Safety assessment of uniaxially loaded stiffened panels

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The aim of the present study is to assess the reliability of uniaxially loaded stiffened panels. Firstly, the Idealized Structural Unit Method (ISUM) is selected to evaluate the nonlinear behavior of stiffened panels for different loading stages, namely, buckling, post buckling, and collapse. Moreover, different modes of buckling concerning plate, stiffener, and stiffened panel are considered. Secondly, the First Order Reliability Method (FORM) is selected to evaluate the safety index of the stiffened panel for the previous modes of failure. A review of the reliability theory is included and the different safety margins or performance functions required in the reliability analysis are outlined. Finally, to determine the most critical failure mode and effects of stiffener dimensions on safety index for stiffened panels, practical examples are studied and the results are presented.

الهدف من هذا البحث هو تحديد معولية الالواح المدعمة والمعرضة لاحمال محورية. تستخدم طريقة الوحدة الانشائية التمثيلية لتقدير مقاومة الالواح بينما تستخدم طريقة المعولية من الدرجة الأولى لحساب دليل الأمان فى مراحل الانبعــاج والانــهيار الكلى. تم دراسة الأنواع المحتملة لأنهيار الانبعاج فيما يخص مكونات الألواح المدعمة، اللوح والدعائم كما تم شرح ملخص لكل من طريقة الوحدة الانشائية التمثيلية والمستخدمة لحساب التحليل الاجهادى الغير خطى، ولطريقة التحليل المعولى مــن الدرجة الأولى. أخيرا، تم اختيار نموذج واقعى لتحددي نوع الانهيار الحرج الذى سيتم أولا وكذلك تمت دراسة تأثير أبعــاد الدعائم على دليل الأمان. وقد عرضت النتائج فى شكل منحنيات تظهرا فيها أيضا علاقة الحمل مع دليل الأمان.

Keyword: Stiffened panel, Buckling strength, Safety index and probability of failure.

1. Introduction

Reliability theory has been applied in naval architecture over the last two decades. Mansour et al. [1], and Faulkner and Sadden [2] carried out earlier works on the application of reliability methods to ship structures. Safety index has been commonly used to judge the safety level of a structure against primary failure modes. The higher the safety index, the lower will be the probability of failure. The reliability analysis of plate panels subjected to uniaxial and biaxial loading was studied in [3] and [4], respectively. Stiffened panels constitute most of the steel structural elements in ship structures. They are exposed to various stresses and lateral pressure due to the stillwater and wave induced bending moments. The reliability of stiffened panels has been studied in various literatures such as [5]. In this study, the strength analysis and the reliability analysis of stiffened panels subjected to uniaxial loads are investigated.

Firstly, the stress analysis of the stiffened panels is performed using the Idealized Structural Unit Method (ISUM)[6]. Secondly, a reliability analysis of the stiffened panels is performed using a First Order Reliability Method (FORM)[7-10]. The effects of initial deflection and residual stresses are neglected. A computer program, REL/ISUM (reliability analysis with ISUM), is used to assess the safety index for different modes of failure. Several examples are carried out and the results are presented.

2. Stress analysis

2.1. Description of ISUM

Ueda and Rashed [11] have proposed ISUM as a model for the ultimate strength analysis of frame and plate structures. In this method, the idealized plate panel element and the idealized stiffened plate element were considered. The nonlinear behavior of each element was idealized and expressed in the form of a set of failure functions defining the necessary conditions for different failures perceived to take place in the element, and a set of stiffness matrices defining the relationship between the nodal force increments and nodal displacement increments until and after the ultimate strength state. The details formulating these structural units are found in [11,12]. Only the theoretical outlines are briefly presented here.

