Excitation requirements for self-excited induction generators

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This paper presents a simple technique to determine both minimum and maximum values of excitation capacitor required for isolated self-excited induction generator at various rotor speeds and under different loading conditions. This technique determines, also, the critical speed below which induction generator fails to excite irrespective of excitation capacitance value. The proposed technique is based on eigen value analysis. To confirm the validity and accuracy of this technique, the theoretical results have been compared with those obtained experimentally.

يقدم هذا البحث طريقة مبسطة لتحديد كل من القيمة الصغرى والقيمة العظمى لمكثفات الإثارة اللازمة لتغذية المولدات التأثيريـــة ثلاثية الأوجه بالقدرة غير الفعالة عند ظروف التشغيل المختلفة. ويشمل البحث دراسة لتأثير تغيير سرعة دوران العضو السدوار للمولد على قيم هذه المكثفات. كما يمكن باستخدام هذه الطريقة تحديد قيمة السرعة الحرجة التي عنده! لا تتم عملية بناء الجهد للمولد بصرف النظر عن قيمة مكثفات الإثارة الموجودة . وتعتمد الطريقة المقترحة على تمثيل المولد مع مكثفات الإثارة والحمل الكهربي باستخدام النظرية العامة للألات في صورة منظومة خطية حرة القوة ثم يتم تحليل قيم جذور ممادلة الخواص لمصفوفــة هذه المنظومة، وقد تم التحقق من صحة الطريقة المقترحة بمقارنة النتائج النظرية بالعملية وقد ظهر توافقا تاما بينهما.

Keywords: Induction generator, Self-excitation, Generalized machine theory, Excitation capacitor value

1. Introduction

Induction generators have been used since early in the twentieth century, but by the 1960s they had largely disappeared from use. However, induction generators have made a comeback since oil price shocks of 1973, due to the growing interest in renewable energy generation systems. Induction generator, due to its high reliability, robustness, low cost, wide speed range and less maintenance requirement, is ideal for wind and small hydroelectric energy conversion [1-2].

Self-Excited Induction Generator (SEIG) is basically a three-phase induction machine driven by a prime mover with an external capacitor bank connected to its terminal to supply the reactive power required by the generator and by any attached load, as shown in fig. 1 To insure self-excitation a proper value of the excitation capacitor must be selected.

The capacitance required to initiate the self-excitation process has been investigated by many authors [3-7]. Chan [3], used a nodal admittance method in which the minimum exciting capacitance can be calculated by solving a 6th order polynomial equation. Harrington et al. [4],

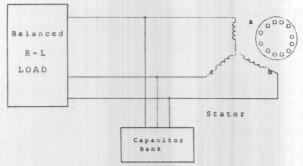


Fig. 1. Schematic diagram for SEIG.

proposed a method based on the analysis of the complex impedance matrix of SEIG. Chakraborty et al. [5] suggested methods derived from loop and nodal analyses to find different criteria for maintaining generator self excitation. Alghuwainem [6], applied Newton-Raphson technique to solve the system non-linear equations at steady state to determine the minimum excitation capacitance required when the generator is driven by regulated and unregulated turbine. Simulation of SEIG using Matlab/Simulink was used by Gastli et al. [7], to calculate both minimum and maximum values of the excitation capacitance.

In this paper SEIG feeding a balanced inductive load has been represented in state-

space format as a linear force-free system (pX = A.X). Based on the analysis of matrix A eigen values, minimum and maximum excitation capacitance values have been determined at various rotor speeds and under different loading conditions. The critical speed below which induction generator fails to excite, irrespective of the value of capacitance used, has been determined.

