Relative precision criteria in geodetic control networks

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In design of engineering control networks, the positional precision of one point relative to another adjacent network point is important than its positional precision with respect to an arbitrary zero-variance reference base. Consequently the common local and global measures of the precision are not usually of prime importance. The main objective of this paper is to study the relative precision between the neighboring points within the control network. Two important relevant criteria were found to describe the relative precision. A schematic local goedetic network has been used as an example via using different variables in adjustment approaches in order to ascertain the study importance.

تستخدم شبكات التحكم الجيوديسية فى مراقبة تحركات القشرة الأرضية وفى مراقبة المنشآت الهندسية الكبيرة مثال ذلك الكباري والسدود والأنفاق. فى مثل هذه الشبكات فإن تعيين دقة العلاقة التى تربط نقطة من نقاط الشبكة بمجموعة النقاط المجاورة لها تكون أكثر أهمية من تعيين دقة موضع هذه النقطة بالنسبة إلى نظام الإحداثيات الإستنادى. ومن ثم فإن ذلك يؤدى إلى معرفة مدى تحرك النقاط المثبتة على جسم المنشأ الهندسي والتي تعرف بنقاط الهدف بالنسبة إلى نقاط الشبكة المرجعية. وعليه فإن هذا البحث يهدف إلى دراسة الدقة النسبية بين نقاط الشبكة بعضها وبعض والتي تعتمد على العلاقات الرياضية الداخلية التى تربط نقطة من نقاط الشبكة بنقطة أخرى مجاورة لها أو نقطتين مجاورتين لها من نقاط الشبكة بهدف الوصول إلى أنسب وأفضل المعايير التى تستطيع أن تصف الدقة النسبية بين نقطتين أو ثلاث نقاط متجاورة فى الشبكة الجيوديسية والتي يمكن أيضاً عن طريقها إجراء المقارنة الصحيحة بين الشبكات الجيوديسية الحرة والمربوطة المستخدمة فى الأعمال الهندسية المختلفة وكذلك معرفة أماكن القوة والضعف فى الشبكات. وقد تم الاستعانة بشبكة جيوديسية محلية كمثال تطبيقي حيث تم ضبطها عدة مرات بمتغيرات مختلفة لتلائم هذه الدراسة.

Keywords: Relative precision, Neighboring points, Datum transformation, Relative error ellipse, Zero-variance reference base.

1. Introduction

Geodetic control networks are widely used for many purposes in civil engineering, such as for setting out an engineering structures, controlling the breakthrough of tunneling, the monitoring of structural deformations, and providing photogrammetric control. In most of these applications, the relative precision between the neighboring points is more important than absolute precision of each point in the network. For example, the accurate position of the object point relative to the reference points is more important than its accurate position relative to the origin of the coordinate system. Relative precision criteria have been derived from the covariance matrix of the coordinates differences, which is the appropriate submatrix of the variance-covariance matrix of the estimated coordinates. The most important measures of the relative precision should satisfy the following conditions [1,5]:

a- Invariance with respect to the choice of zero - variance reference base.
b- Invariance with respect to the datum transformation.
c- Independent of the choice of least squares adjustment techniques.
d- Characterize the relative precision between the neighboring points in the network.
e- Applicable to primary as well as secondary geodetic networks.

The main objective of this paper is to investigate and analyze the relative precision between the netpoints in order to find the most suitable criteria that describe the relative precision between two or three neighboring points within the network.

In this study, the adjustment for the geodetic network is performed by using least squares principle in three approaches:

I- Minimal constrained adjustment approach (holding three coordinates unchanged).
II- Inner constrained adjustment approach (free from any outer constraints).

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III-Over constrained adjustment approach (holding more than three coordinates unchanged).

2. Mathematical model of geodetic networks

In the classical Gauss-Markov model, the unknown parameters x of a linearized least squares model for a parametric adjustment are determined based on the following functional and stochastic models,

$$\left.\begin{aligned} \mathbf{v} &= \mathbf{A}\,\mathrm{x} - \mathbf{l} \\ \mathbf{P} &= \mathbf{Q}_\ell^{-1} = \sigma_0^2\ \mathbf{C}_\ell^{-1} \end{aligned}\right\}, \quad (1)$$

