Spectral analysis of elevated box breakwaters

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This paper introduces a method to find the distribution of wave height in both sides of an elevated box like breakwaters. The proposed method takes into account the random nature of the sea through the concept of the wave spectrum. Depending on the allowed wave heights in the lee side of the breakwater and the wave heights in the seaside, the dimensions of the structure could be selected. The approach is based on mass and energy flux and extends the transmission and reflection coefficients calculated for monochromatic wave to cover the case of infinite number of wave.

يقدم هذا البحث طريقة، أرتفاعات الأمواج على جانبى حاجز أمواج على هيئة مستطيل يوجد أسفله فراغ. الطريقة المقدمة تأخذ فى الأعتبار عشوائية الأمواج. من خلال استخدام المنحنى الطيفى للأمواج. يمكن بمعلومية أرتفاع الأمواج خلف الحاجز أختيار الأبعاد المناسبة للحاجز.

Keywords: Elevated dock, Elevated breakwaters, Wave spectrum, Eigenfunction expansion,

1. Introduction

Mei and Black [1] used the varitional principle to solve the problem of fixed surface or submerged obstacle. Black et al. [2] used the same approach to solve the same problem for a movable obstacle. Sharaki [3] introduced a simplified analytical approach to solve the problem of a fixed body piercing the water surface but not extending to the sea floor. Sharaki [3] considered the case of a single harmonic wave. Sharaki et al. [4] covered the case of heave motion. In this paper the previous work, presented in ref. [3], is extended to cover the case of random sea. The random nature of the sea waves is introduced through the use of the wave spectrum.

Consider a box that has an infinite length, draft d and width 2B. The water depth is h and the gap beneath the box is G, fig. 1.

The transmission and reflection coefficients T, R, for a single harmonic wave, with frequency ω propagating normal to the structure are given by, [3]

$$T(\omega) = \frac{i\,I_o^2}{k_o BG + i\,I_o^2 + k_o \sum_{j=1}^{N} I_j^2 \Big/ k_j}, \tag{1}$$

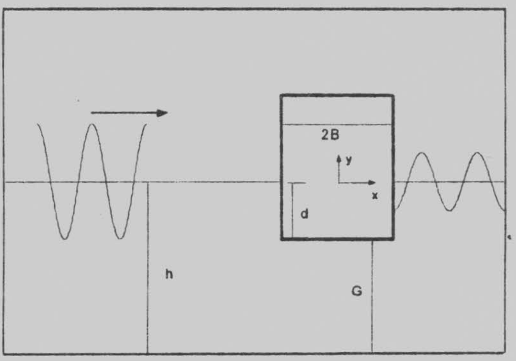

Fig. 1. Definition sketch of the pontoon .

$$R(\omega) = 1 - \frac{i\,I_o^2}{k_o BG + i\,I_o^2 + k_o \sum_{j=1}^{N} I_j^2 \Big/ k_j}, \tag{2}$$

where N is the number of the evanescent modes used in the analysis.

The quantities on the right hand side of eqs. (1) and (2) are functions in ω among other variables. They are given by

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$$I_i = \int_{-h}^{-d} f_i(y)\,dy \,, \tag{3}$$

$$f_o(y) = \frac{\cosh\left[k_{o_o}(y+h)\right]\sqrt{2}}{\left[h + \dfrac{g\sinh^2(k_o h)}{\omega^2}\right]^{0.5}} ,$$

$$f_i(y) = \frac{\cos\left[k_i(y+h)\right]\sqrt{2}}{\left[h - \dfrac{g\sin^2(k_i h)}{\omega^2}\right]^{0.5}} , \tag{4}$$

$$k_O \tanh(k_O h) = \frac{\omega^2}{g} \text{ and } k_j \tan(k_j h) = -\frac{\omega^2}{g} , \tag{5}$$

where g and A_m are the gravity acceleration and the amplitude of the incident wave, and $i = (-1)^{0.5}$

2. Spectral analysis

The surface elevation η of unidirectional random sea is given by

$$\eta = \sum_{n=1}^{\infty} 0.5\, H_n \exp(i k_n x - \omega_n + \theta_n) , \tag{6}$$

$$H_n = 2\sqrt{S_\eta(\omega)\,d\omega} \,, \tag{7}$$

where,
]k_n , ω_n and θ_n are the wave number, radian frequency, and random phase angle of the nth wave component, respectively. S_η (ω) is the surface wave spectrum.

