Comparative study for the computational methods of uplift pressures underneath hydraulic structures

A.A. El- Masry
Faculty of Engineering Mansoura University Mansoura Egypt
Yousry Ghazaw
Faculty of Engineering Alexandria University Alexandria Egypt

This paper introduces a comparative study between five methods to compute uplift pressure underneath a dropped floor with rows of sheet piles. The methods are Bligh, Lane, Fragments, electrical analogue, and the method of finite elements. Results show that. all the applied methods give different values of uplift pressure than the finite elements one. Comparing with the results (for the case study) of finite elements, Bligh differs by about 59%, Lane by 43%, Fragments method by 13%, while electrical analogue method gives only 1.4% deviation from the finite elements one.

في هذا البحث تم حساب قوة التعويم اسفل منشا ذو فرشة متدرجة المستوي ومزودة بثلاث صفوف من الستائر الحديدية وذلك باستخدام خمسة طرق مختلفة. الطرق الخمسة التي تم تطبيقها هي: طريقة بلاي - طريقة لين - طريقة الشرائح - طريقة العناصر المحدودة - طريقة التشابه الكهربي . نظرا لدقة نتائج نظرية العناصر المحدودة تم مقارنة نتائج باقي الطرق بها للحكم مدى دقة نتائج كل منها. من المقارنة وجد أن الفرو قات كانت ٥٩% لطريقة بلاي - ٤٣% لطريقة لين -١٣% لطريقة الشرائح- بينما لم تتجاوز ١.٤% لطريقة التشابه الكهربي.

Keywords: Drop structure, Uplift pressure, Fragments, Seepage, Finite elements.

1. Introduction

The uplift pressure acting along the subsurface contour of water-retaining structures has always been an important design consideration for geotichnical engineers. Furthermore, when hydraulic gradients in the ground are significant, a knowledge of their magnitude is essential to guard against heave or piping, especially at the exit points where the flow direction opposes gravity.

For steady state conditions, the total head potential, h, within a flow domain with known boundary conditions is governed by Laplace's equation

$$k_X \frac{\partial^2 h}{\partial x^2} + k_Y \frac{\partial^2 h}{\partial y^2} = 0 , \qquad (1)$$

where k_X= horizontal permeability, and k_Y= vertical permeability. A wide variety of methods exist for its solution. The methods include analytical solutions using conformal mapping and complex functions [1], analogue methods[2], sand tank model [3], viscous flow model [3], numerical methods [4,5], and stochastic analyses [6]. The best known method in soil mechanics, however, is flow net sketching [3]. Flow nets are a powerful and versatile method in experienced hands, but they can be time consuming, and their accuracy is sometimes difficult to assess. An alternative, simple, although approximate approach for solving problems of confined flow is the method of fragments [7]. Empirical method such as Bligh's method and weighted creep method (Lane) are also available [8,9]. Since the methods of Bligh, Lane, Fragments, Electrical analogue and Finite elements are most common by used in Egypt, these methods will be addressed.

In this paper, finite elements method, Bligh method, weighted creep theory (Lane), electrical analogue method, and the method of fragments are applied to compute uplift pressure for different combination of hydraulic structures with dropped floor. The computed uplift pressures are compared and the observed remarks are listed.

1.1. Bligh's creep theory

The creep path is the name given to the path followed by seepage flow as it percolates through the foundation from headwater to tailwater. If the percolation distance is too short, a steep hydraulic gradient will exist through the foundation, and transport of the particles may result. A progressive loss of foundation material may occur, starting at the point of egress of the seepage flow. This is called a piping failure because a continuos void or "pipe" develops in the foundation. Piping usually results in structure collapse due to loss of support.

Seepage flow will naturally follow the path of least resistance, and this may very well be along the contact line between the structure and the foundation because it is difficult to secure an intimate contact between the plane surface of the structure and the granular soil on which it rests. This was the basis for the original creep-head design method proposed by Bligh in 1912. The length of the creep path along the contact line is designated as L_T. This include the vertical distances on both sides of the cutoff walls as well as the horizontal distances along the base of the structure, i.e. $L_T = L$, and $L+2d_1+2d_2$ in Fig. 1-a and 1-b, respectively.

