Effect of the first local buckling mode on the behaviour of steel I-Section beam-columns

Raafat E. S. Ismail

Department of Engineering Mathematics and Physics, Faculty of Engineering, Alexandria University, Alexandria, Egypt

Hesham Ali Zien El Din

Department of Structural Engineering, Faculty of Engineering, Alexandria University, Alexandria, Egypt.

In this paper, the effect of the first local buckling mode on the strength and post-buckling behavior of steel I-section beam-columns is investigated. The first local buckling mode type and the corresponding critical load of the studied I-beam-columns are determined through bifurcation analysis, using incremental nonlinear finite element approach in the standard eigen problem form. In this approach the elements of the diagonal matrix D, resulting from linear factorization of the overall tangent stiffness matrix K_t, are monitored at each load increment, instead of performing the expensive eigenvalue analysis. At the critical load, where one of the diagonal elements becomes zero or very small positive value, eigenvalue analysis is carried out to obtain the local buckling mode. The ultimate strength and post-buckling behavior of the I-section beam-columns are determined by the finite element method considering both geometric and material nonlinearities. The effect of first local buckling mode type is investigated through a parametric study, where several values of web width-thickness ratio and flange width-thickness ratio, are considered. Moreover, the influence of the axial force ratio with the different geometric parameters is studied.

فى هذا البحث، يتم دراسة تأثير شكل الانبعاج الموضعى الأول على المقاومة والسلوك بعد الانبعاج للأعمدة الكمرية المعدنية ذات قطاع I يتم تحديد نوع شكل الانبعاج الموضعى الاول والحمل الحرج المناظر، للاعمدة الكمرية المعدنية المدروسة، من خلال التحليل التشعبى مستخدما طريقة العناصر المحددة اللاخطية فى الصورة القياسية لمسألة القيمة الذاتية. فى هذه الطريقة، يتم مراقبة عناصر المصفوفة القطرية، الناتجة من الفك الخطى لمصفوفة الجسائة المماسة الكلية، وذلك بدلا من أداء تحليل القيمة الذاتية المكلف، عند الحمل الحرج، حيث يكون أحد العناصر القطرية مساويا للصفر أو قيمة موجبة صغيرة جدا، يتم اجراء تحليل القيمة الذاتية للحصول على شكل الانبعاج الموضعى. يتم تحديد المقاومة القصوى والسلوك بعد الانبعاج للأعمدة الكمرية المعدنية ذات قطاع I باستخدام طريقة العناصر المحددة معتبرة كلا من اللاخطية الهندسية ولأخطية المادة. يتم بحث تأثير نوع شكل الانبعاج الموضعى الاول من خلال دراسة بارامترية، حيث يتم اعتبار عدة قيم لنسبة عرض العصب الى سمكه وعرض الشفة الى سمكها. علاوة على ذلك، يتم دراسة تأثير نسبة الحمل المحورى مع الاختلاف فى العوامل الهندسية.

Keywords: I-section beam-columns, Bifurcation analysis, Nonlinear finite element, Local buckling and Ultimate strength.

1. Introduction

Tendency to use high tensile steel due to the advanced technology of manufacturing, and the desire to get thinner plate elements of structural members, for economical design, increase the possibility of local buckling occurrence under unexpected severe loading. An important characteristic of plates is that they can resist increasing load after reaching the critical load [1]. Therefore, the critical load of a plate is not its failure load, which is certainly dependent on the buckling mode corresponding to the critical load.

Steel members, such as I-section beam-columns, channels and angles, are composed of plate elements. When a plate element only buckles, while the entire member does not, the buckling of this element is referred to as local buckling. For buckling of plate element to take place, it is not necessary that a plate is loaded in axial compression. It is necessary that compression stress exists in some parts of the plate. Thus, instability can occur in a plate loaded by a pure shear. The design methods of plate girder webs are divided into two categories: (1) allowable stress design based on elastic buckling as a limiting condition; and (2) strength design based on ultimate strength. The American Institute of Steel Construction (AISC) specifications account for elastic shear buckling strength and post-buckling strength separately and combine these resisting capacities based on the aspect ratio of the web panel. Although these specifications predict the overall shear strength with reasonable accuracy, they often underestimate the elastic shear buckling strength, due to an underestimation of the rigidity at the flange-web juncture. Based on a parametric study [2], on plate girder web panels under pure shear, using nonlinear finite element analysis, it is shown that the boundary condition at the flange-web juncture is much closer to a fixed support rather than simple support. Also, it is found that the flange rigidity has little effect on the post-buckling strength of web panels, whereas it affects the elastic shear buckling strength. This result has been sustained through a study on curved I-girder web panels under pure shear [3].

