Ship hull surface generation using finite element method: resistance assessment

Y. A. Abdel Nasser and M. A. Kotb
Naval Architecture and Marine Eng. Dept, Alexandria Uiversity, Alexandria, Egypt

A proposed method based on Finite Element Method (FEM) was effectively applied for the preliminary generation of ship hull surfaces. The generated hull forms depend on the principal dimensions and form coefficients. The objective of this paper is to examine the generated hull form with respect to resistance qualities over a range of working speeds. The total resistance is calculated using hull form offsets generated by FEM. The influence of varying hull form coefficients on the total resistance is investigated in order to make better selection regarding minimum total resistance. Different parameters such as the variations in prismatic coefficient, Longitudinal Center of Buoyancy "LCB", loaded water plane area coefficient, and the half angle of entrance of the loaded water line. A fishing vessel is considered and a parametric study is performed and the associated resistance was calculated as a step necessary for selection of an optimum hull.

من الصعوبات التي تواجه مهندسي تصميم السفن في المراحل الأولية من خطوات التصميم هو تحديد خطوط بدن السفينة. استخدمت طريقة العنصر المحدد لاستنتاج خطوط بدن السفينة .عند تطبيق هذه الطريقة تبين لنا وجود عدة خطوط لبدن السفينة الواحدة وذلك نتيجة التغير الواسع لمتغيرات السفينة. لذلك كان الغرض من هذا البحث هو إجراء اختبار لهذه الخطوط لبدن السفينة وذلك بحساب مقاومة السفينة عند كل حالة من هذه المتغيرات مثل معامل الامتلاء المنشوري ومعامل مساحة المقطع المستعرض و الموقع الطولي لمركز الطفو وكذا زاوية خط تحميل المياه و غيرها. تم تطبيق هذه الطريقة على سفينة صيد حيث تم استنباط عدد من خطوط البدن تختلف من حيث نسب البدن والمعاملات و من ثم حسبت المقاومات المناظرة لهذه الخطوط وتم دراسة كل منها ومقارنتها كخطوة لاختيار البدن الأمثل

Keywords: Ship hull surface generation, Finite element method, Ship resistance, Table of offsets, Ship hull parameters

1. Introduction

One of the problems which faces the naval architect at an early stage of ship design is to generate the ship hull form in order to be able to proceed with various preliminary design routine calculations. These calculations include hydrostatics, wetted surface, capacity plans, resistance and propulsive coefficients. Some of these calculations are based on the below-water hull form such as those for resistance and propulsive coefficients. Other calculations are based on the above-water hull form of the ship. This paper discusses applications of the Finite Element Method (FEM) for the preliminary generation of hull surfaces. In general, it is possible to generate several hull forms for the same principal dimensions and hull form coefficients depending on the given constraints and boundary conditions of the numerical ship model. Different criteria such as, minimum total resistance, optimum speed, maximum dead weight, and/or low wake wash may be applied to define the optimum hull form. The objective of this work is to select among those hull forms one that possesses good resistance characteristics over a range of working speeds. In order to achieve such criteria, the influence of varying hull form coefficients on the total resistance is investigated. A number of form parameters were selected to examine their impact on ship's total resistance. These include prismatic coefficient, longitudinal center of buoyancy, loaded water plane area coefficient and half angle of entrance of the load waterline.

Ship resistance was calculated based on schemes given by Holtrop, Schneekluth, and Michell's integral. While both Holtrop and Schneekluth methods require main dimensions and form coefficients; Michell's

Alexandria Engineering Journal, Vol. 40(2001), No. 5, 665-673

integral uses the hull form table of offsets generated by FEM as input.

The proposed method based on FEM was used to generate the hull surface of a fishing vessel with given main dimensions and hull form coefficients. Further, the generated hull forms are examined regarding their resistance characteristics.

2. Hull form generation using FEM

Mathematical representation of a ship form is treated in literature. The objective of the mathematical representation could be used for generation, fairing and hydrostatics/dynamic calculations. Each of the hull longitudinal curves is represented by a polynomial function whose accuracy depends on the degree of the polynomial, whereas least square methods are applied to fit each curve. Examples of this representation are given in [1].