2.1.1. ISUM rectangular plate element [11]

The element is a rectangular plate as shown in fig. 1. Its edges are assumed to remain straight after deformation. The bending stiffness of the element is neglected in comparison with the bending stiffness of the whole structure, and therefore the element is treated as a membrane. The plate has four nodes, one at each corner, each node has three translatory degrees of freedom. The nodal displacements (**U**) and forces (**R**) are expressed, respectively, as follows:

$$\mathbf{U}=[U_1\ U_2\ U_3\ U_4]^T,\ U_i=[u_i\ ,\ v_i\ ,\ w_i\ ]^T, \quad (1)$$

$$\mathbf{R}=[R_1 R_2\ R_3\ R_4]^T,\ R_i=[R_{xi}\ ,\ R_{yi}\ ,\ R_{zi}\ ]^T. \quad (2)$$

In the absence of initial deformations, and under increasing load the element behaves as shown in fig. 2. The relationship between **U** and **R** may be expressed incrementally as follows:

$$\Delta \mathbf{R} = \mathbf{K}\Delta\ \mathbf{U}. \quad (3)$$

K is evaluated depending on the state of the element. The failure- free stiffness matrix, buckling conditions, post- buckling stiffness matrices, plasticity condition and elastic-plastic stiffness matrix are summarized in the next subsections, however, more details are given in [12].

2.1.1.1. Failure- free stiffness matrix: Before buckling occurs the stress distribution is assumed to be linearly. The elastic failure free stiffness matrix ($\mathbf{K^E}$) can be derived as follows:

$$\mathbf{K^E} = \int_v \mathbf{B^T\ D^E\ BdV}. \quad (4)$$

Where

D^E is the stress- strain matrix in the elastic range,

B is the strain- displacement matrix, and

V is the volume of the element,

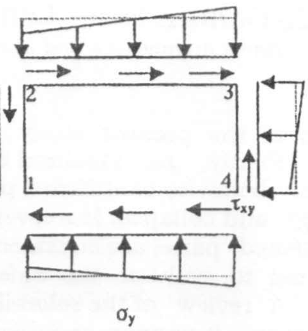

Fig. 1. A rectangular plate element [1].

The stress in the element may be expressed as:

$$\sigma = \mathbf{D^E}\ \Delta\varepsilon = \mathbf{D^E BU}. \quad (5)$$

Where $\Delta\varepsilon$ is the incremental of the strain vector.

2.1.1.2. Post- buckling stiffness matrix: After the element has buckled, the stress distribution in the middle plane of the element becomes non-linear. The post buckling stiffness matrix (K^B) is given as follows:

$$\mathbf{K^B} = \int_v \mathbf{B^T D^B BdV} \quad (6)$$

The relation between the average strain and the average stress may be written as follows:

$$\varepsilon_{xav} = (\sigma_{x\max} - \nu\sigma_{yav})/E,$$

$$\varepsilon_{yav} = (\sigma_{y\max} - \nu\sigma_{xav})/E,$$

$$\gamma_{xyav} = \tau_{xy}/G_e.$$

Where,

σ_{xmax}, σ_{ymax} are the maximum stresses in X and Y directions, respectively.

G_e is the effective shear modulus.

The stress in the element may be expressed as:

$$\sigma = D^{B} \quad \Delta\varepsilon. \qquad (7)$$

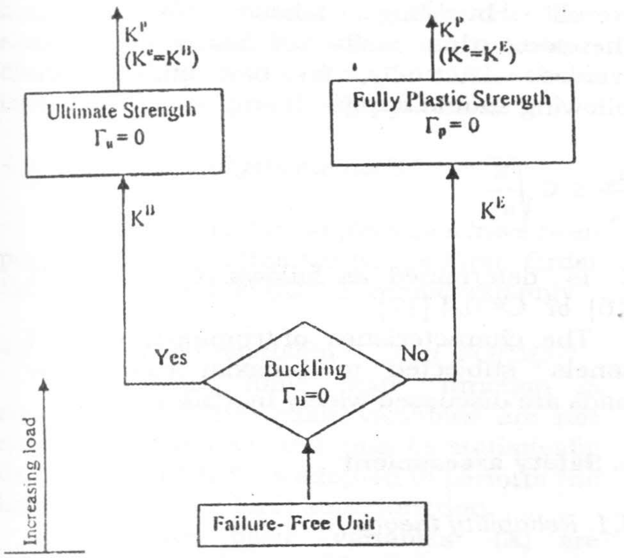

Fig. 2. Local behavior of rectangular plate element [11].

