# Sizing of battery storage for a stand-alone hybrid wind-PV system using a five-event probability density approximation

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A simple technique for sizing a battery bank that is used in a stand-alone hybrid wind/PV system is introduced. A technique using a five-event probability density instead of the two-or three-event probability approximations is developed. Chi-Squared (C-S) method is used to examine the goodness-of-fit criteria for the five-event probability density approximation. A probabilistic approach of non-exponential distribution system based on the stage method is used. The change ability of the battery storage is treated as a case of random walk and is solved using the Markov technique. Long term data of wind speed and irradiance recorded every hour of the day for 30 years are used. A load of a typical house in New England is considered as a load demand of the hybrid system. For these data and a desired Loss of Load Probability (LOLP), an optimum number of batteries is calculated.

في هذا البحث نقدم طريقة مبسطة لتقييم سعة البطاريات المستخدمة لتخزين القدرة الكهربية في نظم التوليد المختلطة من طاقة الرياح والخلايا الفوتوفولتية.هذه الطريقة تعتمد علي تقريب كثافة توزيع احتمالات متغيرات المنظومة كسرعة الرياح وقوة الإشعاع الشمسي وأيضا الحمل الكهربي المطلوب الى خمسة تقريبات بدلا من اثنين أو ثلاثة كما هو مستخدم من قبل، وللتحقق من دقة هذه الطريقة استخدمنا طريقة $\chi^2$ ، وأيضا استخدمنا نظرية الاحتمالات للمنظومات ذات الكثافة اللأسية مستخدمين في ذلك طريقة المرحلة وأيضا طريقة ماركوف.هذا وقد استخدمنا المعدلات الساعية لسرعة الرياح وقوة الإشعاع الشمسي المسجلة بمطار لوجان الدولي ببوسطن لمدة ٣٠ عاما (٦١-١٩٩٠) لتحديد متوسطات تلك المتغيرات بدقة عالية، كما استخدمنا معدلات التحميل المنزلي للقدرة الكهربية للمنازل بقطاع نيو انجلاند بالولايات المتحدة الأمريكية.

**Keywords**: Renewable energy, Wind energy, Photovoltaic, Hybrid system, Reliability

## 1. Introduction

Today, more than ever, renewable energy systems are poised to become significant sources of electricity. The increasing of fuel prices, environmental concerns and renewable energy systems technology leads us to build hybrid wind/PV systems [1]. Given the present cost of renewable energy systems, it is evident that these can better compete with the grid in locations that are remote, where grid is either not feasible or non-existent. A power generation system for such a location would not have any back up of electrical energy and would in effect be left to itself [2]. In this paper, a method is presented to size the battery storage bank for a hybrid renewable energy system, powered by wind and solar energy. Fig. 1 shows the detailed schematic diagram of the system under study. The storage system is sized to meet certain given conditions of reliability assuming that the wind turbine and the PV array have already been selected. The system had been designed using either deterministic approach [3] or probabilistic [4-6].

In the probabilistic approach, the daily surplus (or deficit) energy transferred to (or from) the battery bank (D) is calculated, then converted to a probability density function p(D). The p(D) is approximated into two-event density function [4], and as a three-event density function [5]. The two-event approximation technique assumes that the storage either increases or decreases by a value $\delta$. In effect it replaces the probability density for D by a two-event Markov process. However in reality there is a possibility of the storage staying in the same level over the course of a day [5]. In [5], the resulted number of batteries was found depending on the pre-assumed step of charge. fig. 2, gives the variation of resultant number of batteries as a function of the step of charge. From fig. 2, we can see that the designer of such hybrid system cannot accurately select the number of batteries of bank storage. In the proposed five-event approximation, it is assumed that the storage either increases or decreases by two values of charges $\delta_1$ and $\delta_2$. The fifth event is possibility of the storage staying in the same level. Chi-Squared (C-S) method was used to examine the goodness-of-fit for the five-event probability density approximation. It was found that the five-event approximation gives more accurate and definite result than the others [4-5]. Long term data of wind speed and irradiance recorded for every hour of the day of 30 years were used. A load of a typical house in New England is considered as a load demand of the hybrid system. These data were used to calculate the average daily surplus (or deficit) energy generated by a battery bank. For these data and a desired LOLP, optimum number of batteries is calculated.

