Docks and breakwaters with minimum environmental impact for small boat harbors and marinas

Mahmoud A. Sharaki
Transportation Eng. Dept., Faculty of Eng., Alexandria University, Alexandria , Egypt.

This paper introduces a simplified analytical method to study the effect of a pontoon type dock. The method predicts the forces experienced by the waves, and the flow field in the leeward of the dock. The dimensions of the structure could be resized to minimize the environmental impact resulting from the existence of the dock, such as formation of salients and tombolos, while keeping the breakwater functional criteria. The model eliminates the use of the variational principle and is based on mass and energy flux.

يقدم هذا البحث طريقة لحساب ارتفاع الامواج خلف الحواجز و منشآت التراكى التي تتواجد فتحات بأسفلها و أيضا يقدم طريقة لحساب سرعة الجزيئات قريبا من القاع لاستخدامها فى حسابات النحر و الترسيب. تعتمد الطريقة المقدمة على تمثيل دالة الجهد فى شكل متسلسلة و استخدام السرعة و الضغط عند حدود المنشأ لإيجاد ثوابت حدود المتسلسلة. ايضا تقدم النظرية طريقة لحساب القوى الرأسية و الأفقية و العزوم الواقعة على المنشأ نتيجة الأمواج. يمكن تقديم النتائج في شكل مجموعة من المنحنيات لاستخدامها في التصميم.

Keywords: Elevated dock, Breakwaters with gap, Environmental effect eigenfunction, Expansion

1. Introduction

It is necessary to have calm water along recreation beaches and small boat marinas. Usually, this is achieved by building breakwaters. To minimize the environmental impact, floating breakwaters proved to be an ideal solution for moderate wave conditions. Mei and Black [1] used the varitional principle to solve the problem of fixed surface or submerged obstacle, Black et al [2] used the same approach to solve the same problem for a movable obstacle. In this work we introduce a simplified analytical approach to solve the problem of a fixed body piercing the water surface but not extending to the sea floor. The theory compares well with the previous works and has the advantage that it can be used manually, without the need to any advanced programming.

Consider a pontoon with infinite length, draft d, bridth 2B, water depth h and gap G beneath the pontoon, fig. 1. Also, consider a Cartesian coordinate system with the origin at the intersection of the still water level and the pontoon vertical axis of symmetry. The horizontal axis x is directed to right and the vertical axis y points upward and the horizontal axis z runs along the axis of the pontoon. The pontoon is subjected to harmonic incident wave propagating from left to right.

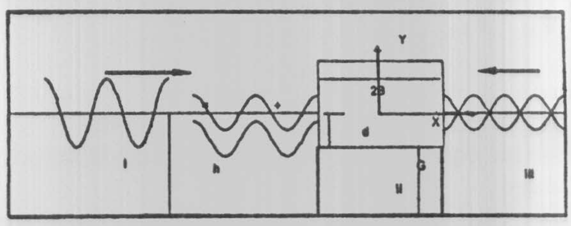

Fig. 1. The definition sketch of the pontoon and wave decomposition.

It is a natural procedure to decompose the incident wave into two waves, symmetric and asymmetric waves. One propagating from left to right and the other propagating from right to left. Each one consists of two waves. The wave propagating to the right consists of two waves in phase, while the one propagating to the left consists of two waves in anti phase, fig. 1. This technique is used widely in the electromagnetic theory of wave guide, [1].

2. Mathematical formulation

The usual assumptions of irrotational flow and incompressible and inviscid fluid are employed. If the flow is assumed to be periodical, then the potential is to be given by, eq. (1)

$$V_x = -2 i k_o f_o(y) S_{oo} \exp(i k_o (x+B)) \exp(-i\omega t) \quad (1)$$

where $i = (-1)^{0.5}$, and Re means the real part

The potential $\varphi(x,y)$ may be decomposed into two parts. A potential due to symmetric flow φ_S and a potential due to asymmetric flow φ_A, [1]

$$\phi = \phi_S + \phi_A \quad (6) \quad (2)$$

Each of the potentials φ_i , φ_s consists of two components, symmetric and asymmetric potentials