2.1.1.3. Ultimate strength condition and elastic-plastic stiffness matrix: The buckled plate will continue to carry further load until yielding starts, and the ultimate strength is reached. After yielding, Eq.3 may be expressed in terms of an elastic- plastic stiffness matrix ($\mathbf{K^P}$):

$$\Delta\mathbf{R} = \mathbf{K}^{P}\Delta\mathbf{U}\,.$$

The stress in the element may be expressed as:

$$\sigma = \mathbf{D}^{Pl}\mathbf{B}\mathbf{K}^{-1} \quad . \qquad (8)$$

D^{Pl} defines the relationship of the maximum stress to the average strains. Yielding is assumed to start at the edges to satisfy the Mises yield equation, that is:

$$\Gamma_u = \sigma_x^2 - \sigma_x\sigma_y + \sigma_y^2 + 3\tau_{xy}^2 - \sigma_o^2 = 2\,. \qquad (9)$$

2.1.2. *ISUM stiffened plate element [6]*

The stiffened plate element is shown in fig. 3. Its edges, similar to the plate element, are assumed to remain straight after deformation. It has (L) parallel and equispaced stiffeners and four nodes, one at each corner, and is also treated as a membrane element. Nodal displacements and forces are expressed by eqs. (1) and (2). In the absence of initial deformation and under increasing load, the element behaves as shown in fig. 4. The relationship between **U** and **R** is expressed by eq. (3), where **K** is evaluated depending on the state of the element. The total stiffness matrix **K** is the assembly of the stiffness matrix of plate and that of stiffener.

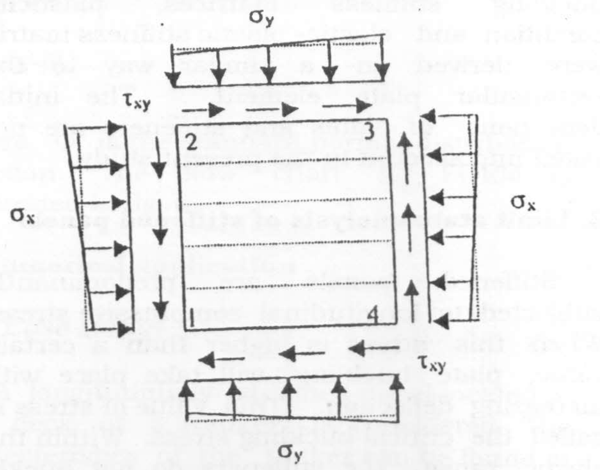

Fig. 3. A rectangular stiffened plate element [6].

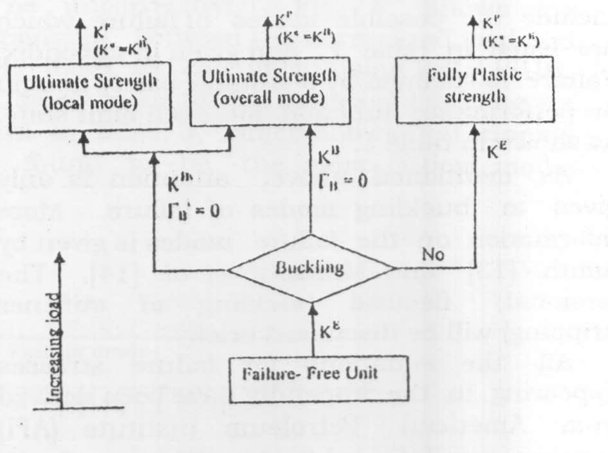

Fig. 4. Local behavior of stiffened plate element [6].

The relationship between the incremental nodal force and the incremental displacement may be expressed in term of beam stiffness matrix (K^b) as follows [12],

$$\Delta \mathbf{R} = \mathbf{K}^b \Delta \mathbf{U}.$$

Then, the stress of the beam (stiffener) may be expressed as,

$$\sigma = R / A_s. \quad (10)$$

Where A_s is the area of stiffener.

The failure-free stiffness matrix, buckling conditions for overall panel buckling and local buckling of plates between stiffeners, post-buckling stiffness matrices, plasticity condition and elastic- plastic stiffness matrix were derived in a similar way to the rectangular plate element. The initial deflections of plates and stiffeners are not taken into account in the present study.

3. Limit state analysis of stiffened panels

Stiffened panels are predominantly subjected to longitudinal compressive stress. When this stress is higher than a certain value, plate buckling will take place with increasing deflection. This value of stress is called the critical buckling stress. Within the elastic range. the stiffeners do not buckle before the plate panels have reached their material yield strength and thus collapse. The limit state analysis of stiffened panels must include the possible modes of failure, which are listed in table 1, and given in appendix. Failure is defined by a suitable safety margin or performance function for each limit state, as shown in table 1.