The maximum differential head, which may exist across the structure, is designated as H. this is the head (headwater minus tailwater) which must be dissipated along the path taken by the underseepage. The ratio L_T/H is called the creep head ratio, this ratio must exceed a certain specified value, depending upon the foundation type, to insure safety against piping. The recommended safe values for the creep-head ratio, C, for different foundation materials are given in Table 1. Punmia and Pande [10].

1.2. Lane's weighted creep theory

In 1935, Lane suggested that in calculating the creep path length, greater weight should be attributed to creep along steep or vertical surfaces than to flat or horizontal surfaces. Lane studied some 278 dams, including many at which piping failures occurred,

Table 1. Creep head ratio (C).

Type of soil	Value of C
1. Light silt and mud (as that of the Nile)	18
2. Fine micaceous sand (as in north India rivers)	15
3. Coarse grained sand (as in central and south India)	12
4. Boulders or shingle, gravel and sand mixed	5-9

and proposed the following weighting factors:

1. Unity for contact surfaces steeper than 45 degrees;
2. One-third for contact surfaces flatter than 45 degrees; and
3. Two for a creep path taken through virgin or undisturbed foundation.

The value of L_T used in calculating the weighted creep-head ratio is then the weighted creep distance and not the actual length of the path.

1.3. The method of fragments

Pavlovsky first proposed the method of fragments in 1935 for the computation of seepage under hydraulic structures incorporating multiple piles. The method was publicized by Harr [3], and more recently described in a standard soil mechanics text by Holtz and Kovacs [11]. The method is a powerful though approximate means of obtaining seepage quantities in confined flow problems. Although the method also has applications in free surface problems [3], the present discussion is confined to configurations in which the boundaries of the flow regime are known a priori.

The method is approximate because it relies on the assumption that the equipotential at certain points within the soil mass are vertical. The zones between these 'vertical' equipotentials are called fragments, and because the fragments are often rectangular, with simple boundary conditions, solutions for the individual fragments can be found, and superposed to give the overall solution. A dictionary of the fragment types and associated form factor (Φ) are presented in Fig. 2.

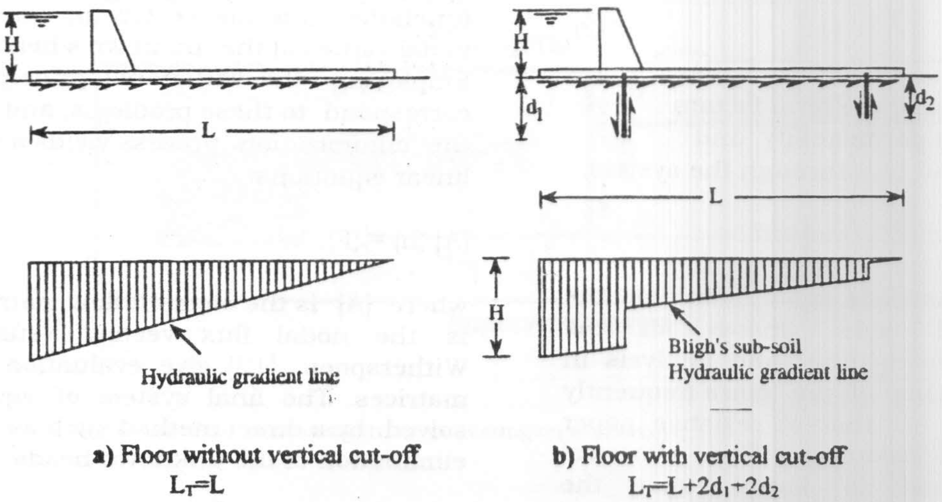
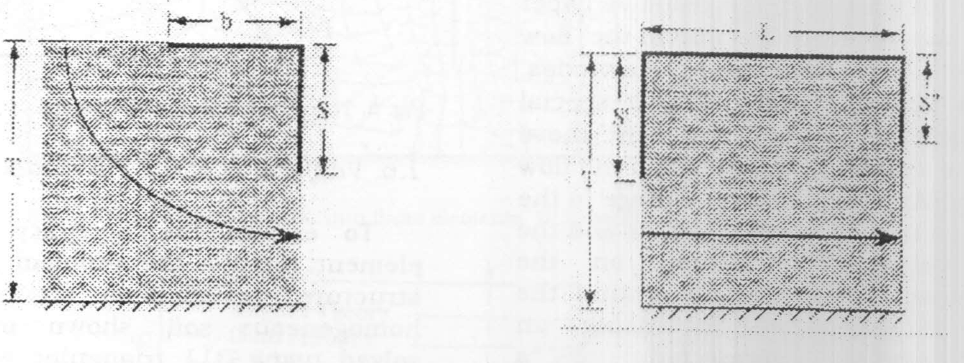


