A three dimensional mathematical model of temporomandibular joint loading during maximum unilateral occlusion

Maha Moustafa Halim [a] and Khaled T. Mohamed [b]
[a] *Proth. Dept, Faculty of Dentists, Tanta University.*
[b] *Mech. Eng. Dept., Faculty of Engineering, Alexandria University.*

Since the clinical studies have indicated that trempromandibular joint (TMJ) dysfunction is correlated to dentoskeletal abnormalities such as mandibular asymmetries, thus it is thought that excessive joint forces may be a result of these effects. The idea that mechanical forces is a primary source of many clinical problems leads to the essence of examination of parameters such as the occlusal loading position within the arch, the occlusal angle and the occlusal load upon resultant joint loads. The main objective of this work is to introduce a generalized biomechanical model that can predict the musculoskeletal forces at the mandible during maximum unilateral occlusion at the first, second and third mandibular molar regions.

مفصل الفك البشري أثناء أقصى إطباق أحادي. هذه الدراسة تتناول حالات الإطباق على كل من الضرس الأول و الثاني و الثالث. استخدم الغرض الأساسي من هذه الدراسة هو تقديم نموذج عام، باستخدام الميكانيكا الحيوية، للتنبؤ بالقوى الناتجة في العضلات التي تتحكم في حركة الباحثان قوانين نيوتن للاتزان الاستاتيكي للحصول على معادلات الاتزان الستة لمفصل الفك البشري. هذه المعادلات الستة تحتوي على ثمانية عشرة مجهولا، ألا و هي ردود الأفعال عند المفصلين (ستة)، بالإضافة إلى القوى الناشئة في العضلات (اثنا عشرة)، و لذلك يجب استخدام إحدى طرق الاختيار الأفضل للحصول على قيم هذه المجاهيل، باستخدام طريقة minimax و تطبيقها على محصلتي رد الفعل عند المفصلين تم الحصول على نتائج مرضية عند المقارنة بنتائج الدراسات السابقة. و من أهم هذه النتائج التي تم التوصل إليها: ١-أوضح البحث أكثر العضلات إجهادا أثناء هذه النوعية من التحميل. ٢ - أثبت البحث قدرة النموذج المقترح على التنبؤ بالقوى المؤثرة على المفصل و العضلات في الحالات الطبيعية، كم أنه يمكن استخدامه في الحالات المرضية بإدخال بعض التعديلات البسيطة.

Key words: Maximum occlusio ,Unilateral occlusion, Temporomandibular joint, Forces Muscle force

1. Introduction

Precise values of forces within the human trempromandibular joint during normal function are unknown. Direct measurements of such forces have been quantitative and confined to animal experiments [1]. These experiments required invasive techniques not applicable to human subjects. Theoretically, human temporomandibular joint forces could be calculated from noninvasive measurements if the bite and muscle forces were known precisely. previous models used to calculate trempromandibular joint forces have usually been assumed that precise values for the magnitude, directions and moment arm lengths of the bite and muscle forces are known [2]. However, considerable uncertainty exists for all of these parameters except bite force magnitude and position. Muscle force magnitudes have been estimated from either cross sectional areas of each muscle [3, 4] or integrated electromyography [4,5]. However, there is no experimental correlation of either of these methods to actual force generated by jaw muscles. In 1992, Richard et al. [6] introduced a three-dimensional finite element model of the mandible. They included muscle loading based on an algorithm that assigns muscle forces in accordance with the muscle cross sectional area during static equilibrium. They studied the cases of unilateral and bilateral bite on the incisors and the second premolar with relatively small loads (40N-100N). The results did not show the force