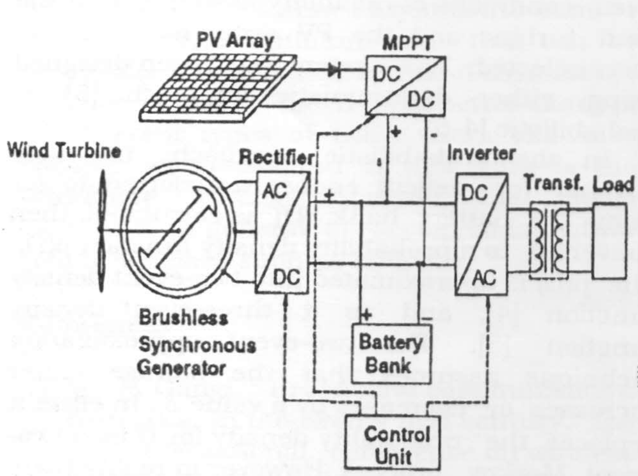


Fig. 1. Schematic diagram of hybrid wind/PV system.

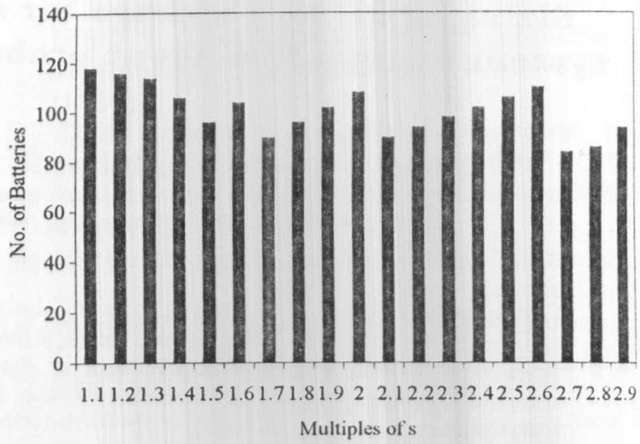


Fig. 2. Number of batteries vs. different step of charge [5].

## 2. Methodology

The developed methodology is described in steps as follows.

### *2.1. Calculations of the probability density function of battery energy*

The energy is stored in batteries when the generated power by both the wind turbine and the PV array is greater than the load demand. When the generated power is less than the load, the energy is taken from the batteries. The case of overcharge may occurs when higher generated power by the wind turbine and PV, or when low load demand exists. In such a case when the state of charge of the batteries reaches the maximum value, $D_{max}$, the control system stops the charging process. On the other hand, if the state of charge decreases to a minimum level, $D_{min}$, the control system disconnects the load. This is important to prevent shortening the life of batteries or even their destruction [3]. The p(D) curve has been drawn for a particular combination of PV array, wind turbine and the load, according the following equation:

$$D = \{E_w(t)\eta_{rect} + N_{pv}E_{pv}(t)\eta_{mppt} - E_L(t)/\eta_{inv}\}\eta_{batt}$$

$$\text{For } D_{min} \le D \le D_{max}. \quad (1)$$

The calculations were repeated for all the days for which the data is available and then made up of a certain number of discrete levels of D to obtain the p(D) curve. The action of the control system can be summarized in the following equation:

$$p(D) = 0.0 \quad , \text{ For } D \le D_{min} \quad \text{and } D \ge D_{max} . \qquad (2)$$

The above eq. have a shortage, that is if D is the whole charge going to (or out of) the battery bank, so the values of $D_{min}$ and $D_{max}$ are unknowns because they are functions of both $n_1$ and $n_2$.

*2.2. Calculations of the events' probabilities*

The five-event distribution has to be equivalent to the actual distribution of D. This section devoted to obtaining a reasonable approximation of the p(D). The evaluation of the probabilities $p_1, p_2, q_1, q_2$, and t is the same as the basic concepts of the probability theory as used in [4] and [5]. The following equation gives the $i^{th}$ moment about the origin of the random variable $\delta$.

$$\mu_i' = \sum_{i=1}^{\infty} \delta^i f(\delta) , \qquad (3)$$

where $f(\delta)$ is the probability of the battery storage changed by $\delta$.