The finite element method is an alternative approach to deal with the mathematical representation of the hull form [2,3]. A scheme based on FEM was applied by the first author [4] to mathematically represent the longitudinal curves and the body plan sections simultaneously. The hull form is generated in two stages. In the first stage, a beam element is used to generate the main longitudinal curves such as sectional area curve and load waterline. In this element a simple polynomial function is used to satisfy the continuity of the curve at any point. In the second stage, the hull surface is generated using the plate element while taking into consideration the generated sectional area and load waterline curves as design constraints. The procedure is fully explained in [4]. A summary is only given here:

1. A simple equation of the hull surface, Y, is defined as:

$$Y = f(x, z) \, , \qquad (1)$$

2. A fairing criterion, F, is applied to measure the surface unfairness. It is defined as the integral of the sum of squares of the curvature in x, y and z directions respectively. The general form of this criterion is written as follows,

$$F = \frac{1}{2} \oiint_A \left[w_1 k_{xx}^2 + w_2 k_{xz}^2 + w_3 k_{zz}^2 \right] dA \, . \qquad (2)$$

Where, k_{xx} is the curvature in x-direction;

$$k_{xx} = \frac{\partial^2 Y / \partial x^2}{\left[1 + \left(\partial^2 Y / \partial x^2 \right)^2 \right]^{3/2}} \, ,$$

k_{xz} is the curvature in xz-direction;

$$k_{xz} = \frac{\partial^2 Y / \partial x \partial z}{\left[1 + \left(\partial^2 Y / \partial x \partial z \right)^2 \right]^{3/2}} \, ,$$

k_{zz} is the curvature in z-direction;

$$k_{zz} = \frac{\partial^2 Y / \partial z^2}{\left[1 + \left(\partial^2 Y / \partial z^2 \right)^2 \right]^{3/2}} \, ,$$

w_i is the weight function in i-direction,

x,z are the coordinates in x and z directions.

A is the domain of analysis, which is the vertical central plane of symmetry of the ship bounded by bow, stern, deck sheer and base line as shown in fig. 1.

3. Eq. (2) is linearized and simplified as afollows,

$$F = \frac{1}{2} \oiint_A \left[w_1 \frac{\partial^2 Y}{\partial x^2} + w_2 \frac{\partial^2 Y}{\partial x \partial z} + w_3 \frac{\partial^2 Y}{\partial z \partial z} \right] dA. \qquad (3)$$

4. The relationship between the displacement function Y and the nodal displacement, d, is written as:

$$\{Y\} = [k] \{d\} \; . \qquad (4)$$

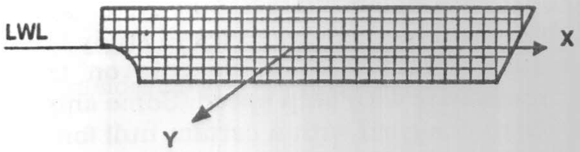

Fig. 1 Finite element domain.

5. The procedure of FEM is then processed to formulate the governing stiffness matrix [k] and to solve eq. (4).

The method was successfully applied to different ships in order to generate their hull surfaces[2]. However, it was found that several hull forms might be generated for the same principal dimensions. This is because there is a wide scope of the variability of the specified constraints and boundary conditions. Different criteria are used to define the optimum hull form such as minimum total resistance, low wake wash, maximum speed and maximum dead weight. In this work, the resistance characteristics of the generated hull forms are examined.

3. Hull form parameters

Ship hull form is specified by a set or a group of independent parameters and coefficients. Certainly the ship hull form is completely specified and defined by the line plans, which in turn can either be specified by the table of offsets or by mathematical expressions (surface equations). However, it is more convenient to represent the line plans by a set of independent form parameters or coefficients of form.

Main dimensional ratios such as L/B (length/breadth),slenderness ratio $L/\nabla^{1/3}$ (length/volume$^{1/3}$),B/T(breadth/ draft) and the longitudinal and vertical displacement distributions can be used to characterize the ship hull surface. The longitudinal distribution of displacement is completely defined by the sectional area curve. Further, the sectional area curve could be characterized by prismatic coefficient "C_p" and longitudinal center of buoyancy "LCB". The vertical distribution of displacement can be characterized by the load water line area coefficient "C_{wp}" and Longitudinal center of Floatation" LCF" and half angle of entrance of the load water line "α". Therefore, L/B or $L/\nabla^{1/3}$, B/T , C_p, LCB , C_w , LCF and half angle of entrance, parameterize the ship hull forms.