Where; v is the n by 1 vector of residuals, A is the n by u configuration matrix, P is the n by n weight matrix of observations, Q_l is the cofactor matrix of observations, σ_0^2 is the a priori variance factor, and C_l is the covariance matrix of observations. The covariance matrix of the estimated parameters (coordinates) can be estimated using the relation,

$$\mathbf{C}_{\hat{x}} = \hat{\sigma}_0^2\ \mathbf{Q}_{\hat{x}} = \hat{\sigma}_0^2\ (\mathbf{A}^T\ \mathbf{P}\ \mathbf{A})^{-1}, \quad (2)$$

where $\hat{\sigma}_0^2$ is the a posteriori variance factor and $Q_{\hat{x}}$ is the u by u cofactor matrix or the weight coefficient matrix of the estimated coordinates $\hat{x}$ [2]. In the design phase, the variance-covariance matrix of the estimated coordinates can be expressed as

$$C_{\hat{x}} = \hat{\sigma}_0^2\ (A^T\ P\ A)_{rs}^-, \quad (3)$$

with $(A^T\ P\ A)_{rs}^-$ representing the reflexive generalized inverse of a matrix. Assume the network is complete, i.e., without configuration defects, then the number of rank defects of A is equal to the number of datum defects d of the network. After a definition of the datum of the network has been given by,

$$G^T\ x = 0, \quad (4)$$

with the rank of matrix G being equal to the number of rank defects of A , then the variance-covariance matrix of the estimated coordinates is given by [9],

$$C_{\hat{x}} = \hat{\sigma}_0^2\ (A^T\ P\ A + G\ G^T)^{-1} - H\ (H^T\ G G^T H)^{-1}\ H^T, \quad (5)$$

where the matrix H spans the null space of matrix A, satisfying the relation

$$A\ H = 0, \quad (6)$$

and for a two dimensional trilateration network with m geodetic points matrix H is expressed as ,

$$H^T = \begin{bmatrix} 1 & 0 & \ldots\ldots & 1 & 0 \\ 0 & 1 & \ldots\ldots & 0 & 1 \\ -x_1^o & y_1^o & \ldots\ldots & -x_m^o & y_m^o \end{bmatrix}, \quad (7)$$

with $(x_i^o\ ,\ y_i^o)$ (i = 1, ..., m) being the approximate coordinates of netpoints. If matrices G and H are equal then the so-called inner constraints are used in the adjustment procedure.

The variance - covariance matrix $C_{\hat{x}}$ contains all required information to compute the variance of any quantity that can be calculated from the estimated parameters $\hat{x}$. The straightforward relationship for any derived quantity is given by,

$$\sigma_{\hat{F}}^2 = f^T\ C'_{\hat{x}}\ f. \quad (8)$$

Where f represents the vector of linearization coefficients relating the estimated parameters to the quantity required, F, and $C'_{\hat{x}}$ is the appropriate submatrix of $C_{\hat{x}}$ [7].

3. Relative precision criteria between two netpoints

3.1. The positional error between two netpoints

In horizontal geodetic network, the vector of coordinate differences for a pair of estimated netpoints $P_i(\hat{x}_i, \hat{y}_i)$ and $P_j\ (\hat{x}_j, \hat{y}_j)$ can be written as follows:

$$\left(\hat{\Delta}x\right)_{ij} = \hat{x}_{p_j} - \hat{x}_{p_i} \tag{9}$$

By applying the law of variance-covariance propagation [2], the weight coefficient matrix $Q_{\hat{\Delta}x}$ of the coordinate differences vector $\hat{\Delta}x$ can be derived as follows:

$$\left(Q_{\hat{\Delta}x}\right)_{ij} = \begin{bmatrix} Q_{\hat{\Delta}x\hat{\Delta}x} & Q_{\hat{\Delta}x\hat{\Delta}y} \\ Q_{\hat{\Delta}y\hat{\Delta}x} & Q_{\hat{\Delta}y\hat{\Delta}y} \end{bmatrix}_{ij} . \tag{10}$$

Where,

$$\left.\begin{aligned} \left(Q_{\hat{\Delta}x\hat{\Delta}x}\right)_{ij} &= Q_{\hat{x}_i\hat{x}_i} + Q_{\hat{x}_j\hat{x}_j} - 2\, Q_{\hat{x}_i\hat{x}_j} \\ \left(Q_{\hat{\Delta}y\hat{\Delta}y}\right)_{ij} &= Q_{\hat{y}_i\hat{y}_i} + Q_{\hat{y}_j\hat{y}_j} - 2\, Q_{\hat{y}_i\hat{y}_j}, \\ \left(Q_{\hat{\Delta}x\hat{\Delta}y}\right)_{ij} &= Q_{\hat{x}_i\hat{y}_i} + Q_{\hat{x}_j\hat{y}_j} + Q_{\hat{x}_i\hat{y}_j} - Q_{\hat{x}_j\hat{y}_i} \\ \left(Q_{\hat{\Delta}y\hat{\Delta}x}\right)_{ij} &= \left(Q_{\hat{\Delta}x\hat{\Delta}y}\right)_{ij} . \end{aligned}\right\} \tag{11}$$

The covariance matrix of the estimated coordinates differences can be computed using the following equation:

$$\left(C_{\hat{\Delta}x}\right)_{ij} = \hat{\sigma}_o^2 \cdot \left(Q_{\hat{\Delta}x}\right)_{ij} \tag{12}$$

The above equation gives all necessary data required for the calculating the positional error between any two netpoints. Also, it gives us the standard errors of both the estimated distance and the estimated direction for any two netpoints as well as the relative error ellipse between two netpoints.

