Numerical modeling of a single span towed marine cable under periodic excitation

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A developed mathematical model of a single span sagged cable subjected to dynamical condition, provides a way of determining the location and tension of any material point of the cable as a function of the time. A code based on finite difference approximation to the differential equations derived from basic dynamics is implemented. The equations of motion are developed for a general three - dimensional case of a single span cable, supported at two different elevations at the end points, with fixed and movable supports. It is subjected to different types of acting loads. Numerical examples are given and comparisons are made between numerical and the available experimental results. This has been done with the aim to demonstrate the validity of the present method. The influence of various factors, such as excitation amplitude, excitation frequency, initial tension and Young's modulus, are also investigated.

يقدم البحث النموذج الرياضي لحل مسائل الكابلات البحرية المعلقة ذات الترخيم والمستخدمة في أغراض الجر والربط وخاصة منها تلك المعرضة لأحمال ديناميكية فى صورة ترددية عند أحد الأطراف او كلاهما. وقدم البحث مقارنة النتائج العددية بتلك المعملية (لأخرين) بغرض التأكد من سلامة ودقة النموذج الرياضي المقترح . ثم استعرض البحث تأثير بعض العوامل على أداء تلك الكابلات وحدود تأثيرها.

Keywords: Marine cable, Numerical, Dynamics, Mooring, Periodic excitation.

1. Introduction

Cables are commonly used in applications that require long, flexible and/or lightweight structural element [1]. Cables are used for towing ships as well as berthing and mooring aids for floating vessels. For such applications, marine cable becomes under periodic environmental excitation of taut-slack conditions. In these applications, the cable's inherent flexibility renders it susceptible to oscillation that may impair its performance. For instance, cables submerged in cross-flows may be excited by vortex shedding which can lead to a resonant response known as strumming [1]. Strumming may increase cable drag forces, promote fatigue, amplify cable acoustic emissions and degrade the performance of attached instruments (e.g., hydrophones) [1].

Prior knowledge of the cable structural dynamics is required in the analysis of these complex fluid/cable interactions. A history of the derivations and solutions of the equations for static and dynamic response of horizontal chord (symmetrical) and inclined chord (asymmetrical) cables is given in Cable Structures [2], Handbook of Coastal and Ocean Engineering [1, 3].

The non-linear vibrations of marine cables have been reported by quite a few authors in the past. Schram and Reyle [4] have employed the method of characteristics to solve the governing equations of a towed cables system using numerical interpolation. Delmer el al. [5] presented a numerical method for the dynamic simulation of towed cable. The model was constructed from a set of generic elements incorporating cable strands, knots, kinks, cable ends and winches. Ahmadi [6] derived an exact mathematical formula for a towed elastic catenary associated with a uniform distributed load. Triantafyllou and Chyrssostomidis [7] developed a procedure for calculating the response of a towed marine cable subjected to a harmonic excitation applied at the up-stream end. The model was based on the slender body approximation.

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Rajagopal [8] adopted the transfer matrix method for developing a closed form for a taut string/mass system. Lin [9], used a hybrid analytical/numerical solution method to solve the non-linear response of a suspended cable supporting an array of discrete masses. Patel [10] investigated the transient behavior of towed marine cables in two dimensions as the first phase towards the objective of developing a reliable prediction model for the cable vibration problem. Srivastava [11] suggested an analytical formula to solve a problem of towed array system of marine cables. Recently an analysis of cable dynamics were presented by Moustafa [12] for predicting the behavior of a marine cable operating under snap load due to the heaving or pitching ship motions.

This paper focuses upon a three-dimensional modeling and a numerical integration scheme that solves the dynamic response of such marine cables. The differential equations of motion were developed for the general case when the cable is supported at two different elevations and subjected to spatial static and dynamic loads as a result of vessel top end oscillation. The non-linear strain-displacement relationship for the cable is used, accounting for the change in cable tension during motion.

The problem defined above is governed by three non-linear coupled partial differential equations. The equations are solved using a robust and stable finite difference method. Numerical results are presented and good agreement is achieved with the available experimental results that help to demonstrate the efficiency and the validity of the present method. The influences of various factors governing the cable motions are also examined.

