Comparative study on precise leveling data using different equipment

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It is generally agreed that leveling networks performed by many typical equipment can produce highly accurate results for many engineering purposes. This paper reviews the current leveling techniques and highlights inadequacies of new leveling laser equipment in comparison with typical leveling equipment especially from the accuracy requirements point of view.

نظرا لأهمية شبكات الميزانيات فى الأعمال الهندسية على وجه العموم والأعمال المدنية والمساحية على وجه الخصوص ونظرا لإزدياد الحاجة إلى تثبيت روبيرات جديدة فى المناطق التى يتم فيها العمران حديثا مثل منطقة توشكى ومنطقة شرق التفريعة ومناطق أخرى كان من الواجب إلقاء الضوء على شبكات الميزانيات وذلك من ناحية إستعمال أجهزة مختلفة مثل أجهزة الليزر التى كثرت وإنتشرت حديثا فى الأعمال المساحية. والغرض الأساسى من هذا البحث هو استعمال أجهزة مختلفة لقياس فروق الإرتفاعات وبالتالى حساب مناسيب روبيرات جديدة من (روبيرات) موجودة. ثم إجزاء عمليات الضبط الخاصة طبقا للنموذج الرياضى الذى تم توضحية فى البحث. والجزء العملى فى هذا البحث تم اجراءة على شبكة من الميزانية معلوم فيها ثلاثة روبيرات وذلك لإيجاد مناسيب ثلاثة نقط جديدة. وقد تم الرصد بإستخدام ثلاثة أنواع من الأجهزة أولا: باستخدام الميزان الدقيق ثم بإستخدام ميزان الليزر ثم باستخدام جهاز المحطة المتكاملة . وقد تم إجراء الضبط لهذه الارصاد المختلفة بنفس النموذج الرياضى وذلك لتثبيت طريقة الضبط. وقد تم الحصول على فروق المناسيب فى خطوط الميزانية قبل وبعد الضبط. ثـم تمت عملية المقارنة بين الطرق الثلاث على أساس أن أدق الطرق وكما هو موضح فى النتائج (جداول واشكال)هـى عندما يكون الرصد بإستخدام الميزان الدقيق. وفى نهاية البحث تم التوصل إلى مقدار الدقة عند النقط الجديدة التى حصلنا على مناسيبها من النقط الثابتة (الروبيرات). وقد أثبت البحث أن الدقة التى تم التوصل إليها من الطرق الأخرى (باستخدام جهاز الليزر أو باستخدام المحطة المتكاملة) ليست بعيدة القيمة عن دقة الوازين الدقيقة.

Keywords: Leveling, Typical leveling equipment, Laser equipment, Total station.

1. Introduction

Within the last decade, a sophisticated class of laser leveling and total station equipment has arisen, characterized by a high degree of portability, rapid and simple operation as well as high accuracy. The use of such equipment for the provision of leveling control of all classes is now very well established. Their use has considerably altered the classical techniques of leveling works.

The development of laser leveling systems in which the optical line of sight of conventional levels is replaced by the narrow beam of a (He Ne) or (Ga As) laser, with automatic readouts on leveling rods using photosensitive centering detectors, has enabled to obtain the same reaching accuracy on leveling rods, at a distance of several hundred meters, as can be obtained with conventional precision methods at the distance of 50 ms. [1,3].

The use of laser rotating equipment is now widespread, and many users have discovered that to establish whether an instrument is performing within specified accuracy is not always as straightforward a task as might be imagined. The task is greatly facilitated by knowledge of its mode of operation and a reasonable insight into the nature of the possible error sources[4].

The development of laser instruments in connection with all phases of survey work especially in precise leveling works is considered as one of the epic advances in the science of surveying and geodesy [5-8].

Using new equipment such as total station levels measurements is taken, presenting an intelligent total station concept that is intended to perform extensive surveying work in concert with onboard data storage and software, as well as measuring angles and distances electronically .

