Cut planning and waste optimization for two-dimensional cutting stock applications

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Dual simplex method was used to solve linear programming cutting stock problem obtained by two combinatoric methods. The two methods generated constraine d cutting patterns by successive horizontal and vertical builds of ordered rectangles. The alternative optima solutions aroused by the use of simplex method were utilized to address various production requirements and constraints. This is done without sacrificing the main objective of producing minimum trim waste. Tests on actual orders from sheet metal manufacturer showed that, from a wide production planning perspective, the developed software is more suitable than just focusing on trim waste minimization.

هـذا البحـث يستكشـف اسـتخدام بدائل الحلول المثلى الناتجة من برمجة الأرقام الصحيحة لحل عمليات قطع المواد الخام بأخذ متطلـبات و محددات الإنتاج بعين الاعتبار. و يتم ذلك مع مراعاة إنتاج أقل قدر ممكن من الفاقد . أثبتت الاختبارات على طلبات حقيقية من أحد المصانع المنتجة أن استخدام هذه الطريقة أكثر ملائمة من التركيز على تقليل الفاقد إذا نظر الى العملية من وجهة نظر التخطيط الشامل للإنتاج .

Keywords: Cutting stock problems, Waste optimization, Trimming problems, Production planning

1. Introduction

A nesting or allocation process usually precedes cutting of two-dimensional shapes from sheets of raw material. The objective of the nesting process is to distribute a batch of pieces to be cut such that maximum utilization of material resources is obtained [1]. Such minimization of waste is a request for various industries including sheet metal, textile, furniture, leather, paper and flat glass [2-4]. Manual or human solution for such a problem may be adequate for small-scale problems. Some industries, especially glass industries, solve the problem by minimizing and standardizing their pieces such that the allocation process is standardized to give adequate results every time [3]. This approach would be acceptable when there are a low variety of the required pieces. Other industries, like sheet metal industries, use different metal rolls, where each roll would have a different width. However, this approach might not be acceptable to some industries from a financial and manufacturing point of view. Furthermore, such a solution would not be feasible if a high variety of pieces exist. Various approaches were proposed to define, model and solve cutting stock problems. These include, restricted mathematical optimization [5], tree search [6,7], neural networks [8], simulated annealing [9], and heuristics [10,11].

In most of the developed approaches, even those which are based on mimicking the approaches adopted by human experts in making the allocations, the optimization was narrowed down to minimize the raw material waste. These approaches overlooked the global manufacturing process perspective. "Optimal" cutting that results from such approaches might not necessarily produce "optimal" outcomes for the total manufacturing cycle. Availability of raw material, cutting machines set-up times, cutting time and various manufacturing constraints should be taken into account if totally acceptable results are desired. Some attempts have taken place to include some of these conditions into the optimization model. A math-based solution that optimizes the order pattern across many feedstock pieces which then determines a cutting schedule to reduce lead-times from order entry to product delivery as well as to minimize waste was considered [12]. However, to include all production conditions in the optimization model is not only difficult but also may lead to producing higher waste and complex software to use. In this work such manufacturing conditions are addressed to some degree in the cutting stock problem by exploring the "alternative solutions" obtained from applying integer programming to the cutting stock problem. The two well-known constrained two-dimensional optimization approaches developed by Wang [1] are used to form the constrained cutting patterns. Comparison of the developed alternative solutions with other automatic and manual solutions to actual examples taken from a sheet metal manufacturing company will be considered.

2. Statement of the problem and the methods of Wang

Let L×W be a rectangular stock sheet having length L and width W and available with unlimited number. Let R be a set of rectangles $\{R_1, R_2, \ldots, R_n\}$ with dimensions $\{L_1 \times W_1, L_2 \times W_2, \ldots, L_n \times W_n\}$. Let P be a set of numbers {P1,P2,...,Pn}, that indicate the maximum duplicate of each rectangle in R per sheet. Let G be defined as a guillotine pattern that cuts a subset of the rectangles $\{R_1, R_2, \ldots, R_n\}$ from the stock sheet. Let m represent the number of all generated patterns $\{G_1, G_2, \ldots, G_m\}$ based on geometrical constraints and contains any generated combination of rectangles R. Then it is required to determine patterns with minimum trim waste that meet the specified demand within certain allowances. The problem can be stated in the form:

$$\text{Minimize} \quad z = \sum_{i=1}^{m} w_i x_i \,, \tag{1}$$

$$\text{subject to; } LD_j \le a_{ij} x_i \le UD_j \,, \quad j=1,2,..,n,\ i=1,2,..,m,$$

$$\text{and } x_i \ge 0 \,. \tag{2}$$

where:

