Determination of natural frequencies of clamped non-homogeneous trapezoidal plates

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Galerkin`s method has been applied to study the free transverse vibration of non-homogeneous clamped trapezoidal plates. The assumed deflection function is exactly satisfying the associated boundary conditions. The non-homogenity considered here is a sort of thickness variation. Three different cases , which are, the linear, the exponential and the parabolic thickness variations are considered. The present results for non-homogeneous clamped trapezoidal plates are entirely new and not available elsewhere. Comparisons can only be made for the special cases of non-homogeneous rectangular and homogeneous trapezoidal plates. In these special cases, the results are found to be in good agreement with those previously published in the available literature. A series of tabulated results , for each case of thickness variation is then presented.

يختص هذا البحث بدراسة الاهتزازات الحرة للالواح ذات الشكل الشبه منحرف و متغيرة السمك و المثبتة تثبيتا تاما على محيطها و ذلك بتطبيق طريقة جاليركين الرياضية. ثلاث حالات مختلفة من تغير السمك للوح و هى على الترتيب, التغير الخطى و التغير الاسى و التغير التربيعى تمت دراستها. و قد استهل البحث بمقدمة موجزة عما تم انجازه فى هذا المجال من دراسات سابقة ثم العرض المختصر للمعادلات الاساسية و الطريقة الرياضية المستخدمة ثم عرض النتائج. و تمت مقارنة النتائج التى تم التوصل اليها فى هذا البحث بنتائج الباحثين السابقين وذلك فى الحالات الخاصة المتاحة حيث ظهر التقارب الواضح بين المجموعتين. هذا وقد خلص البحث الى بعض النتائج الهامة منها ان الترددات الطبيعية للالواح موضوع الدراسة تزيد بزيادة زاوية شبه المنحرف و تقل بنقصانها , كذلك وجد انها تزيد بزيادة سمك اللوح و تقل بنقصانه.

Keywords: Free vibration, Non-homogeneous, Clamped, Trapezoidal plates

1. Introduction

The study of free vibration of non-homogeneous plates is very important in a wide variety of applications in engineering design such as that of space vehicles, modern missiles aircraft wings, etc. Although a certain amount of work has been done on free vibration of non-homogeneous rectangular plates, the authors have found that , there is a little amount of work related to the oscillatory motion of trapezoidal plates that have variable thickness, in particular, for those having clamped edges. The free lateral vibration of rectangular plates that have linear thickness variation has been studied by many researchers. Apple and Byers [1] determined the fundamental natural frequency coefficient for simply supported plates that have linear thickness variation in the span-wise direction. Ashton [2,3] applied the Rayleigh - Ritz method for determining both the natural frequencies and the natural modes, respectively, for clamped plates. Soni and Rao [4] used a spline technique method of solution. Filipich et al. [5] applied Galerkin`s method for the determination of the fundamental natural frequency coefficients of plates that have some different arrangements of boundary conditions. Ng and Araar[6] used Galerkin`s method to study the vibration and buckling of clamped plates. Sanzi et al. [7] applied four different mathematical methods to determine the frequency coefficients of plates that have some mixed boundary conditions. Kukretti et al. [8] used the differential quadrature method to study the free vibration of plates that have some different combinations of edge condition.

For rectangular plates which have parabolic thickness variation, Olson and

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Hazil[9] applied both theoretical and experimental methods to study the free vibration of clamped plates. In [10], the finite element was applied by Mukherjee and Mukhopandhyay to determine the natural frequencies of skew and curved plates that have parabolically varying thickness .They also presented the results for clamped rectangular plates.

The problem of rectangular plate that has exponential thickness variation was considered by Sonzogni et al. [11]. They applied both the finite element method and the optimized Kantrovich approach to determine the natural frequencies for the two cases of CCCF and CSSF boundary conditions. In [12], the finite element method was applied by the first author to determine the frequency coefficients for five, non-uniform thickness, regular polygonal plates that start from the triangular up to the heptagonal one. Both the linear and the exponential thickness variations were considered.

The free vibrations of homogeneous trapezoidal plates that have different arrangements of boundary conditions have been investigated by many researchers [13-17]. For non-homogeneous trapezoidal plates, there is a little amount of work. Laura et al. [18] applied the Rayleigh-Ritz method to study the oscillatory motion of cantilevered plates that have linear thickness variation in the span-wise direction. In [19], the finite element method was used by the first author to determine the frequency coefficients for both rectangular and cantilevered trapezoidal plates that have quadratic thickness variations.

In the present study, the problem of free vibration of non-homogeneous clamped trapezoidal plates is investigated. Galerkin`s method is applied. Three different cases of thickness variations, namely, the linear, the exponential and the parabolic thickness variations are considered. In each case, the thickness is taken to be varying along the x-axis, which is the span-wise direction of the plate. A detailed discussion on the problem formulation and the accuracy of results is presented.