2. Mathematical model

The voltage equations of the three-phase induction machine in actual winding axes may be represented as,

$$\begin{bmatrix} v_a \\ v_b \\ v_c \\ v_r \\ v_s \\ v_t \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{ar} & Z_{as} & Z_{at} \\ Z_{ba} & Z_{bb} & Z_{bc} & Z_{br} & Z_{bs} & Z_{bt} \\ Z_{ca} & Z_{cb} & Z_{cc} & Z_{cr} & Z_{cs} & Z_{ct} \\ Z_{ra} & Z_{rb} & Z_{rc} & Z_{rr} & Z_{rs} & Z_{rt} \\ Z_{sa} & Z_{sb} & Z_{sc} & Z_{sr} & Z_{ss} & Z_{st} \\ Z_{ta} & Z_{tb} & Z_{tc} & Z_{tr} & Z_{ts} & Z_{tt} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_r \\ i_s \\ i_t \end{bmatrix}$$

Connecting three identical impedance, each equals **Z**, in series with phase windings, leads to the following combined voltage equation;

$$\begin{bmatrix} v_{a} \\ v_{b} \\ v_{c} \\ v_$$

Since each impedance **Z** is connected in series with the corresponding phase winding, then they have the same current and the following current transformation is deduced,

$$\begin{bmatrix} i_a \\ i_b \\ \vdots \\ i_{\ell_i} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots \\ i_{\ell_k} \end{bmatrix}, \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_c \end{bmatrix}, \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_c \end{bmatrix}, \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_c \end{bmatrix}, \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_c \end{bmatrix}, \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_c \end{bmatrix}, \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_c \end{bmatrix}, \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_c \end{bmatrix}, \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_c \end{bmatrix}, \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_c \end{bmatrix}, \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}, \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}, \begin{bmatrix} i_a \\ i_b \\ i_c$$

i.e. $I = C_1 I$, consequently the transformed impedance matrix Z can be calculated from the relation $Z = C_1 Z C_1$; [8].

$$\mathbf{Z'} = \begin{bmatrix} Z_{aa} + Z & Z_{ab} & Z_{ac} & Z_{ar} & Z_{as} & Z_{at} \\ Z_{ba} & Z_{bb} + Z & Z_{bc} & Z_{br} & Z_{bs} & Z_{bt} \\ Z_{ca} & Z_{cb} & Z_{cc} + Z & Z_{cr} & Z_{cs} & Z_{ct} \\ Z_{ra} & Z_{rb} & Z_{rc} & Z_{rr} & Z_{rs} & Z_{rt} \\ Z_{sa} & Z_{sb} & Z_{sc} & Z_{sr} & Z_{ss} & Z_{st} \\ Z_{ta} & Z_{tb} & Z_{tc} & Z_{tr} & Z_{ts} & Z_{tt} \end{bmatrix}$$

i.e. $Z' = Z_m + Z_L$

If the actual winding axes are transformed to stationary axes by the transformation matrix C_2 , [8], where

$$C_2 = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & \cos(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{2\pi}{3}) \\ 0 & 0 & \cos(\theta - \frac{4\pi}{3}) & \sin(\theta - \frac{4\pi}{3}) \end{bmatrix},$$

then,
$$\mathbf{z}'' = \mathbf{C_2^T.z'.C_2} = \mathbf{C_2^Tz_m.C_2} + \mathbf{C_2^Tz_LC_2}$$

Now $\mathbf{Z_m}'' = \mathbf{C_2^T} \cdot \mathbf{Z_m} \cdot \mathbf{C_2}$ is the usual machine impedance matrix in stationary reference frame (d, q, D, Q axes),[8],namely,

$$\mathbf{Z_m}'' = \begin{bmatrix} R_s + L_s p & 0 & Mp & 0\\ 0 & R_s + L_s p & 0 & Mp\\ Mp & M\omega & R_r + L_r p & L_r \omega\\ -M\omega & Mp & -L_r \omega & R_r + L_r p \end{bmatrix}, \text{ and}$$

$$\mathbf{Z_L}'' = \mathbf{C_2^T} \cdot \mathbf{Z_L} \cdot \mathbf{C_2} \text{ is found to be:}$$

$$\mathbf{Z_L}'' = \begin{bmatrix} \mathbf{Z} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Z} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