As mentioned above, attention is only given to buckling modes of failure. More information on the failure modes is given by Smith [13] and Mansour *et al.* [14]. The torsional/ flexural buckling of stiffener (tripping) will be discussed briefly.

All the equations for failure stresses appearing in the appendix have been derived from American Petroleum Institute (API) requirements [15], while the ultimate collapse load of plate is calculated with ISUM.

3.1. Tripping of stiffeners

In general, tripping is regarded as collapse because once it occurs the plating is left without stiffening, and consequently the overall buckling follows immediately. Therefore, this mode of failure should be avoided. Actually, for bar stiffeners the following should apply:

$$\frac{h_w}{t_w} \leq C\sqrt{\frac{E}{\sigma_o}} \quad (11)$$

C is determined as follows: C= 0.35-.0.37 [16] or C= 0.4 [17].

The characteristics of tripping in stiffened panels subjected to uniaxial compressive loads are discussed widely by Paik *et al.* [18].

4. Safety assessment

4.1. Reliability theory

The reliability of a structure ($\Re$) is a measure used to decide its ability to fulfill its design purpose during a certain period. This ability depends on a probability of failure (P_f) which is affected and determined by the entire spectrum of demand (D) and capacity (S). In general, the reliability is expressed as [19]:

$$\Re = 1 - P_f.$$

In general, an alternative measure of reliability, commonly known as the safety index (β) is used. A limit state function or performance function g(x) is defined in terms of the set of basic random variables X, representing uncertain quantities such as loads, material properties, structural dimensions, ... etc. These uncertainties are defined by mean values, variances and probability functions. The limit state satisfies:

$$g(X)\begin{cases} <0 & \text{failure} \\ =0 & \text{limit state surface} \\ >0 & \text{survival.} \end{cases}$$

Thus, the probability of failure can be evaluated by the following integral [20]:

$$p_f = P(g(x) \le 0) = \int_{g(X)\le 0} f_X(x)dx \,.$$

Where, $f_X(x)$ is the joint probability density function of the random variable. Because each of the basic variables has a unique distribution and they are correlated, the above integral cannot be practically evaluated.

4.2. Reliability analysis methods

Many probabilistic approaches have been proposed [21]. In this study, the First- Order Reliability Method (FORM) is considered only.

4.2.1. First- order reliability method (FORM)[7]

Usually, the limit state function is nonlinear and the basic variables are not normally distributed and may be statistically dependent. FORM is adopted to perform the linearization of the limit state function.

Firstly, the basic variables (**X**) are transformed to the standard normal vector space (**Z**) [10]. The joint distribution of **X** such that Z is jointly normal is obtained using the Nataf model [8,9]. Then, the distribution model is transformed to the standard normal space (**Y**).

Finally, it is convenient to write this transformation in the form:

$$Y = \Gamma\, D^{-1}(X-M).$$

In which D= $\mathrm{diag}[\sigma_i]$= the diagonal matrix of the standard deviation σ_i, and $\Gamma = L^{-1}$, where L= a lower- triangular matrix obtained from Cholesky decomposition and $M = [m_1, \ldots, m_n]^T$ is the equivalent mean vector for (n) number of variables.The performance function is expressed as:

$$G(x) = g(M+DLY) = G(Y).$$

As mentioned above, FORM replaces the limit state surface, g(x)=0, with a tangent hyperplane at the design point(Y*) in the standard normal space G(Y). Thus, $\beta = |Y^*|$ is as shown in fig. 5. The first-order approximation to probability of failure is given by:

$$P_f = \Phi(-\beta).$$

Where, Φ is the standard normal distribution function. The flow chart for FORM is illustrated in fig. 6.

5. Numerical application

5.1. Case study

A longitudinally stiffened panel located in the deck of a tanker is considered. The characteristics of the tanker can be found in [22]. The initial stiffened panel is shown in fig. 7. The required variables are listed in table 2 [23]. All random variables are assumed to be uncorrelated. Fig. 8. Shows the relationship between the normalized load and safety index as calculated using REL/ ISUM. The safety index for the modes of buckling are shown in table 3, which shows that tripping was found to be the most critical mode.