- m is the number of patterns,
- n is the number of parts,
- w_i is the trime waste in pattern i,
- x_i is the number of repetitions of pattern G_i,
- a_{ij} is the number of times the rectangle R_j appears in pattern G_i, and
- LD_j and UD_j are the lower and upper limit of demand of rectangle R_j.

Wang [1] proposed heuristic approaches that do not guarantee optimal solutions in one invocation. They are based on the observation that all guillotine cutting patterns on the stock sheet can be obtained by means of horizontal and vertical builds. A horizontal build of two rectangles having dimensions $L_1 \times W_1$ and $L_2 \times W_2$ is a rectangle, S_h, that has dimensions $(L_1+L_2, \max.(W_1,W_2))$ while their vertical build would give, S_v, having dimensions of $(\max.(L_1,L_2), W_1+W_2)$ as can be seen in fig. 1. Wang's algorithms use a list to accumulate guillotine rectangles as they get constructed to make guillotine patterns G. The list initially contains one copy of each type of the demanded rectangle in R. The list then is expanded by forming a larger guillotine rectangle from two smaller guillotine rectangles. Any newly formed rectangle must satisfy the following conditions:

1- The two guillotine rectangles forming it have not been considered for combination together before. The dimension of the formed rectangle should not exceed LxW (stock sheet dimensions).

2- The number of duplicates of each demanded rectangle Ri should not exceed the maximum allowed number Pi, where 1<i<n.

3- The rectangle waste < β_1 (algorithm one). Where â1 is defined as the maximum acceptable percentage of trim waste produced by the formed rectangle with respect to the stock sheet area OR the rectangle waste< β_2 (algorithm two). Where β_2 is the maximum percentage acceptable of trim waste produced by the formed rectangle with respect to its area.

It is clear that at early stages of pattern generation, the number of the generated and accepted rectangles will increase. However, at later stages this number will decrease because the number of accepted rectangles will decrease. This is mainly due to the fact that the size of the generated rectangles will be greater than the size of the stock sheet.