4. Resistance calculations

A ship moving on an undisturbed free surface of water with a constant speed, experiences a force, known as the total resistance, so that it opposes its motion and acts parallel to the motion of the ship. The main components of ship resistance are viscous and wave making resistance. Viscous resistance is considered to be equivalent to the sum of the frictional and a pressure resistance component of viscous origin. The frictional resistance is associated with the force required to overcome the tangential stresses developed between the hull surface and the viscous fluid. The viscous pressure resistance is due to the pressure difference between the forebody and the aftbody as a consequence of the growth of the viscous boundary layer. This causes the pressure on the aftbody to be lower than on the forebody, leading to a resultant pressure force on the hull. Since this pressure force is of viscous origin, as distinct from the forces associated with the pressure distribution over a body in potential flow, it is accordingly termed the viscous pressure resistance (or pressure form resistance, or form drag). The viscous resistance R_v at a ship speed V can be expressed as:

$$R_V = \frac{1}{2}\rho S V^2 (1+k) C_F \,, \qquad (5)$$

where ρis the water density, S is the wetted surface area of the body in static condition, and C_F is frictional resistance coefficient assumed to be equivalent of that of a flat plate with the same Reynolds number as that of the ship. The ITTC 1957 Correlation line can be used to determine C_F. The form factor for a given hull-form, k, can be obtained through experiments at very low Froude numbers where the wave-making of the hull can be neglected. Prohaska's plot

[5] and Holtrop's [6] empirical formula may be used to determine the form factor.

The wave resistance is the force required to maintain the energy of the wave pattern created by the ship. An integral equation to calculate ship's wave making resistance is proposed by Michell using an array of volumetric source/sink singularities. According to Michell's integral, wave making resistance R_w is represented as [7]:

$$C_w = \frac{R_w}{0.5\rho SV^2} = \frac{8g^2}{\pi SV^4}\int_{-T}^{0} dz \int_{-L/2}^{L/2} dx f(x,z) \int_{-T}^{0} d\xi f(\xi,\zeta) \int_{0}^{\pi/2} d\theta, \quad (6)$$

where f(x,z) is a function representing the hull form. This requires evaluation of a triple integral, one integral in each of the length-wise (-L/2 to L/2) and draft-wise (0 to-T) co-ordinate directions, and one integral with respect to the angle of propagation θ of the ship-generated waves. The main assumptions in the derivation of Michell's integral are: that the body is thin, (i.e. the beam-to-length ratio is small) and the boundary conditions on both the free surface and the body can be linearised. The fluid is also assumed inviscid. Several simplifications were proposed in the literature to solve the integral. e.g. [8].

It is worth mentioning here that other methods for calculating wave resistance exist in the literature. Examples include non-linear methods such as that described by Raven [9], empirical formulae such as those developed by Holtrop and Scheenkluth [10], and the (extended) Delft series of Gerritsma et al. [11].

Concluding, the total resistance R_t, is expressed as:

$$R_t = R_v + R_w \quad (7)$$

5. Effect of hull form parameters on the total resistance

Quantitative and qualitative trends have been established through many and varieties of model testing on independent systematic variation and methodical series to study the variation of hull form coefficients on the total resistance with ship speed. Some ships have been designed with a certain hull form, which can achieve a high speed in smooth water and a small loss of speed in rough sea. Different parameters such as ratios of principal dimensions and hull form coefficients have drastic influence upon the ship speed and in turn the total resistance. Increase in length will reduce wave resistance but increase frictional resistance. An increase in draft, T is given beneficially for resistance. The increase in the breadth, B is beneficially for the stability, however, the resistance will increase. A full ship may suffer loss of speed and excessive fullness will increase resistance. For a given hull form and at a given speed or $V/\sqrt{L}$, the resistance coefficient is reduced as L/B or $L/\nabla^{1/3}$ is increased.

When the principal dimensions and fullness coefficients have been chosen, the resistance then depends mainly upon the following elements of ship form:

1. Distribution of displacement along the length as typified by the curve of cross-sectional area and longitudinal center of buoyancy.
2. Shape of the load waterline, particularly in the fore body, and the half angle of entrance.
3. Shape of the transverse section, especially near the ends.
4. Midship section area coefficient.
5. Type of stern such as cruiser or transom.

The influence of some of the above hull form coefficients upon the total resistance will be quantitatively investigated through a case study.