Table 1
Limit states for stiffened panels

Safety index	Failure mode
1- Buckling of plate between stiffeners	$M_1 = \sigma_{cr} - \sigma_x$ (eq. (A-1)
2* – Collapse of plate between stiffeners	$M_2 = \sigma_{ul} - \sigma_x$ eqs.(8) and (9)
3* – Column buckling of stiffeners	$M_3 = \sigma_{cr} - \sigma_x$ eq.(A-2)
4*-Torsional/ flexural buckling of stiffeners	$M_4 = \sigma_{cr} - \sigma_x$ eqs (A-3), (A-5)
5- Buckling of stiffened panel	$M_5 = \sigma_{cr} - \sigma_x$ eqs.(A-6)

Note: * collapse, the values of σ_x are calculated from eqs. (7), (8) and (10).

Table 2
Variables of stiffened panel

Variables	Distribution	Mean	Cov
t_p	Lognormal	30 mm	0.01
t_w	Lognormal	32 mm	0.01
E	Normal	206 KN/mm²	0.02
$\overline{\sigma}_o$	Normal	260 N/mm²	0.063
$\overline{\sigma}_x$	Normal	122 N/mm²	0.2
h_w	Fixed	280 mm	–
ν	Fixed	0.3	–

Table 3
Safety index for the modes of buckling

Buckling mode	Plate Buckling	Column buckling	Tripping of stiffener	Overall buckling
Safety index	4.394	4.194	2.582	4.386
Probability of failure	5.558×10^{-4}	1.37×10^{-3}	4.911×10^{-2}	5.782×10^{-4}

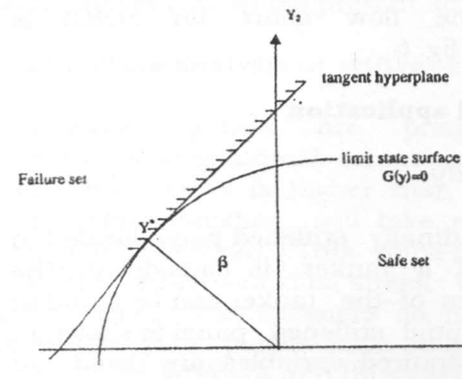

Fig. 5. Illustration of the safety index.

5.2. Effect of stiffener dimensions

The effect of stiffener, dimensions, such as the stiffener height, on safety index is considered. From the safety view point, the larger the height of the stiffener, the smaller the safety index for tripping, as shown in fig. 9, whereas the opposite is true for the column buckling mode, as shown in fig. 10. From figs. 9 and 10 it is seen that, for the same load ($\overline{\sigma}_x = 0.4\,\overline{\sigma}_o$) the safety index for tripping will increase from 2.25 to 3.19 with reducing h_w from 300 mm to 200 mm. These values of safety index correspond to probabilities of failure of 1.22×10^{-2} and 7.114×10^{-4}, respectively.

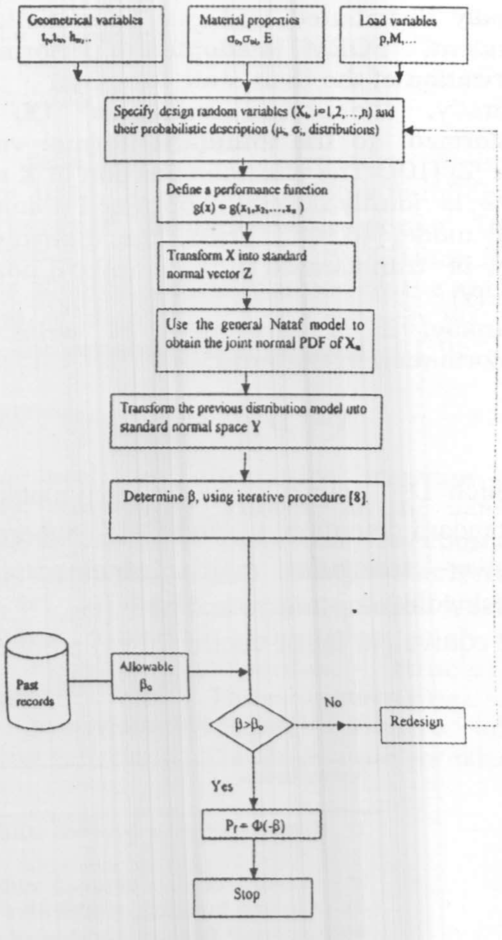

Fig. 6. Flow chart for first order reliability method (FORM).