Fig. 1. Bligh's creep theory.



Fragment type I

$$m = \cos\frac{\pi s}{2T}\sqrt{\tanh^2\frac{\pi b}{2T} + \tan^2\frac{\pi s}{2T}}$$

$$\Phi = K / K'$$

Fragment type II

$$C_1 = \left(1 - \frac{S'}{T}\right)\left(1 - \frac{S''}{T}\right)$$

$$C_2 = \frac{L - (S' + S'')}{T}$$

$$\Phi = (C_2 - \ln C_1) \qquad C_2 > 0$$

$$\Phi = \ln\left(\frac{(2 + C_2)^2}{4C_1}\right) \qquad C_2 < 0$$

Fig. 2. Dictionary of confined flow fragment types and form factors after Harr [3].

The quantity of flow can be obtained when the form factors for all fragments are evaluated. Since the flow through all fragments is a constant, the quantity of flow (Q) can be calculated from the following equation:

$$Q = kH / \sum \Phi_i \, . \qquad (2)$$

Where:

$\Sigma\Phi_i$ is the sum of the form factors

k is the soil permeability, and

H is the head loss through the system.

1.4. Electrical analogue method

Analogue methods are undeniably the most powerful aids known for the instruction, investigation and analysis in ground water. One of the more frequently used methods is that of resistive paper electrical field plotting. The electrical analogue method is based upon the similarity of Ohm's law for the movement of electric current through electrical conductors, and Darcy's law for ground water flow.

In applying this method, the resistive paper represents the medium in which the flow takes place. The silver paint electrodes, drawn to a suitable scale using a special highly conductive paint, represent those parts of the system between which the flow takes place. Applying a low voltage to the electrodes causes a current to flow and the resulting voltage distribution on the resistive paper, between and around the electrodes is measured by using an exploratory probe connected to a galvanometer and potentiometer in the well known Whetstone bridge arrangement.

1.5. Finite element method

In finite element method, the solution of the governing equation with its boundary conditions is sought by dividing the flow domain into a network of elements connected at common nodes, which collectively approximate the shape of the domain. The used three node tringular element is shown in Fig. 3. For each element the total head h is approximated by, Zienkiewicz [4];

$$h = [N] \{h\}. \qquad (3)$$

Where the matrix [N] consists of the shape functions and the vector {h} contains the nodal values of the unknown's heads.

Employing the variational principles, that correspond to these problems, and following the minimization process yields a system of linear equations:

$$[A] \{h\} = \{F\}, \qquad (4)$$

where [A] is the element flow matrix and {F} is the nodal flux vector. Neuman and Witherspoon [12] give evaluation of these matrices. The final system of equations is solved by a direct method such as Gaussian elimination of the unknown heads.

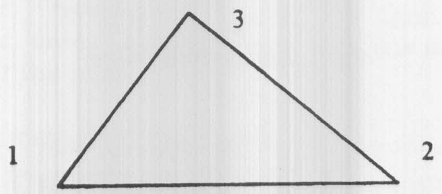


Fig. 3. Three node triangular element.