$$\phi_i = \phi_{iS} + \phi_{iA} \quad (3)$$

$$\phi_s = \phi_{SS} + \phi_{SA},$$

where,

φ_I is the potential of the incident wave

φ_s is the potential of the scattered (diffracted) wave

For the cases of the symmetric and the asymmetric waves we have $\varphi_{iA} = \varphi_{sA} = 0$ and $\varphi_{is} = \varphi_{ss} = 0$, respectively. Since, actually there is no waves propagating from right, then the two hypothetical waves propagating from right must cancel one another. This requires that they must have the same amplitude. Based on this, the heights of the two waves propagating from left are equal one half the height, H_i, of the original incident wave that is, eq. (4)

$$H_{iS} = H_{iA} = 0.5 H_i, \quad (4)$$

where, H_i, H_{is}, and H_{ia} are the heights of the total incident, the symmetric and the asymmetric wave components, respectively. It should be clear that the reflection coefficient R_S, of the symmetric wave, equals unity. This stems from considering the symmetric wave propagating to the left as a result of the reflection of the symmetric wave propagating to the right.

The reflection coefficient of the asymmetric wave R_A needs more elaborated treatment. The flow domain is divided into three sub domains, I, II and III, Fig. 1. The continuity equation, together with the sea bed boundary conditions are to be satisfied in the three sub domains.

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0, \quad (5)$$

$$\frac{\partial \phi}{\partial y} = 0 \quad \text{at } y = -h \quad . \quad (6)$$

The linearized kinematic free surface boundary conditions, eqn., (7), should be satisfied at regions I and III

$$\frac{\partial \phi}{\partial y} = \frac{\omega^2}{g} \phi \quad \text{at} \quad y = 0 \quad |y| > B. \quad (7)$$

The components of the fluid particle velocity normal to the surface of the pontoon should be equal to that of the pontoon, that is.

$$\frac{\partial \Phi_i}{\partial n} + \frac{\partial \Phi_S}{\partial n} = 0. \quad (8)$$

Where

n is a unit vector normal to the surface of the pontoon and directed outward

$\frac{\partial \Phi}{\partial n}$ is the fluid velocity normal to the surface

In region II the boundary condition given by eq. (6) must be satisfied at y = -d, for the fixed body, that is,

$$\frac{\partial \phi}{\partial y} = 0 \quad \text{at } y = -d. \quad (9)$$

To satisfy the continuity equation in addition to the different boundary conditions imposed in regions I, II and III, different mathematical expressions for the potential in regions I, II, and III are introduced. In region I the potential $\varphi_{I,o}$, is given by ,

$$\phi_{I,o} = \phi_{IS,o} + \phi_{IA,o}$$
$$= f_0(y)\, S_{oo}\left[\exp(ik(x+B)) + R_s \exp(-ik(x+B))\right]$$
$$+ \sum_{i=1}^{\infty} f_i(y)\, S_{o,i} \exp(k_i(x+B)) +$$
$$f_0(y)\, A_{oo}\left[\exp(ik(x+B)) + R_a \exp(-ik(x+B))\right]$$
$$+ \sum_{i=1}^{\infty} f_i(y)\, A_{o,i} \exp(k_i(x+B) \qquad x \leq -B, \tag{10}$$

with

$$f_o(y) = \frac{\cosh\left[k_{o_o}(y+h)\right]\sqrt{2}}{\left[h + \frac{g \sinh^2(k_o h)}{\omega^2}\right]^{0.5}} , \tag{11}$$

$$f_i(y) = \frac{\cos\left[k_i(y+h)\right]\sqrt{2}}{\left[h - \frac{g \sin^2(k_i h)}{\omega^2}\right]^{0.5}} . \tag{12}$$

The functions f_i (y), i =0,1,2, are orthogonal in the interval (-h, 0), that is [3],

$$\int_{-h}^{0} f_i(y)\, f_j(y)\, dy = \delta_{ij}$$

where δ is Kronecker delta function (δ_{ij} = 1 i = j , δ_{ij} = 0 i ? j)

This will be important in finding the coefficients of eq. (10). Eq. (10) satisfies the continuity equation and the boundary conditions given by eq. (6). Eq. (7) is satisfied by considering the dispersion relation (13) and eq. (14) [3]

$$k_o \tanh(k_o h) = \frac{\omega^2}{g} , \tag{13}$$

$$k_i \tan(k_i h) = -\frac{\omega^2}{g} , \tag{14}$$

where k_i is the positive real root of eq. (14).