Table 4
Values of β and P_f for tripping and for column buckling

	β		P_f	
H_w	Tripping	Column buckling	Tripping	Column buckling
200 mm	3.19	4.86	7.114×10^{-4}	5.8691×10^{-7}
300 mm	2.25	5.11	1.222×10^{-2}	2.868×10^{-7}

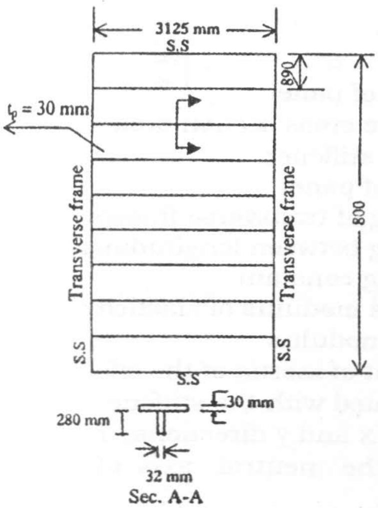

Fig. 7. Longitudinally stiffened panel in center tank of example tanker at a midship [22].

The reduction of β obtained by reducing h_w for column buckling mode from 5.11 to 4.86 corresponding to probabilities of failure 2.86 $\times10^{-7}$ to 5.86 $\times10^{-7}$, respectively, does not constitute a substantial change. Therefore, the height of stiffener must be chosen to avoid tripping, see eq. (11). The previous results are summarized in table 4.

It may be noted that, the design may be made considering the local buckling and collapse of plate between stiffeners to occur at the same level of stresses, while the collapse of stiffener is prior to plate collapse.

5.3. System reliability

In general, stiffened panels are considered as a series system, since they fail immediately if any of their limit states is reached. In this study the probability of failure of the system (stiffened panel) is determined using the simple bounds [24] given by:

$$\max(P_f^{\,j}) \le P_f \le 1-(1-P_f^{\,1})\,(1-P_f^{\,2})\ldots\ldots\ldots(1-P_f^{\,m}).$$

Where $P_f^{\,j}$ is the failure probability of the j *th* failure mode, for j =1, 2, ...,m. Applying the system reliability to the previous case study showed that the upper and lower bounds are identical to the probability of failure for the weakest mode of failure, namely, tripping.

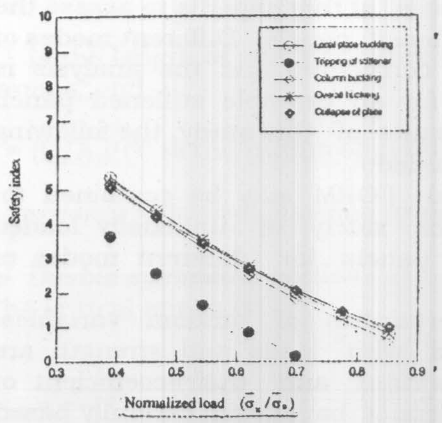

Fig. 8. Relationship between safety index and normalized load for different modes of failure.

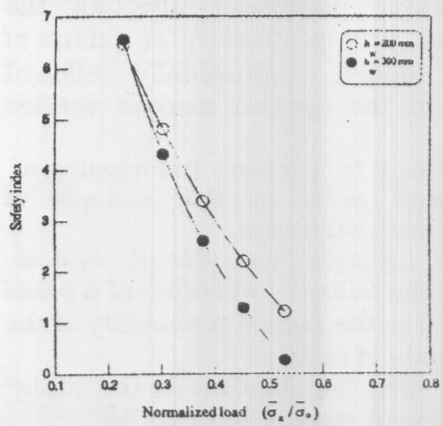

Fig. 9. Effect of stiffener height on safety index for tripping mode of failure.