2. Basic equations

Within the framework of the theory of elasticity, the cable analyzed in this paper falls under the category of problems that are geometrically non-linear, but the materials are linear. It is assumed that the external loads on the cable result in an axial tension along the cable and do not produce any shear or flexural stresses; the material of the cable is assumed to have homogeneous and isotropic elastic properties and the material obeys Hooke's law.

Of special interest in this paper is a cable that hangs initially under its own weight (fig. 1). The initial shape of the cable has the form, [1],

$$z = a \cosh\left(\frac{x}{a} + a_1\right) + a_2, \tag{1}$$

in which $a = H/q$, where H is the horizontal component of initial tension, q is the uniform static load on the cable, (a_1) and (a_2) are the constants to achieve the initial shape of the cable.

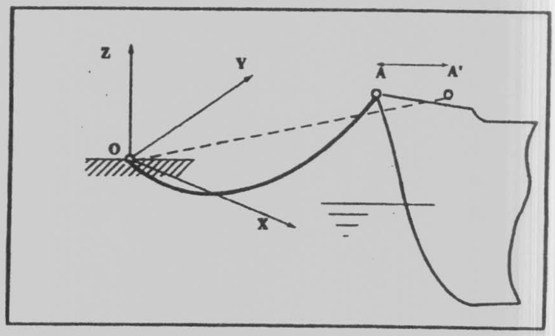

Fig. 1. Mooring line geometry.

The nonlinear strain-displacement relation used in the development of the governing equations of motion, Irvine [2], can be written as:

$$\varepsilon = \left\{\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right\} \operatorname{sech}^2\left(\frac{x}{a} + a_1\right) + \frac{\partial w}{\partial x} \operatorname{sech}^2\left(\frac{x}{a} + a_1\right) \sinh\left(\frac{x}{a} + a_1\right), \tag{2}$$

in which,
u is the the longitudinal in-plane displacement component along the x-axis.
w is the vertical in-plane displacement component along the z-axis.
v is the out-of-plane, or swinging

displacement component along the y-axis. All the above components are measured from the static equilibrium configuration represented by eq. (1).

The stress-strain relation for the cable is [2],

$$\sigma\,(s, t) = \sigma_o\,(s) + E\varepsilon\,(s, t). \tag{3}$$

Where;

- σ is the instantaneous axial stress,
- s is s the coordinate along the length of the cable,
- T is the time in seconds,
- σ_o is the axial stress before the application of the dynamic loads,
- E is the cable modulus of elasticity,

The displaced position of the cable oscillating due to the spatially distributed dynamic load *F*(s, *t*) has the coordinates [1];

$$\bar{x} = x + u(s,t), \tag{4-a}$$

$$\bar{y} = y + v(s,t), \tag{4-b}$$

$$\bar{z} = z + w(s,t). \tag{4-c}$$

During the motion, an initial cable element of length {ds} changes to {dS} following the relation [2];

$$dS = (1 + \varepsilon)\, ds, \tag{5}$$

and the initial cable tension T(s) is changed to T_1 (s, t) according the relationship [2],

$$T_1\,(s, t) = T(s) + AE\varepsilon\,(s, t), \tag{6}$$

in which A is the effective cross-sectional area of the cable.

The equations of motion for the displaced position of the cable are in the form [2];

$$m\frac{\partial^2 u}{\partial t^2} = F_x(s,t)\frac{\partial(x+u)}{\partial s} + \frac{\partial}{\partial s}\left\{(T + AE\varepsilon)\frac{\partial}{\partial s}(x+u) - T\frac{dx}{ds}\right\}, \tag{7-a}$$

$$m\frac{\partial^2 v}{\partial t^2} = F_y(s,t)\frac{\partial(x+u)}{\partial s} + \frac{\partial}{\partial s}\left\{(T + AE\varepsilon)\frac{\partial}{\partial s}(y+v) - T\frac{dy}{ds}\right\}, \tag{7-b}$$

$$m\frac{\partial^2 w}{\partial t^2} = F_z(s,t)\frac{\partial(x+u)}{\partial s} + \frac{\partial}{\partial s}\left\{(T + AE\varepsilon)\frac{\partial}{\partial s}(z+w) - T\frac{dz}{ds}\right\}, \tag{7-c}$$

in which F_x (s, t), F_y (s, t), and F_z (s, t) are the x, y and z components of the dynamic load per unit length of the cable respectively projected on the x-axis and m is the mass per unit length of the cable. Using eq. (1) for the initial shape of the cable and assuming that the cable lies in the vertical plane under the application of the static loads, eqs. (7) become:

$$m\frac{\partial^2 u}{\partial t^2} = F_x\left(1+\frac{\partial u}{\partial x}\right)\operatorname{sec\,h}\left(\frac{x}{a}+a_1\right).$$
$$\frac{\partial}{\partial x}\left\{AE\left(1+\frac{\partial u}{\partial x}\right)\left[\left(\frac{\partial u}{\partial x}\right)^{2} + \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right)\operatorname{sec\,h}^2\left(\frac{x}{a}+a_1\right),\right.$$
$$\left.+\frac{\partial w}{\partial x}\operatorname{sec\,h}^2\left(\frac{x}{a}+a_1\right)\sinh\left(\frac{x}{a}+a_1\right)\right]\times \operatorname{sec\,h}\left(\frac{x}{a}+a_1\right) + H\frac{\partial v}{\partial x}\right\}\operatorname{sec\,h}\left(\frac{x}{a}+a_1\right), \tag{8-a}$$

$$m\frac{\partial^2 v}{\partial t^2} = F_y\left(1+\frac{\partial u}{\partial x}\right)\sec h\left(\frac{x}{a}+a_1\right).$$

$$\frac{\partial}{\partial x}\left\{AE\frac{\partial v}{\partial x}\left[\left(\frac{\partial u}{\partial x}\right)+\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right)^2\sec h^2\left(\frac{x}{a}+a_1\right)\right.$$

$$\left.+\frac{\partial w}{\partial x}\sec h^2\left(\frac{x}{a}+a_1\right)\sinh\left(\frac{x}{a}+a_1\right)\right]\times\sec h\left(\frac{x}{a}+a_1\right)+H\frac{\partial v}{\partial x}\right\}\sec h\left(\frac{x}{a}+a_1\right), \quad (8\text{-b})$$

$$m\frac{\partial^2 w}{\partial t^2} = F_z\left(1+\frac{\partial u}{\partial x}\right)\sec h\left(\frac{x}{a}+a_1\right).$$

$$\frac{\partial}{\partial x}\left\{AE\left(\sinh\left(\frac{x}{a}+a_1\right)+\frac{\partial w}{\partial x}\right)\left[\left(\frac{\partial u}{\partial x}\right)+\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right)^2\sec h^2\left(\frac{x}{a}+a_1\right)\right.$$

$$\left.+\frac{\partial w}{\partial x}\sec h^2\left(\frac{x}{a}+a_1\right)\sinh\left(\frac{x}{a}+a_1\right)\right]\times\sec h\left(\frac{x}{a}+a_1\right)+H\frac{\partial w}{\partial x}\right\}\sec h\left(\frac{x}{a}+a_1\right). \quad (8\text{-c})$$

Eqs. (8) constitute a system of coupled partial differential equations for the displacement components u, v and w, of a sagged cable whose initial form is given by eq. (1).

3. Method of solution

The equations of motions (8) are solved using the finite difference approach. In discretizing the equations of motion in any direction, implicit finite difference expressions are used for the space derivations for the displacement in that direction. The space derivatives for the other two displacements and the time derivation are discretized by the standard finite difference approximation. These discretizations are given bellow:

$$\frac{\partial u}{\partial x} \cong \frac{1}{8h}\left\{\left(u_{m+1}^{n+1}-u_{m-1}^{n+1}\right)+2\left(u_{m+1}^{n}-u_{m-1}^{n}\right)+\left(u_{m+1}^{n-1}-u_{m-1}^{n-1}\right)\right\}, \quad (9\text{-a})$$

$$\frac{\partial^2 u}{\partial x^2} \cong \frac{1}{4h^2}\left\{\left(u_{m+1}^{n+1}-2u_{m}^{n+1}+u_{m-1}^{n+1}\right)+2\left(u_{m+1}^{n}-2u_{m}^{n}+u_{m-1}^{n}\right)+\left(u_{m+1}^{n-1}-2u_{m}^{n-1}+u_{m-1}^{n-1}\right)\right\}, \quad (9\text{-b})$$

$$\frac{\partial v}{\partial x} \cong \frac{1}{2h}\left(v_{m+1}^{n}-v_{m-1}^{n}\right), \quad (9\text{-c})$$