6. Case study

The hull surfaces generated using the FEM described earlier are examined with reference to their resistance characteristics over a range of speeds An existing trawler [12] of a length 69.35m, breadth 11.7 m, draft 5.2m, and prismatic coefficient of 0.66 is selected to demonstrate the influence of variations of hull form coefficients upon the total resistance. Table 1 shows the range of different values of hull form coefficients that were considered in the present study.

Table 1
The range of different values of hull form coeffieients

1	Prismatic coefficient, Cp	0.654 0.632 0.680
2	Longitudinal center of buoyancy, LCB	+0.986%L Modship -1.396%L
3	Water plane area coefficient, C_{wp}	0.792 0.753 0.847
4	Half angle of entrance of LWL	15° 22° 30°

6.1. *Results and discussions*

Resistance values are given as a total resistance coefficient C_T. For each hull form two plots are shown. The first is the body plan showing the different stations along the hull. The second plot compares resistance results versus Froude's number for the range of the parameters studied. The standard ITTC correlation line was used for the viscous resistance. No allowance was made for form drag effects. The wave and/or residuary resistance is calculated using Holtrop ,Schneekluth and Michell's integral in order to conduct a qualitative comparisn between the methods. The numerical method used to calculate wave resistance by Michell's integral is described fully in [13,14]. The hull forms were represented by 21 equally spaced stations, 9 waterlines; and 200 intervals of angular positions were used in the calculation of the integrals.

6.2.1. *Effect of varying Cp*

One of the basic features of the form of a ship, which affects resistance, is the general fullness of the lines which may be represented by either the block coefficient or the prismatic coefficient. For most merchant ships the resistance at a given speed will increase as the block or prismatic coefficient is increased. This is quite understandable since the full ship will be expected to create a greater disturbance than ships of finer form. This was the case with the three hull forms generated each with different C_p value. The line plan for these forms together with their resistance curves are shown on figs. 2-a,2-b, 2-c and 2-d, respectively. When these curves are closely examined a number of observations are made:

1. The three methods used for total resistance estimation show the same trend. However, only in Michell method one can observe the oscillations which characterize the resistance – speed curves obtained. This is obviously due to the fact that the methods takes into account the three dimensional aspects of the hull forms. The only drawback is that the resistance absolute values are relatively higher than their Holtrop and Schneekluth counterparts. Since we are primarily concerned with qualitative comparisons and not the absolute resistance and/or powering predictions, the results obtained by Michell method will be shown for other hull form parameters on resistance.
2. At low Froude's numbers (less than 0.15) ,the waves made by the ship are very small, and the resistance is almost wholly viscous in character. No significant effect on the resistance due to variation in *Cp* With further increase in speed, the value of C_T begins to increase more and more rapidly which are due the effects of wave systems generated by the ship motion. Amplitudes of resistance are higher for forms with higher Cp values.
3. Amplitudes of humps and hollows are higher for forms with higher *Cp* values. However, they are located at the same Froude number regardless *Cp* value. Two major peaks were observed at 0.23 and 0.27 Froude numbers

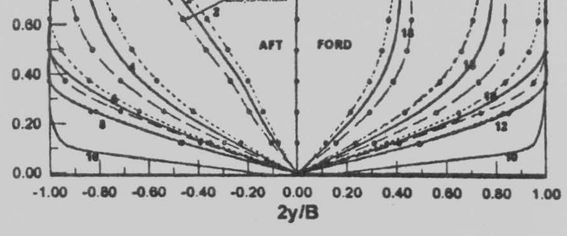

Fig. 2-a. Body plan for three values of prismatic coefficient (Holtrop method).

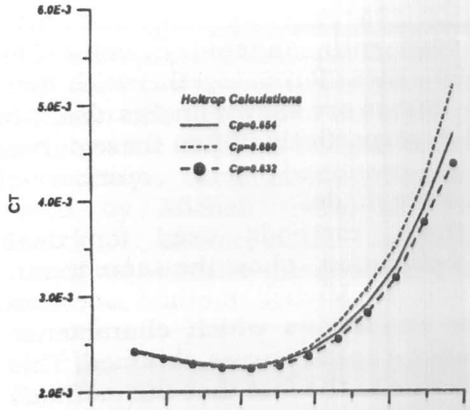

Fig. 2-b. Variation of C_T versus F_n for three values of prismatic coefficient (Holtrop method).