applied by each muscle which is very necessary to know as stated previously. In the same year Osborn J.W. and Baragar F.A. [7], introduced a very good three dimensional model of the tempromandibular joint in which each condyle was modeled as a surface composed of 12 flat facets. The model also, included the tensions of 24 independent muscle elements. They used the optimization technique to find the reaction force and its direction. However, no values were introduced for the muscle forces since the study was, mainly, constructed to explain the effect of the variation of the articular surface shape on the joint force directions. The previous literature review revealed that up till now there is no generalized biomechanical model that can predict the musculoskeletal forces at the mandible region taking the pre-mentioned parameters into consideration. Therefore, the main objective of this paper is to introduce such a model. This model can be used to analyze load sharing between the muscles and the bones of the TMJ at any position under any loading condition. The model was applied to predict the musculoskeletal forces during unilateral occlusion at the first, the second and the third molars. Six static equilibrium equations were obtained using Newton's laws.
Twelve muscle groups and the two temporomandibular joints were considered in the analysis. The problem is statically indeterminate since we have eighteen unknowns (twelve muscle forces and three reactions at each joint) and only six equilibrium equations. The Matlab optimization toolbox was utilized in computing the musculoskeletal forces. Many objective functions and optimization techniques were tested to solve the problem. The results showed a good agreement with the previous researches and the physiological facts. The results indicated that the balancing side condyle is more heavily loaded during unilateral occlusion. Also, the effects of geometric abnormalities, upon the TMJ and muscle forces, were analyzed. The model can also be used to predict the musculoskeletal forces during bilateral occlusion.

2. Material and method

In this model the mandible and the skull are treated as three-dimensional rigid bodies. They are in contact at the bite location and at both condyles. Twelve muscle groups are included in the model, as shown in Table 1. They are considered as stretched strings following the shortest path between there points of origin and insertion [8] Thus. the forces exerted by muscle contraction can be represented by vectors. According to [4], this assumption is reasonable when the muscle is acting as a hole and when it has a homogeneous structure. Fig. 1. is a schematic representation of the coordinate system, occlusal load, condyles reactions and muscle forces in a three dimensional vector forms. The points of application and direction of the muscle force were obtained from the measurements of [9]. The location and angles of each muscle group were measured using a Cartesian coordinate system centered at the apex of the left condylar process as shown in fig. 1. The positions and directions of each muscle well as, the vector positions of the condyles and molars needed to complete the vector description of the mathematical model are given in table 1.

The direction of the occlusal load was assumed to be perpendicular to the occlusal plane. The components of the two condyles reactions are assumed in the directions shown in fig. 1. During contraction of the masticatory muscles the mandible is to be in static equilibrium.

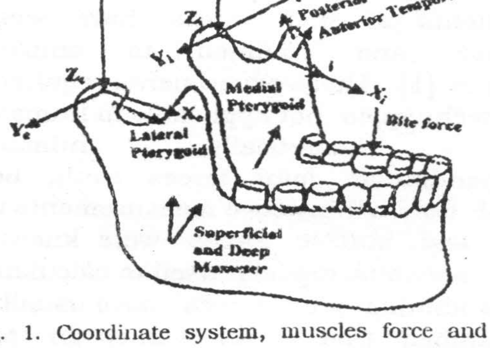

Fig. 1. Coordinate system, muscles force and condylar reac-tions.

According to Newton's law, the conditions of static equilibrium are satisfied when the vector sum of the forces and the vector sum of the moments equal zero. This gives the following force and moment set of equilibrium equations:

1- Forces in X direction:

$$F_{x1} + F_{x2} + F_{x3} + F_{x4} + F_{x5} + F_{x6} + F_{x7} + F_{x8} + F_{x9} + F_{x10} + F_{x11} + F_{x12} + X_i + X_c = 0.0 \qquad (1)$$

2- Forces in Y direction:

$$F_{y1} + F_{y2} + F_{y3} + F_{y4} + F_{y5} + F_{y6} + F_{y7} + F_{y8} + F_{y9} + F_{y10} + F_{y11} + F_{y12} + Y_i + Y_c = 0.0 \qquad (2)$$

3- Forces in Z direction:

$$F_{z1} + F_{z2} + F_{z3} + F_{z4} + F_{z5} + F_{z6} + F_{z7} + F_{z8} + F_{z9} + F_{z10} + F_{z11} + F_{z12} + P + Z_i + Z_c = 0.0 \qquad (3)$$

