# Vertical cylinders wave absorber in shallow water

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A theory is developed to predict the wave heights due to the existence of a vertical slotted barrier. The method takes into account the random nature of the waves through the introduction of the wave spectrum. The method is based upon fulfilling the principles of mass and energy conservation at both sides of the cylinders. It was found that the transmission coefficient decreases with increasing the wave steepness.
يقدم هذا البحث طريقة لإيجاد خواص الأمواج الناشئة عن وجود أسطوانات رأسية لامتصاص الأمواج في المياه الضحلة . يأخذ البحث في الاعتبار عشوائية الأمواج. يعتمد البحث على تحقيق مبدأ حفظ الطاقة والمادة.

**Keywords**: Wave attenuation, Breakwaters, Energy dissipation.

## 1. Introduction

Rubble mound and vertical breakwaters are the common types of breakwaters. Some of the disadvantages associated with them are reducing circulation in the leeward side, generating currents and changing the shoreline (accretion - erosion). A phenomenon generally related to the vertical breakwaters is that the reflected waves may create problems to navigation. The rareness of the construction materials and environmental protection laws are major factors that limit using rubble mound breakwaters. Both of the above breakwaters require special equipment and trained labor.

This paper suggests using vertical array of cylinders as a breakwater. This kind of structure has the following advantages: - a) the amount of the used material is minimum, b) the water circulation is not affected by the existence of the structure, c) the effect of the reflected waves are minimized, d) the area needed for the breakwater is very small compared with any other type of breakwaters.

The only uncertain parameter needed, to determine the transmission and reflection coefficient, in addition to the wave data, is the contraction coefficient Cc. This quantity can be measured experimentally with good accuracy. The other quantities needed in the proposed method are the gap width and the dimensions of the cylinders, which can be determined accurately. This gives the method advances over the rubble mound breakwater where a lot of, hard to calibrate, quantities are needed i.e. the run up, the permeability, porosity and surface roughness

## 2. Mathematical formulation

Consider a line of vertical cylinders (circular or rectangular), extended from the seabed and piercing the water surface. The cylinders are aligned along the Y-axis. The waves propagate in the positive direction of the X-axis. The Z-axis points upward, with the origin of coordinate system located in the still water level. The spacing between the centers of any two adjacent cylinders is 2S and the minimum gap width is 2G. The maximum dimension of each cylinder in the direction of wave propagation is 2r (diameter of the circular cylinder or width of the rectangular cylinder), fig. 1.

The surface elevation $\eta$ of unidirectional random sea is given by;

$$\eta = \sum_{n=1}^{\infty} 0.5\, H_n \exp(i\, k_n\, x - \omega_n + \theta_n) \quad , \tag{1}$$

$$H_n = 2\sqrt{S_\eta(\omega)\, d\omega} \quad , \tag{2}$$

where,

$i = (-1)^{0.5}$, $k_n$ , $\omega_n$ and $\theta_n$ are the wave number, radian frequency, and random phase angle of the $n^{th}$ wave component, respectively . $S_\eta(\omega)$ is the surface wave spectrum.

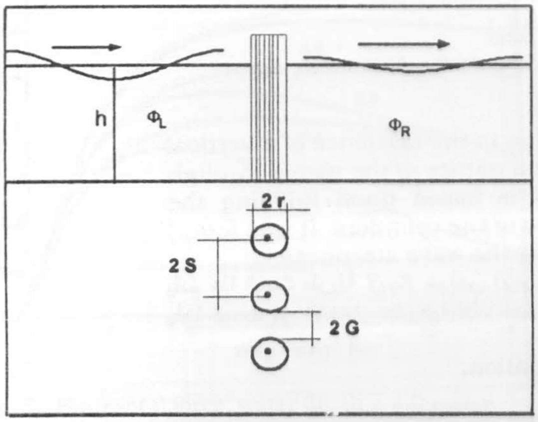


Fig. 1. Definition sketch of the wave absorber system.

The wave number $k_n$ and the radian frequency $\omega^n$ satisfy the dispersion relation

$$k_n \tanh(k_n h) = \frac{\omega_n^2}{g} , \quad (3)$$

where h is the water depth and g is the gravity acceleration.