Usually the local plate buckling is not allowed before the overall buckling occurs. When the design load of a member is far smaller than the allowable load of the overall buckling, the section is still defined by the allowable width-thickness ratio of the plate. This leads to bad design and wasting of material. Therefore, the design of thin-walled beam-columns requires consideration of not only local plate buckling and overall member buckling, but also the interaction between them. Bradford [4] studied the effect of interaction between local buckling and lateral buckling of simply supported beams, beams on seats and beams with continuous tension flange restraint. He found that the effect of interaction is most significant for rolled section beams with incomplete end restraint or with continuous restraint of the tension flange. Ren [5] studied, experimentally and numerically, the interactive buckling behavior and ultimate load of I-section steel columns. His results showed that short columns fail by local buckling of the compressive component plates. The behavior of columns with a low plate width-thickness ratio is dominated by the overall buckling while those with a relatively high plate width-thickness ratio fail due to the interactive buckling.

This paper is concerned with the study of strength and behavior of steel I-section beam-columns, with the presence of local buckling either in the flange or in the web. The effect of first local buckling mode is investigated through a parametric study, considering the web width-thickness ratio, the flange width-thickness ratio, as well as the axial force ratio.

2. Method of analysis

2.1 First local buckling mode and corresponding critical load

The critical load and its buckling mode are determined for a perfect elastic system through a bifurcation analysis. The critical load is the load at which a system in equilibrium passes from a stable to unstable equilibrium. For a stable conservative system in equilibrium, positive energy change is required for any small perturbation of displacement increment vector, δx, about the equilibrium position. Therefore, a necessary condition, for stable equilibrium, is that:

$$\delta x^T \,.\, K_t \,.\, \delta x > 0 \qquad \text{for all } \delta x. \tag{1}$$

Hence, the overall tangent stiffness matrix, K_t, should be positive definite, with only positive eigenvalues [6].

For unstable equilibrium, energy is released for a small perturbation, δx, in a particular direction resulting in negative energy change and consequently such a perturbation will move the system to another equilibrium state with lower energy. Therefore, a necessary condition, for unstable equilibrium is that:

$$\delta x^T \,.\, K_t \,.\, \delta x < 0 \qquad \text{for some } \delta x. \tag{2}$$

Hence, K_t is not positive definite, having at least one negative eigenvalue.

For critical equilibrium, which represents the state of transition from stable equilibrium to unstable equilibrium, zero energy change is needed for a small perturbation, δx, about the equilibrium position. Hence, a necessary condition for critical equilibrium is that:

$$\delta x^T \,.\, K_t \,.\, \delta x = 0 \qquad \text{for some } \delta x. \tag{3}$$

Therefore, the determinant of K_t is zero, having a zero eigenvalue.

The critical equilibrium state may be either a limit point or a bifurcation point [7 - 9]. As we seek for the first critical load and its corresponding buckling mode, we deal with a perfect elastic system and the critical point is defined as a bifurcation point.

The precise definition of a critical equilibrium state is given by means of the standard eigenvalue problem form, as follows:

$$[K_t]\{z\}_i = \omega_i \{z\}_i, \qquad (4)$$

where $\{z\}_i$ denotes the i^{th} characteristic vector, i^{th} eigen vector, and ω_i denotes the i^{th} characteristic value, i^{th} eigenvalue, of the overall tangent stiffness matrix $[K_t]$. At the critical point, bifurcation point, the lowest eigenvalue, ω_1, will be zero, where the necessary condition;

$$[K_t]\{z\}_1 = 0, \qquad (5)$$

is satisfied, and the corresponding eigenvector, $\{z\}_1$, will be the first occurring buckling mode.

To get the first critical load and its corresponding buckling mode, an efficient incremental nonlinear approach, [6], is used as follows:

1. A small load increment is given in successive cycles.
2. The overall tangent stiffness matrix, K_t, of I-section beam-column is calculated at each load increment.
3. Several test values, τ_i, may be checked to detect if the critical load is reached or not, where the test value τ changes its sign when passing a critical load. The value τ may be one of the following:

$$\tau = \det(K_t), \qquad (6)$$

$$\tau = D_{min}, \qquad (7)$$

$$\tau = \text{Product of } D_{max} \text{ and } D_{min}, \qquad (8)$$

$$\tau = \omega_1, \text{ minimum eigenvalue of } [K_t]. \qquad (9)$$

Where D_{min} and D_{max} are the minimum and maximum values of the diagonal elements in the matrix [D], resulting from the linear factorization of the overall tangent stiffness matrix $[K_t]$.

The test value $\tau=\det(K_t)$, does not work with a multiple bifurcation. The test value $\tau=\omega_1$, is potentially the most effective but is the most expensive. Hence, the value $\tau=D_{min}$ is the test value which is used at each load increment, in this paper.