The first term in both $\phi_{is,o}$ and $\phi_{iA,o}$ eq. (10), corresponds to the incident wave, while the second term corresponds to the reflected wave. The terms under the summation sign, the third term, give the evanescent modes, which decay rapidly with x. As was stated before, the potential is decomposed into the sum of symmetric and asymmetric potentials around the vertical axis passing through the center of the pontoon. Based on this, the potential in region III is given by

$$\phi_{III}(x,y) = \phi_{IS}(-x,y) - \phi_{IA}(-x,y) . \tag{15}$$

Since all the equations used in the analysis are linear, then one can solve the equations separately for the symmetric and the asymmetric cases, and then summing up those separate solutions

The potential in region II, the gap beneath the pontoon is given by,

$$\phi_{ii,o} = \phi_{iiS,o} + \phi_{iiA,o}$$
$$= U_{GS,o} + \sum_{i=1}^{\infty} A_{GS,i} \cosh\frac{i\pi x}{G} \cos\frac{i\pi(y+h)}{G} +$$
$$+ U_{GA,o}\, x + \sum_{i=1}^{\infty} A_{GA,i} \sinh\frac{i\pi x}{G} \cos\frac{i\pi(y+h)}{G} . \tag{16}$$

Eq. (16) satisfies the continuity eq. (5) and the seabed boundary condition eq. (6). Since, there is no mass flux.

$$\int_{-h}^{-d} \cos\frac{i\pi(y+h)}{G}\, dy = 0 , \tag{17}$$

and the energy flux of the evanescent modes

$$\int_{-h}^{-d} P_{evan}\, V_{x,evan}\, dy = \sum_{i=1}^{\infty} \int_{-h}^{-d} \frac{\partial\phi}{\partial t}\frac{\partial\phi}{\partial x}\, dy , \tag{18}$$

where

$$\int_{-h}^{-d} \sin\frac{i\pi(y+h)}{G} \cos\frac{j\pi(y+h)}{G}\, dy = 0 . \tag{19}$$

Using eqs. (17) and (19), one can conclude that there is neither mass flux nor energy flux along the boundary between regions I and II, then the eignfunction expansion terms in eq. (16) may be neglected, and eq. (16) reduces to,

$$\phi_{ii,o} = \phi_{iiS,o} + \phi_{iiA,o} = U_{GS,o} + U_{GA,o}\ x. \tag{20}$$

Making use of eq. (4), it may be concluded that the potential of both the symmetric and the asymmetric components of the incident waves are equal to one half of the potential of the incident wave. This requires that $A_{oo} = S_{oo}$ in eq. (10).

Equating the velocity at the boundary between regions I and II, we get

$$\frac{\partial \phi_{I,o}}{\partial x} = \frac{\partial \phi_{II,o}}{\partial x} \quad \text{at} \quad x = -B \quad . \tag{21}$$

Using eqs. (10) and (20) multiplying by $f_i(y)$ and integrating along the interval (-h,0), noting that the functions $f_i(y)$; i=0,1,2,... are orthogonal , this yields,

$$R_S = 1 , \tag{22}$$

$$R_a = 1 + \frac{i\,U_{GA,o}\,I_o}{k_o\,S_{o\,o}} , \tag{23}$$

$$S_{o,i} = 0 \qquad i = 1,2,\dots , \tag{24}$$

$$A_{o,i} = \frac{I_i\,U_{Ga,o}}{K_i} \qquad i = 1,2,\dots , \tag{25}$$

where

$$I_i = \int_{-h}^{-d} f_i(y)\,dy . \tag{26}$$

To find S_{oo} and A_{oo} we take into account eq. (4), and the surface elevation η given by

$$\eta = -\frac{1}{g}\frac{\partial \Phi}{\partial t} \quad \text{at } y = 0 . \tag{27}$$