Hence the matrix representing SEIG in stationary reference frame is:

$$\mathbf{Z}'' = \begin{bmatrix} R_{S} + L_{S}p + Z & 0 & Mp & 0 \\ 0 & R_{S} + L_{S}p + Z & 0 & Mp \\ Mp & M\omega & R_{r} + L_{r}p & L_{r}\omega \\ -M\omega & Mp & -L_{r}\omega & R_{r} + L_{r}p \end{bmatrix}$$

The voltage equations in stationary reference frame will be:

$$\begin{bmatrix} v_d \\ v_q \\ v_D \\ v_Q \end{bmatrix} = \begin{bmatrix} R_s + L_s p + Z & 0 & Mp & 0 \\ 0 & R_s + L_s p + Z & 0 & Mp \\ Mp & M\omega & R_r + L_r p & L_r \omega \\ -M\omega & Mp & -L_r \omega & R_r + L_r p \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_D \\ i_Q \end{bmatrix}$$

The voltages of the a, b, c circuits, which are closed, are zero. Consequently the voltages v_d and v_q are both zero. Also, v_D and v_Q are both zero, since the rotor is short-circuited. Therefore, the voltage equation may be rewritten as:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_{S} + L_{S}p + Z & 0 & Mp & 0 \\ 0 & R_{S} + L_{S}p + Z & 0 & Mp \\ Mp & M\omega & R_{r} + L_{r}p & L_{r}\omega \\ -M\omega & Mp & -L_{r}\omega & R_{r} + L_{r}p \end{bmatrix} \begin{bmatrix} i_{d} \\ i_{q} \\ i_{D} \\ i_{Q} \end{bmatrix}$$

i.e.

$$\begin{bmatrix} -Zi_d \\ -Zi_q \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 \\ 0 & M\omega & R_r & L_r\omega \\ -M\omega & 0 & -L_r\omega & R_r \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_D \\ i_Q \end{bmatrix} +$$

$$\begin{bmatrix} L_s & 0 & M & 0 \\ 0 & L_s & 0 & M \\ M & 0 & L_r & 0 \\ 0 & M & 0 & L_r \end{bmatrix} \cdot p \begin{bmatrix} i_d \\ i_q \\ i_D \\ i_Q \end{bmatrix}$$

Thus the voltage equation can be written as, v = R.i + L.pi.

From which the current derivatives $p_{\underline{i}}$ is given by,

$$pi = L^{-1}.v - L^{-1}Ri.$$
 (1)

It should be noted that all machine parameters are constant except the magnetizing inductance, which is affected by magnetic saturation.

3. State-space representation

Define the following state-variables;

$$x_1 = i_d, x_2 = i_q, x_3 = i_D, and x_4 = i_O.$$

To write the derivative equations in the form pX = A.X, let us consider first the terms $Z.i_d$ and $Z.i_q$.

The impedance **Z** is the per-phase load impedance in parallel with the per-phase excitation capacitance, i.e,

$$Z = \frac{(R + Lp)(\frac{1}{Cp})}{R + Lp + \frac{1}{Cp}} = \frac{R + Lp}{LCp^2 + RCp + 1},$$

$$R + Lp + \frac{1}{Cp} = \frac{(R + Lp) \cdot x_1}{(R + Lp) \cdot x_1} = \frac{(R + Lp) \cdot x_1}{(R + Lp) \cdot x_1}$$

$$Z.i_d = \frac{(R + Lp).x_1}{LCp^2 + RCp + 1} = (R + Lp)x_5,$$

= $Rx_5 + Lpx_5 = Rx_5 + Lx_6,$

where;

$$x_5 = \frac{x_1}{LCp^2 + RCp + 1}$$
 and $x_6 = px_5$,

from which

$$px_6 = \frac{1}{LC}x_1 - \frac{1}{LC}x_5 - \frac{R}{L}x_6.$$
 (2)
Similarly;

$$Z.i_d = \frac{(R + Lp).x_2}{LCp^2 + RCp + 1} = Rx_7 + Lx_8$$

where,

$$x_7 = \frac{x_2}{LCp^2 + RCp + 1}$$
 and $x_8 = px_7$

from which,

$$px_8 = \frac{1}{LC}x_2 - \frac{1}{LC}x_7 - \frac{R}{L}x_8$$
 (3)