4. At the load increment, where one or more of the diagonal elements of the matrix [D] becomes zero or small negative values, indicating the occurrence of one or more buckling modes, an iterative procedure is used to reduce this load increment. During these iterations, the overall tangent stiffness matrix, $[K_t]$, is recalculated, until only one of the diagonal elements of the matrix [D] becomes zero or very small positive value, getting the first critical load.
5. At the critical load, eigenvalue analysis is performed on the corresponding overall tangent stiffness matrix, $[K_t]$, to obtain the first local buckling mode, the resulting eigenvector corresponding to the lowest eigenvalue which is almost zero.

A finite element program Finite Element Bifurcation Analysis of Plate and Shell Structures, "FEBAPS", has been developed based on the above-mentioned incremental nonlinear approach. The validity of the program was checked through a numerical example shown in Fig 1. The exact buckling stress of a rectangular plate subjected to biaxial stresses is given as [10]:

$$\frac{\sigma_{xcr}}{\sigma_Y} = \frac{\pi^2 \cdot D}{b^2 \cdot t \cdot \sigma_Y}\left(\frac{m}{C} + n^2 \cdot \frac{C}{m}\right)^2 - \frac{\sigma_y}{\sigma_Y} \cdot C^2 \cdot \frac{n^2}{m^2}, \qquad (10)$$

	$\sigma_{x\,cr}/\sigma_Y$	
	Exact	"FEBAPS"
Uniaxial $\alpha = 0.0$	.6106	.6208
biaxial $\alpha = 0.2$	.4106	.4065

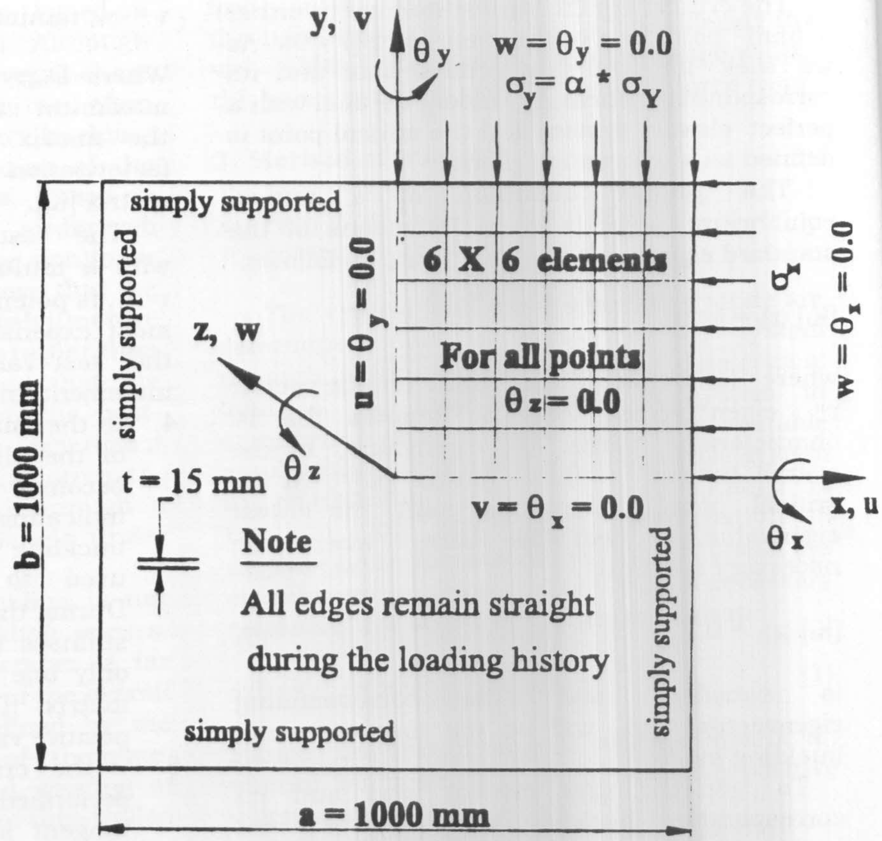

Fig. 1. FEM mesh division and boundary conditions of the plate used for the comparison

Where, δ_{xcr} is the critical axial stress in x-direction, σ_y is the axial stress in y-direction and σ_Y is the Yield stress. C, D, b, t are the aspect ratio, rigidity, width and thickness of the plate, respectively. m and n are the number of buckling waves in x and y directions, respectively.

2.2 Ultimate load and post-ultimate behavior

The finite element method, considering both geometric and material non-linearities, is used to determine the ultimate load and the post-ultimate behavior of the studied steel I-section beam-columns. A quadrilateral isoparametric flat-shell element is used, based on the Mindlin plate theory [11], in which the transverse shear deformations are considered. Selective reduced integration scheme is adopted to prevent the transverse shear and membrane locking. Large deformation, but small strain, is considered on the bases of Total Lagrangian formulation, which uses the Green strain tensor and the second-Piola Kirchhoff stress tensor. The plastic flow theory is applied considering the Mises's yield function as a plastic potential. The material is assumed to follow the kinematic hardening law. An incremental predictor with Newton-Raphson iterations is adopted with the aid of displacement control to trace the entire equilibrium path. The non-proportional loading can be applied. The modified version of the finite element program "CYNAPSS1" [12], based on the above-mentioned features. is employed in this research to trace the complete equilibrium path, getting the ultimate load and the post-ultimate behavior.