$$\frac{\partial^2 v}{\partial x^2} \cong \frac{1}{h^2}\left(v_{m+1}^{n}-2v_{m}^{n}+v_{m-1}^{n}\right), \quad (9\text{-d})$$

$$\frac{\partial w}{\partial x} \cong \frac{1}{2h}\left(w_{m+1}^{n}-w_{m-1}^{n}\right), \quad (9\text{-e})$$

$$\frac{\partial^2 w}{\partial x^2} \cong \frac{1}{h^2}\left(w_{m+1}^{n}-2w_{m}^{n}+w_{m-1}^{n}\right), \quad (9\text{-f})$$

$$\frac{\partial^2 u}{\partial t^2} \cong \frac{1}{k^2}\left(u_{m}^{n+1}-2u_{m}^{n}+u_{m}^{n-1}\right), \quad (9\text{-g})$$

Where h and k are the grid sizes in the distance and time coordinates, respectively, and the difference eqs. (9) refer to the grid points x=mh, t=nk, and m and n are integers.

For the equations of motion in the other two directions, the space and time derivatives are re-placed by the corresponding finite difference approximations, in a similar manner. The finite difference replacement of the equations of motion gives a system of non-linear algebraic equations that can be solved by the Newton-Raphson method of iteration. At each time step, the tension in the cable is determined from eq. (5) using the standard finite difference relations for the derivatives in the strain-displacement equation. However, for m=1 and m=n+1, which correspond to the ends of the cable, forward and backward finite differences are used respectively, instead of the central differences.

The formulation is completed by specifying a set of boundary and initial conditions. Different types of boundary conditions may occur depending upon the types of physical conditions at the two ends of the cable. For a sagged cable with one fixed end and the other subjected to known motion, the boundary conditions are;

$$u(0,t) = v(0,t) = w(0,t) = 0,$$

and

$$u(x_s,t) = U(t) \;, v(x_s,t) = V(t) \text{ and } w(x_s,t) = W(t).$$

Where x_s is the projection of the cable free end on the x axis: $U(t)$, $V(t)$ and $W(t)$ are to be specified corresponding to the case under investigation.

4. Numerical example

To demonstrate the capability and accuracy of the present solution technique, tests models on a small-sagged marine cable performed in Ship Research Institute of Norway [13] are chosen for comparison with the numerical results. On the other hand, to investigate the influence of the different parameters governing such kind of cable motions a parametric study of each parameter is cared out. The tests consider a uniform small-sagged cable suspended at the two ends on the same level. One end of the horizontally placed sagged cable is fixed while the other end is subjected to horizontal excitation given by,

$$U(t) = u_o \sin(\omega_f t).$$

The principal parameters of the cable and test are as follow:

Cable diameter	(0.01 m)
Cable length	(10.9774 m)
Cable span	(10.7920 m)
Sag-span ratio	(0.0812)
Young's modulus	(10E11 n/sq.m)
Mass distribution	(0.61 Kg/m)
Range of amplitude	(0.025 -0.1 m)
Range of frequency	(0.0 – 1.2 Hz)
ω_f	(0.55 Hz)

Figs. 2-5 show the maximum and minimum values of tension in the cable under excitations at different amplitudes for the range of (0.025 m to 0.1 m) and frequencies for the range (0.1 Hz to 1.2 Hz). Both for the numerical results and experimental data given by Ship Research Institute of Norway [13].

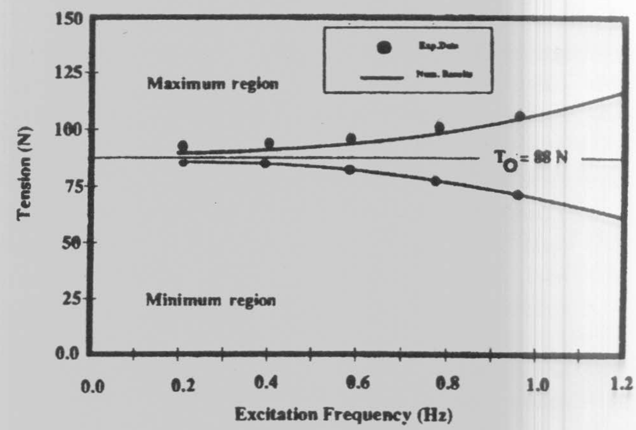

Fig. 2. Comparison between the numerical and experimental results [excitation amplitude = 0.025 m].