2. Mathematical formulation

The plate middle surface is assumed to have a symmetric trapezoidal platform which, for $\theta = 0$, it becomes a rectangle and for certain values of the plate aspect ratio and the trapezoid angle θ, it tends to be an isosceles triangle. The plate geometry is shown in Fig. 1. In the following formulation, the x,y – coordinates and the deformations of the plate are non-dimensionalized by a characteristic length ($L = 2a$), which is the chord of the plate at its root ($x = 0$).

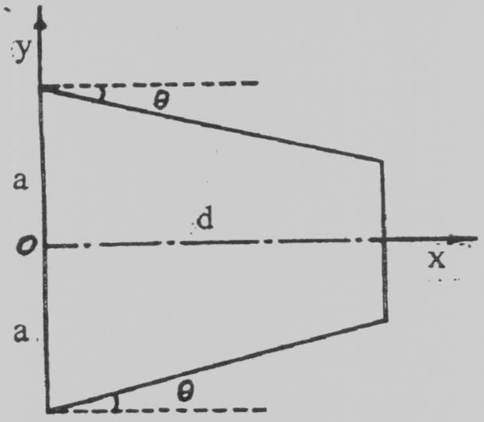

Fig. 1. The geometry of the plate.

The partial differential equation which governs the free oscillatory motion of isotropic plates, after the assumption of simple harmonic motion, is given by:

$$D[w_{xxxx}+2w_{xxyy}+w_{yyyy}]-\omega^2L^4\rho h.w = 0 , \quad (1)$$

where $w_{xxyy} = \partial^4 w/\partial x^2\partial y^2$, ω is the frequency of oscillations , ρ is the density of the plate material , h is the plate thickness and $D = Eh^3 / 12(1 - \nu^2)$ is the bending rigidity with E is the Young`s modulus of elasticity and $\nu$ is the Poisson`s ratio.

Three different cases, which are, the linear, the exponential and the quadratic thickness variations will be considered. The thickness of the plate is assumed to vary along the x-axis (the span-wise direction) as follows:

$$h = h_o f(x) \quad (2)$$

where h_o is the plate thickness at its base(x = 0) and $f(x)$ is the function that represents the thickness variation. The function $f(x)$ will be taken as follows:

$$f(x) = 1 + \alpha x \quad \text{for linear thickness variation} \tag{3-a}$$

$$f(x) = e^{\alpha x} \quad \text{for exponential thickness variation} \tag{3-b}$$

$$f(x) = 1 + \alpha x + \beta x^2 \quad \text{for quadratic thickness variation,} \tag{3-c}$$

where α and β are the parameters which govern the thickness variation.
The substitution from eq(2) into eq (1) gives

$$[f(x)]^2[\, w_{xxxx} + 2w_{xxyy} + w_{yyyy} \,] - \lambda^2 w = 0, \tag{4}$$

where $\lambda = \omega L^2 \sqrt{\rho h_o / D_o}$ is the non-dimensional natural frequency coefficient with $D_o = D(h_o/h)^3$.

The solution of eq (4) will be assumed in the following form:

$$w(x,y) = \sum_{n=1}^{N} A_n \varphi_n(x,y) = G(x,y) \sum_{n=1}^{N} A_n \psi_n(x,y), \tag{5}$$

where G(x,y) is the part of the deflection function that satisfies the essential boundary conditions. In the case of a clamped edge, the associated boundary conditions are w = 0 and $\partial w/\partial n = 0$, where n is the direction normal to the edge. The function G(x,y) satisfies both these two kinematical boundary conditions. The explicit expression for this function is given by:

$$G(x,y)=x^2(x -\gamma)^2(y + \mu x - 0.5)^2(y - \mu x +0.5)^2, \tag{6}$$

where $\mu = \tan\theta$ and γ is the plate aspect ratio($\gamma = d/L$).

The function $\psi_n(x,y)$ represents the distribution of the deflection w(x,y) over the domain of the plate with respect to that of the point($\gamma/2$,0). It is constructed by following Laura et al. [20] with the modifications concerning the origin location.

$$\psi_n(x,y)=(x-b)^{2(n-1)}+y^{2(n-1)} \quad n = 1,2,\ldots,N, \tag{7}$$

where $b = \gamma/2$.

After carrying out the multiplication of the function G(x,y), it is then expressed in the following form:

$$G(x,y) = \sum_{i=1}^{15} a_i \, x^{m_i} \, y^{n_i}, \tag{8}$$

where the a_i's are constants which depend on the parameters μ and γ , m_i ,n_i are the powers of x and y, respectively, in the i th term. m_i takes the values 8,7,6, ...,2 and n_i = 0,2,4. The explicit form of G(x,y) will be given in the appendix.