3. Numerical studies

The effect of the first local buckling mode on the ultimate load capacity and the post-buckling behavior of steel plate I-section beam-columns is investigated through a parametric study. The structural model and loading condition are indicated in Fig. 2. Both the axial force, P, and the lateral load, H, are concentrated loads applied to the beam-column model at the free end, which has free lateral translations and rotations. A rigid part is attached to the model at the free end, to avoid local yielding caused by the applied concentrated loads. The axial load, P, is kept constant while the lateral displacement is increased. To initiate the local deformations during the numerical computations, the actual first local buckling mode, obtained previously, is imposed as initial imperfection. The height of the model, L, the height of the section web, h, and the flange width, b, are kept constants with the dimensions indicated in Fig. 2. The web panel aspect ratio is constant, (L/h=5). The stress-strain curve of steel material used here is shown in Fig. 3. Several values of web width-thickness ratio, flange width-thickness ratio and axial load ratio are studied as shown in Tables 1, 2 and 3, respectively. The axial load ratio is defined as P/P_Y, where P is the applied axial load and P_Y is the yield axial load.

4. Discussion of results

A typical first local buckling mode, which occurred in the web, is shown in Fig. 4 and a typical one, which formed in the flange, is shown in Fig. 5. It is worth noting that the local buckling in web is associated with out of plane deformations in the flange caused by the web local buckling itself. While, as expected, both the flange and web local buckling formed near the fixed end due to the existing maximum stresses.

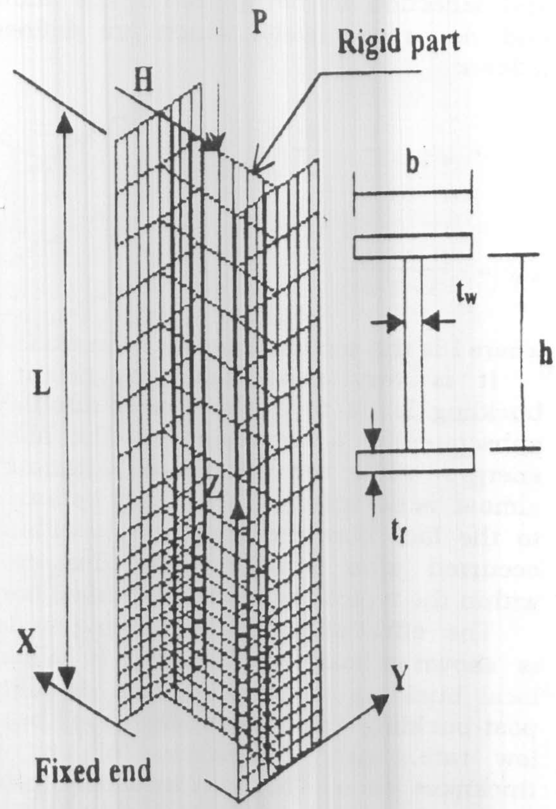

Fig. 2. Structural model.

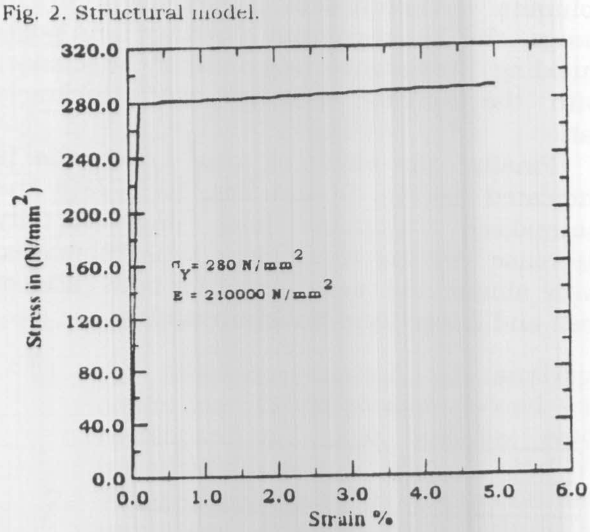

Fig.3. Stress-Strain curve of the used material.

Both the elastic normalized lateral load-deflection relation, till the first critical load, and the full elasto-plastic normalized lateral load-deflection relation for the different cases, are shown in Fig. 6. Where the lateral load and deflection are normalized by the values H_Y and δ_Y, respectively, which are defined as follows:

$$H_Y = \sigma_Y \cdot \frac{I}{L} \cdot \left(\frac{h}{2} + t_f \right) \tag{11}$$

$$\delta_Y = \frac{H_Y \cdot L^3}{3 \cdot E \cdot I} \tag{12}$$

where I is the section moment of inertia.