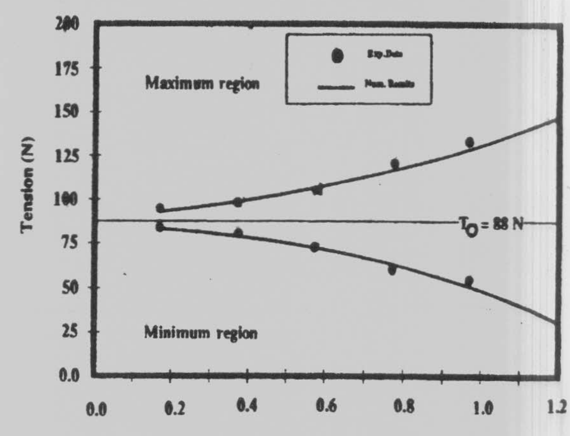

Fig. 3. Comparison between the numerical and experimental results [excitation amplitude = 0.05 m].

The quantitative and qualitative agreement between both results is acceptable. Numerical results follow the physical expected behavior of the tension in the cable especially for small excitation amplitude as well as for small values of the frequency. It is clear from these figures that; when the excitation are not severe, i.e. the excitation frequency is low or the amplitude is small or both, the maximum and minimum tensions are symmetrical with a constant mean value which is equal to the static tension. When the excitation becomes severe, however, this symmetry can no longer hold. As the minimum tension approaches the zero level, that is the cable starts to operate in an alternating taut-slack condition, the maximum tension increases significantly, this is manifested in figs. 4, 5.

The effect of the initial tension on the total tension is illustrated in fig. 6. It is noted that as the initial tension increases, the sag decreases. In this case, it is believed that the cable dynamics is influenced mainly by the elastic stiffness, resisting the applied load through stretching. Fig. 7 shows the effects of Young's modulus on the cable dynamics. It can be concluded that, when the frequency is low, the force is small and the system employs mainly the catenary stiffness to resist the applied excitation. As the excitation frequency becomes greater, the elastic stiffness becomes more important.

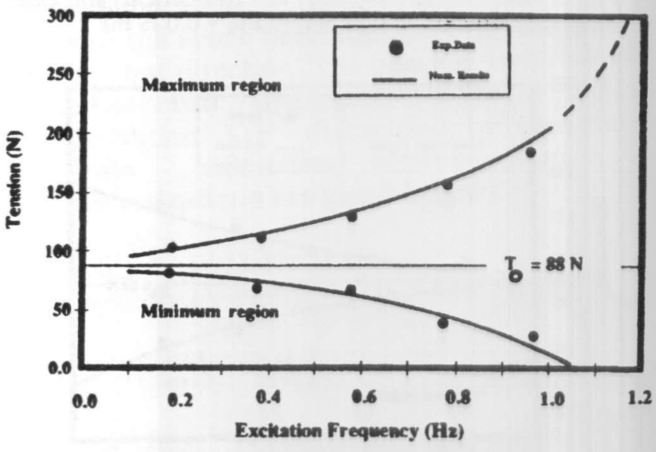

Fig. 4. Comparison between the numerical and experimental results [excitation amplitude = 0.075 m].

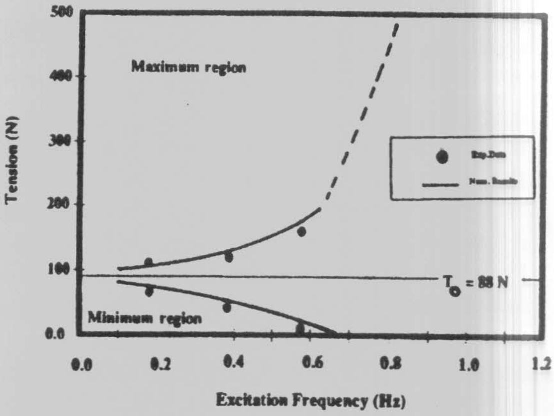

Fig. 5. Comparison between the numerical and experimental results [excitation amplitude = 0.1 m].