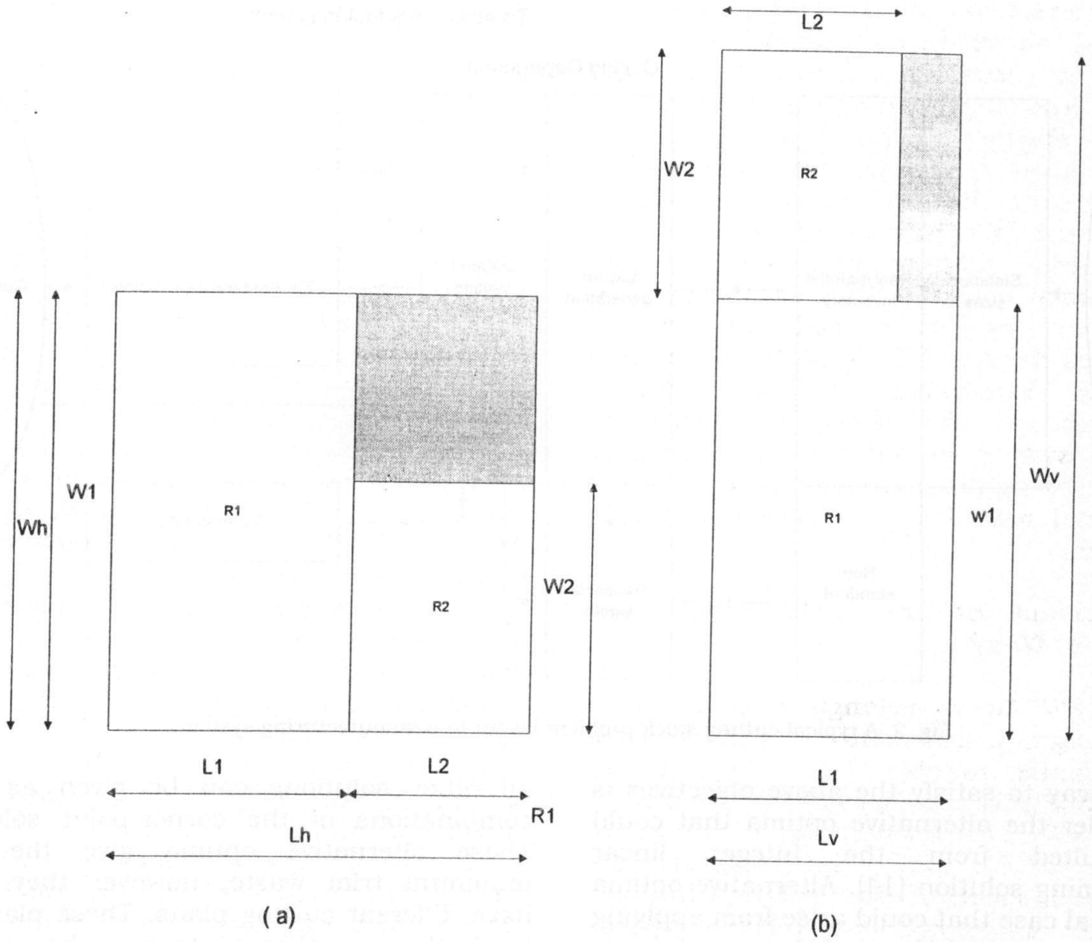

Fig.1. Guillotine rectangle (S), a- Horizontal Bulid S_h b- Vertical Build S_v.

The dual simplex method is used in this work to solve a linear programming model that is represented by eqs. (1) and (2). Since the methods developed to solve integer linear programming are not totally reliable from a computational standpoint, It was decided to solve the model as a continuous linear programming model then round the solution to the closest feasible integer values [1, 13].

3. Alternative optima

As mentioned before isolating the cutting stock unit activity from other activities in the manufacturing process may lead to results, which are not optimal in the overall manufacturing sense. A typical cutting stock layout unit in a manufacturing system is shown in fig. 2. As it can be seen, waste is usually classified as reusable and scrap wastes. So if the minimization process gives more reusable waste it would be preferable to one that gives more scrap as long as the total waste is the same in both cases. In some cases production time is a critical factor to meet a certain delivery schedule, hence minimizing the number of cuts is a critical factor. In other cases operators would like to have the minimum number of cut patterns since it affects the speed of their operation. The previously mentioned, and many other similar manufacturing, situations indicate that the software which should be developed should help planning engineers and operators to choose a cutting stock allocation that will cater for other time-variant manufacturing constraints and conditions. However, this advantage shouldn't be at the expense of not having minimum trim waste.