It is very clear that early elastic local buckling leads to a decrease of ductility and subsequently a decrease in the absorbed energy, while the normalized ultimate load almost is not affected. This may be attributed to the fact that the real first local buckling occurred after reaching the ultimate load, within the typical dimensions studied here.

The effect of first local buckling mode type is shown in both Fig. 7 and Fig. 8. When first local buckling happens in web, ductility and post-buckling resistance decrease, but with low rate, with the increase of web width-thickness ratio. This may be attributed to the main role of flange in the behavior of beam-columns. When first local buckling occurs in flange, the beam-column ductility and post-buckling resistance significantly decrease, with the increase of flange width-thickness ratio.

Finally, the effect of axial force ratio is indicated in Fig. 9 and Fig. 10. Both the normalized ultimate load and ductility decrease as the axial force ratio increases, with almost the same rate for both cases of web and flange local buckling modes.

Table 1. Studied cases with different web width-thickness ratios

h/t_w	b/t_f	L/h	P/P_Y
30	20	5	0.2
45	20	5	0.2
60	20	5	0.2

Table 2. Studied cases with different flange width-thickness ratios.

h/t_w	b/t_f	L/h	P/P_Y
30	25	5	0.2
30	35	5	0.2
30	45	5	0.2

Table 3. Studied cases with different axial load ratios.

h/t_w	B/t_f	L/h	P/P_Y
60	20	5	0.1
60	20	5	0.2
30	45	5	0.1
30	45	.5	0.2

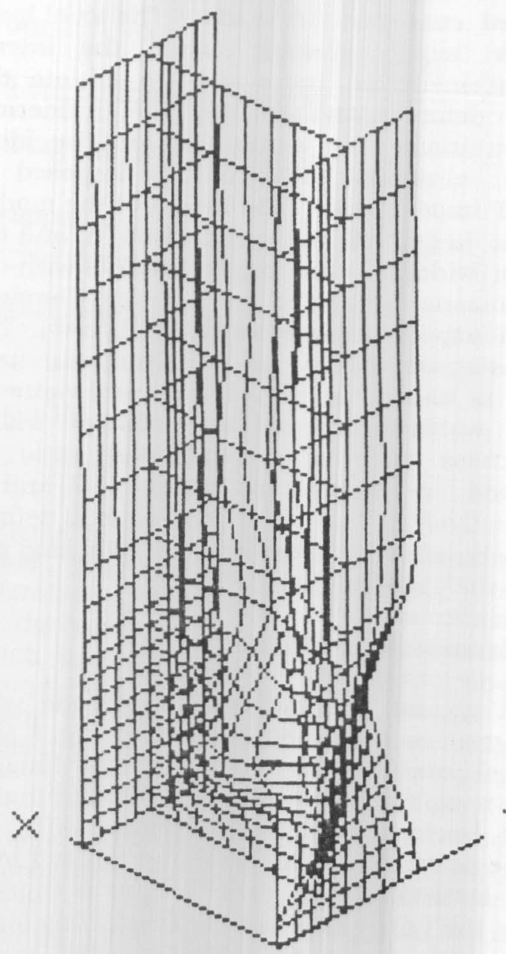

Fig. 4. First local buckling mode (by FEBAPS) in case of h/t_w=60, b/t_f=20 and P/P_Y=0.2.