The wave number k_n and the radian frequency ω_n satisfy the dispersion relation

$$k_n \tanh(k_n h) = \frac{\omega_n^2}{g} . \tag{8}$$

For each harmonic component, there is a set of associated evanscent wave. They are related together with

$$k_{n,j} \tan(k_{n,j} h) = -\frac{\omega_n^2}{g} \quad \pi(j-0.5) < k_{n,j} < \pi j. \tag{9}$$
$$j = 1,2,\ldots,M$$

The spectrum of the reflected and transmitted waves are given by, Goda [6] and Sharaki [5]

$$S_{\eta,R}(\omega) = |R(\omega)|^2 \, S_\eta(\omega) , \tag{10}$$

$$S_{\eta,T}(\omega) = |T(\omega)|^2 \, S_\eta(\omega) , \tag{11}$$

where, $|X|^2 = X \cdot X^*$ is the square of X and X^* is the conjugate of X

The root mean square η_{rms} of the surface elevation is given by

$$\eta_{rms} = \sqrt{m_o} \,, \tag{12}$$

where, m_o is the zero order moment of the spectrum, given by ;

$$m_o = \int_0^{\infty} S_\eta(\omega)\, d\omega \,. \tag{13}$$

With the definition of the transmission (reflection) coefficient as the ratio between the transmitted (reflected) and the incident wave heights, one can write, Goda [6], Sharaki [5]

$$T = \sqrt{(m_{o,T}/m_{o,I})} , \tag{14}$$

$$R = \sqrt{(m_{o,R}/m_{o,I})} , \tag{15}$$

with $m_{o,I}$ $m_{o,R}$ and $m_{o,T}$ are the zero moments of the incident, reflected and transmitted spectrums, respectively

In this study the P-M spectrum is used. In terms of the significant wave height H_S it has the form

$$S_\eta(\omega) = 8.1 \;\, 10^{-3} \;\, \omega^{-5} g^2 \exp(-B\,\omega^{-4}), \quad \text{with} \quad B = 3.11\, H_S^{-2}. \tag{16}$$

3. Results and discussion

A dock located in water with depth 3.0 m, is subjected to a wave train represent with a P-M spectrum with significant wave height = 2.m. The dock has a total width of 6.m and draft 2.1 m. Fig. 2 shows the transmission and reflection coefficients and the transfer functions for the reflected and incident waves. From the figure, one may conclude that when the width of the dock is of the same order of the wavelength, the dock acts as a totally reflecting structure. When the wavelength increases, the transmission coefficient increases and the reflection coefficient decreases.

From fig. 2, one may conclude that the dock acts as a non-linear filter. The efficiency of the dock as a breakwater decreases with increasing the wavelength. Since there is no energy losses in the proposed method, then it must be expected that:

$$|T|^2+|R|^2=1.$$

This has been verified numerically, and may be checked in fig. 2.

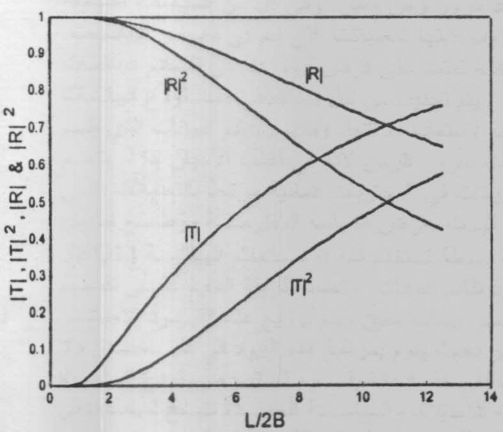

Fig. 2. Variation of the transmission, reflection coefficients and transfer functions with the wavelength.

Fig. 3 shows the spectra for the incident transmitted and reflected waves. The peak period for the incident, transmitted and reflected wave spectra are 7.125, 7.5938 and 6.8125 seconds, respectively. This means that the dock shifts the peak period of the transmitted wave to a higher value, hence to a longer wave. Such a property is desired, since the disturbance resulting from a long wave is less than that resulting from a shorter wave, with the same height

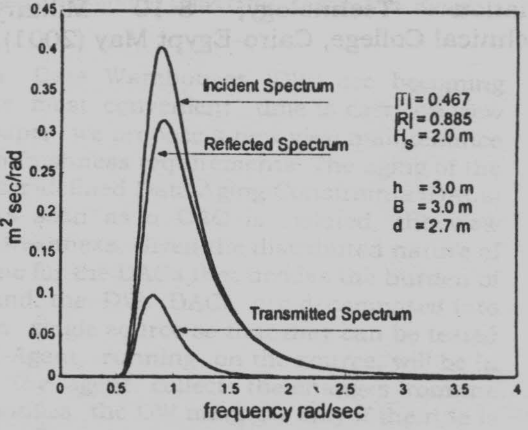

Fig. 3. Wave spectrum for the incident, transmitted and reflected waves.

4. Conclusions

A method is introduced to obtain the properties of the wave on both sides of an elevated box. The box may work as a breakwater, and as a dock or both. The proposed method takes into account the random nature of the wave via the wave spectrum. In this paper the P-M spectrum is used. The dock acts as a non-linear filter that shifts the peak frequency of the transmitted wave spectrum to a value less than that of the incident wave spectrum. This property is a desired for harbors and recreational beaches.

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