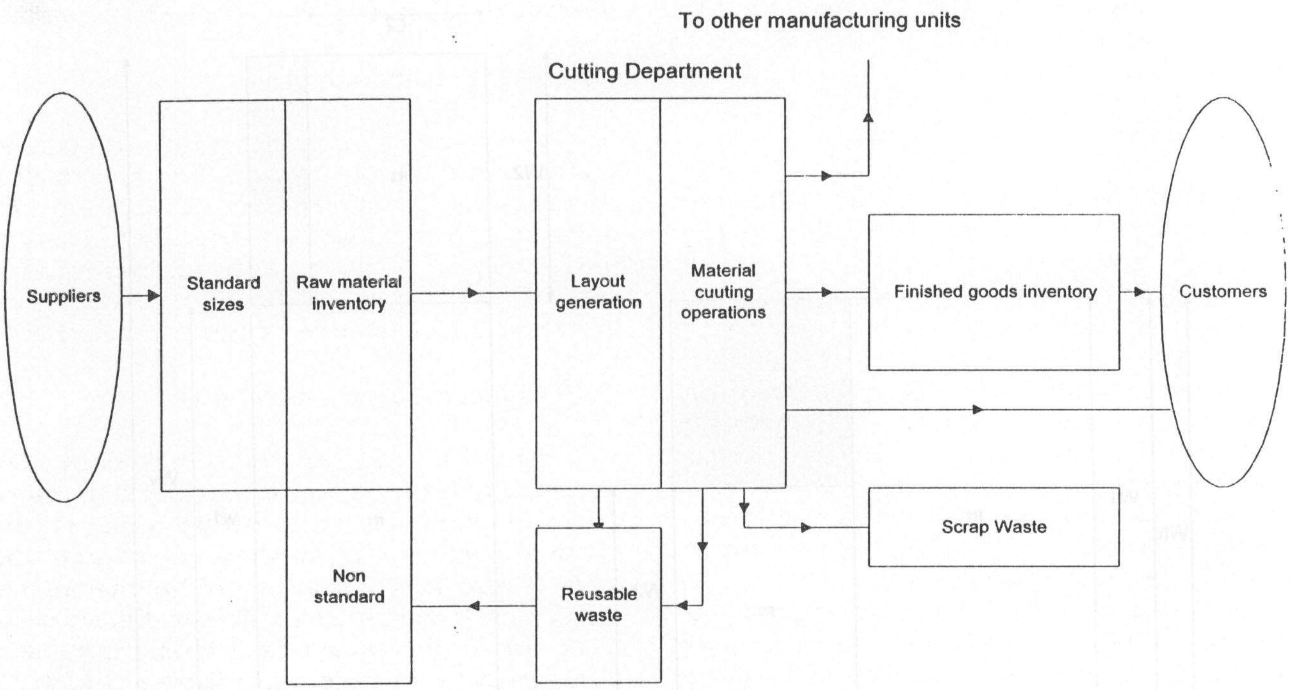

Fig. 2. A typical cutting stock problem layout in a manufacturing system.

One way to satisfy the above objectives is to consider the alternative optima that could be resulted from the integer linear programming solution [14]. Alternative optima is a special case that could arise from applying the simplex method if one of the constraint is parallel to the minimized objective function as well as binding (i.e. passes through the optimum solution point(s)). In mathematical terms if a constraint, j, is given as;

$$LD_j \le \sum_{i=1}^{m} a_{ij} x_i = \frac{1}{\beta}\sum_{i=1}^{m} w_i x_i = \frac{1}{\beta} z \le UD_j \,. \quad (3)$$

Where β is a constant, and if X^1 and X^2 represent two solutions points in the input *m*-dimensional space, then every solution given by:

$$X^{'} = \lambda X^1 + (1-\lambda) X^2 \,, \quad (0 \le \lambda \le 1) \,, \quad (4)$$

is an optimal solution to problem and assumes the same optimal value. Normally there are an infinite number of such solutions. However, algebraically the simplex method is capable of encountering corner-point solutions only and all other solutions can be given as linear combinations of the corner-point solutions. These alternative optima give the same minimum trim waste, however they would have different cutting plans. These plans can be further investigated to give the one that satisfy the requirements of the manufacturing situation at a time.

After consultations with various sheet metal and glass manufacturing industries, it was decided that the developed software should give the planning engineer or operator the option to request the cutting plan (out of the alternative optima) that satisfy one or more of the following criterions.

3. 1. Maximum reusable waste percentage

The user should specify the area dimension were if the waste area is greater than it, then it would be considered as a non-standard raw material and can be used at a later stage of manufacturing.

3. 2. Minimum number of cuts

Sometimes it is required that the cutting process can be accomplished with the minimum number of cuts in order to save total cutting time, especially when the parts are needed just in time.

3. 3. Minimum number of patterns

In some cases operators prefer to cut fewer number of patterns since it speeds up their operation. Assume that it is required to allocate the ordered parts shown in table. 1 on a stock sheet, which has a (2mx1m) dimension.