If A_m is the amplitude of the incident wave, then using eqs. (4), (10) and (27), one gets, using the linear wave theory

$$0.5\,A_m = -\frac{1}{g}\frac{\partial \phi}{\partial t} = \frac{i\,\omega\,f(0)\,S_{o\,o}}{g} = \frac{i\,\omega\,f(0)\,A_{o\,o}}{g} ,$$

or

$$S_{oo} = A_{oo} = -\frac{i\,A_m\,g}{2\,w\,f_o(0)} . \tag{28}$$

The reflection coefficient, R_f for the fixed pontoon, is given by

$$R_f = 0.5\,(R_S + R_A) . \tag{29}$$

Since the incident wave is decomposed into two waves of equal amplitude, each one equals one half of the incident wave, then

$$\begin{aligned} R_f\,A_i &= R_S\,(0.5\,A_i) + R_A(0.5\,A_i) \\ &= 0.5\,(R_S + R_A)\,A_i \end{aligned} \tag{30}$$

This leads directly to eq. (29). The transmission coefficient is given by,

$$T_f = 0.5\,(R_S - R_A) . \tag{31}$$

T_f is the reflection coefficient of the two waves propagating from right. They are treated exactly as before. The minus sign appears because there is a phase difference, π, between the two waves.

To find $U_{GA,o}$, it could be noted that the velocity under the pontoon within region II, is constant, eq. (20). So one can apply Newton second law, by equating mass times acceleration to the difference of the forces applied at the two ends of region II,

$$2BG\rho\,\frac{\partial^2 \Phi_{II,o}}{\partial x\,\partial t} = \int_{-h}^{-d} P(-B,y) - P(B,y)\,dy . \tag{32}$$

Noting that, the pressure P is given by

$$P = -\rho\frac{\partial \Phi}{\partial t} = i\,\omega\,\rho\,\phi , \tag{33}$$

and ,the acceleration is given by

$$\frac{\partial^2 \Phi_{II,o}}{\partial x \partial t} = -i\omega\, U_{GA,o}, \qquad (34)$$

and

$$P(-B,y) = -\rho \frac{\partial \phi_{I,o}}{\partial t} = -i\omega\rho \left[f_o(y)\,(A_{oo} + A_{oo} R_a) + \sum_{i=1}^{\infty} f_i(y)\, A_{o,i} \right], \qquad (35)$$

$$P(-B,y) = -\rho \frac{\partial \phi_{III,o}}{\partial t} = -i\omega\rho \left[f_o(y)\,(A_{oo} + A_{oo} R_a) + \sum_{i=1}^{\infty} f_i(y)\, A_{o,i} \right]. \qquad (36)$$

Substituting from eqs. (23), (25), (26), (35) and (36) to eq. (32) yields,

$$U_{GA,o} = -\frac{2 A_{oo} I_0}{GB + i I_o^2 / k_o + \sum_{i=1}^{\infty} I_i^2 / k_i}. \qquad (37)$$

To find $U_{GS,o}$, the total pressure has been equated on both sides of the vertical line separating regions I and II along the gap. This leads to,

$$U_{GS,o} = 2 S_{oo} I_o / G. \qquad (38)$$

Using (22), (23), (28),(29) and (37), yields,

$$R_f = 1 - \frac{i I_o^2}{k_o BG + i I_o^2 + k_o \sum_{i=1}^{\infty} I_i^2 / k_i}, \qquad (39)$$

and

$$T_f = \frac{i I_o^2}{k_o BG + i I_o^2 + k_o \sum_{i=1}^{\infty} I_i^2 / k_i}. \qquad (40)$$

2.1. The Exciting forces and moments

The forces and moments on the body due to the fluid pressure acting on the submerged part of the body consist of two parts. The first part is due to the hydrostatic pressure and is a function of the instantaneous position of the body. The second is due to the hydrodynamic pressure, which is given by

$$F_j = \rho \int_S \frac{\partial \phi}{\partial t} n_j \, dS = -i\omega\rho \int_S (\phi \; n_j \, dS)\, \alpha_j \quad j = 1,2,3. \qquad (41)$$

Where;

j = 1,2,3 represent the vertical force, horizontal force and the moment , respectively

S is the wetted area of the dock.