1.6. Verification of the F.E model

To ensure the accuracy of the finite element results, the problem of an overfall structure resting on an isotropic and homogeneous soil, shown in Fig. 4, is solved using 311 triangular elements with total of 185 nodes, Fig. 5.

The upstream and downstream reaches, X are considered equal to the depth of the permeable strata,T. Fig. 5. shows the finite element mesh for the domain for case of ($S/P=2.$; $L_1/P = L_2/P = 4.$, and $S/T = 0.5$). Fig. 6 shows the comparison between the results of the finite element method and that obtained by Gutti [13] using Pavlovsky's method, which involves the use of conformal transformations. Bligh's solution is also included. From this comparison, it is noted that, there is a good agreement between finite element solution and the analytical one. Also, it is observed that Bligh's method gives a lesser uplift pressure at the critical section, point 4 in Fig. 6.

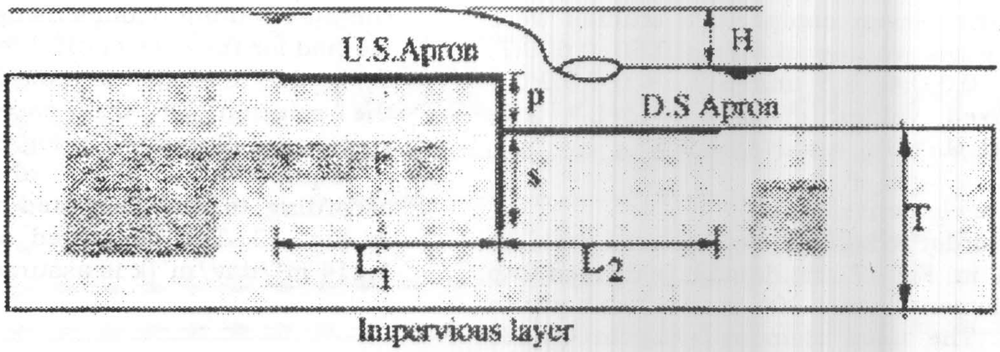


Fig. 4. Layout of an overfall structure resting on an isotropic soil.

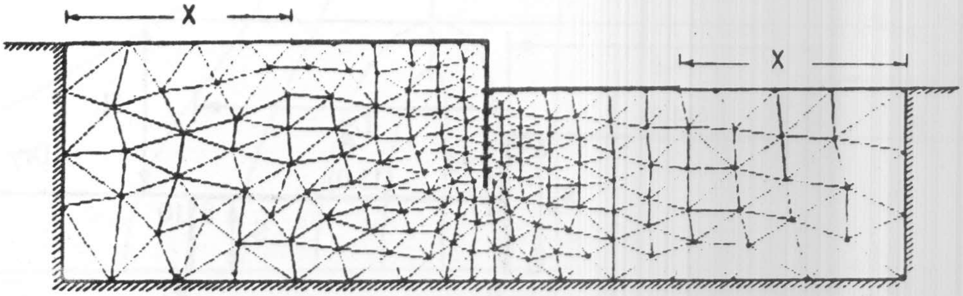


Fig. 5. Division of the region into finite elements, (F.E. Mesh).

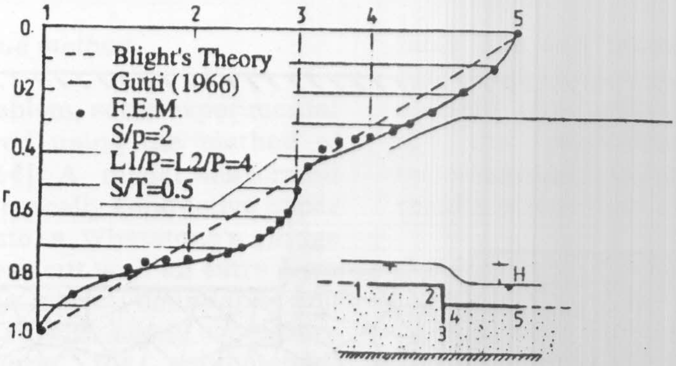


Fig. 6. Uplift pressure distributions for an overfall structure.