After running the developed software, fifteen cutting plans were found to meet the demand requirements with the zero waste percentage. Two plans, shown in fig. 3 and 4 are used to illustrate the present selection criterion. Table 2 shows the resultant performance for the two plans.

As it can be seen from table 2, plan one had two cutting patterns while the other plan had three. Usually, less number of patterns reflects less set-up time and easy implementation if cutting is done by humans.

3. 4. Best demand matching

In many manufacturing situations, meeting the lower demand limits is a goal worth striving to achieve. It has various implications regarding raw material inventory and costs. As it can be deduced from table.2, for the previous example, plan needed fewer sheets and met the lower demands exactly.

3. 5. Minimum number of set-ups

Minimum number of cuts does not necessarily mean minimum cutting time, since in most industries the set-up time of the cutting machine is much greater than the cutting time itself. A set-up of the machine is needed each time a new different dimension is to be cut. To illustrate this criterion, assume it is required to nest the parts shown in table. 3 on a stock sheet of dimension (2m×1m).

Table 1
Ordered parts data

Part	Length (mm)	Width (mm)	Lower demand limit	Upper demand limit	Duplications
1	300	400	1000	1050	None
2	500	400	1500	1050	None
3	400	350	2000	2100	None

Table 2
Cutting performance for two plans

	No. of cuts	No. of patterns	Total No. of sheets	Generated quantity Part 1, 2 and 3
Plan one	4150	2	350	1000, 1500,2000
Plan two	4284	3	362	1062, 1580, 2004

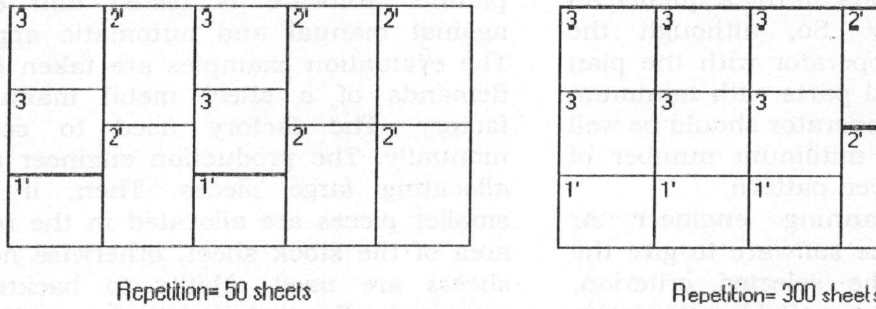

Fig. 3. Cut patterns obtained from plan one.

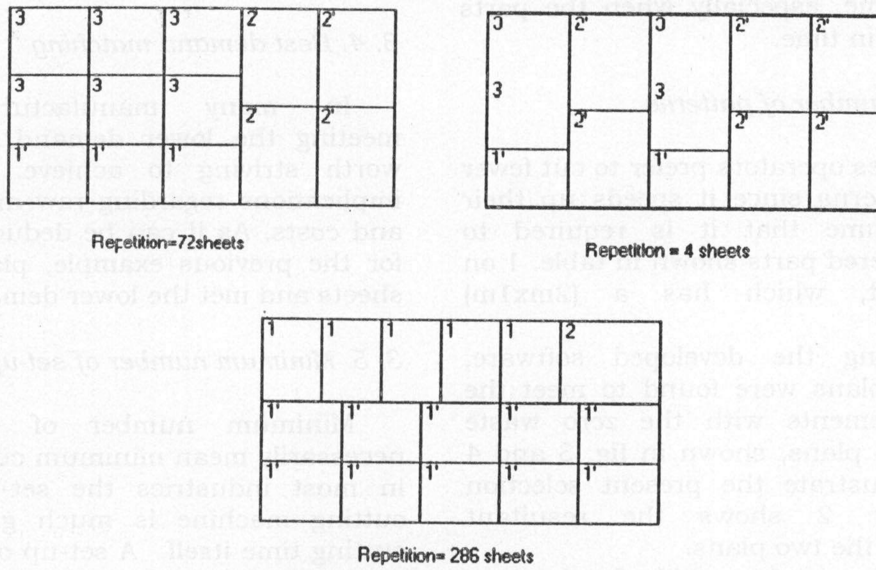

Fig. 4. Cut patterns obtained from plan two.