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2. Description of the practical leveling network

Figure 1. illustrates a local level network in which A, B and C are three benchmarks(B.M.) with known levels at Nasser hospital (A), Bab sharki police dept. (B), and Faculty of Eng. (C). P_1, P_2 and P_3 are three new level points with unknown levels. This leveling network was established as a test network and connected all the points with seven leveling lines. The lengths $L_1, L_2, \ldots, L_7$ between the points were measured and ranged from 0.450 km to 1.415 km as shown in Fig. 1 .

A local second-order precise leveling network of six points and seven leveling lines was established as a basis for comparison

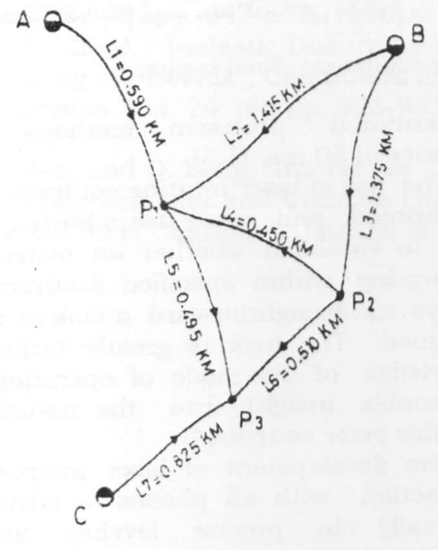

Fig. 1. The out line of the precise leveling net.

The difference in levels for each of these leveling lines was first obtained by using the KONI 007 precise level. The leveling was performed two-way and the observations were then averaged. The error did not exceed the permissible second-order $8\sqrt{K}$. Results of the observed level differences $h_1, \ldots, h_7$ for the conventional KONI 007 precise level, regardless of their adjustment, is given in Table 1.

As for laser leveling, all the possible systematic errors influencing the field performance of the laser LP3A level were reduced to negligible small values also by double (two-way) leveling (the error also not exceeding $8\sqrt{K}$. Results of the observed level differences (averaged), regardless of their adjustment, is given in Table 1.

Using electronic total station POWR SET 3010 the difference in levels for each of these leveling lines was obtained. The leveling was performed also two-way and the observations were then averaged. The error also not exceeding the permissible second-order $8\sqrt{K}$.

3. Adjustment model

The mathematical model for the least square adjustment by the method of observation equations is composed of functional model, which is given in linear form by [2,4]:

$$V = A\,\bar{x} - l\,, \qquad (1)$$

and the stochastic model

$$\Sigma l = \sigma_o^2 . P^{-1}\,, \qquad (2)$$

where:

- V is the n-vector of residual,
- A is the n by u configuration matrix,
- $\bar{X}$ is the u-vector of the unknown parameters,
- l is the n-vector of the absolute terms,
- P is the n by n weight matrix,
- σ_o^2 is the apriori variance factor, and finally
- Σl is the n by n variance-covariance matrix of the observed quantities.

By implementing requirements of the method of least squares

$V^T PV = \min$,

one gets to the system of normal equations

$$A^T P A \bar{X} - A^T P l = 0 , \quad (3)$$

or

$$N\bar{X} - U = 0 . \quad (4)$$

The system (4) can be solved easily if r = u and r = rank (A) = rank (N) and the best estimates for the weight coefficient matrix is:

$$Q_{\bar{x}} = N^{-1} . \quad (5)$$

In case of free network r <u the matrix of normal equations N is singular and the cayley inverse N^{-1} does not exist. Such a problem can be practically solved by using the so-called the Moore-Penrose Pseudo inverse, or some other simple approaches like the inner constraints procedure [2,4].

The best estimates for the solution vector $\bar{x}$ and their corresponding variance-covariance matrix $\Sigma \bar{X}$ are obtained as:

$$\bar{X} = Q_{\bar{X}} U , \quad (6)$$

and

$$\Sigma \bar{X} = \bar{\sigma}_o^2 . Q_{\bar{X}} , \quad (7)$$

where:

$\bar{\sigma}_o^2$ is the a posteriori variance factor which is computed from:

$$\bar{\sigma}_o^2 = (V^T P V) / r , \quad (8)$$

with r = n –u (in case of fixed network),
or r = n – u +d (in case of free network),
where:

r is the number of degrees of freedom (redundancies),
n is the number of the observations,
u is the number of the unknown parameters(point levels),
d is the datum defect.