To find the moment, horizontal force and vertical force experienced by the fixed pontoon due to the incident and diffracted waves (the exiting potential), one has to integrate the pressure, given by eq. (33), along the wetted surface of the pontoon. For the horizontal force, F_2 only the flow in regions I and III, must be considered. To obtain the moment, F_3, the flow in regions I, II and III must be considered. While for the vertical force, F_1, only, the flow in region II must be considered. Using eqs. (23), (25), (32) and (36) we get.

$$F_1 = \left[\int_{-d}^{0} P(-B,y) - P(B,y)\, dy \right] = 2 i\omega\rho \left[\bar{I}_o A_o (1 + R_a) + \sum_{i=1}^{\infty} \bar{I}_i A_{o,i} \right] \exp(-i\omega t), \qquad (42\text{-a})$$

or

$$F_1 = 4 i\omega\rho A_{oo} \left[\bar{I}_o - T_f \left(\bar{I}_o - i \frac{k_o}{I_o} \sum_{i=1}^{\infty} \bar{I}_i I_i / k_i \right) \right] \exp(-i\omega t) \qquad (42\text{-b})$$

and

$$F_3 = 2 i\omega \left[\begin{array}{l} A_{oo}(1 + R_A)(d \bar{I}_o + \hat{I}_o) + \\ + \sum_{i=1}^{\infty} A_{o,i} (d \bar{I}_i + \hat{I}_i) + \frac{1}{3} U_{GA,O} B^3 \end{array} \right] \exp(-i\omega t) \qquad (43\text{-a})$$

or

$$F_3 = 4i\omega\rho A_{o,o}\left[\hat{I}_0 + T_f\left(\hat{I}_0 - \frac{ik_o}{I_0}\left\{\sum_{i=1}^{\infty}\hat{I}_i\, I_i / k_i + B^3/3\right\}\right)\right] \exp(-i\omega t). \tag{43-b}$$

The term containing B^3, is the result of the pressure under the pontoon, region II.

The vertical force under the pontoon is given by,

$$F_2 = i\omega\rho \int_{-B}^{B} U_{GS}\, dx = 2 i\omega\rho B\, U_{GS,o} \exp(-i\omega t), \tag{44}$$

where;

$$\hat{I}_i = \int_{-d}^{0} y\, f_i(y)\, dy, \text{ and} \tag{45}$$

$$\bar{I}_i = \int_{-d}^{0} f_i(y)\, dy. \tag{46}$$

2.2. Environmental impact

Any man-made structure is supposed to change the natural equilibrium of the beach. This may results in accretion or erosion. One of the major factors that controls the sediment transport, which causes accretion and erosion, is the horizontal velocity of the water particles near the bottom. The less the deviation between the horizontal velocity before and after the construction of any coastal structure, the less is the changes in the beach profile. The potential before the construction of the dock is given by twice the potential due to the symmetric, potential of the incident wave.

$$\phi_I = 2 f_o(y)\, S_{oo} \exp(i k_o(x+B)) \cdot \tag{47}$$

The velocity V_x in the horizontal direction is given by,

$$V_x = -2ik_o f_o(y)\, S_{oo} \exp(ik_o(x+B)) \,.\, \exp(-i\omega t) \tag{48}$$

The horizontal velocity under the dock V_{dock},, is given by differentiating eq. (20)

$$V_{dock} = U_{GA,o} \exp(-i\omega t). \tag{49}$$

The closer the ratio V_{dock} / V_x to unity, the lesser is the environmental impact resulting from the dock

3. Results and discussion

Fig. 2 shows the Transmission coefficient as calculated using numerical methods versus the present suggested approach. From this figure, one may conclude that there is a good agreement between the two methods for large values of d/h and small gap. However there is some deviation for the case with smaller d/h. This shows that the present approach can be used for small gap in shallow water. This is the dominant condition for docks and breakwaters

A series of design curves can be introduced for different depths and different values of B/h and B/d. For each depth the set contains, the transmission coefficient, relative velocity, horizontal force, vertical force and moment. The forces are normalized with. respect to ρ g h A_m, , while the moment is normalized with respect to ρ g h^2 A_m Based on the local wave height and the required wave height in the leeward of the dock, the suitable transmission coefficient is selected. Using the grain size, we select the accepted change in the water particle velocity near the bottom. With this in hand the dimension of the dock (or breakwater) and the water depth to locate the breakwater in, are selected. The charts for the forces and moment are used to design the supporting system. Figs. 3 through 7, show the design curves for water depth 4.0 m, for the cases B/h = 1.0 and 2.0, respectively.