Table 3
Ordered parts data

Part	Length (mm)	Width (mm)	Lower demand Limit	Upper demand limit	Duplications
1	250	250	1000	1050	None
2	500	500	1000	1050	None
3	400	400	1000	1050	None

After solving this problem, 15 cutting plans were obtained which gave the same amount of percentage waste. Two of the plans are shown in fig. 5 and fig. 6. Table 4 shows the performance of the two plans. As it can be seen the first two cutting patterns are similar for the two plans. However, in plan one the third pattern can be cut using only one set-up of the machine while the corresponding one in plan two needs four set-ups. Of course, the number of setups needed to cut any pattern is not unique and it depends on the sequence an operator would follow. So, although the software provides the operator with the plan that cuts the demanded parts with minimum number of set-ups, the operator should be well trained to deduce the minimum number of machine set-ups in a given pattern.

Although, the planning engineer or operator can request the software to give the plan, which satisfy the selected criterion, reviewing all plans is an available alternative option. Fig. 7 shows a flowchart of the developed software. A cutting plan report is provided for each plan, which summarizes all related cutting parameters. These include reusable percentage, scrap percentage, number of sheets, generated demands as well as displaying the patterns to be cut with internal parts dimensions.

4. Results

The developed cutting optimizer and planner software is tested and evaluated against manual and automatic approaches. The evaluation examples are taken from real demands of a sheet metal manufacturing factory. The factory used to solve CSP manually. The production engineer starts by allocating large pieces. Then, if possible, smaller pieces are allocated in the remaining area of the stock sheet, otherwise new stock sheets are used. Ability to backtrack the previously allocated pieces was very limited.

Table 4
Cutting performance for two plans

	Total waste %	No. of cuts	No. of patterns	Total No. of sheets
Plan one	7.6	2903	3	263
Plan two	7.6	2992	3	264

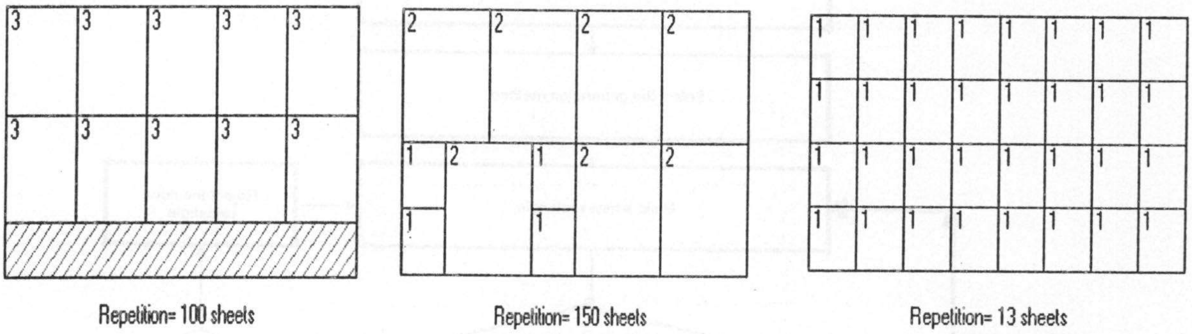

Fig. 5. Cut patterns obtained from plan one.