To the left of the wall, $(x \le 0)$, the surface elevation $\eta_L$ is composed of the incident and reflected waves. In this case eq. (1) takes the form

$$\eta_L = \sum_{n=1}^{\infty} 0.5\, H_n \{\exp(i k_n x) + R_n \exp(-i k_n x)\} \exp(-\omega_n + \theta_n). \quad (4\text{-a})$$

To the right of the wall, $(x \ge 0)$, the surface elevation $\eta_R$ due to the transmitted waves, is given by,

$$\eta_R = \sum_{n=1}^{\infty} 0.5\, H_n\ T_n \exp(i k_n x)\ \exp(-\omega_n + \theta_n) \cdot \quad (4\text{-b})$$

It should be noted that eqs. (4) have no evanscent components. Fugazza and Narale[1], showed that the evanscent modes are not required to satisfy the boundary conditions on both sides of the wall, as long as the dissipation coefficient $D_c$ is constant. The surface elevation is given by,

$$\eta = -\frac{1}{g}\frac{\partial \Phi}{\partial t}, \quad (5)$$

where $\Phi$ is the total potential.

For shallow water, a potential function that satisfies eq. (5) is given by,

$$\Phi_L = \sum_{n=1}^{\infty} 0.5 \frac{g H_n}{\omega_n} \phi_{n,L} \exp(-i\omega_n t + \theta_n), \quad (6\text{-a})$$

$$\Phi_R = \sum_{n=1}^{\infty} 0.5 \frac{g H_n}{\omega_n} \phi_{n,R} \exp(-i\omega_n t + \theta_n), \quad (6\text{-b})$$

with

$$\phi_{n,L} = \exp(i k_n x) + R_n \exp(-i k_n x), \quad (7\text{-a})$$

$$\phi_{n,R} = T_n \exp(i k_n x). \quad (7\text{-b})$$

Eq. (6) satisfy the bottom and the free surface boundary conditions and the conservation of mass equations, given by,

$$\frac{\partial \phi}{\partial y} = 0 \text{ at } y = -h \qquad \frac{\partial \phi}{\partial y} = \frac{\omega^2}{g}\phi \text{ at } y = 0$$

and $\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0.$

It may be noted that the hyper cosine terms do not appear in eqs. (6). This is because for shallow water, we have $kh \ll 1$ and $\cosh(kh) = \cosh(k(h+z)) = 1$.

### 3. Spectral analysis

The spectrum of the reflected and transmitted waves are given by,

$$S_{\eta,R}(\omega) = |R(\omega)|^2 S_\eta(\omega), \quad (8\text{-a})$$

$$S_{\eta,T}(\omega) = |T(\omega)|^2 S_\eta(\omega) \cdot \quad (8\text{-b})$$

where,
$R(\omega)$ and $T(\omega)$ are the overall reflection and transmission coefficients, respectively,
$|B|^2 = B.B^*$ is the square of B and $B^*$ is the conjugate of B.

The root mean square $\eta_{rms}$ of the surface elevation is given by

$$\eta_{rms} = \sqrt{m_o} \ ,$$

where, $m_o$ is the zero order moment of the spectrum, given by ,

$$m_o = \int_0^\infty S_\eta(\omega)\, d\omega \tag{9}$$

With the definition of the transmission (reflection) coefficient as the ratio between the transmitted (reflected) and the incident wave heights, we can write, Goda [2]

$$T = \sqrt{(m_{o,T}/m_{o,I})} \ , \tag{10-a}$$

$$R = \sqrt{(m_{o,R}/m_{o,I})} \ , \tag{10-b}$$

with $m_{o,I}$ $m_{o,R}$ and $m_{o,T}$ are the zero moments of the incident, reflected and transmitted spectra, respectively.