It should be noted that the vertical force is insensitive to the variation in the ratio B/d, so a single curve is sufficient to represent the different values of B/d. Also, the vertical force shown is due to the dynamic pressure only. To find the total vertical force, the effect of the hydrostatic water pressure must be added.

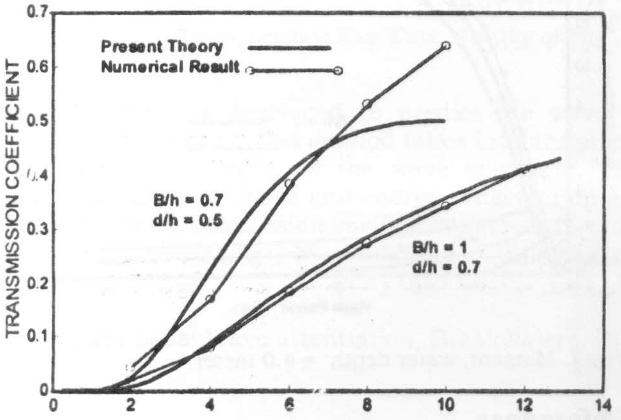

Fig. 2. Transmission coefficient.

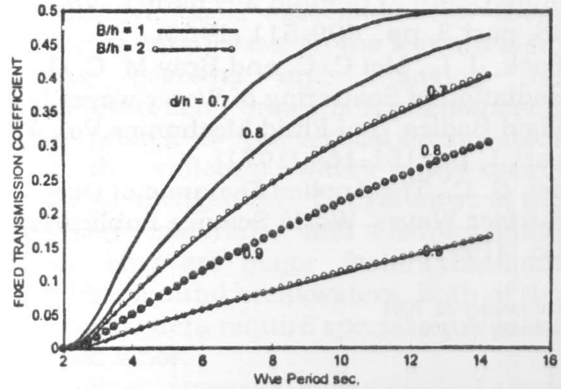

Fig. 3. Transmission coefficient, water depth = 4.0 meter.

As an example to see how to use the charts, assume that the incident wave period and height are 6 seconds and 1.5 meter, respectively. The deep-water wave length L_o is found to be 56.2 meter. For water depth 4.0 meter, the wave length is 34.75 meter and L/h = 8.687. From fig. 3, one finds that the transmission coefficients are 0.12, 0.22 and 0.32, for d/h = 0.9, 0.8 and 0.7, respectively, for the case with B/h = 1. For the case with B/h = 2.0, the transmission coefficients are 0.06, 0.12 and 0.17, for the cases with B/d = 0.9, 0.8 and 0.7 respectively. If the accepted height of the waves in the leeward of the structure for example is 0.35 meter, then one has to select a transmission coefficient of 0.22. This corresponds to d/h = 0.8 and B/h = 2.0.

The next step is to check the relative velocity from fig. 4. In this case one finds a value of 0.7. Depending on the size of the sand particles, it could be decided either to accept this value or not. If both the wave height in the leeward and the relative velocity are accepted, then figs. 5, 6 and 7 are used to determine the forces and moment needed to design the supporting system. However, if either of them is rejected, then the structure should be moved to water with different depth using the appropriate charts. An alternative approach is to use the charts for the same depth with different values of B/h and B/d.

4. Conclusions

A simple analytical method is introduced to select the optimum dimensions of a breakwater or a dock for marinas. The dimensions are selected such that the wave height in the leeward of the structure is within the accepted range and at the same time no accretion or erosion problems will occur. The proposed approach compared well with the more sophisticated ones. A series of design charts for different water depths, draft/water depth and width/draft may be introduced to eliminate the need of using any software. The proposed method handles the cases of fixed structure in a monochromatic sea. It can be extended to cover the cases of random sea and floating objects.