Finally, The adjusted values of the observations, as well as, their estimated variance-covariance matrix can be computed as:

$$\bar{L} = L + V , \quad (9)$$

and

$$\Sigma \bar{L} = \bar{\sigma}_o^2 . Q_L , \quad = \bar{\sigma}_o^2 (A Q_{\bar{x}} - A^T) . \quad (10)$$

4. The precision concept

In fixed leveling network, the mathematical model for the estimated level difference from the point P_i to any fixed point P_f in an arbitrary datum system is given by [4]:

$$\Delta h_i = H_i - H_f . \quad (11)$$

in which H_f is the errorless level of the fixed point P_f. By applying the error propagation concepts using the variance law on Eq. (11), it was found that [4]:

$$\sigma_{\Delta \bar{h}i} = \delta \bar{H}_i . \quad (12)$$

Therefore, it can be concluded that, in fixed leveling network the standard error of any new point level $\delta \bar{H}_i$ is identical with the standard error of the level difference between it and the level of any fixed point. Where the standard error $\sigma_{\Delta \bar{h}i}$ can be obtained from:

$$\delta_{\Delta \bar{h}i} = \sigma_o \sqrt{(A Q_{\bar{x}} A^T)} . \quad (13)$$

All these equation are new in form that is easily programmable, thus allowing us to graphically see the precision concept of each feeling.

5. Results and analysis

As for least squares adjustment of the leveling network, the method of network, the method of the observation equation was applied for the observed data. The a priori variance factor was assumed to be unity for simplicity.

The weights are established inversely proportional to the distances of the leveling. lines. The adjusted values h_1, h_2,...h_7 for both precise laser and total station leveling are given in Table 2. Table 3 shows the levels of the known benchmarks (A, B, C), as well as the levels of the new points (P_1, P_2, P_3) calculated from the adjusted aforementioned level differences for both methods in Table 2.

Table 1. The results of the practical leveling in network.

Difference In levels	I	II	III	Δh=II-I	Δh=III-I
	With KONI 007 precise leveling	With LP 3A Laser leveling	With POWER SET 300 Tatal Station leveling	Discrepancy Between I, II mm	Discrepancy Between I, III mm
h_1	1.63207	1.6396	1.6440	7.53	11.93
h_2	9.22407	9.2306	9.241	6.53	16.93
H_3	6.17584	6.1865	6.192	10.66	16.16
H_4	3.04023	3.0515	3.061	11.27	20.77
h_5	2.12093	2.1318	2.138	10.87	17.07
h_6	5.16818	5.1829	5.191	14.74	22.84
h_7	0.99875	1.0098	1.012	11.05	13.25

Table 2 .The adjusted differences in leveling lines.

Difference In levels	I	II	III	Δh=II-I	Δh=III-I
	Precise leveling (M)	Laser leveling (M)	Total Station Leveling (M)	Discrepancy Between I,II mm	Discrepancy Between I, III mm
h_1	1.63307	1.6396	1.644	6.53	10.93
h_2	9.23007	9.2366	9.241	6.53	10.93
h_3	6.17734	6.1865	6.1835	9.16	6.16
h_4	3.05274	3.041	3.0575	1.36	4.76
h_5	2.12093	2.1318	2.125	14.87	8.07
h_6	5.16966	5.1819	5.1825	12.24	12.84
h_7	0.9930	1.0144	1.012	21.4	19.00

Table 3 .The adjusted levels.

Station	I	II	III	Δh=II-I	Δh=III-I
	Precise leveling (M)	Laser leveling (M)	Total station Leveling (M)	Discrepancy Between I,II mm	Discrepancy Between I, III mm
A	16.038	16.038	16.038	0.0	0.0
B	8.440	8.440	8.440	0.0	0.0
C	18.794	18.794	18.794	0.0	0.0
P_1	17.67007	17.6766	17.681	6.53	10.93
P_2	14.61734	14.6265	14.6235	9.16	6.16
P_3	19.7870	19.8084	19.806	21.40	19.00

Table 4. The standard error (σ) at new points.