The positional error between two new netpoints can be computed as follows:

$$\sigma p_{ij} = \sqrt{\mathrm{trace}\left(C_{\hat{\Delta}x}\right)_{ij}} = \hat{\sigma}_o \cdot \sqrt{\left(Q_{\hat{\Delta}x\hat{\Delta}x}\right)_{ij} + \left(Q_{\hat{\Delta}y\hat{\Delta}y}\right)_{ij}}$$

$$\sigma p_{ij} = \hat{\sigma}_o \cdot \sqrt{\left(Q_{pp}\right)_{ii} + \left(Q_{pp}\right)_{jj} - 2\left(Q_{x_ix_j} + Q_{y_iy_j}\right)}$$

$$\sigma p_{ij} = \sqrt{\sigma_{p_i}^2 + \sigma_{p_j}^2 - 2(\sigma_{x_ix_j} + \sigma_{y_iy_j})} . \tag{13}$$

3.2. The standard distance error and the standard direction error

In horizontal geodetic network, the estimated distance and the estimated direction between two new netpoints $P_i\ (\hat{x}_i, \hat{y}_i)$ and $P_j\ (\hat{x}_j, \hat{y}_j)$ can be calculated using the following equations:

$$\hat{S}_{ij} = \sqrt{\left(\hat{x}_j - \hat{x}_i\right)^2 + \left(\hat{y}_j - \hat{y}_i\right)^2} ,$$

and

$$\hat{t}_{i,j} = \arctan\left(\frac{\hat{y}_j - \hat{y}_i}{\hat{x}_j - \hat{x}_i}\right).$$

By applying the law of variance-covariance propagation [2], the standard error of the estimated distance (standard distance error) and the standard error of the estimated direction (standard direction error) can be computed using the following equations:

$$\sigma_{\hat{S}_{ij}} = \hat{\sigma}_o \sqrt{\cos^2 \hat{t}_{ij}\left(\sigma^2_{\hat{\Delta}x}\right)_{ij} + \sin^2 \hat{t}_{ij}\left(\sigma^2_{\hat{\Delta}y}\right)_{ij} + \sin 2\,\hat{t}_{ij}\left(\sigma_{\hat{\Delta}x\Delta y}\right)_{ij}} \tag{14}$$

and

$$\sigma_{\hat{t}_{ij}} = \frac{\hat{\sigma}_o \cdot \rho''}{\hat{S}_{ij}} \sqrt{\sin^2 \hat{t}_{ij}\left(\sigma^2_{\hat{\Delta}x}\right)_{ij} + \cos^2 \hat{t}_{ij}\left(\sigma^2_{\hat{\Delta}y}\right)_{ij} + \sin 2\,\hat{t}_{ij}\left(\sigma_{\hat{\Delta}x\hat{\Delta}y}\right)_{ij}} \tag{15}$$

From the standard error of the estimated direction $\sigma_{\hat{t}_{ij}}$, the standard cross error can be calculated as follows:

$$\sigma_{q_{ij}} = \frac{\hat{S}_{ij} \cdot \sigma_{\hat{t}_{ij}}}{\rho''} . \tag{16}$$

Based on the Eqs. (14) and (16), the following equation can be obtained:

$$\pm\sqrt{\sigma^2_{\hat{S}_{ij}} + \sigma^2_{\hat{q}_{ij}}} = \pm\sqrt{\left(\sigma^2_{\hat{\Delta}x}\right)_{ij} + \left(\sigma^2_{\hat{\Delta}y}\right)_{ij}} = \sigma_{p_{ij}} . \tag{17}$$

From Eq. (17), it can be concluded that the positional error between any new two netpoints is identical with the square root of summation of the variances of the coordinate differences between those netpoints. Furthermore, the positional error between any new two netpoints is identical with the geometric addition for the standard error of the estimated distance and the standard cross error. obviously the standard cross error can be computed from the standard error of the estimated direction between these new two netpoints.

3.3. The relative error ellipse between two netpoints

The relative positional precision between any pair of points within the network can be graphically represented by the relative error ellipse [1,6,7]. Fig 1 illustrates the relative error ellipse for the pair of points P_i and P_j, whose coordinates are $P_i(\hat{x}_i, \hat{y}_i)$ and $P_j(\hat{x}_j, \hat{y}_j)$ This ellipse is conventionally drawn on the mid-point of the line joining the two points P_i and P_j.