The substitution of the solution (5) into eq.(4) results in the error function which is known as the generic residual ε (x,y). For example, for N=3 , it will take the following form:

$$\begin{aligned}\varepsilon(x,y)= & [f(x)]^2[A_1(F_1+2G_1+H_1) + A_2(F_2+ 2G_2 +H_2) \\ & + A_3(F_3 + 2G_3 + H_3)] \\ & -\lambda^2\{A_1.2G(x,y)+A_2.G(x,y)[(x-b)^2+y^2]+A_3 \\ & G(x,y)[(x-b)^4+y^4]\},\end{aligned} \tag{9}$$

where F_i,G_i,H_i, i = 1,2,3, are functions of x,y , which are obtained , for N = 3 ,from the following relations:

$$w_{xxxx} = \sum_{i=1}^{3} A_i F_i(x,y) \tag{10-a}$$

$$w_{xxyy} = \sum_{i=1}^{3} A_i G_i(x,y) \tag{10-b}$$

$$w_{yyyy} = \sum_{i=1}^{3} A_i H_i(x,y) \tag{10-c}$$

The explicit expressions of these elements of $\varepsilon(x,y)$, for N = 3, as an example, are given in the appendix.

According to Galerkin's method, it is required that the residual $\varepsilon(x,y)$ to be orthogonal to each of the deflection functions $\varphi_n(x,y)$ over the area A of the plate. i.e.

$$\iint_A \varepsilon(x,y)\,\varphi_n(x,y)\,dA = 0 \quad n = 1,2,..,N. \qquad (11)$$

Substituting from Eq (5) and (9) into Eq (11), a number N of characteristic equations is obtained from which the natural frequency coefficient λ is determined.

The substitution from Eq (5) and (9) into Eq (11) results in integrals of the following type:

$$I = \iint_A x^p y^q dA = \int_0^{\gamma} \int_{-(0.5-\mu x)}^{(0.5-\mu x)} x^p y^q \, dxdy. \qquad (12)$$

The evaluation of such integral gives the following:

$$I = 0 \qquad \text{for odd number,} \qquad (13-a)$$

and

$$I = \int_0^{\gamma} [x^p/2^q(q+1)]\,(1 - 2\mu x)^{(q+1)}dx \qquad \text{for q even.} \qquad (13-b)$$

The term $(1-2\mu x)^{q+1}$ is expressed in its expanded form and the integral I is given by the following algorithm:

$$I = f(\mu,\gamma,p,q) \;/\; 2^q(q+1), \qquad (14)$$

Where;

$$f(\mu,\gamma,p,q) = \gamma^{(p+1)}/(p+1) - 2\mu(q+1)\gamma^{(p+2)}/(p+2) + (2\mu)^2.(q+1).q.\gamma^{(p+3)}/2!.(p+3) - -(2\mu)^3(q+1).q.(q-1).\gamma^{(p+4)}/3!.(p+4) + ...$$

It must be mentioned that the number of the non-zero terms in the expansion given in Eq (14), that correspond to a certain value q is (q+2). A computer program is constructed to determine such integrals and the resulting characteristic equations are solved to determine the natural frequency coefficients.

3. Numerical examples and discussion

To check the convergence of the solutions, calculations which correspond to three successive values of N (N=1,2,3) are performed. The fundamental natural frequency coefficients for tapered clamped square plates, as a test case, are given in Table. 1. Five different values of the thickness variation parameter α are considered. For all calculations, the Poisson`s ratio is taken to be 0.3. The results are compared with most of those given in the available literature. Trials show that good convergence and acceptable accuracy could be obtained for N=2, since the maximum percentage difference between the present results and those obtained by other different mathematical methods is about 1%.

Tables. 2-4, indicate the variation of the natural frequency coefficients with both the angle of the trapezoid θ and the thickness variation parameter α, for clamped trapezoidal plates which have linear thickness variations. The plate angle θ varies from 0° (rectangular or square plate) to 25° or 15° (according to the aspect ratio) at an interval of 5°. The parameter α is allowed to vary from 0.5 to -0.5 ,at a step of 0.1, to study the effect of both thickness increasing and thickness reduction, along the span of the plate, on the frequency coefficients. Three different values of the aspect ratio γ are considered. In Table. 2., where γ =1 , the values of the fundamental natural frequency coefficient λ_1 , which correspond to θ = 0 (square plate) are compared with those given in [12] by using the finite element method . The results of Ref. [12] are given between brackets. The agreement between the two sets of results is found to be acceptable and the maximum percentage difference between them is about 3% , which is the solution for α = - 0.5. It is also found that, for any value of θ, the fundamental natural frequency λ_1 increases with the increase of the thickness variation parameter α and for any value of α in its range of variation, the value of λ_1 increases with the increase of θ. Such variation of λ_1 with both of θ and α is nearly the same for each of the three different aspect ratios.