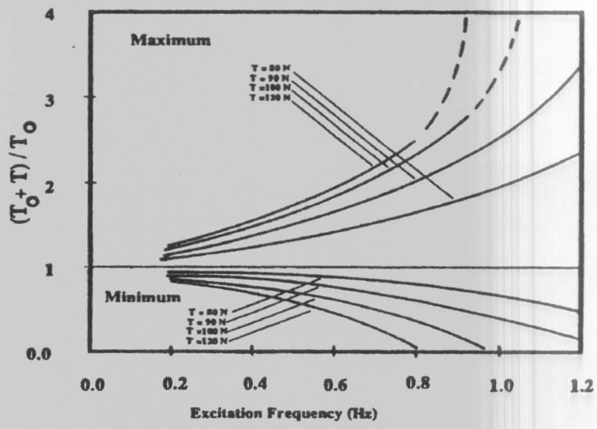

Fig. 6. Effect of initial tension on both max. and min. tension in the cable the excitation is fixed at 0.075 m.

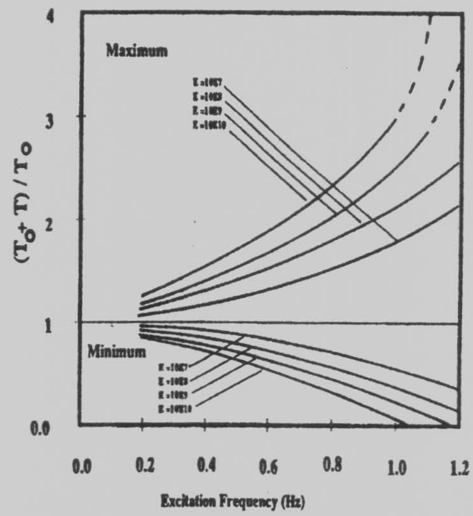

Fig. 7. Effect of Young's modulus on both max. and min. tension in the cable the excitation amplitude is fixed at 0.075 m.

The displacement is characterized by sharp troughs and flat crests, with flat troughs and sharp crests for the velocity. The magnitude of the acceleration becomes much larger as the transition from the slack to the taut state involves a sudden change in velocity. As the excitation further increases, (amplitude 0.1 m, frequency 1Hz), the magnitudes of displacement velocity and acceleration all become much greater, and the distortion is even more pronounced. For the velocity, the so-called free-falling phenomenon appears. As the cable moves upwards, characterized by the sharp increase in velocity, then reaches its highest level and starts to fall downwards, a loss of tension caused by slack renders its motion controlled by its own weight in water and the fluid drag force. Subsequently, the cable becomes taut again, characterized by the large magnitude of acceleration. However, this great acceleration is not sustainable, depicted by the sharp oscillations at its peaks, possibly due to the development of elastic waves.

5. Conclusions

A numerical solution of the nonlinear dynamic response of a single sagged cable under the action of time-dependent loads is presented. A code is written to compute the motion of a towed cable, based on a robust and stable finite difference approximation to the differential equations derived form basic dynamics.

An implicit finite difference technique is used as the method of solution. The derivation of the equations of motion of the cable element using non-linear strain-displacement relation leads to a formulation that is applicable to problems of small strains and large displacements. The equations of motion are solved to determine the displacements and the tension in the cable, assuming that Hooke's law is valid. Applications of the proposed numerical technique to a single span sagged cable operating under towing loads is solved for a cable with fixed ends as well as for the case where movable ends are used.

Furthermore, kinematics behaviors of the cable mid-point are investigated in order to arrive at a qualitative and quantitative analysis of the dynamic behavior of the cable. Figs. 8–10 illustrate the vertical displacements, velocities and accelerations of the mid-point of the cable under three different situations. In these figures the positive direction corresponds to upward motion, and negative direction to the downward. In fig. 8, where the cable is under small excitation (amplitude 0.05 m and frequency 0.75 Hz), the responses are almost sinusoidal with small distortion. In fig. 9, the cable is under far more severe excitation (amplitude 0.075 m and frequency 1 Hz). The responses become apparently distorted.

Good agreement is noted between the numerical and some published experimental results, which demonstrates the accuracy and the validity of the proposed method. It is found that, when the cable is under small excitation, the non-linear effects are not important. This is not the case when the excitation is severe, especially when the cable is operating in an alternating taut-slack condition.