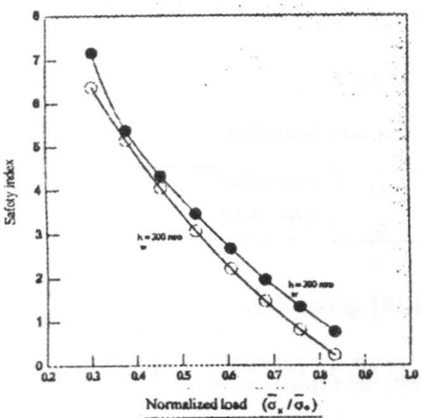

Fig. 10. Effect of stiffener height on safety index for Column buckling mode of failure

6. Conclusions and recommendations for future work

The purpose of this paper is to assess the safety of stiffened panels. Different modes of failure are considered and the analysis is carried out for an example stiffened panel. From the results of this study, the following may be concluded:

1- ISUM and FORM may be combined to assess the safety of uniaxially loaded stiffened panels for different modes of buckling.

2- The uncertainties of random variables associated with loads and strength are very important and their coefficient of variation must be chosen carefully based on existing data.

3- It is well known that the stiffener height is an important parameter affecting the safety index and probability of failure of uniaxially loaded longitudinally stiffened panels, and the applied method verifies this.

4- It is important to prevent the tripping of stiffener web prior to the collapse of plating between stiffeners.

5- Using the simple bounds of system reliability, the failure probability of a panel is identical to the failure probability of the weakest mode of failure.

6- This study was established for uniaxially loaded stiffened panels, but may be extended to include biaxial loading effects, shear stress and lateral pressure.

7- The influence of initial imperfection and residual stress are not included in the present study, but further work is underway to take them into account.

It is to be noted that the proposed procedure should be extended so as to assess the safety of the ship hull girder using REL/ISUM.

Nomenclature

A	Length of panel
A_e	Effective cross section area
A_s	Area of stiffener
B	Width of panel
a	Spacing of transverse frames
b	Spacing between longitudinal stiffeners
c_w	Warping constant
E	Young's modulus of elasticity
G	Shear modulus
I_{px} , I_{py}	Moment of inertia of the effective plate associated with the stiffeners extending in the x and y directions, respectively; bout the neutral axis of the entire section
I	Moment of inertia (including effective plate)
J	St Venant's torsion constant
S_x, S_y	Spacing of the stiffeners extending in the x and y directions, respectively
t_p	Plate thickness
t_w	Web thickness
t_e	Equivalent thickness = $(t_p + A_s/b)$
h_w	Height of web stiffener
ν	Poisson' s ratio
$\bar{\sigma}_o$	Yield stress of material (mean)
σ_p	Proportional limit stress in compression, taken as 60 % of σ_o
$\bar{\sigma}_x$	Axial load (mean)
σ_{cr}	Critical buckling stress
β	Safety index
P_f	Probability of failure
μ	Mean
σ	Standard deviation
Γ_B	Local buckling function
$\Gamma_B\backslash$	Overall buckling function
Γ_u	Ultimate function

Appendix

Failure of plating between stiffeners [15]:

The serviceability and ultimate limit states of plates are considered.

Serviceability limit state: the critical buckling equations for the determination of the elastic stability of plates are as follows:

$$\sigma_{cr} = K_c \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t_p}{b}\right)^2, \qquad \text{if} \quad \sigma_{cr} < \sigma_p$$

$$\sigma_{cr} = \frac{c_1 \sigma_o}{c_1 + 1},$$

for $\frac{a}{b} \geq 1$ and $\sigma_{cr} > \sigma_p$ (A-1)

$$\sigma_{cr} = \sigma_o - \frac{1}{c_s},$$

for $\frac{a}{b} < 1$ and $\sigma_{cr} > \sigma_p$

and

$K_c = 4$ for $\frac{a}{b} \geq$

$K_c = \left(\frac{a}{b} + \frac{b}{a}\right)^2$ for $\frac{a}{b} <$

$$C_1 = \frac{\sigma_1^2}{\sigma_p(\sigma_o - \sigma_p)}, \quad \sigma_1 = \frac{4\pi^2 E}{12(1-\nu^2)}\left(\frac{t_p}{b}\right)^2$$

$$C_s = \frac{\sigma_s}{\sigma_p(\sigma_o - \sigma_p)}, \sigma_s = \left(\frac{a}{b} + \frac{b}{a}\right)^2 \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t_p}{b}\right)^2$$

Failure of stiffeners with effective plating [15]:

In the following, only the ultimate limit state is considered due to the fact that, when column buckling occurs the ultimate strength is immediately reached.