## 4. The transmission and reflection coefficients

One may write the transmission and reflection coefficients, due to the n[th] wave component in complex form, that is

$$R_n = R_{n,R} + i\, R_{n,C} \ , \tag{11-a}$$

$$T_n = T_{n,R} + i\, T_{n,C} \ . \tag{11-b}$$

The conservation of mass requires that the velocity at both sides of the wall to be equal, that is

$$\frac{\partial \Phi_L}{\partial x} = \frac{\partial \Phi_R}{\partial x} \quad \text{at } x = 0 \ . \tag{12}$$

Substituting eqs. (7) and (11) into eq. (12)and equating the real and imaginary parts, gives

$$R_{n,R} + T_{n,R} = 1 \ , \tag{13-a}$$

$$R_{n,C} + T_{n,C} = 0 \ , \tag{13-b}$$

Mei [3] applied the principle of momentum conservation, and obtained the following equation.

$$\eta_L - \eta_R = (f/2g)\ u_L\,|u_L| + (L/g)\frac{\partial u_L}{\partial t} , \tag{14}$$

where $u_L$ is the horizontal velocity to the left of the wall, f is the head(energy ) loss coefficient and L is the effective orifice length. The first term in the right hand side of eq. (14) is caused by energy dissipation due to flow separation, while the second term is due to inertia resistance. The second term is called the inertia term and is in phase with the acceleration. The first term is in phase with the velocity and may be called the friction loss term. The velocity squared term in eq. (14) may be linearized by writing

$$0.5\, f\, u_L\ |u_L| = A(z,t)\, u_L \tag{15}$$

The dependency of A(z,t) on time and elevation follows from the fact that u, the velocity changes with both time and elevation. In this paper we consider shallow water waves, where the horizontal velocity is uniformly distributed with depth. This means that A depends on time only. A(t) is to be replaced with a constant value $\bar{A}$, the linear dissipation coefficient. $\bar{A}$ is selected such that the error E , given by

$$E = (\bar{A} - 0.5\, f\, |u_L|)\, u_L \ ,$$

is minimized in the least square sense. That is

$$\langle E^2 \rangle = \langle (0.5\, u_L\, |u_L|)^2 \rangle - f\,\bar{A}\,\langle u_L^2\, |u_L| \rangle + \langle u_L^2 \rangle$$

is minimum, where < > means the time average. This gives

$$\left\langle \frac{\partial E^2}{\partial \bar{A}} \right\rangle = f \left\langle u_L^2\, |u_L| \right\rangle + 2\,\bar{A} \left\langle u_L^2 \right\rangle = 0,$$

that is

$\bar{A} = 0.5 f \langle u^2 |u| \rangle / \langle u^2 \rangle$,

If u is a Gaussian process with zero mean, then

$\langle u^2 |u| \rangle = \sqrt{8/\pi}\, \sigma_u^3$ and $\langle u^2 \rangle = \sigma_u^2$,

and Ā is given by, Brebbia [7]

$$\bar{A} = 0.5 f \sqrt{8/\pi}\, \sigma_u , \tag{16}$$

where $\sigma_u$ is the standard deviation of u .

The standard deviation of any process u having spectrum $S_u(\omega)$ is given by

$$\sigma_u^2 = \int_0^\infty S_u(\omega)\, d\omega. \tag{17}$$

Noting that,

$$u = \frac{\partial \varphi}{\partial x} \qquad \frac{\partial u}{\partial t} = \frac{\partial^2 \varphi}{\partial x\, \partial t} = -i\omega \frac{\partial \phi}{\partial x}. \tag{18}$$

Substituting eqs. (5), (15) and (16) into eq. (14), gives

$$\phi_{n,R} - \phi_{n,L} = L \frac{\partial \phi_{n.L}}{\partial x} + \frac{i \bar{A}}{\omega_n} \frac{\partial \phi_{n.L}}{\partial x}. \tag{19}$$

Substituting eq. (7) and (11) into eq. (19), and separating the real and imaginary parts gives

$$(W_n + 1) R_{n,R} + C_n R_{n,C} - T_{n,R} = W_n - 1, \tag{20-a}$$

$$C_n R_{n.R} - (W_n + 1) R_{n,C} + T_{n,C} = C_n, \tag{20-b}$$

with

$$W_n = \frac{\bar{A} k_n}{\omega_n} \quad \text{and} \quad C_n = L k_n .$$

Solving eqs. (13) and (20) gives

$$R_{n,R} = \left\{ W_n(W_n + 2) + C_n^2 \right\} / V_n, \tag{21-a}$$

$$R_{n.C} = 2 C_n / V_n, \tag{21-b}$$

$$T_{n,R} = 2 (W_n + 2) / V_n , \tag{21-c}$$

$$T_{n.C} = 2 C_n / V_n , \tag{21-d}$$

with

$$V_n = (W_n + 2)^2 + C_n^2 . \tag{21-e}$$

Substituting eqs. (21) into eqs. (11), gives the reflection and transmission coefficients.