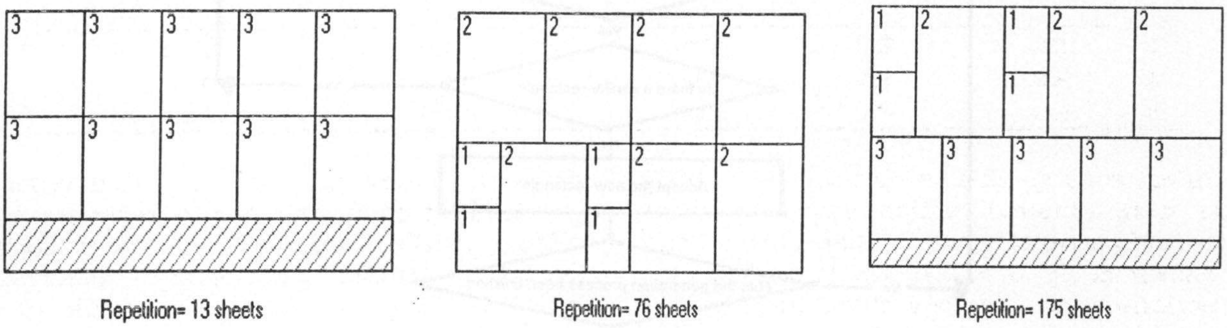

Fig. 6. Cut patterns obtained from plan two.

4.1. Example one

A typical stock sheet specifications and cost are shown in table 5, while a typical order is shown in table 6.

Table 7 shows the cutting results obtained manually by the production engineer and the ones obtained by the developed software. The software was run using β_2=20% with a 5% demand allowance.

As it can be deduced a considerable reduction in waste percentage was achieved by the software, which resulted in fewer stock sheets and consequently lower cost. Furthermore, the cutting time was reduced considerably.