Since the methods of Bligh, Lane, and Fragments are approximate, the method of finite element will be considered as a reference of the computations.

2. Computation of uplift pressure by different methods

To make a comparison between the above mentioned methods, the uplift pressures underneath a dropped floor with three rows of sheet piles are calculated, Fig. 7. Twenty seven comparitive studies were made using values of S_1/L_1= 0.50, 0.6, 0.7, S_2/L_1= 0.3,0.4, 0.5 and S_3/L_1= 0.1,0.2,0.3 and fixed values of L_2/L_1= L_3/L_1= 1.0, d_1/L_1=d_2/L_1= 0.1, and H/L_1= 0.4.

1) Finite element method

In order to solve the considered problem shown in Fig. 7, the domain is divided into 332 elements with 206 nodes as shown in Fig. 8. The used element is the three-node triangular one, Zienkiewicz [4]. The computed uplift values using finite element method for the case of (S_1/L_1=0.6, S_2/L_1=0.4, S_3/L_1=0.2) are listed in table 2. As a result of the solution for this case, the velocity vectors are illustrated in Fig. 9.

Equipotential lines are also plotted according to the computed heads as shown in Fig. 10. The computed seepage flow is 1.114 m^3/day/m' (k is assumed 1.0 m/day).

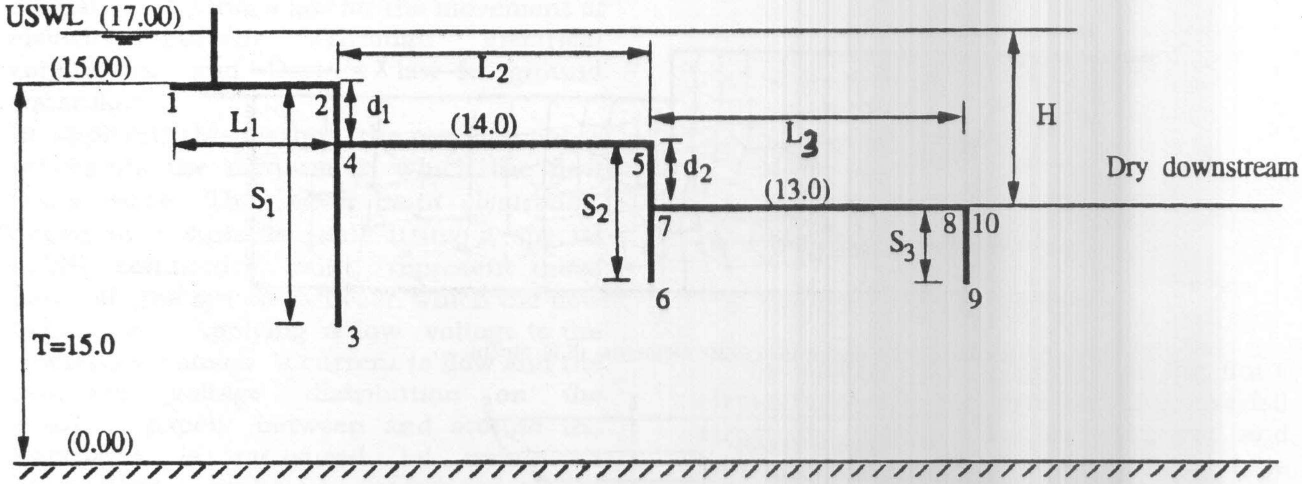


Fig. 7. Equipotential lines for the studied case.