It is clear, from eqs. (21), that the reflection and transmission coefficients depend on the dissipation coefficient Ā through the parameter $W_n$. At the same time the dissipation coefficient is controlled by the horizontal velocity, which is a function in the coefficients. This means that iteration methods must be used to find Ā.

## 5. The dissipation coefficient and the effective orifice length

The head loss coefficient, f, is calculated using plate orifice for large opening ($G/r > 0.5$). For small opening the pipe formula is used, Fugazza and Natale [1]. The head loss coefficient, for a rectangular orifice is given by

$$f = \left(\frac{1}{PC_C} - 1\right)^2, \tag{22}$$

where, $C_C$ is the contraction coefficient and P ( = G/S) is the porosity.

It was shown by Kirchhoff, more than a century ago, that $C_C = \pi/(\pi + 2) \simeq 0.6$, for a plate orifice, see Rouse [4]. For all practical applications $C_C$ does not depends on Renolds number, and can be taken constant, Fugazza and Natale [1]. For sharp edged orifice, the empirical formula for $C_C$ is $C_C = 0.6 + 0.4 P^2$. For thick or rounded edges $C_C$ is much closer to unity. According to the above formula $C_C$ varies between 0.6 and 1.0, Mei [3]. This range of variation is mentioned by Shaes [5].

For a rectangular opening the porosity is constant, while for a circular cylinder, the porosity varies with distance along the direction of wave propagation. In this case a mean value for the head loss coefficient is needed. Expanding the head loss coefficient using Taylor series, we get

$$f = (\frac{1}{P C_c})^2 - 1 + \cdots$$

This shows that a mean value for $P^{-2}$ must be used. From fig. 1, we can write

$$P^2(x) = \frac{(S - r\cos\theta)^2}{S^2} \qquad -\pi \le \theta \le \pi$$

The mean value $P^{-2}$ is given by

$$\frac{1}{\bar{P}^2} = \frac{S^2}{2\pi} \int_{-\pi}^{\pi} \frac{d\theta}{(S - r\cos\theta)^2} . \qquad (23)$$

The mean value of the head loss is given by

$$\bar{f} = (\frac{1}{\bar{P} C_c} - 1)^2 . \qquad (24)$$

Mei et al [6], and Mei [3] reported that for flow with no separation the effective orifice length $L_o$ is an upper bound for the effective orifice length, L, with separation. They gave the following formula for $L_o$

$$L_0 = \frac{T - (1 + R)}{k T} ,$$

where T, R and k are the transmission coefficient, reflection coefficient and the wave number , respectively. Since the linearized potential theory is in analog with the acoustics theory, then the effective length, $L_o$, calculated from acoustic theory may be used in this study. Based on this argument, Mei [3] used the following two limiting formula for large and small porosity.

$$L_o \cong (2S/\pi) \ln(\frac{2S}{\pi G}) \qquad G/S \ll 1, \qquad (25\text{-a})$$

and

$$L_o \cong (S\pi/4)(\frac{S-G}{S})^2 \qquad (S-G)/S \ll 1. \qquad (25\text{-b})$$

The order of magnitude of the inertia term and head loss (friction) term are given by

$$\mu = O\left(\frac{k L}{\frac{f}{2}\frac{0.5 H}{h}}\right),$$

where 0.5 H is the wave amplitude.

Mei [3] used this relation to show that the effect of the inertia term is small with respect to the head loss term, for the cases of steep waves and long waves.