In the case of exponential thickness variation, the function $f(x) = e^{\alpha x}$ which is given in Eq (3-b) is expressed in a Maclaurin expansion form and calculations are performed by considering the first five terms of the expansion. The results for the trapezoidal plates that have exponential thickness variation are given in Tables. 5-7. In Table. 5, where $\gamma = 1$, the present results for the square plate ($\theta = 0$) are compared with those given in [12] by using the finite element method. A wide range of variation of the tip to root thickness ratio (h_1/h_o) is covered. The thickness variation parameter α is related to h_1/h_o, according to Eq(3-b), as follows: $\alpha = (1/\gamma)\ln(h_1/h_o)$. As could be shown, the results are in good agreement with those given in [12]. The maximum percentage difference between these two sets of results is about 1.8%. For trapezoidal plates ($\theta \neq 0$), most of the results presented here are new in literature and can not be compared as no other results are available. The fundamental natural frequency coefficient λ_1 is calculated for the same range of variation of θ as in the case of linear thickness variation. It could be concluded that, the same behavior of λ_1 with both α and θ, for each of the three different aspect ratios is nearly the same as that of the linear thickness variation.

The solutions for the case of the quadratic thickness variation are presented in Table. 8-10. The values of the parameters α and β, which govern the parabolic thickness variation of the plate can not be arbitrary chosen. They are related to two other parameters: the aspect ratio γ and the tip to root thickness ratio h_1/h_o. According to Eq (2) and (3-c):

$$\alpha x + \beta x^2 = h/h_o - 1.$$

If $h = h_1$(where h_1 is the thickness at the tip of the plate), then $x = x_1 = \gamma$ and the relation between α and β will be given by:

$$\alpha\gamma + \beta\gamma^2 = h_1/h_o - 1.$$

For assumed values of γ and h_1/h_o, the two parameters α and β will be linearly dependant and then, for any arbitrary value of one of them, the other can be determined. In table.8, the results for clamped square plate ($\gamma = 1$ and $\theta = 0$) are presented and compared with those given in [12].The agreement between the two sets of results is acceptable since the maximum percentage difference between them is about 3%. As in the two preceding cases of thickness variation ,for trapezoidal plates that have quadratic thickness variation, decreasing the thickness of the plate along the span results in corresponding decrease of the natural frequency coefficients. Also, the effect of variation of the trapezoid angle θ on the frequency coefficient is the same as that of the two preceding cases of thickness variations.

The effects of the parameters α and β, which govern the parabolic thickness variation, on the fundamental natural frequency coefficient λ_1, could be explained as follows: The parameter α represents the linear thickness variation of the plate while the parameter β governs its quadratic thickness variation. For any value of $h_1/h_o < 1$, and $\gamma = 1$, the two parameters are dependant according to the relation:

$$\alpha + \beta = h_1/h_o - 1.$$

If $\beta = 0$, the variation of the thickness is purely linear and the thickness of the plate at any position x is determined from the relation $h = h_o[1-(1-h_1/h_o)x]$. If $\alpha = 0$, the thickness of the plate along the span is given by $h = h_o[1-(1-h_1/h_o)x^2]$. The term $1-h_1/h_o$ represents the reduction of the thickness in one side of the plate middle surface along the span. Since the aspect ratio is unity, then the non-dimensionalized coordinate x is governed by the inequality $x \leq 1$ and hence, $x^2 \leq x$. In accordane, the reduction of the thickness in the case of its linear variation is larger than that corresponding to its pure quadratic variation. Since the rigidity D of the plate is a function of its thickness, then the case when $\beta = 0$ will result in a value of the rigidity which is less than that corresponding to $\alpha = 0$. Therefore, the value of λ_1 for any ratio of $h_1/h_o < 1$ and $\beta = 0$, is expected to be less than that corresponding to the same value of h_1/h_o and $\alpha = 0$. As an example , for $\gamma = 1$, $h_1/h_o = 0.5$, $\beta = 0$,$\alpha = -0.5$, the value of λ_1 is 26.94 and for the same values of γ and h_1/h_o , but $\alpha = 0$, $\beta = -0.5$, the value of λ_1 is 30.16.

5. Conclusions

The free transverse vibration of non-homogeneous trapezoidal plates has been analyzed by using Galerkin's method. Three different cases, which are, the linear, the exponential and the quadratic thickness variations are considered. It is possible from the preceding study to draw the following conclusions:

1. The suggested deflection function is exactly satisfying the boundary conditions which associate the edges of the models.
2. The convergence and the accuracy of the present solutions are demonstrated through comparisons with most of the available results which are obtained by different mathematical methods.
3. The effects of variation of both the trapezoid angle θ and the thickness variation parameters α and β, for plates of three different aspect ratios, on the frequency coefficients have been investigated. The fundamental natural frequency coefficient that corresponds to certain values of the aspect ratio γ and the thickness variation parameters α and β is found to be monotonically increasing with the increase of the trapezoid angle θ . It is also found that , for certain values of γ and θ , the increase of the thickness along the span of the plate leads to a corresponding increase in the value of λ_1.