The first through the fourth moments are as follows:

$$\mu = \delta_1(p_1 - q_1) + \delta_2(p_2 - q_2) \qquad (4)$$

$$\mu^2 + \sigma^2 = \delta_1^2(p_1 + q_1) + \delta_2^2(p_2 + q_2) \qquad (5)$$

$$3\mu\sigma^2 + \mu^3 = \{\delta_1^3 - (\delta_1 + \mu)^3\}(p_1 - q_1) + \{\delta_2^3 - (\delta_2 + \mu)^3\}(p_2 - q_2) \qquad (6)$$

$$6\mu^2\sigma^2 + \mu^4 = \{\delta_1^4 - (\delta_1 - \mu)^4 - 4\mu(\delta_1 - \mu)^3\}p_1 + \{\delta_1^4 - (\delta_1 + \mu)^4 + 4\mu(\delta_1 - \mu)^3\}q_1 + \{\delta_2^4 - (\delta_2 - \mu)^4 - 4\mu(\delta_2 - \mu)^3\}p_2 + \{\delta_2^4 - (\delta_2 + \mu)^4 + 4\mu(\delta_2 - \mu)^3\}q_2 \qquad (7)$$

$$p_1 + q_1 + p_2 + q_2 + t = 1.0 \qquad (8)$$

*2.3. Calculations of the number of batteries*

In the present method, it assumes that the number of possible events is five. The first is the transition of the storage $\Delta$ to the next lower level by charge $\delta_1$ with the probability $q_1$. The second is the transition to next higher with a probability $p_1$. The third is the transition of the storage $\Delta$ to the next lower level by charge $\delta_2$ with the probability $q_2$. The forth is the transition to the next higher with a probability $p_2$. The fifth is staying of the charge at its level $\Delta$ with a probability t.

Suppose that the transition process having $n_1$ steps of $\delta_1$ and $n_2$ steps of $\delta_2$. Let that $\pi_{11}$ is the probability that the storage would contain $\delta_1$ charge, and $\pi_{21}$ is the probability that the storage would contain $\delta_2$ charge. The LOLP is defined as the following equation:

$$LOLP = \pi_{11}q_1 + \pi_{21}q_2 - \pi_{11}q_1 \times \pi_{21}q_2 . \qquad (9)$$

The negative term represents the intersection of the first and the second terms of eq. [9].

Using the same basic concepts mentioned in [5], which consider the process to be a Markov process, the probabilities of the battery bank containing at least one level of $\delta_1$, or $\delta_2$ are:

$$\pi_{11} = \frac{1-\lambda_1}{1-\lambda_1^{n_1}} , \text{ and} \qquad (10)$$

$$\pi_{21} = \frac{1-\lambda_2}{1-\lambda_2^{n_2}} , \qquad (11)$$

where $\lambda_1 = p_1/q_1$, and $\lambda_2 = p_2/q_2$.
By solving eqs. (9), (10) and (11), we get:

$$n_2 = \frac{\ln(1+x/y)}{\ln\lambda_2}, \qquad (12)$$

where

$$x = q_1(\lambda_1 - 1)(\lambda_1^{n_1} - 1) - q_1 q_2(\lambda_1 - 1)(\lambda_2 - 1), \text{ and} \qquad (13)$$

$$y = \text{LOLP}(1-\lambda_1^{n_1}) - q_2(\lambda_1 - 1). \qquad (14)$$

The solution of eq. (12) must satisfy the following condition,

$$(\delta_1 + \delta_2) \geq D_{min}, \qquad (15)$$

where,

$$D_{min} = 0.2(n_1\delta_1 + n_2\delta_2). \qquad (16)$$

The condition in eq. (15) will be reached by restarting the program with increasing the value of $n_1$ by 1.

Using the above definitions, the number of batteries is calculated as follows:

$$N_b = \frac{n_1\delta_1 + n_2\delta_2}{C_b \eta_{batt}}. \qquad (17)$$

## 3. Algorithm

The program for sizing the battery bank was written in PASCAL programming language. The steps of the solution are as follows:

1. Input the data required for the technique, as follows: $\mu, \sigma$ and depth of the battery discharge.
2. Randomize the values of $\delta_1$ and $\delta_2$.
3. Calculate the probabilities $p_1, q_1, p_2, q_2, t$, using eqs. (4-8).
4. Use the C-S method to examine the goodness of fit of the current five-event approximation.
5. Initialize $n_1$=0.
6. Let $n_1 = n_1 + 1$.
7. Calculate $n_2$, using eq. (12).
8. If the condition, eq. (15), is not satisfied, repeat steps 6-8.
9. Calculate the number of batteries, using eq. (17).