4- Moments about X axis:

$$M_{x1} + M_{x2} + M_{x3} + M_{x4} + M_{x5} + M_{x6} + M_{x7} + M_{x8} + M_{x9} + M_{x10} + M_{x11} + M_{x12} + P * r_y - Z_c * r_y = 0.0 \qquad (4)$$

5- Moments about Y axis:

$$M_{y1} + M_{y2} + M_{y3} + M_{y4} + M_{y5} + M_{y6} + M_{y7} + M_{y8} + M_{y9} + M_{y10} + M_{y11} + M_{y12} + P * r_x = 0.0 \qquad (5)$$

6- Moments about z axis:

$$M_{z1} + M_{z2} + M_{z3} + M_{z4} + M_{z5} + M_{z6} + M_{z7} + M_{z8} + M_{z9} + M_{z10} + M_{z11} + M_{z12} + X_c * r_y = 0.0 \qquad (6)$$

where;

F_{xi}, F_{yi} and F_{zi} are the components of the muscle force number "i" in the X, Y and Z directions respectively , and

$$M_{xi} = -F_{yi} r_{zi} + F_{zi} r_{yi},$$
$$M_{yi} = -F_{zi} r_{xi} + F_{xi} r_{zi}$$
$$M_{zi} = -F_{xi} r_{yi} + F_{yi} r_{xi},$$
$$r_{xi} = x_{muscle} - x_{origin},$$
$$r_{yi} = y_{muscle} - y_{origin},$$
$$r_{zi} = z_{muscle} - z_{origin}$$

Since the equilibrium equations are only six, while we have eighteen unknowns (twelve muscle forces and three reaction components at each condyle), the system is statically indeterminate. So, to find a unique solution an optimization technique must be utilized. Possible merit criterion function can be formulated as one or as a weighed combination of several objectives [10]. Mathematically speaking, in our case, this means the minimization of $\sum_{i=1}^{12} \left(\frac{F_i}{A_i}\right)^n$, (7)

where n is given the values of 1,2,3,or 4.

The equality constraints in this case are the static equilibrium equations (equations 1~6). It is reasonable to assume the existence of the following regional (inequality) constraint:

$$0.0 \le F_i \le F_{i,max}, \qquad (8)$$

Eq. (8) limits the muscle force, of muscle number (i), to be a tensile force and not to exceed a maximum value that is forced by the maximum muscle strength.

According to [11] the maximum muscle strength is in the range of 0.4~1.0 MPa.

Thus, the maximum allowable force for each muscle can be obtained from:

$$F_{i,max} = SA_i \qquad (9)$$

In the last equation S is the muscle strength or the maximum allowable muscle stress.

In this work S is assumed, for all the muscles in this study, to be 0.4 MPa. for normal occlusal loads and 1.0 MPa. for very high occlusal loads. The variable A_i is the physiological cross-sectional area of the muscle number (i). Table 2 gives the physiological cross-sectional areas of the muscles used in this study. These values

Table 1
Position and direction of model elements

Element	Position vector (mm)	Unit force vector
Left posterior temporalis muscle(L.P.Temp.)	34.3i-4.57j-0.00k	-0.76i+0.10j+0.64k
Left anterior tempralis muscle (L.A.Temp.)	37.6i-5.08j-6.60k	-0.34i-0.07j+0.94k
Left deep masseter muscle (L.D.Mas.)	31.8I+0.00j-44.5k	-0.18i+0.27j+0.94k
Left superficial masseter muscle (L.S.Mas.)	31.8I+1.27j-44.5k	+0.15i+0.27j+0.95k
Left medial pterygoid muscle (L.M.Pter.)	22.4i-6.35j-44.5k	+0.03i-0.32j+0.94k
Left lateral pterygoid muscle (L.L.Pter.)	6.35I+0.00j-6.35k	-0.94i-0.25j-0.25k
Right posterior temporalis muscle(R.P.Temp.)	34.3I -86.9j-0.00k	-0.76i-0.10j+0.64k
Right anterior tempralis muscle (R.A.Temp.)	37.6i-86.4j-6.60k	-0.34i-0.07j+0.94k
Right deep masseter muscle (R.D.Mas.)	31.8i-91.4j-44.5k	-0.18i+0.27j-0.94k
Right superficial masseter muscle (R.S.Mas.)	31.8i-92.7j-44.5k	+0.15i+0.27j+0.95k
Right medial pterygoid muscle (R.M.Pter.)	22.4i-92.7j-44.5k	+0.03i-0.32j+0.94k
Right lateral pterygoid muscle (R.L.Pter.)	6.35i-91.4j-6.35k	-0.94i-0.25j-0.25k
Left condyle	0.00i +0.00j+0.00k	------------------
Right condyle	0.00I-91.4j+0.00k	------------------
First left molar (M1)	73.1i-21.8j-24.1k	------------------
Second left molar (M2)	31.8i-19.8j-26.2k	------------------
Third left molar (M3)	6.35i-17.8j-28.2k	------------------