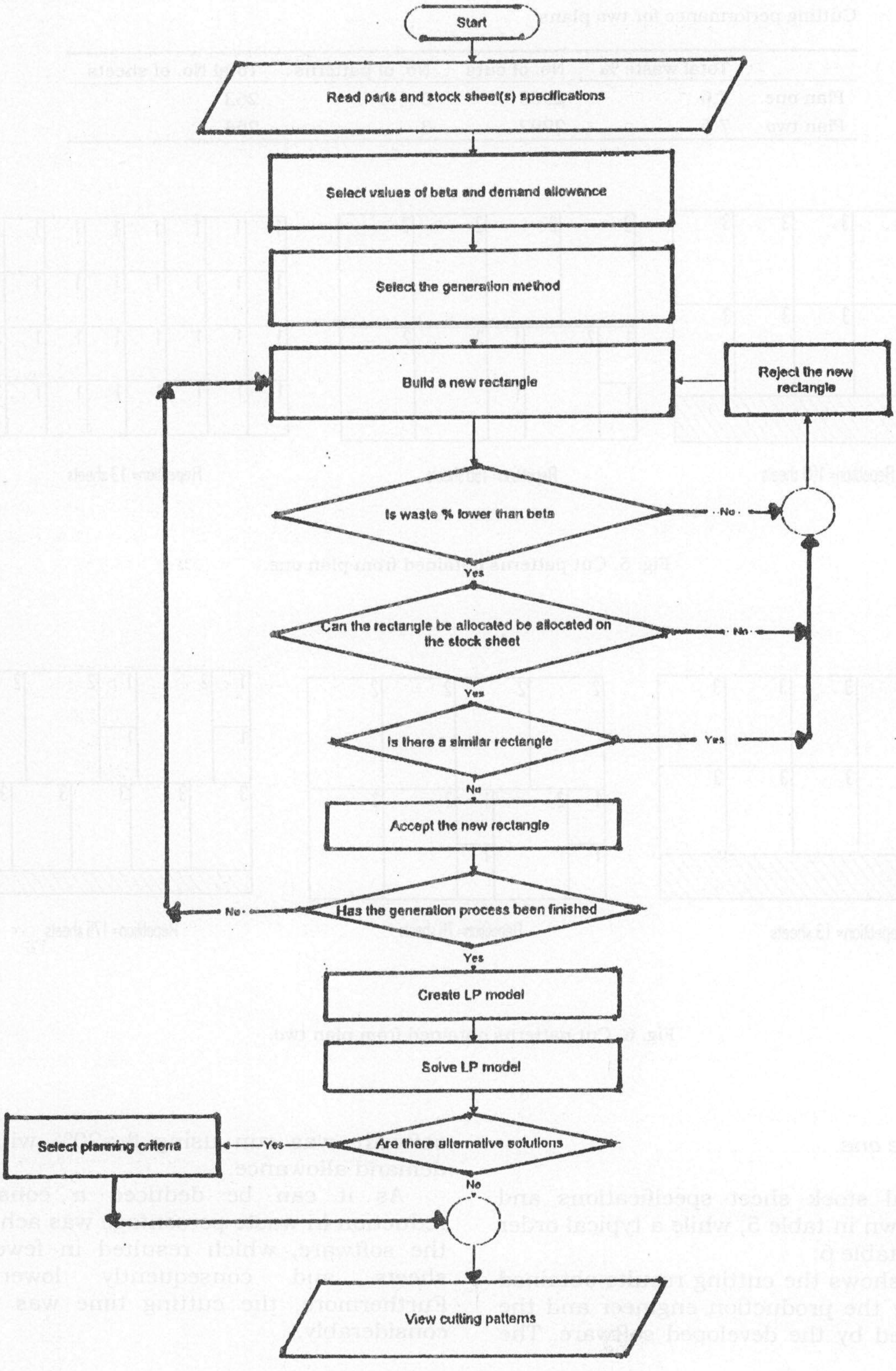

Fig. 7. Optimizer and software flowchart.

Table 5
Galvanized stock sheets specifications

Length (mm)	Width (mm)	Thickness (mm)	Weight (kg/sheet)	Cost/ton ($)
2500	1250	1.4	34.125	1000

Table 6
Parts specifications

Part	Length (mm)	Width (mm)	Demand
1	670	520	1001
2	900	560	1720
3	1000	1000	508

4. 2. Example two

In this example the developed software was compared against two automatic approaches. Ritmo 4 and PoOptimizer are two commercial software packages which use heuristic allocation approaches. Table 8 shows a typical order to be cut at the Factory. The developed software was run with β_2= 5% and a 5% demand allowance. Table 9 shows the results obtained by the three software programs.

It can be seen that the programs that used heuristic approaches were always faster than the developed program which depended on mathematical optimization. However, a much higher percentage waste (almost 100% higher) was obtained from these packages with more number of patterns. Although the PoOptimizer met demands exactly, but that was at the expense of having high percentage waste. The developed software gave five alternative cutting plans that have the same total waste. In fact that gave the production staff the opportunity to select plan 3 which suited their production conditions at the time since it had the minimum scrap percentage.

Table 7
Cutting performance results obtained manually and by the software

Method	Total waste %	Scrap %	Reusable %	Needed stock sheets	Cost($)
Manual	25.6	4.7	20.9	741	25286
Software	16.47	3.62	12.85	662	22590
				Saving	2696

Table 8
Specifications of parts to be cut for example two

Part	Length (mm)	Width (mm)	Demand
1	1000	1000	100
2	500	430	315
3	880	440	500
4	600	600	220
5	900	300	150

Table 9
Cutting performance of the three automatic methods for example two

Method		Waste %	Scrap %	Reusable %	No. of sheets	Time to solve (sec)	No. of patterns	Average demand allowance
Ritmo 4		16.47	NA	NA	141	NA	7	3.09
Prooptimizer		21	NA	NA	181	7	7	0
Developed software plan 1	1	10	4.73	5.27	136	1517	5	2.36
plan 2	2	10	5.16	4.84	136	1517	5	2.36
plan 3	3	10	3.71	6.29	136	1517	5	2.36
plan 4	4	10	5.16	4.84	136	1517	5	2.36
plan 5	5	10	5.03	4.97	136	1517	5	2.36

5. Conclusions

Developing an optimization approach to solve the cutting stock problem might not lead to an optimum solution from a wider manufacturing planning point of view. In this work the alternative optima solutions generated from the integer linear programming in solving the cutting stock problem was utilized to address this shortcoming. Production planners can select the solution that best suits their production conditions without experiencing any deterioration in waste optimization. Applying the developed software to real orders had demonstrated its ability to minimize waste and to address various production constraints.

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Received July16, 2001
Accepted May 13, 2002