Now, L-1v is calculated:

$$\mathbf{L}^{-1}\mathbf{v} = \frac{1}{\Delta} \begin{bmatrix} -L_r(Rx_5 + Lx_6) \\ -L_r(Rx_7 + Lx_8) \\ M(Rx_5 + Lx_6) \\ M(Rx_7 + Lx_8) \end{bmatrix}, \tag{4}$$

where $\Delta = \mathbf{L_r L_s} - \mathbf{M^2}$.

From eqs. (1-4) the state-space derivative equations can be written as:

$$pX = A.X$$
,

where,

$$\mathbf{A} = \begin{bmatrix} -L_r R_s & M^2 \omega & MR_r & MI_r \omega & -L_r R & -L_r L & 0 & 0 \\ -M^2 \omega & -L_r R_s & -MI_r \omega & MR_r & 0 & 0 & -L_r R & -L_r L \\ MR_s & -ML_s \omega & -L_s R_r & -L_s L_r \omega & MR & ML & 0 & 0 \\ ML_s \omega & MR_s & L_s L_r \omega & -L_s R_r & 0 & 0 & MR & ML \\ 0 & 0 & 0 & 0 & 0 & \Delta & 0 & 0 \\ \frac{\Delta}{LC} & 0 & 0 & 0 & \frac{\Delta}{LC} & \frac{\Delta}{L} & 0 & 0 \\ 0 & \frac{\Delta}{LC} & 0 & 0 & 0 & 0 & \frac{\Delta}{LC} & \frac{\Delta}{L} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\Delta}{LC} & L \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{\Delta}{LC} & L \end{bmatrix}$$

The system $p\mathbf{X} = \mathbf{A}.\mathbf{X}$ is a linear force-free system, so it has a unique solution which is: $\mathbf{X} = \phi(\mathbf{t}). \mathbf{X}_0$, where $\phi(\mathbf{t})$ is the state-transition matrix, $\phi(\mathbf{t}) = \exp(\mathbf{A}.\mathbf{t})$.

If λ_i and y_i are the $i^{\underline{m}}$ eigen value and its corresponding eigen vector of matrix **A**, respectively, then [9]

$$\phi$$
 (t) = Y . F(t) . Y-1,

where

$$\mathbf{Y} = [y_1 \ y_2 ... y_8], \text{ and }$$

$$\mathbf{F}(\mathbf{t}) = \text{diag.}[\exp(\lambda_1 \mathbf{t}) \exp(\lambda_2 \mathbf{t}) \dots \exp(\lambda_8 \mathbf{t})]$$

Therefore, for the currents and

consequently the voltages to build up, the induction generator must have residual magnetism (initial conditions Xo) and the matrix A should have at least one eigen value with a positive real part (dominant eigen value). In this case the currents will increase until the magnetizing current reaches the saturation region, hence M begins to decrease and the positive real part of the dominant eigen value starts to decrease until it reaches zero. At this particular point the currents and consequently the voltage remains constant and the induction generator reaches the steady-state operating point. It should be noted that the angular frequency of the stator currents and voltages is the imaginary part of the dominant eigen value. On the other hand if all eigen values of the matrix A have a negative real part, there will be no voltage build up and excitation fails.

4. Numerical results

The machine used in this study was 1.5 kW, 415 V, 4.2 A, 4 pole, 50 Hz, three-phase star-connected induction motor having the following parameters:

$$R_s$$
 = 1.37 Ω , R_r = 3.39 Ω , M = 158 mH (unsaturated value), L_s = L_r = M + 13.3 mH.