were obtained from refs. [12,13]. Another regional constraint is used to limit the reaction forces at both condyles to lie in the range of $-6*10^3 \sim 6*10^3$ to avoid the destructive effect of these forces on the condylar neck [14].

This constraint can be expressed as follows:

$$-6*10^3 \le R_i \le 6*10^3$$
$$-6*10^3 \le R_c \le 6.*10^3 \tag{10}$$

In the last equation R_i and R_c are the resultant reaction forces at the ipsilateral and contralateral condyles respectively.

Many combinations of optimization techniques [15]. and criterion functions have been tested by the authors to choose the one with the most logical, comparable and reliable results. First the authors tried to minimize the sum of the muscles stress, $\sum_{i=1}^{12} \frac{F_i}{A_i}$, with the equilibrium equations eqs. (1- 6) as functional constrains and eqs. (8-10) as regional constraints using the linear programming technique [16]. The results were not either comparable or logical. Then the criterion function was changed to minimize the sum of the square of the muscles stress,

$$\sum_{i=1}^{12} \left(\frac{F_i}{A_i}\right)^2$$
(11)

So, this criterion function is proportional to the strain energy, per unit volume of the muscle, [17]. The sequential quadratic programming [18]. was utilized to solve the last problem. This gave a better results but still not all the results are comparable. So, the authors tried to change the power "n" to be 3 and 4 but the results did not seem to differ much or significantly, which agrees with the previous studies of refs. [19,20] So, we had to choose another optimization technique with more than one criterion function. Thus, we chose the minimax optimization technique [21]. Three criterion functions were used; the sum of the squares of muscle stresses eq. (11), the sum of the square of the reaction components at the ipsilateral condyle;

$$X_i^2 + Y_i^2 + Z_i^2 = R_i^2 \tag{12}$$

and the sum of the square of the reaction components at the contralateral condyle:

$$X_c^2 + Y_c^2 + Z_c^2 = R_c^2 . \tag{13}$$

Table 2
Physiological cross sectional area of the masticatory muscles in mathematical model

Muscle	Physiological cross-sectional area (cm²)
Posterior temporalis	4.5
Anterior temporalis	3.5
Deep masseter	2.3
Superficial masseter	5.7
Medial pterygoid	4.4
Lateral pterygoid	2.2

Finally, we have three criterion functions, eqs. (11-13) subject to the equality constraints, eqs. (1-6), and the regional constraints 8 and 10. The MATLAB optimization toolbox was utilized to solve the optimization problem.

3. Results and discussion

The results of the optimization program yielded the magnitude of the muscle forces and both the magnitude and the direction of the reaction forces at the two condyles, due to different occlusal loads. Figs. 2,3 illustrate how the increase of the occlusal load , on the first molar, affects the muscle forces of the ipsilateral (working) and contralateral (balancing) sides respectively. The occlusal load ranged from 0.0 to 200N (normal range for bite force). In Figs. 4-5 the same effect is shown but for very high occlusal loads to simulate the effect of the maximum bite force. In all of the previous figs. 2-5 there is an obvious linear relationship between the occlusal load and the muscle forces which agrees with the results of Gaylord and Throckmorton [1]. Also, it is clear that the lateral pterygoid muscles (left and right) have no share during this process. This can be explained by the fact that these muscles, among all the mandible muscles, have the shortest moment arms and the minimum physiological cross-sectional areas that make their work very expensive. This result agrees with the results of the previous studies that neglected these muscles. Also, this result compares well with the results of the electromyography (EMG) recordings of Faulkner et al., [9], which showed that the EMG activity of the left and right pterygoid muscles are very small compared with the EMG activities of the other mandible muscles.