The three parameters of the relative error ellipse can be derived from the variance-covariance matrix of the coordinate differences $C_{\hat{\Delta} x}$ as follows [1]:

$$\left.\begin{aligned} A_r &= \hat{\sigma}_o \cdot \sqrt{\frac{\left[\left(Q_{\hat{\Delta}x\hat{\Delta}x}\right)_{ij} + \left(Q_{\hat{\Delta}y\hat{\Delta}y}\right)_{ij} + K\right]}{2}} \\ &\text{in direction } \theta_r \\ B_r &= \hat{\sigma}_o \cdot \sqrt{\frac{\left[\left(Q_{\hat{\Delta}x\hat{\Delta}x}\right)_{ij} + \left(Q_{\hat{\Delta}y\hat{\Delta}y}\right)_{ij} - K\right]}{2}} \\ &\text{in direction } \theta_r + 100 \text{ gon} \\ 2\,\theta_r &= \arctan\left(\frac{2\left(Q_{\hat{\Delta}x\hat{\Delta}y}\right)_{i.j}}{\left(Q_{\hat{\Delta}x\hat{\Delta}x}\right)_{ij} - \left(Q_{\hat{\Delta}y\hat{\Delta}y}\right)_{ij}}\right), \\ &\text{in which,} \end{aligned}\right\} \quad (18)$$

$$K = \sqrt{\left[\left(Q_{\hat{\Delta}x\hat{\Delta}x}\right)_{ij} - \left(Q_{\hat{\Delta}y\hat{\Delta}y}\right)_{ij}\right]^2 + 4\left(Q_{\hat{\Delta}x\hat{\Delta}y}\right)_{ij}^2}\,.$$

Where Ar and B_r are the semi-major and semi-minor axis of the relative error ellipse and θ_r is the angle between the semi - major axis of the relative error ellipse and the x-axis.

Also in Fig. 1 the tangents to the relative ellipse, which are parallel and perpendicular to the line P_i - P_j, have been drawn.

From the relative error ellipse, the standard error of the estimated distance between the netpoints ($\sigma_{\hat{S}_{ij}}$) as well as the standard error of the estimated direction between them, ($\sigma_{\hat{t}_{ij}}$) can also be obtained as follows:

$$oc = \sigma_{\hat{S}_{ij}} \quad \text{and} \quad ok = \frac{\hat{S}_{ij} \cdot \sigma_{\hat{t}_{ij}}}{\rho''} \qquad (19)$$

The relation between the semi-major and semi-minor axis of the relative error ellipse and the positional error between two netpoints can be derived as follows:

$$\sigma_{P_{ij}} = \sqrt{A_r^2 + B_r^2} \qquad (20)$$

Eq. (20) can be used as a check in the computation process [3].

4. Relative precision criterion between three netpoints

In addition to the description of the relative precision of any two adjacent points in geodetic control network, it is also of great importance to describe the relative precision between three neighboring points in the network. This precision criterion should be derived from the geometrical relationship between these netpoints. The standard error of the estimated angle can be considered as a reasonable and rational measure for description of the relative precision between any three neighboring points in geodetic control network [5].

4.1. The standard angle error

In horizontal geodetic network, the estimated angle $\hat{\alpha}$ can be computed from the difference between the estimated directions

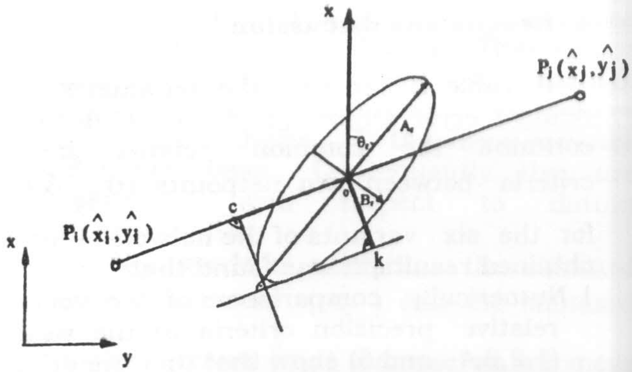

Fig. 1. The relative error ellipse.

$\hat{t}_{ik}$ and $\hat{t}_{ij}$ as given in the following equation:

$$\hat{\alpha}=\arctan\left(\frac{\hat{y}_k-\hat{y}_i}{\hat{x}_k-\hat{x}_i}\right)-\arctan\left(\frac{\hat{y}_j-\hat{y}_i}{\hat{x}_j-\hat{x}_i}\right).$$

By applying the law of variance-covariance propagation [2], the standard error of the estimated angle (standard angle error) can be calculated as follows:

$$\hat{\sigma}_\alpha=\hat{\sigma}_o\sqrt{Q_{\hat{\alpha}\hat{\alpha}}}\,. \tag{21}$$