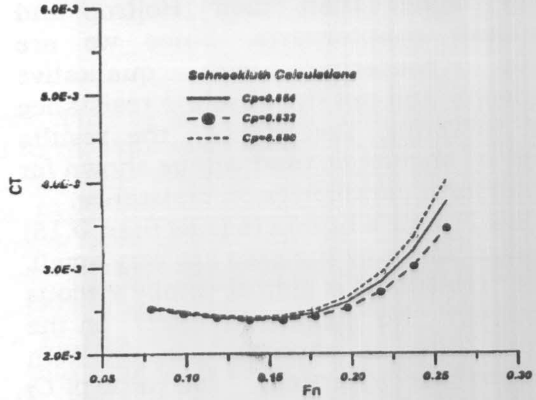

Fig. 2-c. Variation of C_T versus F_n for three values of prismatic coefficient (Schnnekluth method).

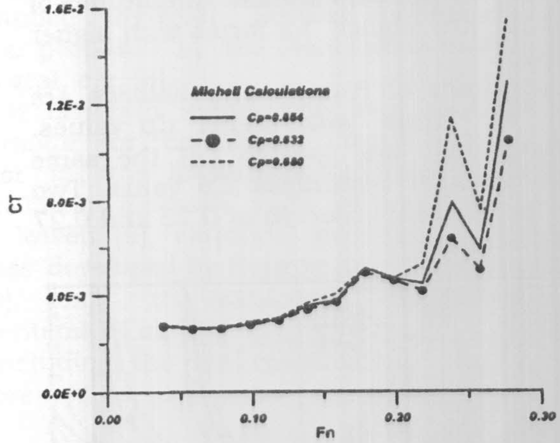

Fig. 2-d. Variation of C_T versus F_n for three values of prismatic coefficient (Michell 's method).

6.1. 2. Effect of varying LCB

The longitudinal position of the center of buoyancy of the underwater form indicates the longitudinal distribution of displacement. The influence of this parameter on resistance arises because for a given block coefficient, the position of the center of buoyancy governs the relative fullness of ends. Fining the fore end will reduce its resistance but this is done at the expense of filling the after end, the resistance of the later will be increased. There is some optimum position of the center of buoyancy where the resistance will be minimal and on either side of which the resistance will increase. For the given hull form, two *LCB* values are taken. One aft of the midship (-1.396%L) and one forward of the midship (0.986%L), in addition to the case of LCB at the midlength.

The generated body plan and the calculated total resistance are shown in figs. 3-a and 3-b,respectively. Consulting fig. 3-b, the following observations were made:

1. At Froud's number below 0.15 there is no effect on the resistance due to the *LCB* variation. When Froude's number is greater than 0.15, the resistance coefficient increases. Humps and hollows characterize the resistance curves for the three LCB's, all occurring at the same Froude number regardless LCB value.
2. The rate of C_T variation with LCB position is not as sensitive as that due to change in Cp
3. Moving center of buoyancy aftward results in fining the forward body and filling the aft body. When Fn is higher than 0.15, the resistance decreases due the finning of the forebody overweights the increase of the resistance due the filling of the aft body. This is clearly reflected on fig. 3-b.

6.1.3. Effect of varying load water plane area coefficient

A rough indication of the shape of the section can be obtained by comparing the shape of the load waterline in relation to the sectional area curve.If the two are plotted non dimensionally, then both lie close together the sections will be of U-form whilst if they lie well apart V-shaped sections will be obtained. Another factor, which might be used to give

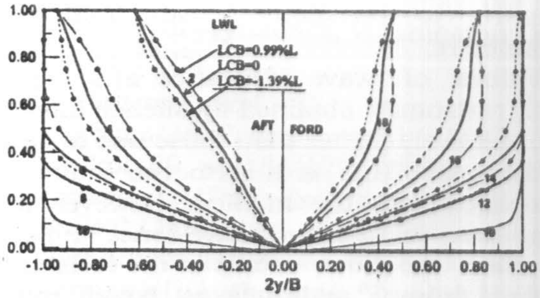

Fig. 3-a. Body plan for three values of longitudinal center of buoyancy.