Figs. 4,5 show that after an occlusal load of 800N the linear relationship starts to fall since the maximum muscle force has been reached for the superficial masseter although, for these high occlusal loads, the maximum allowable tensile stress for all the muscles has been raised to 1MPa.

However, the linearity can be maintained again by increasing the maximum allowable tensile stress, a little, as found by refs. [3,4]. Figs. 6-10 illustrate the effect of the occulsal load on the mandible, left and right, muscle forces. These figures are for the posterior temporalis muscle, the anterior temporalis muscle, the deep masseter muscle, the superficial masseter muscle and the middle pterygoid muscle respectively. In all of the figures, case 1 means that the occlusal load was on the first molar while for case 2 the load was on the second molar and it was on the third molar for case 3.

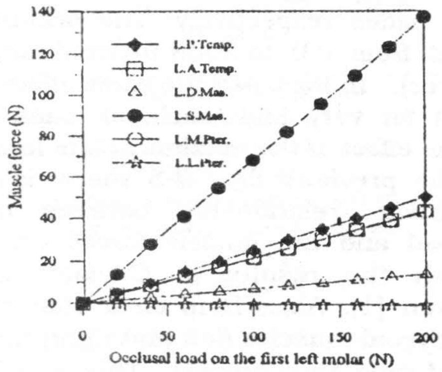

Fig. 2. Effect of occlusal load on the muscle force (ipsilateral side).

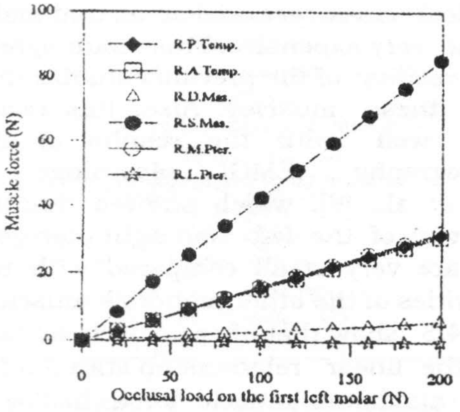

Fig. 3. Effect of occlusal load on the muscle force (contralateral side).

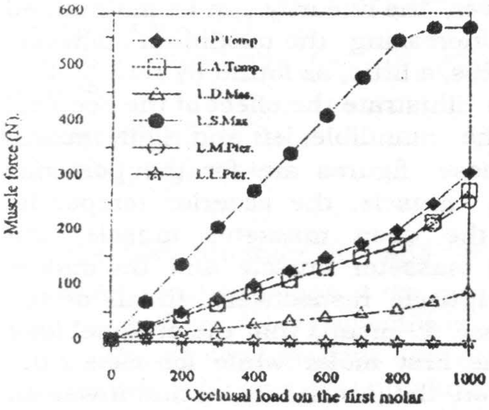

Fig. 4. Effect of high occlusal load on the muscle force (ipsilatcral side).

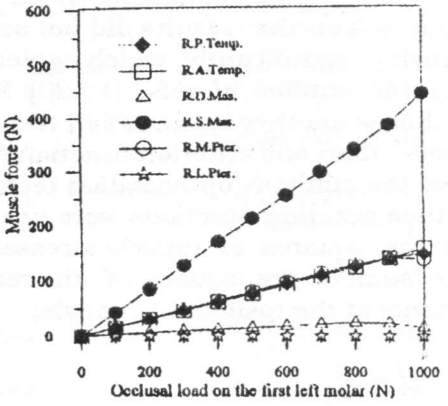

Fig. 5. Effect of high occlusal load on the Muscle (contralatcrl side).