Table. 1
Convergence and comparison of results for tapered square plates

Ref.	α				
	0.4	0.2	0.0	- 0.2	- 0.4
Present: N=1	43.52	40.12	35.99	32.71	29.02
N=2	43.25	39.61	35.99	32.42	28.87
N=3	43.29	39.68	35.99	32.48	28.92
[2]	42.93	39.52	36.01	-	-
[7]	42.91	39.51	35.99	-	-
[8]	42.94	39.55	36.01	-	-
[12]	42.91	39.51	36.00	32.30	28.38

Table. 2
Results for trapezoidal plates of linearly varying thickness ($\gamma = 1$)

α	θ					
	0.0	5	10	15	20	25
0.5	45.07(45.52)	48.81	54.94	64.32	77.17	93.50
0.4	43.25(42.91)	47.03	53.16	62.44	75.12	91.23
0.3	41.43(41.21)	45.26	51.38	60.65	73.05	88.97
0.2	39.61(39.51)	43.49	49.60	58.67	70.99	86.71
0.1	37.80(37.77)	41.73	47.82	56.79	68.93	84.45
0.0	35.99(36.00)	39.97	46.05	54.91	66.86	82.19
- 0.1	34.20(34.17)	38.22	44.28	53.03	64.80	79.93
- 0.2	32.42(32.30)	36.48	42.52	51.15	62.73	77.67
- 0.3	30.64(30.37)	34.74	40.76	49.26	60.67	75.41
- 0.4	28.87(28.38)	33.02	39.00	47.38	58.61	73.15
- 05	27.11(26.30)	31.30	37.25	45.50	56.55	70.91

Table. 3
Results of trapezoidal plates of linearly varying thickness (γ = 0.75)

α	θ					
	0	5	10	15	20	25
0.5	60.72	62.60	66.69	74.03	85.50	101.5
0.4	58.78	60.83	65.06	72.45	83.83	99.61
0.3	56.85	59.07	63.43	70.86	82.17	97.74
0.2	54.92	57.32	61.81	69.28	80.49	95.88
0.1	52.99	55.57	60.18	67.69	78.81	94.01
0.0	51.08	53.82	58.56	66.10	77.13	92.14
-0.1	49.17	52.08	56.94	64.51	75.44	90.26
-0.2	47.26	50.34	55.33	62.91	73.75	88.39
-0.3	45.36	48.62	53.71	61.31	72.05	86.50
-0.4	43.48	46.90	52.10	59.71	70.35	84.62
-0.5	41.60	45.18	50.49	58.10	68.64	82.73

Table. 4
Results for trapezoidal plates of linearly varying thickness (γ = 1.5)

α	θ			
	0.0	5	10	15
0.5	37.28	45.18	55.58	67.43
0.4	35.23	42.87	53.04	64.81
0.3	33.17	40.57	50.50	62.17
0.2	31.12	38.27	47.96	59.50
0.1	29.08	35.98	45.42	56.81
0.0	27.05	33.71	42.88	54.10
- 0.1	25.02	31.44	40.33	51.37
- 0.2	23.01	29.19	37.80	48.62
- 0.3	21.02	26.96	35.26	45.86
- 0.4	19.05	24.74	32.74	43.09
- 0.5	17.11	22.55	30.23	40.33

Table. 5
Results for trapezoidal plates of exponentially varying thickness (α = 1)

H_1/h_o	θ					
	0	5	10	15	20	25
0.5	27.52 (27.67)*	29.74	35.45	43.41	54.09	68.05
0.6	28.03 (28.29)	32.09	37.94	46.15	57.17	71.48
0.7	30.19 (30.13)	34.25	40.20	48.62	59.92	74.54
0.8	32.23 (32.18)	36.27	42.29	50.88	62.42	77.31
0.9	34.16 (34.11)	38.17	44.23	52.97	64.73	79.84
1.0	35.99 (36.00)	39.97	46.05	54.91	66.86	82.19
1.2	39.46 (39.41)	43.32	49.41	58.46	70.74	86.42
1.4	42.69 (42.53)	46.42	52.47	61.66	74.21	90.19
1.6	45.72 (45.39)	49.31	55.30	64.58	77.36	93.60
1.8	48.59 (48.01)	52.04	57.94	67.28	80.25	96.71
2.0	51.33 (50.43)	54.62	60.43	69.79	82.94	99.59

Table. 6
Results for trapezoidal plates of exponentially varying thickness (γ = 0.75)

H1/ho	θ					
	0.0	5	10	15	20	25
0.5	36.58	40.27	45.53	52.88	62.90	76.28
0.6	39.81	43.37	48.61	56.08	66.39	80.18
0.7	42.86	46.24	51.40	58.94	69.48	83.62
0.8	45.74	48.92	53.96	61.53	72.26	86.72
0.9	48.47	51.44	56.34	63.90	74.80	89.54
1.0	51.08	53.82	58.56	66.10	77.13	92.14
1.2	55.99	58.27	62.66	70.08	81.32	96.79
1.4	60.58	62.37	66.38	73.63	85.02	100.9
1.6	64.91	66.22	69.81	76.84	88.32	104.6
1.8	69.02	69.84	73.01	79.79	91.33	107.9
2.0	72.94	73.28	76.02	82.52	94.08	110.9

Table. 7
Results for trapezoidal plates of exponentially varying thickness (γ = 1.5)