New Stations	I	II	III
	Precise leveling σmm	Laser leveling σmm	Total station σmm
P_1	1.51	2.21	2.65
P_2	1.43	2.16	2.54
P_3	1.48	2.19	2.61

The following sources of errors in case of laser equipment may by minimized to a great extend or, partially or totally eliminated, namely:

- Effect of imperfect adjustment of the instrument.
- Parallax, which produces an accidental error.
- Variations in temperature.
- Atmospheric refraction, which causes systematic errors.
- Earth's curvature.
- Number of set-ups of the instrument.

The effect of most of these sources of errors is not of much consequence in leveling using laser equipment.

From the analysis just discussed, it will appear that under normal conditions. The important errors which are majority accidental will be minimized to a great extend in case of using laser equipment.

Experiences in general bears out these conclusions, and for this reason it is customary to express limiting errors of leveling in terms of the square root of the distance, usually in kms.

From the previous results Tables 1-4 and Figs 2-5 it is readily seen that, in leveling network the standard error (σ) of any new point is identical with the standard error of the level difference between it and the average of all the levels of network points.

Thus any leveling procedure could be done as an individual process or possibly incorporated within the other methods.

In such case the accuracy would be maintained.

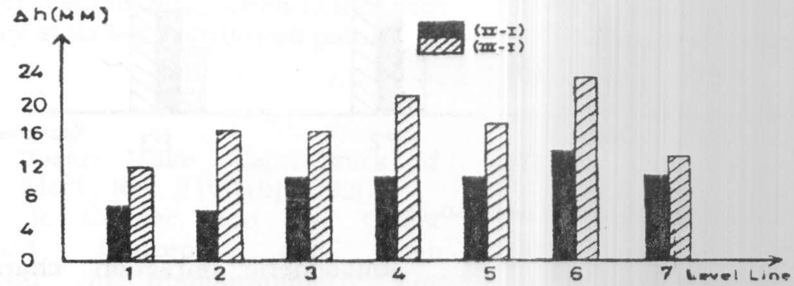

Fig. 2. Crude observations.

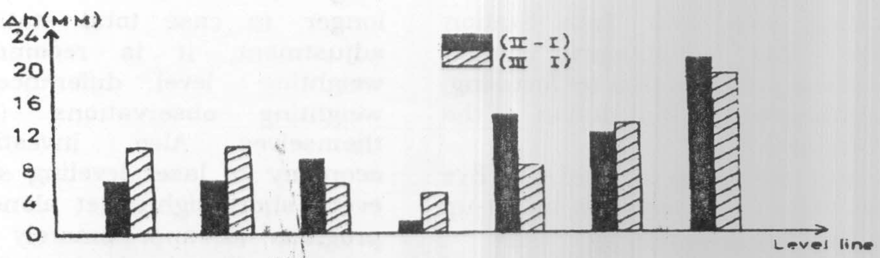

Fig. 3. Adjusted observations.

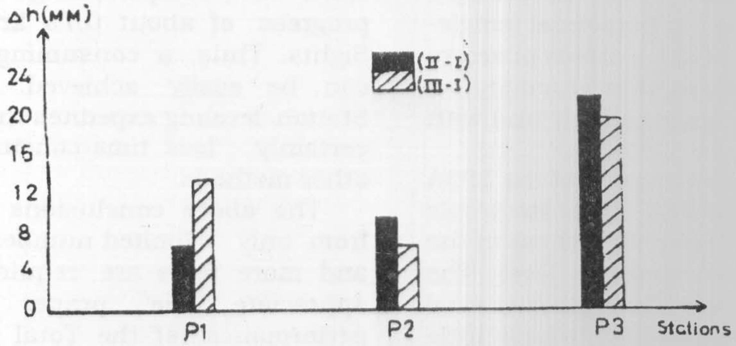

Fig.4. Adjusted levels.