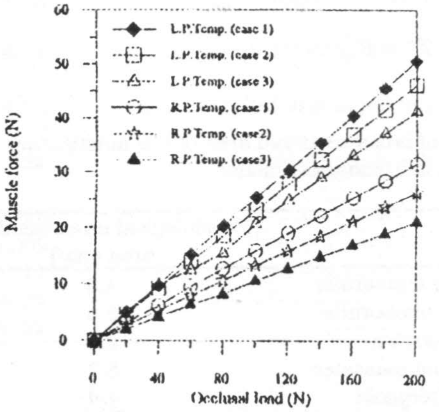

Fig. 6. Effect of the position of the occlusal load on the left and right posterior temporalis muscles.

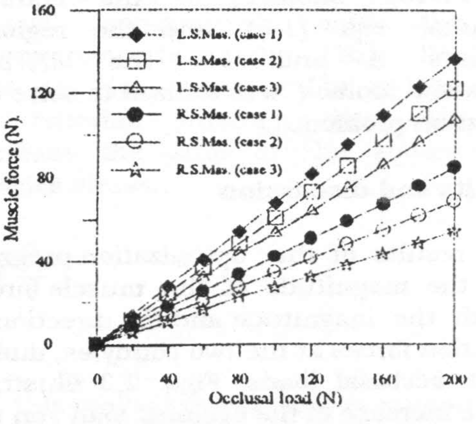

Fig.7. Effect of the position of the occlusa load onTheLeft and right anterior tmporalis muscles.

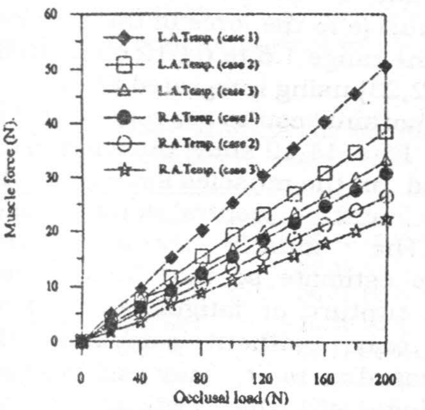

Fig. 8. Effect of the position of the occlusal load on the Left and right deep masseter muscles.

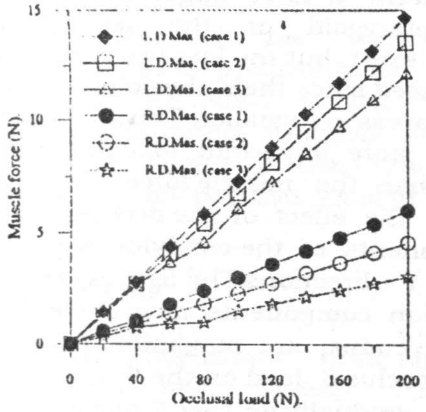

Fig. 9. Effect of the position of the occlusal load on the left and right superficial masseter muscles.

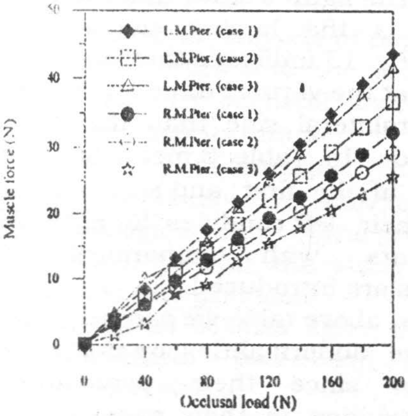

Fig. 10. Effect of the position of the occlusal load on the left and right medial pterygoid muscle.