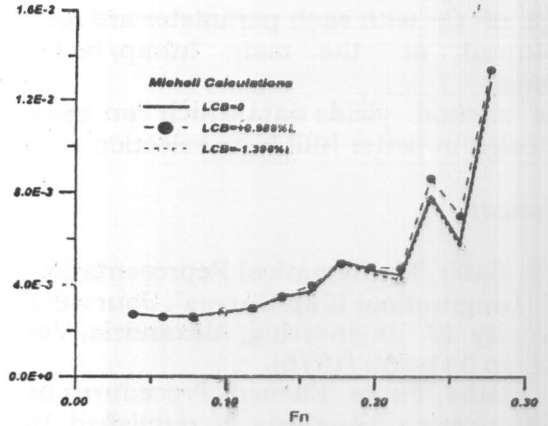

Fig. 3-b. Variation of C_T versus F_n for three values of longitudinal center of buoyancy (Michell's method).

A higher vertical prismatic coefficient will tend to give U- sections whilst a low value will give V-type sections. Variations of the transverse section are attained when coefficients of the load waterline coefficient C_{wp} changes. Three values for the load water line coefficients are considered. These are 0.75, 0.792 and 0.847, respectively.

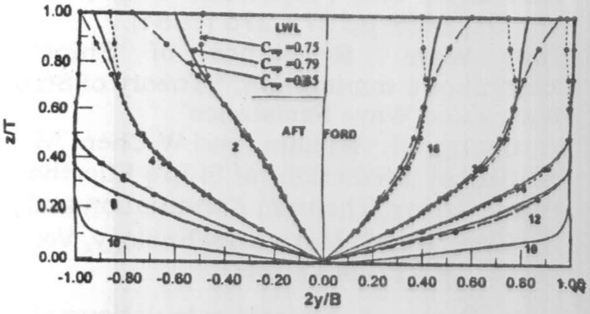

Fig. 4-a. Body plan for three values of water plane area coefficients.

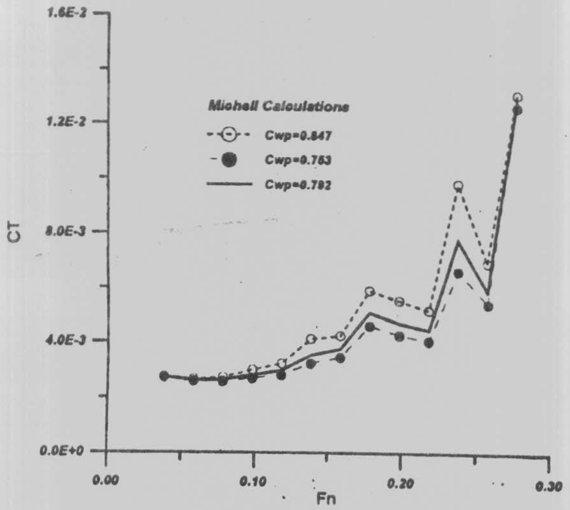

Fig. 4-b. Variation of C_T versus F_n for three values of water plane area coefficients (Michell's method).

The influence of varying of load water line coefficients on the total resistance coefficient is more pronounced at the humps locations. For the case under considerations, at Froud's number below 0.10 there is no effect on the resistance due to varying of C_{wp}. However, the total resistance coefficient C_T rapidly increases when shifting from U-shape to V-shape at Froud's number higher than 0.10

6.1.4. Effects of varying half angle of entrance of the LWL

Three values of the half angle of entrance at the load waterline are considered. These are 15° ,22° and 30° ,respectively. Body plans of these forms are shown on fig. (5a). For Froud's number less than 0.10, the effect of the variations of the half angle of entrance is not significant. Humps and hollows are observed at Froude's numbers greater than 0.15. C_T is higher for forms with higher half entrance angle. The trend is reversed for Froud's number greater than 0.22.

This because at small speeds, the effect of frictional resistance component is more significant than that of wave making resistance, therefore small angle is preferred whereas the effect of wave resistance is more significant at high speeds, hence a large angle of entrance is preferred. The results of resistance analysis are shown in fig. 5-b.