H1/ho	θ			
	0.0	5	10	15
0.5	19.32	24.90	32.76	42.92
0.6	21.05	26.91	35.12	45.59
0.7	22.68	28.77	37.28	48.00
0.8	24.21	30.52	39.27	50.20
0.9	25.66	32.16	41.13	52.22
1.0	27.05	33.71	42.88	54.10
1.2	29.65	36.60	46.10	57.51
1.4	32.06	39.28	49.03	60.56
1.6	34.34	41.79	51.74	63.32
1.8	36.49	44.15	54.26	65.84
2.0	38.54	46.38	56.64	68.17

Table. 8
Results for trapezoidal plates of quadratic thickness variation (γ = 1)

h1/ho	α	β	θ			
			0	5	15	25
1.0	0.0	0.0	35.99(36.00)	39.97	54.91	82.19
	0.0	- 0.5	30.16(29.04)	35.87	51.43	78.85
0.5	- 0.25	- 0.25	28.19(27.66)	33.59	48.47	74.87
	- 0.5	0.0	26.94(26.29)	31.30	45.50	70.91
	0.0	- 0.25	33.10(32.71)	37.90	53.17	80.51
0.75	- 0.125	- 0.125	32.26(32.03)	36.76	51.69	78.53
	- 0.25	0.0	31.42(31.34)	35.61	50.20	76.54
	0.0	1.0	45.94	48.61	61.95	88.92
2.0	0.5	0.5	50.01	53.19	67.84	96.85
	1.0	0.0	54.23	57.76	73.70	104.8

Table 9
Results for trapezoidal plates of parabolic thickness variation ($\gamma = 0.75$)

H1/ho	α	β	θ			
			0	5	15	25
1.0	0.0	0.0	51.08	53.82	66.10	92.14
	0.0	- 8/9	44.29	48.41	62.51	88.74
0.5	- 1/3	- 4/9	41.39	45.38	58.96	84.16
	- 2/3	0.0	38.50	42.35	55.41	79.58
	0.0	16/9	65.41	65.58	73.11	98.64
2.0	2/3	8/9	71.19	71.36	80.15	107.8
	4/3	0.0	76.99	77.45	87.17	116.9

Table. 10
Results for trapezoidal plates of parabolic thickness variation ($\gamma = 1.5$)

H1/ho	α	β	θ		
			0	5	15
1.0	0.0	0.0	27.04	33.71	54.10
	0.0	- 2/9	23.50	30.09	50.63
0.5	- 1/6	- 1/9	21.93	28.15	47.79
	- 1/3	0.0	20.36	26.21	44.94
	0.0	4/9	34.44	41.31	60.97
2.0	1/3	2/9	37.58	45.18	66.38
	2/3	0.0	40.72	49.04	71.72

Appendix

The explicit forms of the elements of the residual function $\varepsilon(x,y)$ which is given in eqn. (9), for N=3 , are given by:

$G(x,y) = a_1x^8 + a_2x^7 + a_3x^6y^2 + a_4x^6 + a_5x^5y^2 + a_6x^5 + a_7x^4y^4 + a_8x^4y^2 + a_9x^4 + a_{10}x^3y^4 + a_{11}x^3y^2 + a_{12}x^3 + a_{13}x^2y^4 + a_{14}x^2y^2 + a_{15}x^2$

Where:

$a_1 = \mu^4$ $\quad a_2 = -2\mu^3 - 2\gamma\mu^4$ $\quad a_3 = -2\mu^2$ $\quad a_4 = 1.5\,\mu^2 + 4\gamma\mu^3 + \gamma^2\mu^4$.

$a_5 = 2\mu + 4\gamma\mu^2$. $a_6 = -0.5\,\mu - 3\gamma\mu^2 - 2\gamma^2\mu^3$. $a_7 = 1$

$a_8 = -0.5\,\mu - 4\gamma\mu - 2\gamma^2\mu^2$. $\quad a_9 = (1/16) + \gamma\mu + 1.5\gamma^2\mu^2$. $a_{10} = -\gamma$.

$a_{11} = \gamma + 2\gamma^2\mu$., $a_{12} = -1/8\gamma - 0.5\gamma^2\mu$., $a_{13} = \gamma^2$. $a_{14} = -0.5\,\gamma^2$., $a_{15} = (1/16)\,\gamma^2$.

$F_1 = 3360a_1x^4 + 1680a_2x^3 + 720\,a_3x^2y^2 + 720\,a_4x^2 + 240\,a_5xy^2 + 240\,a_6x + 48\,a_7y^4 + 48a_8y^2 + 48\,a_9$.

$G_1 = 120\,a_3x^4 + 80a_5x^3 + 288a_7x^2y^2 + 48a_8x^2 + 144a_{10}xy^2 + 24a_{11}x + 48a_{13}y^2 + 8a_{14}$.