Mei et al [6], reported that Hayashi neglected the inertia effect altogether. It may be noted, Mei [3], the inertia term is equivalent to an added mass with length L and cross sectional area 2G added at the *vena contracta*. According to Fugazza and Natale [1], L may be taken equal to the thickness of the wall. This is due to the very small effect of the inertia term, as stated by Mei. In this paper we will use eqs. (25) to calculate L

## 6. Solution procedures

To find Ā, we need first to calculate the horizontal velocity to the left of the wall, $u_L$. Using eqs. (6-a), (7-a), (11-a) and (12), one gets

$$u_{n,L} = \frac{i g k_n}{\omega_n} \frac{H_n}{2} \frac{2\{(W_n + 2) - iC_n\}}{(W_n + 2)^2 + C_n^2} \exp(-i\omega_n t + \theta_n).$$

The transfer function $T_u$ of $u_L$ is given by

$$T_u = \frac{i g k_n}{\omega_n} \frac{H_n}{2} \frac{2\{(W_n + 2) - iC_n\}}{(W_n + 2)^2 + C_n^2}. \qquad (26)$$

The spectrum $S_{u,L}(\omega)$ of the velocity to the left of the wall is given by

$$S_{u,L}(\omega) = |T_{u,L}(\omega)|^2 S_\eta(\omega), \qquad (27)$$

where,

$$|T_{u,L}(\omega)|^2 = T . T^* = 4(\frac{g k_n}{\omega_n})^2 ((W_n + 2)^2 + c_n^2)^{-1}$$
$$= (\frac{4 g}{h})((W_n + 2)^2 + c_n^2)^{-1} . \qquad (28)$$

To write the above equation, we used the dispersion equation, for shallow water waves, which is given by, $\omega^2/k^2 = g h$.

Ā is found by using eqs. (28), (27), (17) and (16) in order. The process is repeated until the difference between the initial value introduced to eq. (28) and the final value given by eq. (16) is accepted.

In this study the P-M spectrum is used. In terms of the significant wave height Hs it has the form

$$S_{\eta}(\omega) = 8.1 \quad 10^{-3} \quad \omega^{-5} \, g^2 \exp(-B\,\omega^{-4}),$$

with (29)

$$B = 3.11\, H_S^{-2}.$$

## 7. Case study

To study the effect of the effective orifice length on the reflection and transmission coefficients, different values for Cc are used (0.6, 0.8, 1.0) with different values for S/G (2.5 to 6.). The significant wave heights used in the analysis are 1,2,3 and 4 to take the effect of wave steepness into account. Neglecting the orifice length all together or taking it equal Lo, as given by eq. (25-a) has minimum effect on the coefficients. This changes the transmission coefficient by less than 1% and the reflection coefficient with less than 7% at most. This is in agreement with the finding of Mei et al. [6],

Fig. 2 shows the variation of the square of the transfer functions, eq. (27), for the transmitted and reflected waves and the linear dissipation coefficient with the significant wave height. The figure shows that the transfer function for the transmitted waves decrease with increasing the significant wave height, while the transfer function for the reflected wave increase with increasing the significant wave height. This can be explained as follow. If we neglect the effective orifice length, then the transfer function for the transmitted waves takes the form, using eq. (21). with W = Ā / (gh)$^{0.5}$ for shallow water..

$$|T|^2 = \frac{4}{(W+2)^2} \text{ and } |R|^2 = \left(\frac{4}{(W+2)^2}\right)^2.$$

Since Ā is proportional to the root mean square of the wave then Ā increase with increasing Hs.

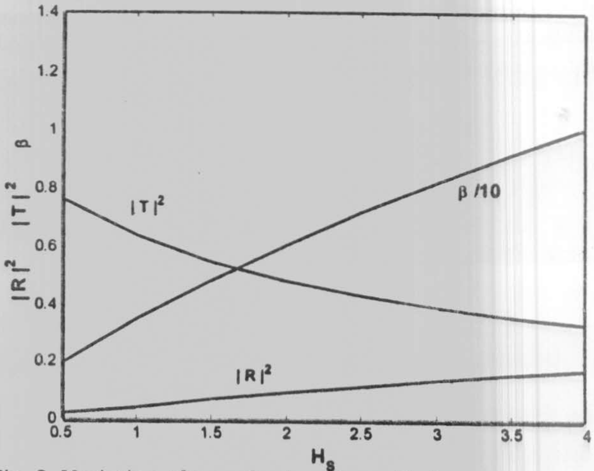


Fig. 2. Variation of transfer function with hs.