## 4. Model and results

### 4.1. Model

The system under study as shown in fig. 1, is a stand-alone wind/photovoltaic hybrid system that is capable of generating electrical power from the alternative energy sources produced by wind and solar radiation. This system was installed on the roof and in the laboratory of the University of Massachusetts-Lowell. The system was equipped with a data acquisition system that records all variables, i.e., voltages, currents, wind speed, generator frequency, and irradiance level.

The output power $P_w$ of the wind turbine calculated from the following eq. (3):

$$P_w = \frac{1}{2} C_p \rho A V_w^3, \qquad (18)$$

where;

$\rho$ is the air density in kg/m³.

$V_w$ is the wind speed m/s.

A is the frontal area of the wind turbine in m².

$C_p$ is the power coefficient given in manufacturer's specifications.

The rated power of the PV array $P_{pv}$ is calculated as follow [3]:

$$P_{pv} = V(S)I(S). \qquad (19)$$

Where $V(S), I(S)$ are the array voltage and current as a function of irradiance, respectively. More details are given in [3].

The batteries available for this system are lead acid type of 110 Ah, 12 V, 80% round trip efficiency and the depth of discharge is 80%. Conditioning power devices: a static inverter with 92% efficiency, maximum power point tracker 95% efficiency, and controlled rectifier with 92% efficiency.

A typical New England house whose load characteristic is determined by the utility company.

*4.2. Results*

The five-events stated above are shown in fig. 3. Table 1 gives all possible results for present technique. December month was used for calculations. The corresponding data are as follows: $\mu = 4.335$ kwh, and $\sigma = 11.3$ kwh.

Taking the value of the minimum charge is $\delta_1 + \delta_2$ as 20% of the total capacity of battery bank, it is found that the number of batteries obtained from this technique varied according to the minimum charge. As shown in table 1, the number of batteries obtained from this technique is about 4.5% less than that the three-event method, and about 12% less than the two-event method.

## 5. Conclusions

A new technique for the sizing battery storage bank for a stand-alone hybrid wind/photo-voltaic system has been introduced. This technique exhibits the following advantages:

1. The five-event approximation method increases the accuracy of the sizing technique.
2. The sizing of the battery bank depends upon the control system strategy and not upon a pre-assumed charge
3. The technique is very useful to determine the optimum relationship between the number of PV modules, and the number of storage batteries.

Table 1
Results for five-event approximation technique

| No. | $\delta_1$ | $\delta_2$ | $p_1$ | $q_1$ | $p_2$ | $q_2$ | t | $n_1$ | $n_2$ | $N_b$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.70 $\sigma$ | 1.54 $\sigma$ | 0.0 | 0.25 | 0.375 | 0.125 | 0.25 | 14 | 1 | 121 |
| 2 | 0.59 $\sigma$ | 1.42 $\sigma$ | 0.063 | 0.5 | 0.344 | 0.063 | 0.031 | 15 | 1 | 110 |
| 3 | 0.63 $\sigma$ | 1.51 $\sigma$ | 0.125 | 0.25 | 0.375 | 0.031 | 0.219 | 15 | 1 | 117 |
| 5 | 0.62 $\sigma$ | 1.61 $\sigma$ | 0.281 | 0.375 | 0.297 | 0.016 | 0.031 | 16 | 1 | 123 |
| 6 | 0.58 $\sigma$ | 1.57 $\sigma$ | 0.188 | 0.375 | 0.344 | 0.063 | 0.031 | 16 | 1 | 116 |
| 7* | 0.51 $\sigma$ | 1.43 $\sigma$ | 0.125 | 0.25 | 0.375 | 0.125 | 0.125 | 17 | 1 | 108 |
| 8 | 0.59 $\sigma$ | 1.65 $\sigma$ | 0.125 | 0.438 | 0.297 | 0.016 | 0.125 | 16 | 1 | 119 |
| 9 | 0.55 $\sigma$ | 1.60 $\sigma$ | 0.063 | 0.375 | 0.375 | 0.063 | 0.125 | 17 | 1 | 117 |
| 10 | 0.52 $\sigma$ | 1.61 $\sigma$ | 0.063 | 0.5 | 0.375 | 0.375 | 0.047 | 18 | 1 | 117 |
| 11 | 0.55 $\sigma$ | 1.71 $\sigma$ | 0.188 | 0.375 | 0.297 | 0.031 | 0.109 | 18 | 1 | 124 |
| 12 | 0.52 $\sigma$ | 1.66 $\sigma$ | 0.25 | 0.375 | 0.25 | 0.063 | 0.063 | 15 | 2 | 119 |
| 13 | 0.54 $\sigma$ | 1.73 $\sigma$ | 0.188 | 0.438 | 0.25 | 0.063 | 0.063 | 15 | 2 | 124 |