Analysis of the dynamic behavior of the cable. Figs. 8–10 illustrate the vertical displacements, velocities and accelerations of the mid-point of the cable under three different situations. In these figures the positive direction corresponds to upward motion, and negative direction to the downward. In fig. 8, where the cable is under small excitation (amplitude 0.05 m and frequency 0.75 Hz), the responses are almost sinusoidal with small distortion. In fig. 9, the cable is under far more severe excitation (amplitude 0.075 m and frequency 1 Hz). The responses become apparently distorted.

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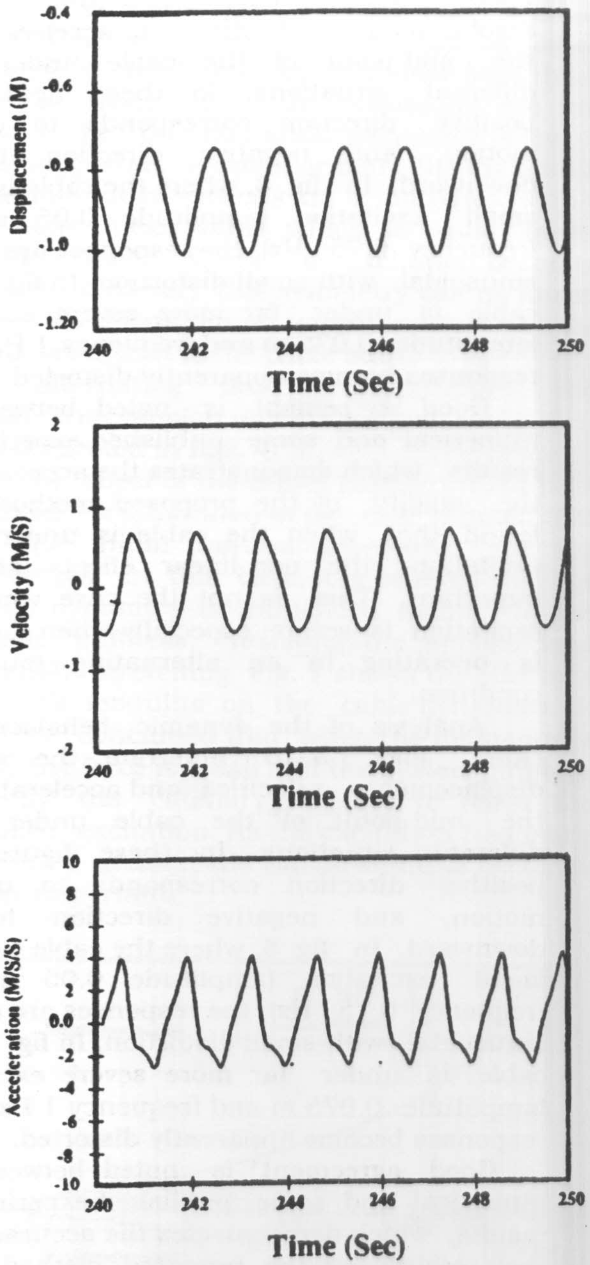

Fig. 8. Time record of the vertical responses of the mid-point of the cable (excitation amplitude 0.05 m, frequency 0.75 Hz).