This means that the transmission coefficient decrease fast with increasing the wave steepness. In contrast the reflection transfer function increase slowly with increasing Hs.

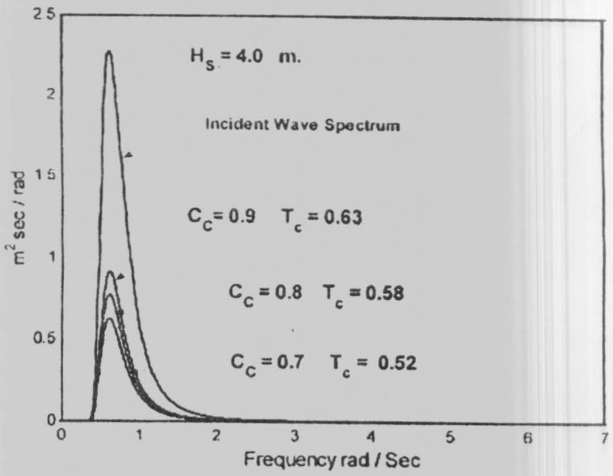


Fig. 3-a. Variation of Tc with Cc .

Fig. 3 show the variation of the transmission coefficient with the contraction coefficient. For all values of Hs, variation in the transmission coefficient is proportional to the variation in the contraction coefficient. Taking into account the uncertainty in the wave data, one may conclude that the model is valid for the practical applications, using approximate value for the contraction coefficient.

Fig. 4 shows the variation of both the transmission and reflection coefficients with the gap width. From the figure, it may be clear that changing the gap width has a pronounced effect on both the transmission and reflection coefficients.

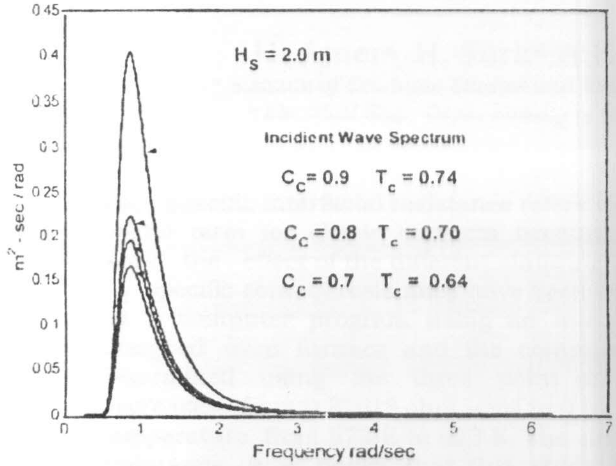


Fig. 3-b. Variation of $T_c$ with $C_c$ .

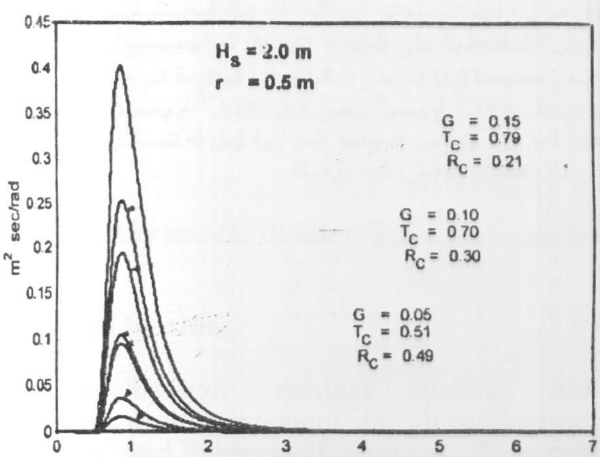


Fig. 4. Variation of $T_c$ and $R_c$ with gap width.

## 8. Conclusions

The proposed approach uses the conservation of mass and energy principles to find the wave climate at both sides of slotted array of vertical cylinders. The method takes into account the random nature of the sea. Besides, it depends mainly on one uncertain parameter, the contraction coefficient. This coefficient can be determined experimentally with good accuracy. The theory shows that the efficiency of this kind of structures increases with increasing the wave steepness. This is an advantage over the other types of breakwaters. Besides the proposed structure needs minimum space and material.

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