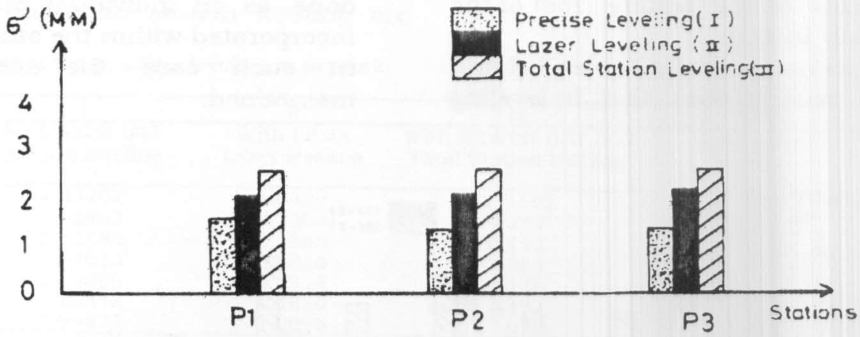

Fig. 5. Standard error at new stations.

6. Conclusions

In some countries and due to various basic reasons such as cost, maintenance facilities, calibration, etc... Laser and total station surveying instruments have unfortunately not been completely integrated into all the various facets of surveying.

Laser rotating level and Total Station Systems have the advantages against conventional optical methods that no focusing, instrument adjustment or instruction of the rod carrier is required.

Also, the unit need not be Set-Up at Eye Level and the reference planes can be set-up directly where they are needed.

Further, and w.r.t construction, several workers can use the same exact reference plane at the same time. These advantages yield substantial saving in personnel (single-operator), diminution of the sources of errors and a substantial increase of productivity, as work can be performed more quickly and with ease.

The accuracy versus economy of the LP3A laser level and POWER SET 3010 electronic total station were compared with those of the conventional KONI 007 precise level. The differences between the results of laser or total station and precise leveling show to be of little significance and seen to be due to atmospheric refraction changes as well as eventual laser and total station drift.

The above preliminary results, though too few, show that one can expect to obtain the same accuracy with the laser and total station systems as with the precise conventional. Levels using lengths of sights (3 to 4 times longer in case of the laser) or (5 to 6 times longer in case total station). Concerning adjustment, it is recommended to use weighting level differences rather than weighting observations (Residual levels) themselves. Also, investigation on the economy of laser leveling shows that using even short sights, let alone long ones, the progress is approximately 1.5 km/hr. And using total station is about 2.5 km/hr. This is a bit more than the progress of the precise KONI 007 compensating level that achieves a progress of about 0.75 km/hr and 50 ms. Sights. Thus, a consuming ratio 1.67:1:3.34 can be easily achieved. Accordingly, Total Station leveling expedites the field work and is certainly less time-consuming than in the other methods.

The above conclusions have been drawn from only a limited number of measurements and more tests are required in order to fully appreciate the proper evaluation and performance of the Total Station and laser level systems. Generally, the application of Total Station and laser equipment in differential leveling has confirmed their full usefulness, and the results are consistent and advantageous, both accuracy-wise and economical.

The effect of imperfect adjustment of the instrument is minimized in case of laser equipment and eliminated entirely in case of Total Station.

Therefore laser equipment Total Station can be used instead of typical precise leveling equipment for establishing bench marks with great accuracy at widely distributed points.

References

[1] Anon., "laser Make Light work of leveling", Elect., Rev.,210 (16)(1982).

[2] M. A. R. Cooper, and P.A. Gross "Statistical Concepts and their application in photogrammetry and surveging", Photagrammetric Record12 (71), pp. 645-677(1991).

[3] K. Fila, "The use of laser in levelling",M.SC. Thesis, dept. of Surveying Engineering UNB, Fredeiction (1971).

[4] E. Gotthardt, "Einfiihrung in die Ausgleich-ungsrechnung", 2-Auflage. Karlsruhe, Herbert Wichmann Verlag (1978).

[5] M. Hosny, A. Rahim and M. Nabil Aly Shukry, "Laser Revolution and Its Perspective Dimensions", Alexandria Engineering Journal. Vol. 30,(3), pp. C173-c188 (1991).

[6] W.F. Price, and J.Uren, "Laser Surveying", VNR International, London (1989).

[7] J. Uren, "Laser Surveying I-current Equipment and User Survey", C.E.S, 12(8), 20 (1987).

[8] J. Uren, and E.Bullock, "Laser Surveying III-attitudes to Safety", C.E.S., 12(10), 20 (1987).

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