Where,

$$Q_{\hat{\alpha}\hat{\alpha}}=\left(Q_{\hat{t}\hat{t}}\right)_{ik}+\left(Q_{\hat{t}\hat{t}}\right)_{ij}-2Q_{\hat{t}_{ik}\hat{t}_{ij}}\,. \tag{22}$$

in which,

$$\left(Q_{\hat{t}\hat{t}}\right)_{ik}=\left(\frac{\rho''}{\hat{S}_{ik}}\right)^2 \begin{bmatrix}\cos^2\hat{t}_{ik}\cdot\left(Q_{\hat{\Delta}y\hat{\Delta}y}\right)_{ik}\\ +\sin^2\hat{t}_{ik}\cdot\left(Q_{\hat{\Delta}x\hat{\Delta}x}\right)_{ik}-\sin 2\hat{t}_{ik}\cdot\left(Q_{\hat{\Delta}x\hat{\Delta}y}\right)_{ik}\end{bmatrix}. \tag{23}$$

$$\left(Q_{\hat{t}\hat{t}}\right)_{ij}=\left(\frac{\rho''}{\hat{S}_{ij}}\right)^2 \begin{bmatrix}\cos^2\hat{t}_{ij}\cdot\left(Q_{\hat{\Delta}y\hat{\Delta}y}\right)_{ij}\\ +\sin^2\hat{t}_{ij}\cdot\left(Q_{\hat{\Delta}x\hat{\Delta}x}\right)_{ij}-\sin 2\hat{t}_{ij}\cdot\left(Q_{\hat{\Delta}x\hat{\Delta}y}\right)_{ij}\end{bmatrix}, \tag{24}$$

$$\begin{aligned}Q_{\hat{t}_{ik}\hat{t}_{ij}}=\frac{\rho''^2}{\hat{S}_{ik}\hat{S}_{ij}}(&\cos\hat{t}_{ik}\cdot\cos\hat{t}_{ij}\cdot Q_{\hat{\Delta}y_{ik}\hat{\Delta}y_{ij}}\\ &+\sin\hat{t}_{ik}\cdot\sin\hat{t}_{ij}\cdot Q_{\hat{\Delta}x_{ik}\hat{\Delta}x_{ij}}\\ &-\cos\hat{t}_{ik}\cdot\sin\hat{t}_{ij}\cdot Q_{\hat{\Delta}y_{ik}\hat{\Delta}y_{ij}}\\ &-\sin\hat{t}_{ik}\cdot\cos\hat{t}_{ij}\cdot Q_{\hat{\Delta}x_{ik}\hat{\Delta}x_{ij}}),\end{aligned} \tag{25}$$

and

$$\left.\begin{aligned}Q_{\hat{\Delta}y_{ik}\hat{\Delta}y_{ij}}&=Q_{\hat{y}_i\hat{y}_i}+Q_{\hat{y}_k\hat{y}_j}-Q_{\hat{y}_i\hat{y}_j}-Q_{\hat{y}_k\hat{y}_i}\\ Q_{\hat{\Delta}x_{ik}\hat{\Delta}x_{ij}}&=Q_{\hat{x}_i\hat{x}_i}+Q_{\hat{x}_k\hat{x}_j}-Q_{\hat{x}_i\hat{x}_j}-Q_{\hat{x}_k\hat{x}_i}\,,\\ Q_{\hat{\Delta}y_{ik}\hat{\Delta}x_{ij}}&=Q_{\hat{y}_i\hat{x}_i}+Q_{\hat{y}_k\hat{x}_j}-Q_{\hat{y}_i\hat{x}_j}-Q_{\hat{y}_k\hat{x}_i}\,,\\ Q_{\hat{\Delta}x_{ik}\hat{\Delta}y_{ij}}&=Q_{\hat{x}_i\hat{y}_i}+Q_{\hat{x}_k\hat{y}_j}-Q_{\hat{x}_i\hat{y}_j}-Q_{\hat{x}_k\hat{y}_i}\end{aligned}\right\} \tag{26}$$

5. Numerical example

The schematic two-dimensional geodetic network is used to demonstrate the above derived results. This local network is composed of eight new points $P_1, P_2,\ldots P_8$ with unknown coordinates. The netpoints P_1, P_2, P_7 and P_8 are connected to eight fixed points A, B, C, D, F, G and H as shown in Fig. 2. The approximate coordinates of the new netpoints are listed in Table 1. with respect to the selected local horizontal coordinate system. The network was adjusted by the least squares method using the parametric technique. Three approaches of adjustment have been applied as follows:

I- Over constrained adjustment approach (more than three fixed coordinates).

II- Minimal constrained adjustments approach (only three fixed coordinates).

III- Inner constrained adjustment approach (free network adjustment).

For the clarity of this study, the local geodetic network illustrated in Figure (2) should be adjusted in six variants. In the first and second variants, the network was adjusted as a fixed trilateration network using the over constrained adjustment approach with eight and four fixed points.