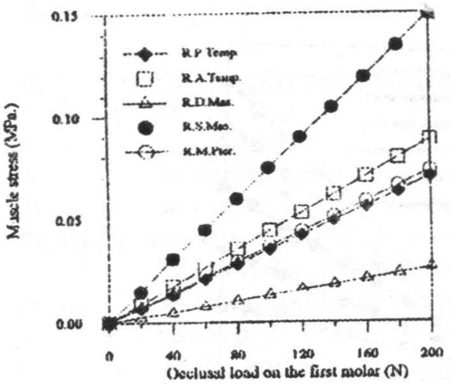

Fig. 11. Effect of occlusal load on the muscle stress (ipsilateral side)

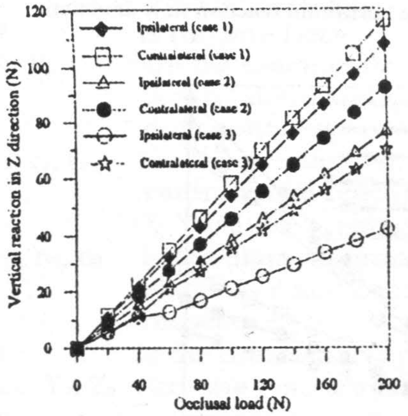

Fig. 12. Effect of occlsal load on the muscle stress (contralateral side).

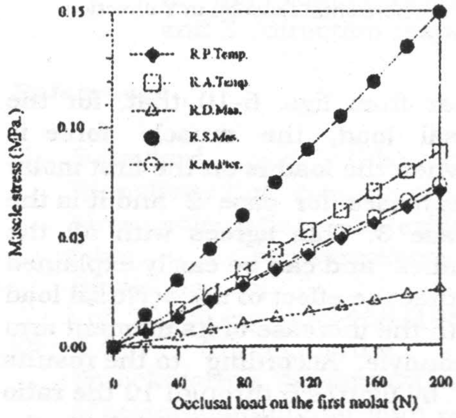

Fig. 13. Effect of the magnitude and position of the cclusal load on the vertical reaction in Z direction

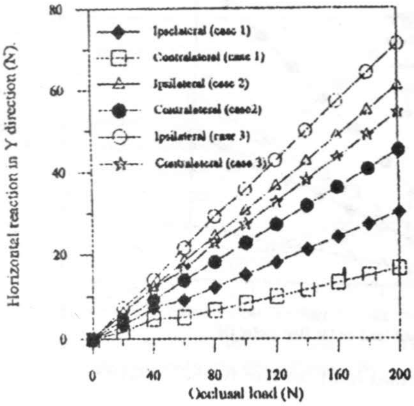

Fig. 14. Effect of the magnitude and position of the Occlusa load on the horizontal reaction in X direction.

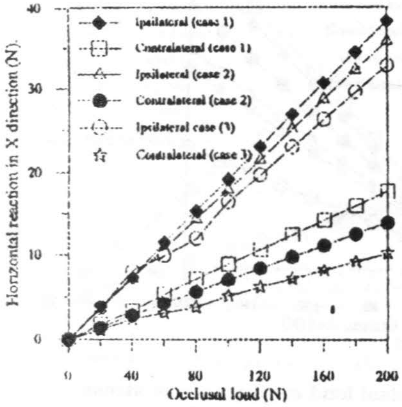

Fig. 15. Effect of the magnitude and position of the occlusal load on the horizontal reaction in Y direction.

It is clear from figs. 6-10 that, for the same occlusal load, the muscle force is maximum when the load is on the first molar and then decreases for case 2 and it is the lowest for case 3. This agrees with all the previous studies and can be easily explained by the fact that the effect of the occlusal load increases with the increase of its moment arm about the condyle. According to the results represented by figures 6 through 10 the ratio of the muscle force of the temporalis to the masseter and the medial pterygoid together in all cases falls in the range 0.65 to 0.9 obtained by Carlsoo, [3] Pruim et al., [4] and Weijs and Hillen, [12]. which was estimated according to the muscle cross sectional area. Also, the ratio of the muscle force of the temporalis muscle to the force of the masseter musle is in the range 1.5 to 0.612 obtained by Pancherz ([22,23] using integrated EMG.

The most important results are represented in figs. 11-15. Figs. 11,12 show the effect of the occlusal load on the muscles stresses for the ipsilateral and contralateral sides respectively.The muscle stress is very important to estimate so that we can avoid the muscle rupture or fatigue.Muscle force estimation only, without calculating the muscle stress due to it, may lead to a false understanding of the critically loaded muscles.