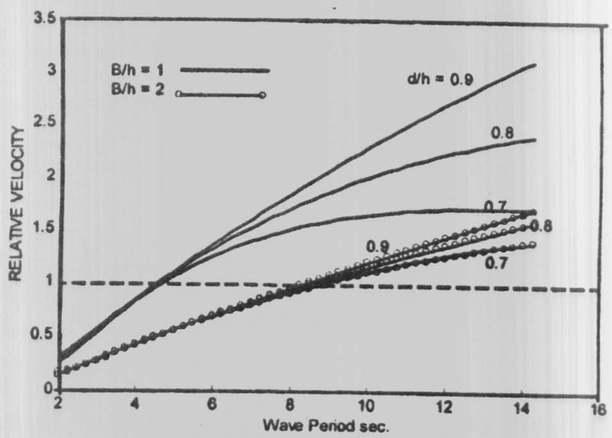

Fig. 4. Relative velocity, water depth = 4.0 meter.

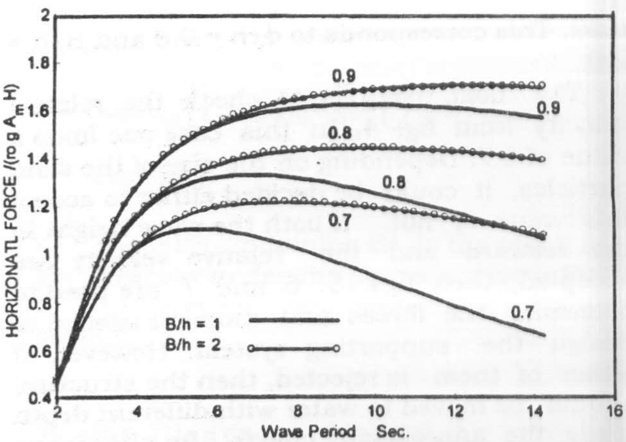

Fig. 5. Horizontal force, water de,th = 4.0 meter.

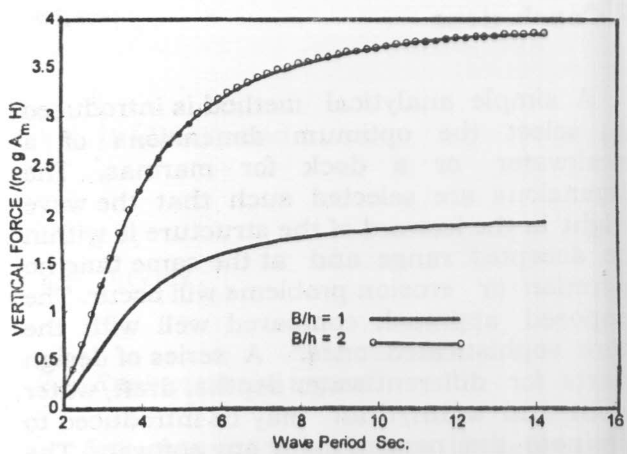

Fig. 6. Vertical force, water depth = 4.0 meter.

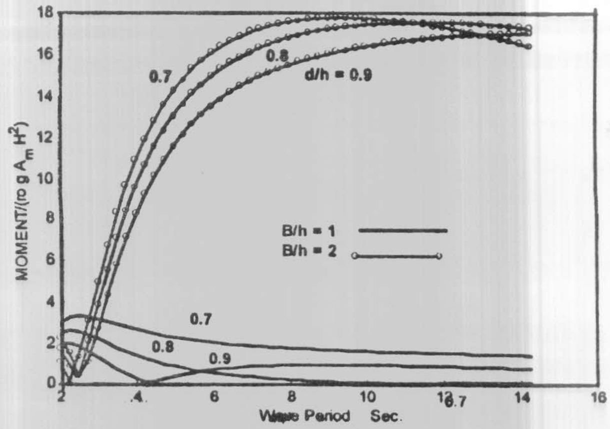

Fig. 7. Moment, water depth = 4.0 meter.

References

[1] Mei, C. C. and Black, J. L. , Scattering of Surface Waves by Obstacles in Waters of

[2] Finite Depth. J of Fluid Mechanics, Vol. 38, part 3, pp. 499-511 (1969).

[3] Black, J. L., Mei C. C. and Bray M. C. G., Radiation of Scattering of Water waves by Rigid Bodies, J of Fluid Mechanics Vol. 46, part 1, pp. 151-164 (1971).

[4] Mei, C. C., The Applied Dynamic of Ocean Surface Waves, World Science Publication Co. (1992).

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