$H_1 = 48a_7x^4 + 48a_{10}x^3 + 48a_{13}x^2$

$F_2 = e_1x^6 + e_2x^5 + e_3x^4y^2 + e_4x^4y^2 + e_5x^3y^2 + e_6x^3 + e_7x^2y^4 + e_8x^2y + e_9x^2 + e_{10}xy^4 + e_{11}xy^2 + e_{12}x + e_{13}y^6 + e_{14}y^4 + e_{15}y^2 + e_{16}$.

where;

$e_1 = 5040\,a_1$, $\quad e_2 = 3024\,(a_2 - 2a_1b)$, $\quad e_3 = 1680\,(a_1 + a_3)$, $\quad e_4 = 1680\,(a_4 - 2a_2b + a_1b^2)$

$e_5 = 840\,(a_2 + a_5 - 2a_3b)$., $e_6 = 840\,(a_6 - 2a_4b + a_2b^2)$, $e_7 = 360\,(a_3 + a_7)$

$e_8 = 360\,(a_8 + a_4 - 2a_5b + a_3b^2)$. $e_9 = 360\,(a_9 - 2a_6b + a_4b^2)$. $\quad e_{10} = 120\,(a_{10} + a_5 - 2a_7b)$.

$e_{11} = 120(a_{11} + a_6 - 2a_8b + a_5b^2)$. $\quad e_{12} = 120\,(a_{12} - 2a_9b + a_6b^2)$. $e_{13} = 24\,a_7$.

$e_{14} = 24(a_{13} + a_8 - 2a_{10}b + a_7b^2)$. $\quad e_{15} = 24(a_{14} + a_9 - 2a_{11}b + a_8b^2)$, $e_{16} = 24(a_{15} - 2a_{12}b + a_9b^2)$.

$G_2 = g_1x^6 + g_2x^5 + g_3x^4y^2 + g_4x^4 + g_5x^3y^2 + g_6x^3 + g_7x^2y^4 + g_8x^2y^2 + g_9x^2 + g_{10}xy^4 + g_{11}xy^2 + g_{12}x + g_{13}y^4 + g_{14}y2 + g_{15}$.

Where:

$g_1 = 112(a_1 + a_3 \quad g_2 = 84\,(a_2 + a_5 - 2a_3b). \quad g_3 = 360\,(a_3 + a_7), g_4 = 60(a_4 + a_8 - 2a_5b + a_3b^2).$

$g_5 = 240(a_5 + a_{10} - 2a_7b), g_6 = 40(a_6 + a_{11} - 2a_8b + a_5b^2), g_7 = 360\,a_7.$

$g_8 = 144(a_8 + a_{13} - 2a_{10}b + a_7b^2), g_9 = 24(a_9 + a_{14} - 2a_{11}b + a_8b^2), g_{10} = 180\,a_{10}.$

$g_{11} = 72(a_{11} - 2a_{13}b + a_{10}b^2). g_{12} = 12(a_{12} - 2a_{14}b + a_{11}b^2), g_{13} = 60\,a_{13}.$

$g_{14} = 24(a_{14} + a_{13}b^2), g_{15} = 4(a_{15} + a_{14}b^2).$

$H_2 = h_1x^6 + h_2x^5 + h_3x^4y^2 + h_4x^4 + h_5x^3y^2 + h_6x^3 + h_7x^2y^2 + h_8x^2.$

where

$h_1 = 24(a_3 + a_7), h_2 = 24(a_5 + a_{10} - 2a_7b), h_3 = 360\,a_7, h_4 = 24(a_8 + a_{13} - 2a_{10}b + a_7b^2).$

$h_5 = 360\,a_{10}, h_6 = 24(a_{11} - 2a_{13}b + a_{10}b^2), h_7 = 360\,a_{13}, h_8 = 24(a_{14} + a_{13}b^2).$

$F_3 = f_1x^8 + f_2x^7 + f_3x^6y^2 + f_4x^6 + f_5x^5y^2 + f_6x^5 + f_7x^4y^4 + f_8x^4y^2 + f_9x^4 + f_{10}x^3y^4 + f_{11}x^3y^2 + f_{12}x^3 + f_{13}x^2y^6 + f_{14}x^2y^4 + f_{15}x^2y^2 + f_{16}x^2 + f_{17}xy^6 + f_{18}xy^4 + f_{19}\,xy^2 + f_{20}x + f_{21}y^8 + f_{22}y^6 + f_{23}y^4 + f_{24}y^2 + f_{25}.$

where:

$f_1 = 11880a_1, f_2 = 7920(a_2 - 4a_1b), f_3 = 5040a_3, f_4 = 5040(a_4 - 4a_2b + 6a_1b^2), f_5 = 3024(a_5 - 4a_3b),$

$f_6 = 3024(a_6 - 4a_4b + 6a_2b^2 - 4a_1b^3). f_7 = 1680(a_1 + a_7)., f_8 = 1680(a_8 - 4a_5b + 6a_3b^2).$

$f_9 = 1680(a_9 - 4a_6b + 6a_4b^2 - 4a_2b^2 + a_1b^4), f_{10} = 840(a_{10} + a_2 - 4a_7b).$