For example the left posterior temporalis muscle exerts a force larger than the left middle pterygoid or the left anterior temporalis exert, but the last two muscles are more stressed since their physiological cross-sectional areas are smaller. Thus, the muscle stress is more important to calculate and compare than the muscle force. Figs. 13-15 illustrate the effect of the occlusal load on the components of the condyles reactions in Z, X and Y directions.The figures show that the reaction components magnitudes, in Z and X directions, are maximum for case 1 with the occlusal load on the first molar and decreases gradually for case 2 and 3 with the occlusal load on the second and the third molars respectively and vice versa for the Y component. The figures show that the vertical component is the largest component in magnitude. Fig. 13 indicates that the reaction component, at the vertical direction, is higher for the contralateral side than that for the ipsilateral side. In Table 3 measurements of the position of the first and second molars, relative to their left condyles, for eight adult male patients with temporomandibular abnormalities are introduced.

From the above table we can see that the most effective abnormalities are in the X and Z directions since their percentage of standard deviation to their mean are high (14.55% in X direction and 14.8% in Z direction for the first molar). This may cause an inclination about ±10° to the normal force applied to the molar, (which agrees with the results of Faulkner et al., (9) which will add a horizontal force component in the X direction.

Table 3
Position of the first and second molars in 8 patients with tempromandibular abnormalities

First left molar	Second left molar
52.8i-16.2j-9.5k	38.4i-15.5j-14.2k
60.2i-15.1j-8.2k	32.4i-16.4j-12.4k
45.5i-14.2j-12.5k	36.5i-13.4j-10.5k
65.8i-15.1j-8.5k	30.5i-13.5j-13.3k
69.8i-15.3j-9.3k	38.2i-12.5j-10.8k
69.8i-12.5j-9.4k	36.5i-11.4j-10.5k
65.4i-14.3j-8.4k	30.5i-10.5j-14.2k
70.5i-17.5k-10.5k	38.5i-10.2j-13.8k
Mean = 62.4i-15.02j-10.5k	Mean = 35.1i-12.8j-12.4k

The effect of this horizontal force component can be predicted using this model by adding its effect to the equilibrium equations, criterion function and to the constraints. Thus we can represent any abnormality as position change using this model.

5. Conclusions

1- A three dimensional static model of the TMJ has been introduced. The model is the first in which 12 muscles and the six reaction components of the two condyles are involved as unknowns.

2- The model is capable of prediction of the muscle forces and the reaction in the TMJ for any occlusal load utilizing the optimization technique.

3- The number of muscles, points of origin and insertions of the muscles, and all the physiological data of the program can be adjusted to suit the study of the TMJ abnormality problems.

4- The model results agree well with the physiological facts and the previous studies results.

5 The model showed that the posterior temporalis muscle is the most stressed muscle among the TMJ muscles during biting.

Nomenclature

A_i	is the physiological cross-section of muscle i
E	is the muscle modulus of elasticity
F_i	is the muscle force of muscle i
F_{xi},F_{yi},F_{zi}	are the muscle force components in X,Y and Z directions espectively.
$F_{i,max}$	is the maximum possible force of muscle i
i	is the muscle number (1~12)
M_i	is the moment of muscle force I
M_{xi},M_{yi},M_{zi}	are the moment of muscle force components about X, Y and Z axes respectively
P	is the bite force
R_c,R_I	are the reaction forces at contralateral and ipsilateral directions respectively.
r_x, r_y, r_z	are the moment arms of contralateral reaction force bout X, Y and Z axes respectively.
r_{xi}, r_{yi}, r_{zi}	is the moment arms of muscle i about X, Y and Z axes respectively.
S	is the muscle strength.
X_c, Y_c, Z_c	are the reaction force components at the contralateral condyle in X, Y and Z direction respectively.
X_i,Y_i,Z_I	are the reaction force components at the ipsilateral condyle in X, Y and Z direction respectively.

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