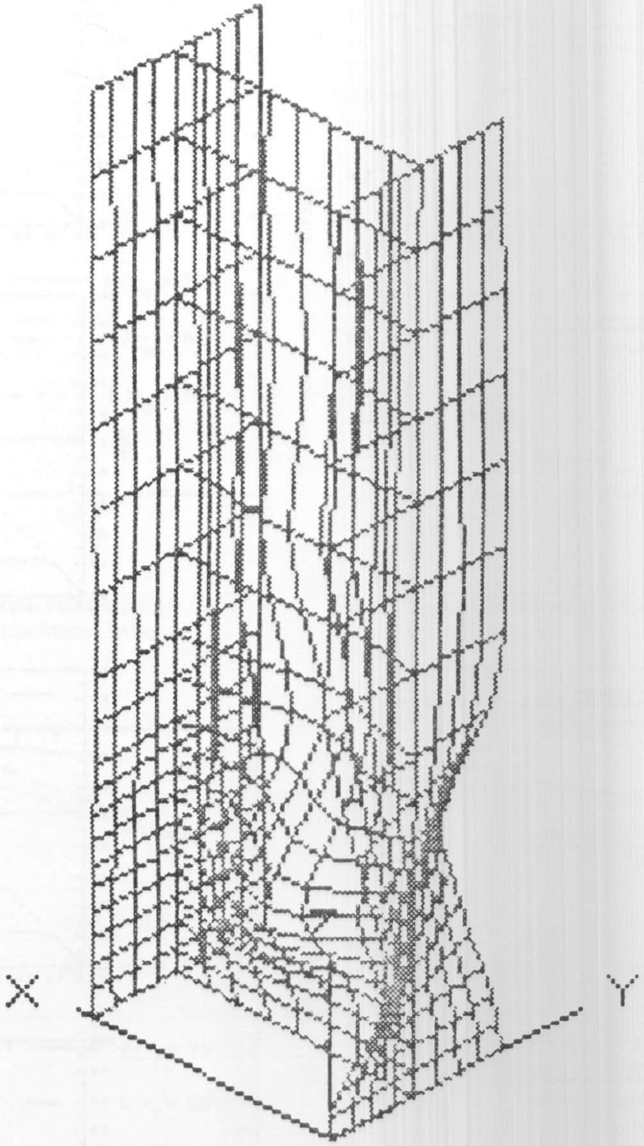

Fig. 5: First local buckling mode (by FEBAPS) in case of h/t_w=30, b/t_f=45 and P/P_Y=0.2.

5. Conclusions

The effect of the first local buckling mode on the ultimate strength and post-ultimate behavior of steel I-section beam-columns was investigated through a parametric study using the finite element method considering both geometric and material nonlinearity. As to the several parameters and dimensions studied here, the following are concluded:

1. An efficient method of analysis to determine the first critical load and its corresponding local buckling mode is introduced in detail.
2. Flange local buckling is more critical than web local buckling, where both the beam-column ductility and post-buckling resistance decrease, with high rate, for the increase of flange width-thickness ratio.