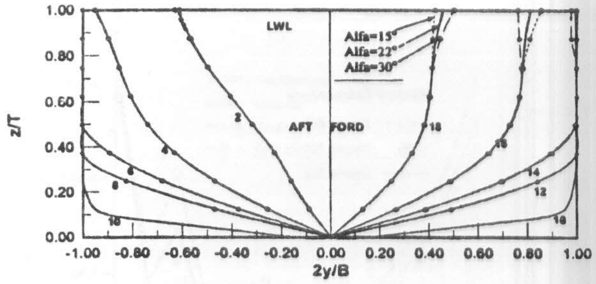

Fig. 5-a. Body plan for three values of half entrance

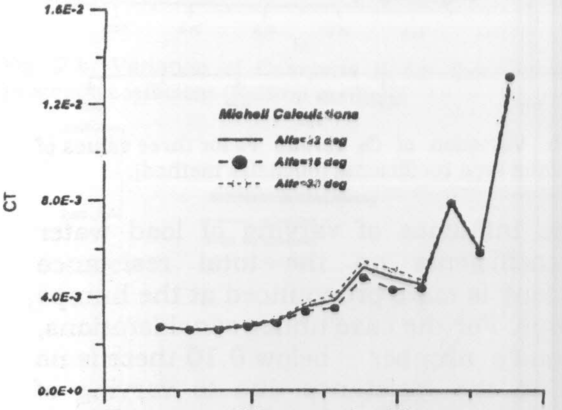

Fig. 5-b. Variation of C_T versus F_n for three values of half entrance angles(Michell's method).

7. Conclusions

In this work, hull surfaces generated using a proposed FEM were examined based on their resistance characteristics. The generated forms are used as input in table of offset format to a resistance routine and the results were discussed. The influence of varying hull form parameters upon the ship resistance is investigated. Current study revealed the followings:

1. FEM can be effectively used to generate ship hull forms. However the generated forms need be examined regarding their resistance characteristics. Hence, FEM coupled with resistance routine may constitute a powerful tool for design and optimization purposes.
2. It is worth mentioning here that the methods used for calculating total resistance generally predict the same behavior. However, Michell integral is more sensitive to variations of ship hull form parameters.
3. Values of wave resistance and hence total resistance obtained by Michell integral are relatively higher than those due to other predictions. This is due to the limitation associated with the method. However it is used here in a comparative sense and also to reflect the finer details in the resistance curve (humps and hollows) which might arise due to variation in hull form parameters.
4. For all parameters studied the rate of change of C_T with each parameter are more pronounced at the main hump/hollow locations.
5. The method yields data which can assist ship design in better hull form selection.

References

[1]A. S. Sabit,"Mathematical Representation of Longitudinal Ship Curves", Journal of Faculty of Engineering, Alexandria, Vol. 12, pp 341-369 (1976).

[2] K. Bathe,"Finite Element Procedures in Engineering Analysis ", published by Prentice –Hall Inc., NJ(1982).

[3] A. Pramila, "Ship Hull Surface Using Finite Elements ", Journal of Ship Research, Vol.16 (1), pp. 97-107 (1976).

[4] Y. A. Abdel-Nasser, "Preliminary Hull Surface Generation Using Finite Element Method", E. J. Alex, Vol.40 (4), (2000).

[5] C. W. Prohaska"A Simple Method for the Evaluation of the Form Factor and Low Speed Wave Resistance". Proceeding of the 11th ITTC

[6] J. Holtrop, "A Statistical Re-analysis of Resistance and Propulsion Data", ISP, Vol. 31(363), pp 272-276 (1984).

[7] "The Wave Resistance of Ships", Kostyukov's monograph, " Theory of Ship Waves and Wave Resistance"

[8] W. Cheng, G. William and W.Chen, "A Statistical Prediction of Ship's Effective Power Using Theorem Formulation and Historic Data", Marine Technology, Vol. 24 (3), pp 237-245 (1987).

[9] H. C. Raven, " Inviscid calculations of ship wave making- capabilities,

limitations and ", 22nd Symposium on Naval Hydrodynamics, Washington DC, U.S.A.(1998).

[10]J. Holtrop and G. Mennen, "An Approximate Power Prediction Method", ISP, Vol. 29, pp 166-172 (1982).

[11]Gerritsma , "Delft Extended Systematic Series", Published by Delft univevsty, Netherland, (1993).

[12]A group of Authorities, "Trawler Model Resistance Data Sheets ", The Society of Naval Architecture and Marine Engineers, N.Y.(1954).

[13] E.O. Tuck, "The wave Resistance of This Ships and Catamarans Report T89701, Dept. of Apphed Mathematics, University of Adelaide Australia (1987).

[14]L. Laz, "Total Resistance Calculations for the 5415 Hull", Published by Dept. of Applied Mathematics, University of Adelaide Autralia, 5005 (1999).

Received April 4, 2001
Accepted August 19, 2001