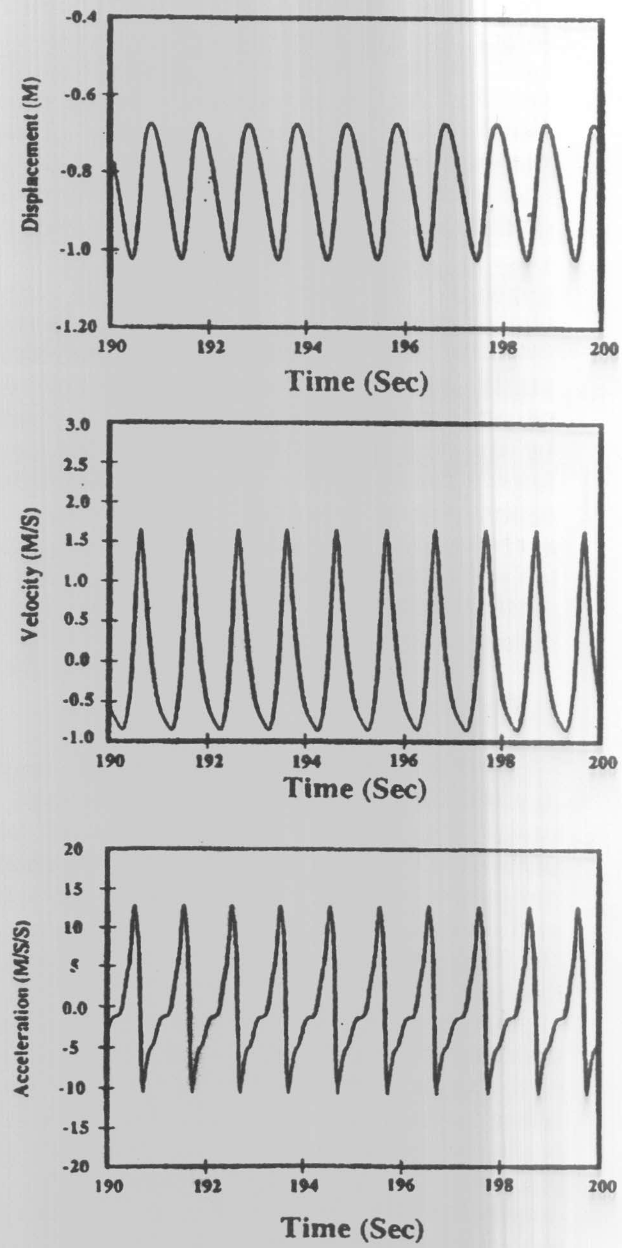

Fig. 9. Time record of the vertical responses of the mid-point of the cable (excitation amplitude 0.075 m, frequency 1Hz).

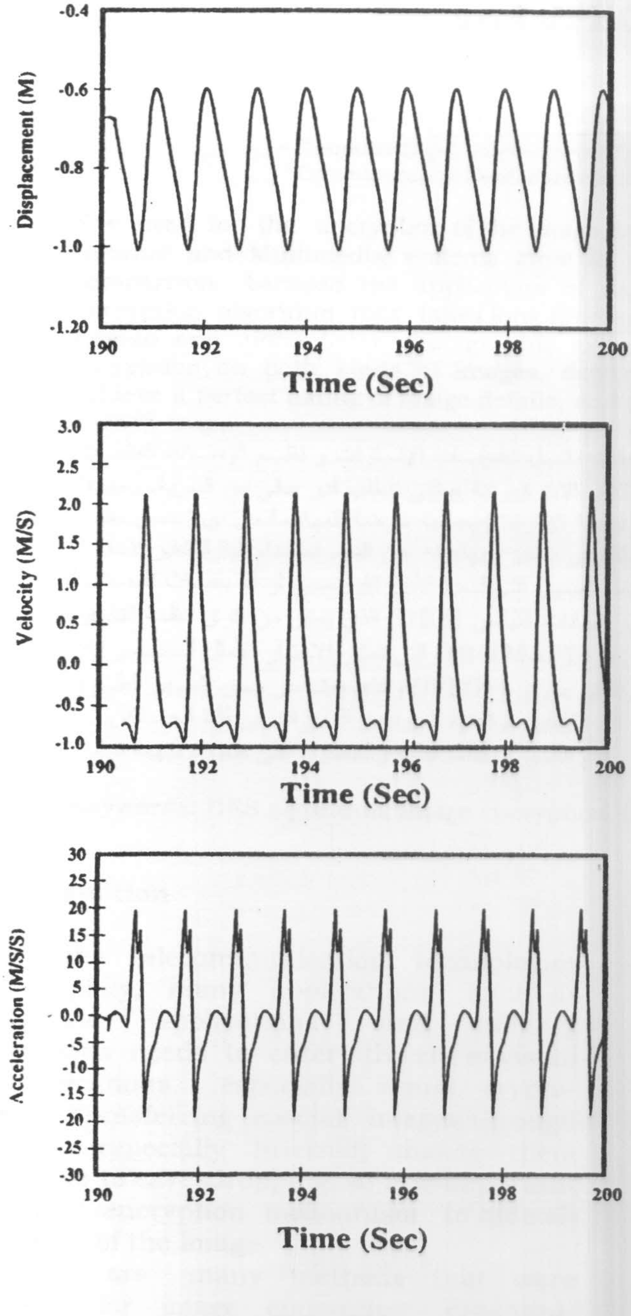

Fig. 10. Time record of the vertical responses of the mid-point of the cable (excitation amplitude 0.1 m, frequency 1Hz).

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