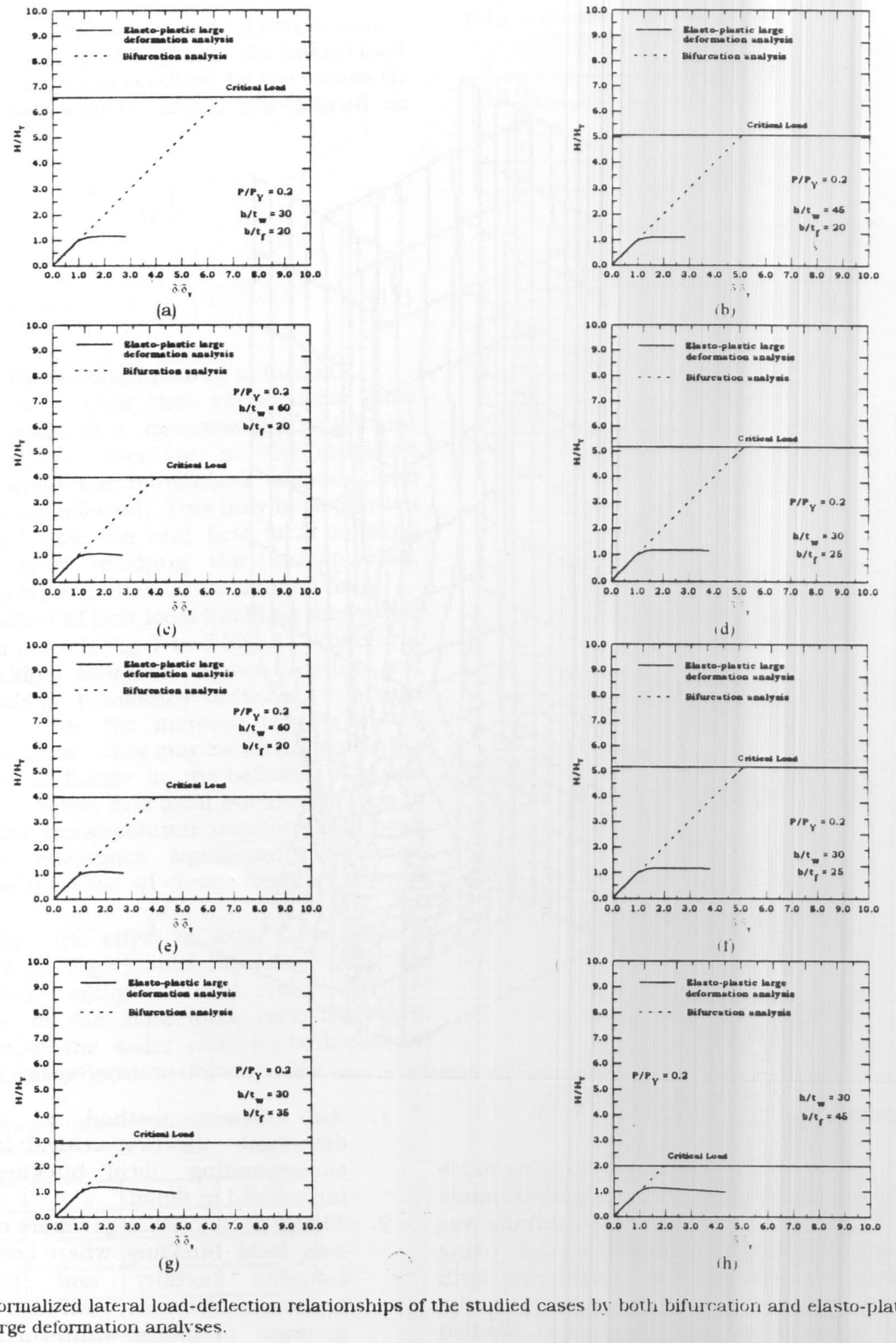

Fig. 6. Normalized lateral load-deflection relationships of the studied cases by both bifurcation and elasto-platic large deformation analyses.

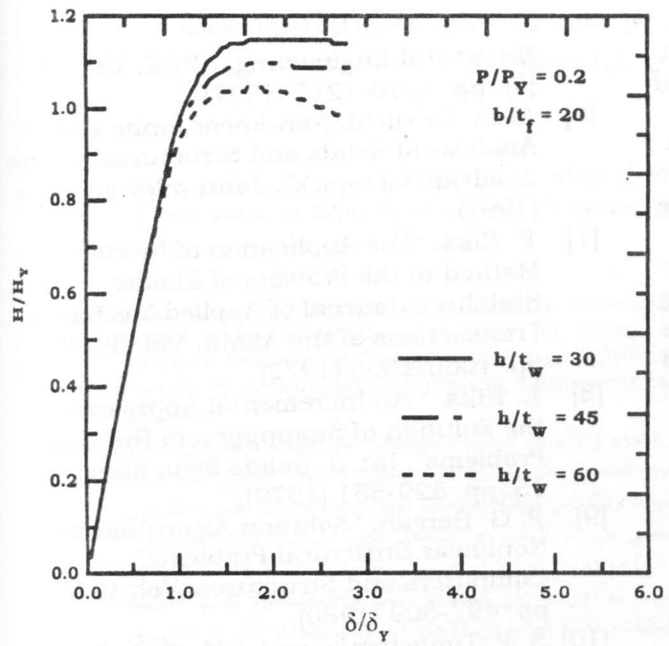

Fig. 7. Normalized lateral load-deflection relationships for different values of web width-thickness ratio.

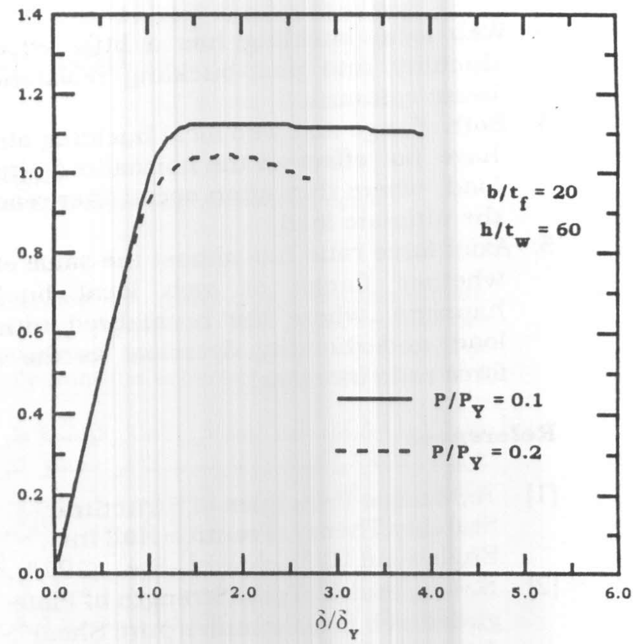

Fig. 9. Normalized lateral load-deflection relationships for different values of axial load ratio.

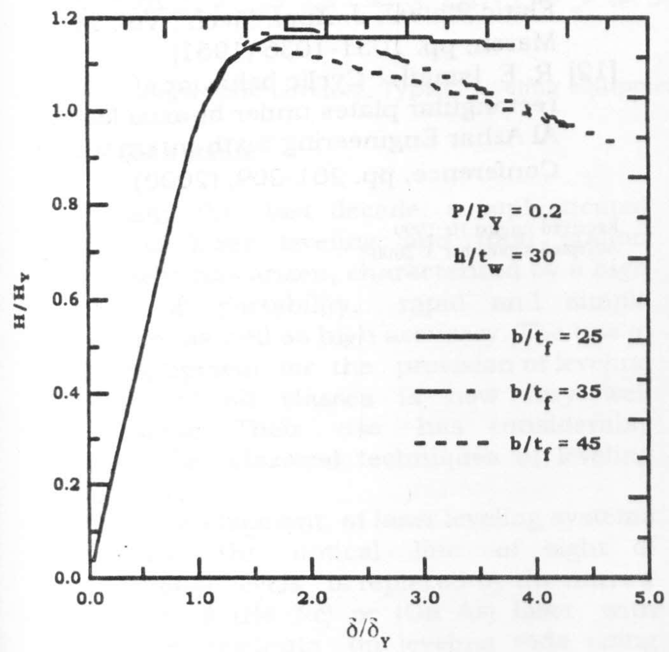

Fig. 8. Normalized lateral load-deflection relationships for different values of flange width-thickness ratio.