Table 1
The approximate coordinates of the net points

Points	x (m)	y (m)
P_1	800.000	400.000
P_2	400.000	400.000
P_3	400.000	800.000
P_4	800.000	800.000
P_5	400.000	1200.000
P_6	800.000	1200.000
P_7	400.000	1600.000
P_8	800.000	1600.000

To study the change effect of the zero-variance reference base and the datum transformation on the above mentioned relative precision criteria, the trilateration network should be adjusted in the third and fourth variants using the minimal constrained approach. In the third variant, the coordinates of P_2 and the direction from P_2 to P_1 being fixed, while in the fourth variant we keep the coordinates of P_3 and the direction from P_3 to P_6 unchanged.

By deleting all the fixed points, the network became free from any fixed points and it can be adjusted as a free network using the inner constrained approach. In the fifth variant, the network was adjusted as a free trilateration network. In addition, the network was adjusted as a free combined network in the sixth variant.

All the computations were performed using PC computer. The computer program developed by the author was used to adjust the geodetic network using the above mentioned approaches and to estimate the relative precision criteria.

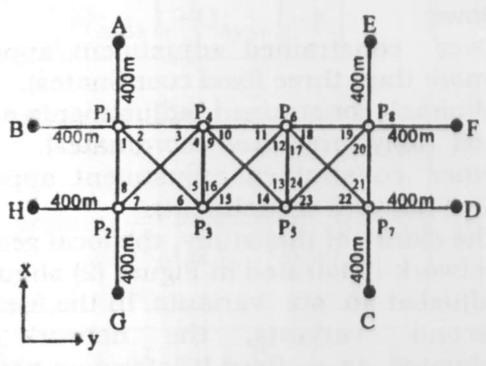

Fig. 2. A schematic geodetic network.

6. Results and discussion

I- Table 2 contains the parameters of the relative error ellipse (A_r & B_r), while table 3 contains the common relative precision criteria between two netpoints ($\sigma_{q_{ij}}$ & $\sigma_{p_{ij}}$) for the six variants of the network. From the obtained results, it was found that:

1-Numerically comparison of the common relative precision criteria in the variants (1,2,3,4 and 5) show that they are different from each other. This illustrates that the covariance matrix of the coordinates differences $C_{\hat{\Delta}x}$ and the common relative precision criteria (A_r, B_r, $\sigma_{q_{ij}}$ & $\sigma_{p_{ij}}$) depend upon the choice of the zero variance reference base and the choice of the least squares adjustment approaches. Consequently, they are not invariant with respect to datum transformation.

2- The common relative precision criteria (A_r, B_r, σq_{ij} & σp_{ij}) gave the highest numerical values of the farthest member from fixed network points (see variants 1,2,3, & 4).

3- In free geodetic networks, relative precision criteria increase as we move towards external boundaries of the network, while they are minimum around the network centroid (see variants 5 & 6).

II- Table 4. shows the standard errors of the estimated distances ($\sigma_{\hat{S}_{ij}}$)while table (5) shows the standard errors of the estimated angles ($\sigma_{\hat{\alpha}}$) for the six network variants. Comparison of the numerical values for both relative precision criteria showed that:

1- As the number of fixed points for geodetic networks decreases, the numerical values of both ($\sigma_{\hat{S}_{ij}}$) and ($\sigma_{\hat{\alpha}}$) increase (see variants 1,2,3 and 4).

2- Free geodetic network adjustment using the inner constrained approach (variant 5) gave the same numerical values of the standard distances errors ($\sigma_{\hat{S}_{ij}}$) and the standard angles errors ($\sigma_{\hat{\alpha}}$) as those obtained by a minimum constrained approach (variants 3 & 4). This means that, they are independent of the choice of the least squares adjustment approaches and the choice of the zero-variance reference base. Consequently, they are invariant with respect to datum transformation.

3- In free geodetic networks, the standard distances errors ($\sigma_{\hat{S}_{ij}}$) and the standard angles errors ($\sigma_{\hat{\alpha}}$) increase as we move towards external boundaries of the network, while they are minimum around the network centroid (see variants 5 & 6).

4- Combined geodetic network (variant 6) gave the smallest numerical values of the common relative precision criteria (A_r, B_r, $\sigma_{q_{ij}}$ & σp_{ij}) as well as the standard errors of both the estimated distances ($\sigma_{\hat{S}_{ij}}$) and the estimated angles ($\sigma_{\hat{\alpha}}$).

7. Conclusions and recommendations

The main conclusions deduced from the analysis and discussion of this study can be summarized as follows:

1- The standard errors of both the estimated distances and the estimated angles are independent of the choice of the least squares adjustment approaches and of the choice of the zero - variance reference base. Consequently, they are invariant with respect to datum transformation.

2- Therefore, the standard errors of both the estimated distances and the estimated angles give better description for the relative internal precision of the geodetic control networks.