$f_{11} = 840(a_{11} - 4a_8b + 6a_5b^2 - 4a_3b^3), f_{12} = 840(a_{12} - 4a_9b + 6a_6b^2 - 4a_4b^3 + a_2b^4).$

$f_{13} = 360\,a_3, f_{14} = 360(a_{13} - 4a_{10}b + 6a_7b^2 + a_4), f_{15} = 360(a_{14} - 4a_{11}b + 6a_8b^2 - 4a_5b^3 + a_3b^4).$

$f_{16} = 360(a_{15} - 4a_{12}b + 6a_9b^2 - 4a_6b^3 + a_4b^4), f_{17} = 120a_5, f_{18} = 120(a_6 - 4a_{13}b + 6a_{10}b^2 - 4a_7b^3).$

$f_{19} = 120(-4a_{14}b + 6a_{11}b^2 - 4a_8b^3 + a_5b^4), f_{20} = 120(-44a_{15}b + 6a_{12}b^2 - 4a_9b^3 + a_6b^4).$

$f_{21} = 24a_7, f_{22} = 24a_8, f_{23} = 24(a_9 + 6a_{13}b^2 - 4a_{10}b^3 + a_7b^4), f_{24} = 24(6a_{14}b^2 - 4a_{11}b^3 + a_8b^4).$

$f_{25} = 24(6a_{15}b^2 - 4a_{12}b^3 + a_9b^4).$

$G_3 = q_1x^8 + q_2x^7 + q_3x^6y^2 + q_4x^6 + q_5x^5y^2 + q_6x^5 + q_7x^4y^4 + q_8x^4y^2 + q_9x^4 + q_{10}x^3y^4 + q_{11}x^3y^2 + q_{12}x^3 + q_{13}x^2y^6 + q_{14}x^2y^4 + q_{15}x^2y^2 + q_{16}\,x^2 + q_{17}xy^6 + q_{16}x^2 + q_{18}xy^4 + q_{19}xy^2 + q_{20}x + q_{21}y^6 + q_{22}y^4 + q_{23}y^2 + q_{24}.$

where:

$q_1 = 180\,a_3, q_2 = 144(a_5 - 4a_3b), q_3 = 672(a_1 + a_7), q_4 = 112(a_8 - 4a_5b + 6a_3b^2).$

$q_5 = 504(a_2 + a_{10} - 4a_7b), q_6 = 84(a_{11} - 4a_8b + 6a_5b^2 - 4a_3b^3), q_7 = 900\,a_3.$

$q_8 = 360(a_4 + a_{13} - 4a_{10}b + 6a_7b^2), q_9 = 60(a_{14} - 4a_{11}b + 6a_8b^2 - 4a_5b^3 + a_3b^4), q_{10} = 600\,a_5.$

$q_{11} = 240(a_6 - 4a_{13}b + 6a_{10}b^2 - 4a_7b^3), q_{12} = 40(-4a_{14}b + 6a_{11}b^2 - 4a_8b^3 + a_5b^4), q_{13} = 672\,a_7.$

$q_{14} = 360\,a_8, q_{15} = 144(a_9 + 6a_{13}b^2 - 4a_{10}b^3 - a_7b^4), q_{16} = 24(6a_{14}\,b_2 - 4a_{11}b^3 + a_5b^4), q_{17} = 336\,a_{10}.$

$q_{18} = 180\,a_{11}, q_{19} = 72(a12 - 6a13b^3 + a_{10}b^4), q_{20} = 12(-4a_{14}b^3 + a_{11}b^4), q_{21} = 112\,a_{13}.$

$q_{22} = 60\,a_{14}, q_{23} = 24(a_{15} + a_{13}b^4), q_{24} = a_{14}\,b^4.$

$H_3 = t_1x^8 + t_2x^7 + t_3x^6y^2 + t_4x^6 + t_5x^5y^2 + t_6x^5 + t_7x^4y^4 + t_8x^4y^2 + t_9x^4 + t_{10}x^3y^4 + t_{11}x^3y^2 + t_{12}x^3 + t_{13}x^2y^4 + t_{14}x^2y^2 + t_{15}x^2.$

where:

$t_1 = 24(a_1 + a_7, t_2 = 24(a_2 + a_{10} - 4a_7b), t_3 = 360\,a_3, t_4 = 24(a_4 + a_{13} - 4a_{10}b + 6a_7b^2).$

$t_5 = 360\,a_5, t_6 = 24(a_6 - 4a_{13}b + 6a_{10}b^2 - 4a_7b^3), t_7 = 1680\,a_7, t_8 = 360\,a_8.$

$t_9 = 24(a_9 + 6a_{13}b^2 - 4a_{10}b^3 + a_7b^4), \quad t_{10} = 1680\,a_{10}, t_{11} = 360\,a_{11}.$

$t_{12} = 24(a_{12} - 4a_{13}b^3 + a_{10}b^4), t_{13} = 1680\,a_{13}, t_{14} = 360\,a_{14}, t_{15} = 24(a_{15} + a_{13}b^4).$

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