r^* \le \overline{R}_v \text{ and } z^* = (\overline{H} + 1) \text{ OR}$$

$$\overline{R}_o \le r^* \le \overline{R}_v \text{ and } z^* = (\overline{H} + \overline{h})$$

$$\frac{\partial \varphi_1}{\partial z^*} = \left(\frac{R}{\ell}\right) \frac{1}{\sqrt{r^*}} \frac{\partial^2 \hat{W}_r}{\partial z^{*2}}$$

$$\begin{split} \frac{\partial \phi_2}{\partial z^*} &= \left(\frac{R}{\ell}\right) \frac{1}{\sqrt{r^*}} \frac{\partial^2 \hat{W}_i}{\partial z^{*2}}. \\ \text{At} & \quad \overline{R}_i \leq r^* \leq 1 \text{ and } z^* = 0 \\ \phi_1 &= \sqrt{r^*} \qquad \text{and} \qquad \phi_2 = 0 \,. \\ \text{At} & \quad \overline{R}_0 \leq r^* \leq \overline{R}_d \text{ and } z^* = \left(\overline{H} + \overline{L}\right), \end{split}$$

The solution for the pressure distribution inside the valve will be in the form,

$$p^* = \frac{\lambda}{\sqrt{r^*}} \operatorname{Real} \left[ (\phi_1 + i \phi_2) e^{i \operatorname{ReS} t'} \right]$$
This is 
$$p^* = -\frac{\lambda}{\sqrt{r^*}} \sqrt{\phi_1^2 + \phi_2^2} \sin(\operatorname{ReS} t' - \gamma_p).$$
(35)

The velocity components may be obtained from,

$$w' = \lambda \operatorname{Real} \left[ \left( \hat{W}_r + i \, \hat{W}_i \right) e^{i \operatorname{Re} S t'} \right]$$

and

$$v' = \lambda \operatorname{Real} \left[ (\hat{V}_r + i \hat{V}_i) e^{i \operatorname{Re} S t'} \right].$$
 (36)

That is

$$\begin{aligned} w' &= -\lambda \sqrt{\hat{W}_r^2 + \hat{W}_i^2} \sin(\text{ReSt'} - \gamma_w), \\ v' &= -\lambda \sqrt{\hat{V}_r^2 + \hat{V}_i^2} \sin(\text{ReSt'} - \gamma_v) \end{aligned}$$

Where.

$$\begin{split} \gamma_{p} &= \tan^{-1}\!\!\left(\frac{\phi_{1}}{\phi_{2}}\right), \gamma_{w} = \tan^{-1}\!\!\left(\frac{\hat{W}_{r}}{\hat{W}_{i}}\right), \text{and} \\ \gamma_{v} &= \tan^{-1}\!\!\left(\frac{\hat{V}_{r}}{\hat{V}_{i}}\right) \end{split}$$

To obtain the net axial force on the valve, integration of the pressure distribution over the disc upper and lower solid walls;

$$F = 2\pi \int_{R_i}^{R_v} r p[t, r, (H + \ell)] dr$$

$$-2\pi \int_{R_o}^{R_v} r p[t, r, (H + \ell + h)] dr, \text{ and}$$

$$F = 2\pi R^2 \rho U^2 \int_{\overline{R}_i}^{\overline{R}_v} r^* p^* [t', r^*, (\overline{H} + 1)] dr^*$$

$$-2\pi R^2 \rho U^2 \int_{\overline{R}_o}^{\overline{R}_v} r^* p^* [t', r^*, (\overline{H} + 1 + \overline{h})] dr^*.$$

And in dimensionless form as:

$$F^{*} = \frac{F}{R^{2}\rho U^{2}}$$

$$= 2\pi \begin{pmatrix} \int \frac{\overline{R}}{R_{i}} v r^{*} p^{*} \left[ t', r^{*}, (\overline{H} + 1) \right] d r^{*} \\ -\int \frac{\overline{R}}{R_{o}} v r^{*} p^{*} \left[ t', r^{*}, (\overline{H} + 1 + \overline{h}) \right] d r^{*} \end{pmatrix}. (38)$$

#### 3. Results and discussion

Preliminary process of parameter elimination was performed here in order to reduce the number of parameters affecting the amplitude of the hydrodynamic exciting forces on the valve disc. It was found that the most influential ones were L/R, H/R and l/R, the dimensionless outlet length, dimensionless inlet length and dimensionless valve opening respectively. The remaining parameters Ro/R,  $R_v/R$ ,  $R_d/R$ , and h/l were considered constants and having practical values of 0.25, 1.25, 2.25 and 0.25, respectively. Fig. 3 shows the variation of the amplitude of the exciting force on valve disc with the dimensionless valve opening I/R at different values of ReS. The results show that the larger the opening the smaller the amplitude of the exciting force. An asymptotic value is clearly demonstrated for any value of ReS close to 1/R-1. The larger the value of ReS the smaller the exciting force. This may be attributed to the fact that the larger the value of ReS the larger the energy to be converted to the so-called steady streaming motion on the account of the periodic variation of the exciting force. For a moderate value of ReS=60 the variation of the dimensionless exciting force with the dimensionless valve body outlet length L/R for different values of dimensionless valve opening is shown in fig. 4. It can be seen that the larger the dimensionless outlet length the

smaller is the exciting force. An asymptotic value is reached as we approach the value of hb/R-10 for all values of 1/R. This means that icincreasing L/R over approximately 10 will produce no appreciable effect on the exciting force. Fig. 5 shows the variation of the dimensionless exciting force with dimensionless valve body inlet length H/R for different values of l/R. It is clear that the larger the value of H/R the smaller is the exciting force. This is true for all values of dimensionless valve opening. An asymptotic value for the exciting force is reached when H/R~5. There is no practical importance for the dimensionless inlet height to exceed approximately 5.

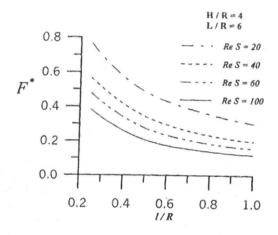


Fig. 3. The variation of the dimensionless exciting force with the dimensionless valve opening for different values of *Re S*.

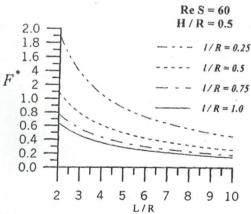


Fig. 4. The variation of the dimensionless exciting force with the dimensionless valve body outlet length L/R at different values of dimensionless valve opening  $\ell/R$ .

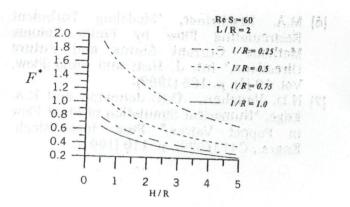


Fig. 5. The variation of the dimensionless exciting force with the dimensionless valve body inlet length H/R at different values of dimensionless valve opening  $\ell/R$ .

# 4. Conclusions

It is concluded that longer inlet and outlet valve body lengths are recommended for reducing the exciting forces causing valve vibration. They should not unnecessarily exceed asymptotic values where no appreciable reduction in exciting force takes place. Values of  $H/R\sim5$  and  $L/R\sim10$  are recommended for all values of valve opening l/R and  $R_eS$ .

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