3- As the number of fixed netpoints or outer constraints increases the relative precision criteria represented by the standard errors of both the estimated distances and the estimated angles of the geodetic network decrease.

4- In fixed geodetic networks, the netpoints farthest away from the fixed points will obviously have the largest relative error ellipse and the highest values of both the positional error between two adjacent netpoints and the standard cross error.

5- In free geodetic networks, all relative precision criteria increase as we move towards external boundaries of the network, while they are minimum around the network centroid.

6- The positional error between two netpoints and the standard cross errors, which can be obtained from the standard errors of the estimated directions between two neighboring netpoints, are dependent upon the choice of the least squares adjustment approaches and the choice of the zero - variance reference base. Consequently, they are not invariant with respect to datum transformation.

7- Relative error ellipses are dependent upon the choice of the zero-variance reference base and the choice of the least squares adjustment approaches. Consequently , they are not invariant with respect to the datum transformation.

8- The rate of improvement of the overall precision of the network can be monitored through the rate of improvement of the standard distance error and the standard angle error. Therefore, the standard distance error and the standard angle error can be used as a base index for assessing the user's accuracy requirements.

9- Finally, more consideration is now given to the standard distance error and the standard angle error which are considered as the function measures of precision used for optimization of network design. Therefore, they are usually necessary as a component of an objective function.

Table 2
The parameters of the relative error ellipse

	Sides	Variant (1)	Variant (2)	Variant (3)	Variant (4)	Variant (5)	Variant (6)
		8- Fixed points A,B,C,D,E F,G & H	4-Fixed points A,B,C & D	x_2, y_2, y_1 are fixed	x_3, y_3, y_6 Are fixed	Free Trilateration network	Free Combined network
n		24	20	16	16	16	40
u		16	16	13	13	16	16
d		-	-	-	-	3	3
r		8	4	3	3	3	27
	1,2	1.03	1.52	0.93	1.69	1.48	0.79
Ar	1,3	1.32	1.63	1.47	1.99	1.60	0.81
[cm]	1,4	1.23	1.44	1.47	1.69	1.44	0.72
	2,3	1.23	1.54	1.47	1.75	1.44	0.72
	2,4	1.32	1.80	1.47	1.92	1.60	0.81
	3,4	0.96	1.09	1.41	0.97	0.92	0.57
	3,5	1.15	1.21	2.00	1.39	1.06	0.66
	3,6	1.17	1.30	2.45	1.22	1.03	0.64
	4,5	1.17	1.31	2.45	1.64	1.03	0.64
	4,6	1.15	1.22	2.00	1.39	1.06	0.66
	5,6	0.96	1.09	2.00	0.97	0.92	0.57
	5,7	1.23	1.44	2.45	1.69	1.44	0.72
	5,8	1.32	1.80	3.16	1.92	1.60	0.81
	6,7	1.32	1.63	3.16	1.99	1.60	0.81
	6,8	1.23	1.54	2.45	1.75	1.44	0.72
	7,8	1.03	1.52	2.45	1.69	1.48	0.79
	1,2	0.78	0.93	0.00	0.93	0.93	0.69
Br	1,3	0.80	0.83	0.76	0.81	0.85	0.61
[cm]	1,4	0.79	0.83	0.83	0.86	0.88	0.61
	2,3	0.79	0.86	0.83	0.87	0.88	0.61
	2,4	0.80	0.85	0.76	0.82	0.85	0.61
	3,4	0.87	0.87	0.88	0.84	0.88	0.50
	3,5	0.83	0.91	0.89	0.86	0.93	0.47
	3,6	0.81	0.85	0.84	0.00	0.82	0.51
	4,5	0.81	0.81	0.84	0.85	0.82	0.51
	4,6	0.83	0.91	0.89	0.86	0.93	0.47
	5,6	0.87	0.87	0.88	0.84	0.88	0.50
	5,7	0.79	0.83	0.91	0.86	0.88	0.61
	5,8	0.80	0.85	0.85	0.82	0.85	0.61
	6,7	0.80	0.82	0.85	0.81	0.85	0.61
	6,8	0.79	0.89	0.91	0.87	0.88	0.61
	7,8	0.78	0.93	0.93	0.93	0.93	0.69