4.1. Locus of dominant eigen value

Fig. 2 shows the locus of one of the two conjugate dominant eigen values, when the excitation capacitance changes from 20 to 9000 µF. The generator is at no load and driven at constant speed of 1500 rpm. It is obvious that if the excitation capacitance is less than 59.4 µF, or greater than 3600 µF, the dominant eigen value has a negative real part and consequently there will be no voltage build up. Therefore at this particular speed the minimum capacitance for self-excitation is 59.4 μF and the maximum one is 3600 μF. It should be noted that the stator current angular frequency varies from 313.57 rad/s (49.9 Hz) with the minimum capacitance to 101.77 rad/s (16.2 Hz) with the maximum capacitance.

4.2. Capacitance and speed limits

Fig. 3. shows the variation of both minimum and maximum capacitance as a function of rotor speed with different load impedances at u.p.f. From this figure, the critical speed below which the induction generator fails to excite, irrespective of the value of capacitance used, is found to be 159 rpm.

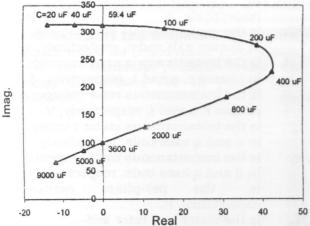


Fig. 2. Locus of the dominent eigen value.

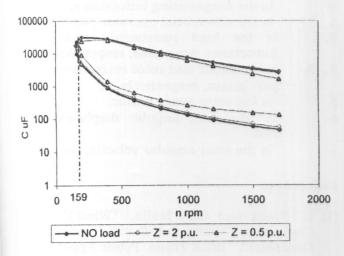


Fig. 3. Variation of capacitance limits as a function of rotor speed.

Fig. 4 shows set of curves of magnetizing inductance versus excitation capacitance for various rotor speeds under no load condition. To sustain self-excitation, the magnetizing inductance should be less than or equal to the

unsaturated magnetizing inductance. It shows clearly in fig. 4 that each curve, for rotor speed higher than 159 rpm, intersecting with the line M=158 mH at two points. These points are the minimum and the maximum excitation capacitance at this particular speed. The curve for rotor speed of 159 rpm is tangent to the line M=158 mH at C= 16000 μ F. For rotor speed less than 159 rpm their will be no intersections and self-excitation will not be developed.

The variation of the minimum excitation capacitance with respect to the load power factor is shown in fig. 5 for load impedance of 4, 2, 1 and 0.5 p.u., and with constant rotor speed of 1500 rpm. It can be seen that -for impedance of 0.5 p.u.- the excitation capacitance should be greater than 206 μF to insure self-excitation irrespective of the load power factor.

5. Experimental results

In order to verify the validity of the proposed technique, experiments were performed on the above-mentioned induction machine and the minimum excitation capacitance at different speeds were measured. Fig. 6 shows the calculated and the measured values of the minimum excitation capacitance required to initiate self-excitation at no load under various rotor speeds. It is obvious that there is a good agreement between measured and calculated results.

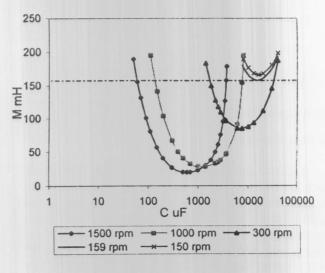


Fig. 4. M versus C curves.

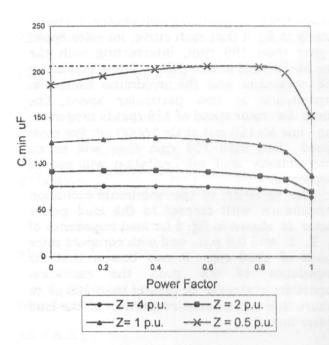


Fig. 5 Variation of minimum capacitance with the load power factor.