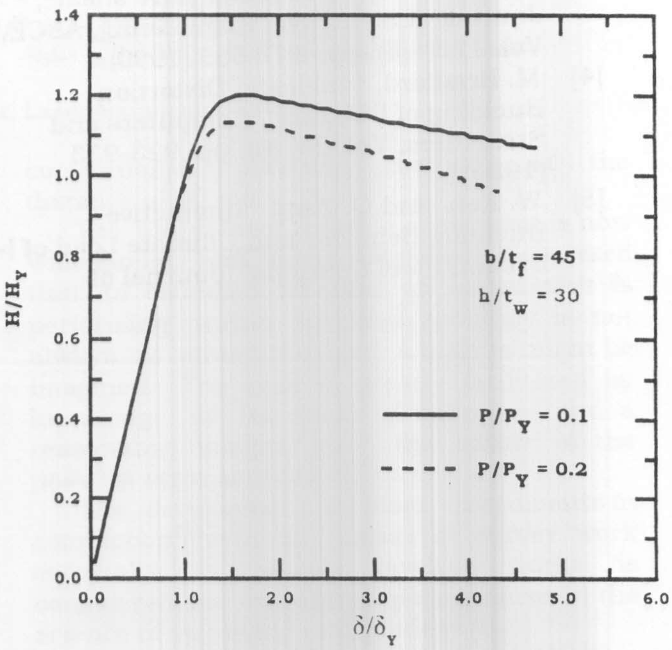

Fig. 10. Normalized lateral load-deflectionrelationships for different values of axial load ratio.

3. Web local buckling has a little effect on ductility and post-buckling resistance of beam-columns.
4. Both flange and web local buckling almost have no effect on the normalized ultimate load, where they often occur after reaching the ultimate load.
5. Axial force ratio has almost the same effect whether flange or web local buckling happens, where the normalized ultimate load and ductility decrease as the axial force ratio increase.

References

[1] A., Chajes Principles of Structural Stability Theory, Prentice-Hall Inc., Englewood Cliffs, N.J., U.S.A., (1974).

[2] S. Lee, and C. Yoo, "Strength of Plate girder web panels under pure Shear", Journal of Structural Engineering, ASCE, Vol. 124 (2), pp. 184-194 (1998).

[3] S. Lee, and C. Yoo, " Strength of curved I-girder web panels under pure shear", Journal of Structural Engineering, ASCE, Vol. 125 (8), pp. 847-853 (1999).

[4] M. Bradford, " Inelastic Distortion Buckling of I-Beams", Computers and Structures, Vol. 24 (6), pp. 923-933 (1986).

[5] W. Ren, and Q. Zeng, "Interactive Buckling Behavior and Ultimate Load of I-Section Steel Columns", Journal of Structural Engineering, ASCE, Vol. 123 (9), pp. 1210-1217 (1997).

[6] M. A. Crisfield, Non-linear Finite element Analysis of Solids and Structures- volume 2: advanced topics", John wiley and sons, (1997).

[7] E. Riks, "The Application of Newton's Method to the Problem of Elastic Stability", Journal of Applied Mechanics, Transactions of the ASME, Vol. 39, Dec., pp. 1060-1066 (1972).

[8] E. Riks, " An Incremental Approach to the Solution of Snapping and Buckling Problems", Int. J. Solids Structures, Vol. 15, pp. 529-551 (1979).

[9] P. G. Bergan, "Solution Algorithms for Nonlinear Structural Problems", Computers and Structures, Vol. 12 (6), pp. 497-509 (1980).

[10] S. P. Timoshenko and J.M. Gere, Theory of Elastic stability, Second Edition, McGraw-Hill Kogakusha (1963).

[11] R. D. Mindlin, "Influence of rotary Inertia and Shear in Flexural Motion of Isotropic, Elatic Plates", J. Appl. Mech., Vol. 18, March, pp. 1031-1036 (1951).

[12] R. E. Ismail, " Cyclic behavior of rectangular plates under bi-axial loading" Al Azhar Engineering Sixth International Conference, pp. 201-209, (2000).

Received August 10, 1999
Accepted September 3, 2000