Table 3
The common relative precision criteria between two netpoints

	Sides	Variant (1)	Variant (2)	Variant (3)	Variant (4)	Variant (5)	Variant (6)
		8- Fixed points A,B,C,D,E F,G & H	4-Fixed points A,B,C & D	x_2, y_2 , y_1 are fixed	x_3 , y_3 , y_6 Are fixed	Free Trilateration network	Free Combined network
σq_{ij} [cm]	1,2	1.03	1.52	0.00	1.73	1.48	0.79
	1,3	1.30	1.63	1.41	2.05	1.58	0.80
	1,4	1.23	1.42	1.40	1.64	1.41	0.65
	2,3	1.23	1.52	1.40	1.79	1.41	0.65
	2,4	1.30	1.79	1.41	1.92	1.58	0.80
	3,4	0.96	1.09	1.41	0.99	0.92	0.57
	3,5	1.15	1.22	1.98	0.96	1.06	0.47
	3,6	1.15	1.30	2.44	0.00	1.00	0.56
	4,5	1.15	1.29	2.44	1.37	1.00	0.56
	4,6	1.15	1.22	1.98	0.96	1.06	0.47
	5,6	0.96	1.09	2.00	0.99	0.92	0.57
	5,7	1.23	1.42	2.44	1.64	1.41	0.65
	5,8	1.30	1.79	3.16	1.92	1.58	0.80
	6,7	1.30	1.63	3.16	2.05	1.58	0.80
	6,8	1.23	1.52	2.44	1.79	1.41	0.65
	7,8	1.03	1.52	2.45	1.73	1.48	0.79
σp_{ij} [cm]	1,2	1.29	1.78	0.93	1.96	1.75	1.05
	1,3	1.54	1.83	1.65	2.23	1.81	1.01
	1,4	1.47	1.66	1.69	1.89	1.69	0.94
	2,3	1.47	1.78	1.69	2.02	1.69	0.94
	2,4	1.54	1.99	1.65	2.11	1.81	1.01
	3,4	1.29	1.40	1.67	1.33	1.27	0.76
	3,5	1.42	1.52	2.19	1.34	1.41	0.82
	3,6	1.42	1.56	2.59	0.86	1.32	0.82
	4,5	1.42	1.54	2.59	1.62	1.32	0.82
	4,6	1.42	1.52	2.19	1.34	1.41	0.82
	5,6	1.29	1.40	2.19	1.33	1.27	0.76
	5,7	1.47	1.66	2.62	1.89	1.69	0.94
	5,8	1.54	1.99	3.28	2.11	1.81	1.01
	6,7	1.54	1.83	3.28	2.23	1.81	1.01
	6,8	1.47	1.78	2.62	2.02	1.69	0.94
	7,8	1.29	1.78	2.62	1.96	1.75	1.05

Table 4
The standard errors of the estimated distances

	Sides	Variant (1)	Variant (2)	Variant (3)	Variant (4)	Variant (5)	Variant (6)
		8- Fixed points A,B,C,D,E F,G & H	4- Fixed points A,B,C & D	x_2 , y_2 , y_1 are fixed	x_3 , y_3 , y_6 are fixed	Free trilateration network	Free combined network
$\sigma_{\hat{S}_{ij}}$ [cm]	1,2	0.78	0.93	0.93	0.93	0.93	0.69
	1,3	0.83	0.84	0.86	0.86	0.86	0.62
	1,4	0.80	0.87	0.93	0.93	0.93	0.68
	2,3	0.80	0.93	0.93	0.93	0.93	0.68
	2,4	0.83	0.86	0.86	0.86	0.86	0.62
	3,4	0.87	0.87	0.88	0.88	0.88	0.50
	3,5	0.83	0.91	0.93	0.93	0.93	0.66
	3,6	0.84	0.86	0.86	0.86	0.86	0.59
	4,5	0.84	0.84	0.86	0.86	0.86	0.59
	4,6	0.83	0.91	0.93	0.93	0.93	0.66
	5,6	0.87	0.87	0.88	0.88	0.88	0.50
	5,7	0.80	0.87	0.93	0.93	0.93	0.68
	5,8	0.83	0.86	0.86	0.86	0.86	0.62
	6,7	0.83	0.84	0.86	0.86	0.86	0.62
	6,8	0.80	0.93	0.93	0.93	0.93	0.68
	7,8	0.78	0.93	0.93	0.93	0.93	0.69

Table 5
The standard errors of the estimated angles [mgon]

Angles No.	Variant (1)	Variant (2)	Variant (3)	Variant (4)	Variant (5)	Variant (6)
	8-Fixed points A,B,C,D,E F,G & H	4- Fixed points A,B,C & D	x_2, y_2, y_1 are fixed	x_3, y_3, y_6 are fixed	Free trilateration network	Free combined network
10=11=14=13	0.93	2= 8=0.93 10=14=0.93	0.96	0.96	0.96	0.53
2=7=19=22	0.92	20=22=0.93 7=11=0.96	0.96	0.96	0.96	0.54
9=12=13=16	0.91	15=19=0.96 12=16=0.96	0.99	0.99	0.99	0.59
4=5=17=24	0.91	5=17=0.98 1= 6=0.99	1.00	1.00	1.00	0.61
1=8=20=21	0.84	9=13=0.99 18=21=0.99	1.00	1.00	1.00	0.64
3=6=18=23	0.87	3= 4=1.00 23=24=1.00	1.00	1.00	1.00	0.65

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