-Column buckling

The critical stress is:

$$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{l}{r}\right)^2} \qquad \text{if} \quad \sigma_{cr} \leq \sigma$$

$$\sigma_{cr} = \sigma_o - \frac{1}{c_s} \qquad \text{if} \quad \sigma_{cr} \geq \sigma_p, \quad \text{(A-2)}$$

where

$$c_s = \frac{\sigma_s}{\sigma_p(\sigma_o - \sigma_p)}, \quad \sigma_s = \frac{\pi^2 E}{(l/r)^2}$$

and

l is the length of stiffener between transverse frames, and

$r = (I_e/A_e)^{1/2}$ is the radius of gyration

- Torsional / flexural buckling (tripping)

a- Double symmetric section

The critical stress is:

$$\sigma_{ct} = \frac{1}{I_o}\left(GJ + \frac{\pi^2 E C_w}{l^2}\right) \qquad \text{if} \quad \sigma_{cr} \leq \sigma_p$$

$$\sigma_{cr} = \sigma_o\left[1 - \frac{\sigma_p\left(1 - \frac{\sigma_p}{\sigma_o}\right)}{\sigma_t}\right] \qquad \text{if} \quad \sigma_{cr} > \sigma_p, \quad \text{(A-3)}$$

where

$$\sigma_t = \frac{1}{I_o}\left(GJ + \frac{\pi E C_w{}^2}{l^2}\right). \qquad \text{(A-4)}$$

b- Section with one plane of symmetry

i) Elastic range $\sigma_{tfe} \leq \sigma_p$

σ_{tfe} is the smallest root of the following equation

$$\frac{I_c}{I_o}\sigma_{tfe}^{\,2} - \sigma_{tfe}(\sigma_{cr} + \sigma_t) + \sigma_{cr}\sigma_t = 0, \qquad \text{(A-5)}$$

where σ_{cr} is as given by eq. (A-1), and σ_t is as given by eq. (A-4).

ii) Plastic range $\sigma_{tfe} > \sigma_p$

$$\sigma_{tfe} = \sigma_o\left[1 - \frac{\sigma_p\left[1 - \dfrac{\sigma_p}{\sigma_o}\right]}{\sigma_{tfe}}\right].$$

Overall buckling of stiffened panel [15]:

The elastic buckling stress can be written in the form:

$$\sigma_{cr} = \sigma_o - \frac{1}{C_s} \qquad \text{for } \frac{A}{B} < 1 \quad \text{if } \sigma_{cr} > \sigma_p,$$

where

$$\sigma_{cr} = \frac{C_1\sigma_o}{C_1 + 1} \qquad \text{for } \frac{A}{B} \geq 1 \quad \text{if } \sigma_{cr} > \sigma_p, \qquad \text{(A-6)}$$

$$\sigma_{cr} = K\frac{\pi^2\sqrt{D_x D_y}}{t_e B^2} \qquad \text{if } \sigma_{cr} \leq \sigma.$$

where

$$K = 4 \qquad \text{for } \frac{A}{B} \geq 1$$

$$K = \left[\frac{1}{\rho^2} + 2\eta + \rho^2\right] \qquad \text{for } \frac{A}{B} < 1$$

$$C_1 = \frac{\sigma_1^{\,2}}{\sigma_p(\sigma_o - \sigma_p)}, \qquad \sigma_1 = \frac{4\pi^2\sqrt{D_x D_y}}{t_e B^2},$$

$$C_s = \frac{\sigma_s}{\sigma_p(\sigma_o - \sigma_p)},$$

$$\sigma_s = \left(\frac{1}{\rho^2} + 2\eta + \rho^2\right)\frac{\pi^2\sqrt{D_x D_y}}{t_e B^2},$$

$$\rho = \frac{A}{B}\left(\frac{D_y}{D_x}\right)^{1/4}, \qquad \eta = \sqrt{\frac{I_{px} I_{py}}{I_x I_y}},$$

$$D_x = \frac{EI_x}{S_y(1 - v^2)}, \text{ and } D_y = \frac{EI_y}{S_x(1 - v^2)}.$$

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