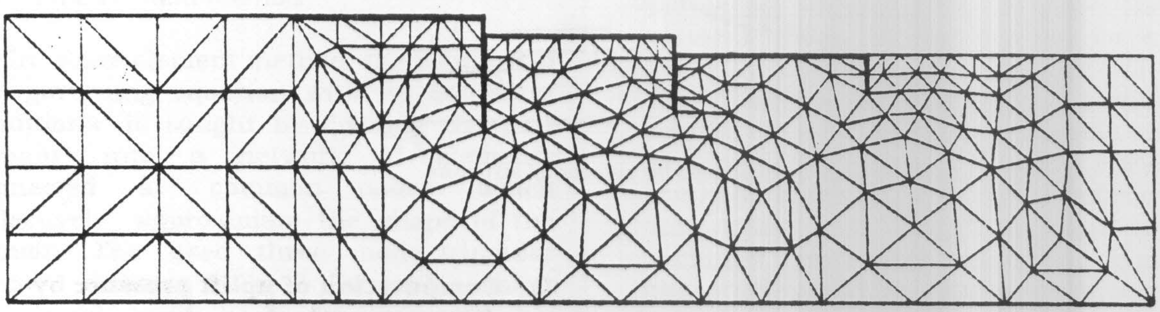


Fig. 8. Layout of the considered case of a dropped floor with three rows of sheet piles.

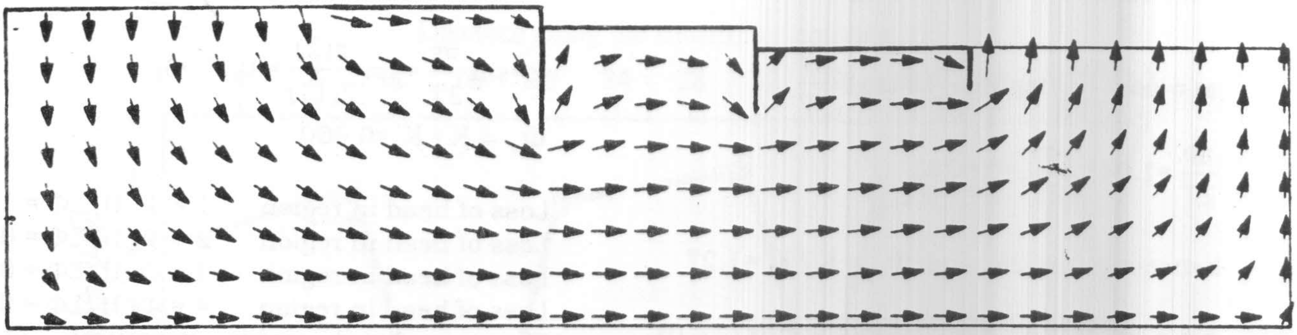


Fig. 9. Division of the domain into finite elements (F.E. mesh).

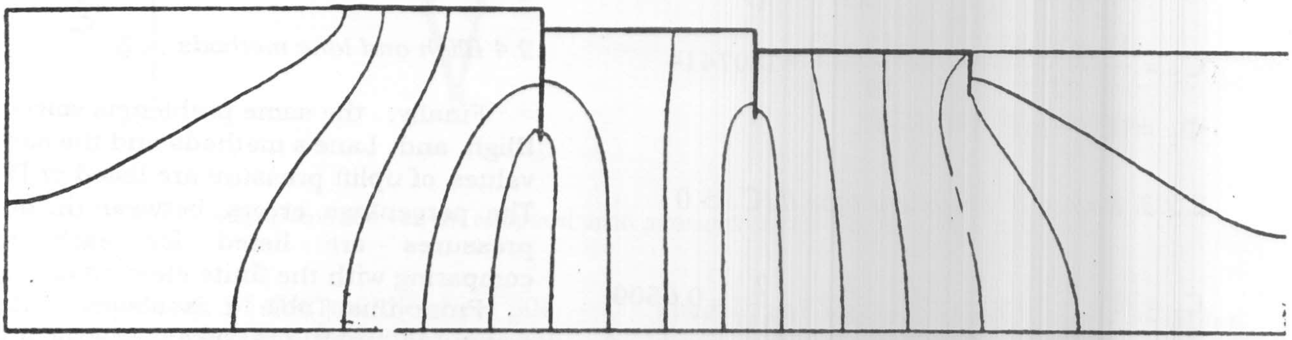


Fig. 10. Seepage velocity vectors according to F.E. results (S_1/L_1=0.6, S_2/L_1=0.4, S_3/L_1=0.2).