*The most suitable solution which satisfy conditions (15) and C-S test.

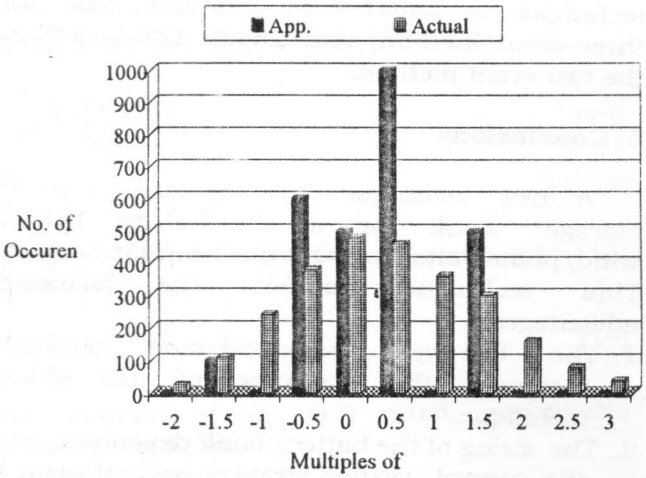


Fig. 3. Actual and approximated probability densities.

**Nomenclature**

$C_b$ — the maximum capacity of each battery.

D — the daily surplus (or deficit) energy generated by the system.

$D_{max}$ — the maximum allowable energy of battery bank.

$D_{min}$ — the minimum allowable energy of battery bank.

$E_L(t)$ — the load energy on the system.

$E_{pv}(t)$ — the energy generated by a PV module.

$E_w(t)$ — the energy generated by wind turbine.

$n_1, n_2$ — the number of charge $\delta_1, \delta_2$ respectively that the battery storage is included.

$N_b$ — number of batteries required.

$p_1, p_2$ — the probability of the storage increased by $\delta_1$ and $\delta_2$ respectively.

p(D) — the probability density for required battery bank energy.

t — the probability of the storage staying in the same level.

$q_1, q_2$ — the probability of the storage decreased by $\delta_1$ and $\delta_2$ respectively.

$\Delta$ — intermediate storage level.

$\delta_1, \delta_2$ — two level of charge by which the battery storage is considered to be changed randomly.

$\eta_{batt}$ — round-trip efficiency of the batteries.

$\eta_{inv}$ — efficiency of the inverter.

$\eta_{mppt}$ — efficiency of maximum power point tracker.

$\eta_{rect}$ — efficiency of the rectifier.

$\mu$ — the mean value of D.

$\sigma$ — the standard deviation of D.

**References**

[1] M.M. El-Wakil, "Power plant technology", McGraw Hill Publishing Company, (1984).

[2] B. Borowy, and Z. Salameh, "Optimum photovoltaic array size for a hybrid wind/PV system", IEEE Trans. on Energy Conversion, Vol. 9, pp. 91-102 (1994).

[3] B. Borowy, "Design and performance analysis of wind/PV hybrid system", Ph.D. Thesis, University of Massachusetts-Lowell, (1996).

[4] L. Bucciarelli, "Estimating loss of power probabilities of stand-alone photovoltaic solar energy systems", Solar Energy, Vol. 32, pp. 584-596 (1984).

[5] A. Bagul, Z. Salameh, and B. Borowy, "Sizing of a stand-alone hybrid wind-photovoltaic system using three-event probanility density approximation", Solar Energy, Vol.56, pp. 325-335 (1996).

[6] A.R. Abdelaziz, and Z. Salameh, "A new statistical distribution function sensitive to renewable energy systems", Electrical machines and power systems, Vol. 26, pp. 659-667 (1998).

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