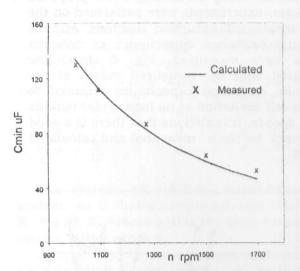


Fig. 6 Experimental results.

6. Conclusions

A direct and simple technique to determine both minimum and maximum values of the excitation capacitor required for stand alone self-excited induction generator has been presented. This technique is based on examining the dominant eigen value of an 8th order linear matrix representing the SEIG. The effect of rotor speed as well as loading

conditions on the excitation capacitance limits has been studied. The critical speed below which induction generator fails to excite, irrespective of the value of capacitance used, has been determined. The analytical results have been verified experimentally.

is the instantaneous stator currents

phases a, b and c,

List of symbols

	respectively, A.
V_a, V_b, V_c	is the instantaneous stator voltages
	in phases a, b and c, respectively, V.
1r, 1s, 1t	is the instantaneous rotor currents
	in phases r, s and t, respectively, A.
V_s, V_s, V_t	is the instantaneous rotor voltages in
	phases r, s and t, respectively, V.
i _d , i _q	is the instantaneous stator currents
-u, -q	in d and q axes coils, respectively.
i _D , i _Q	is the instantaneous rotor currents
1D, 1Q	
0	in d and q axes coils, respectively.
C	is the per-phase excitation
	capacitance, F.
Ls, Lr	is the stator and rotor self-
	inductances per-phase respectively,
	H.
M	is the magnetizing inductance, H.
р	is the differential operator, d/dt.
R, L	is the load resistance, and self-
	inductance per-phase, respectively.
R _s , R _r	is the stator and rotor resistances
148, 147	
	per- phase, respectively, Ω .
X	is the state-space vector.
θ	is the rotor angular displacement,
	rad.
ω	is the rotor angular velocity, rad/s.

References

- G. Raina and O.P. Malik, "Wind Energy Conversion Using Self-Excited Induction Generator", IEEE Trans. Power Apparatus and Systems, Vol. 102 (12), pp. 3933-3936 (1983)
- [2] P.S. Khan and J.K. Chatterjee, "Modeling and Control Design for Self-Excited Induction Generator with Solid-State Lead-Lag Var Compensator in Micro-Hydro Energy Conversion Scheme", TENCON'98, IEEE International Conference on Global

- Connectivity in Energy, Computer and Control, Vol. 2, pp. 398-401(1998).
- [3] T.F. Chan, "Capacitance Requirements of Self-Excited Induction generators", IEEE Trans. on Energy Conversion, Vol. 8 (2), pp. 304-311(1993).
- [4] R.J. Harrington and F.M. Bassiouny, "New Approach to Determine the Critical Capacitance for Self-Excited Induction Generator", IEEE Trans. on Energy Conversion, Vol. 13 (3), pp. 244-249 (1998).
- [5] C. Chakraborty and S. Bhadra "Excitation Requirements for Stand Alone Three-Phase Induction Generator", IEEE Trans. on Energy Conversion, Vol. 13 (4), pp. 358-365(1998).
- [6] S.M. Alghuwainem, "Steady-State Analysis of an Isolated Self-Excited Induction Generator Driven by Regulated and Unregulated Turbine" IEEE Trans. on Energy Conversion, Vol. 14 (3), pp. 718-723 (1999).
- [7] A. Gastli, M. Akherraz and A. El-Badi, " Modeling and Simulation of a Variable Speed Self-Excited Induction Generator", International Journal of Renewable Energy Engineering, Vol. 2 (3) (2000).
- [8] N.N. Hancock, "Matrix Analysis of Electrical Machinery"; Pergamon Press (1974).
- [9] L.K. Timothy and B.E. Bona, "State-Space Analysis", McGraw Hill (1968).

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