2.2. Electrical analogue method

For the same problem, some experimental results were acquired using the method of electrical analogy [14]. A model was formed from a sheet of electrically conductive paper and incorporated into a Whetstone's bridge circuit. The model was cut with an extra 1 cm deep strip along the upper boundaries and this strip was coated with highly conductive silver paint to form the equipotential boundaries. Wires were soldered directly to these boundaries to avoid the problem of variable contact resistance. Three comparative studies were made using values of S_1/L_1= 0.50, 0.6, 0.7 and fixed values of L_2/L_1= L_3/L_1= 1.0, S_2/L_1= 0.4, S_3/L_1= 0.2 ,d_1/L_1= d_2/L_1=0.1, and H/L_1= 0.4. The thickness of the floor was neglected. For each S_1/L_1 value, the relative potentials at the key point were measured. A set of results is listed in Table 2. Upon observing the results in this table one can immediately see that a good agreement between experimental and finite element results prevails. The maximum value of the percentage error between the experimental results and the finite element results is less than 2%.

2.3 Fragments method

The same problem have been solved using the methods of fragments, Fig. 11. shows how the region beneath the structure could be fragmented. Assuming vertical equipotentials at significant changes in boundary geometry, the problem is reduced to the superposition of four fragments. When the form factor, Φ, for all the fragments has been obtained, the total seepage quantity, Q, can be computed using Eq. (2). Form factor Φ is a function of the fragment type and its dimensions as shown in Fig. 2.

2.3.1 Region 1: Fragment type I

$$m = \cos\frac{\pi s}{2T}\sqrt{\tanh^2\frac{\pi b}{2T} + \tan^2\frac{\pi s}{2T}}$$

$$\frac{\pi b}{2T} = 1.047, \frac{\pi s}{2T} = 0.6283,$$

hence m= 0.8628, and $\Phi_1 = K/K' = 1.27$

2.3.2. Region 2: Fragment type II $(C_2 > 0)$

$$C_1 = \left(1-\frac{S'}{T}\right)\left(1-\frac{S''}{T}\right) = \left(1-\frac{5}{14}\right)\left(1-\frac{4}{14}\right) = 0.4592$$

$$C_2 = \frac{L-(S'+S'')}{T} = \frac{10-(5+4)}{14} = 0.07414$$

$$\Phi_2 = (C_2 - \ln C_1) = 0.8497.$$

2.3.3. Region 3 Fragment type II $(C_2 > 0)$

$$C_1 = \left(1-\frac{S'}{T}\right)\left(1-\frac{S''}{T}\right) = \left(1-\frac{3}{13}\right)\left(1-\frac{2}{13}\right) = 0.6509$$

$$C_2 = \frac{L-(S'+S'')}{T} = \frac{10-(3+2)}{13} = 0.3846$$

$$\Phi_3 = (C_2 - \ln C_1) = 0.814$$

2.3.4. Region 4: Fragment type I (b=0)

$$m = \sin\frac{\pi s}{2T} = \sin\frac{\pi(2)}{2(13)} = 0.2393$$

$$\Phi_4 = K/K' = 0.560$$

Loss of head in region 1 = $\Phi_1.H/\Sigma\Phi$ = 1.454
Loss of head in region 2 = $\Phi_2.H/\Sigma\Phi$ = 0.973
Loss of head in region 3 = $\Phi_3.H/\Sigma\Phi$ = 0.932
Loss of head in region 4 = $\Phi_4.H/\Sigma\Phi$ = 0.641

The computed uplift pressures at the numbered points are listed in Table 2.

Seepage computation:
Quantity of seepage = $k.H/\Sigma\Phi$ = 1.145 m^3/day

2.4 Bligh and lane methods

Finally, the same problem is solved using Bligh and Lane's methods and the computed values of uplift pressure are listed in Table 2.. The percentage errors between the obtained pressures are listed for each method comparing with the finite element one.

From this Table it is observed that the computed uplift pressure using the fragment method is the closest one to the finite element result. Maximum difference lies at point 5 where the flow tends to move towards

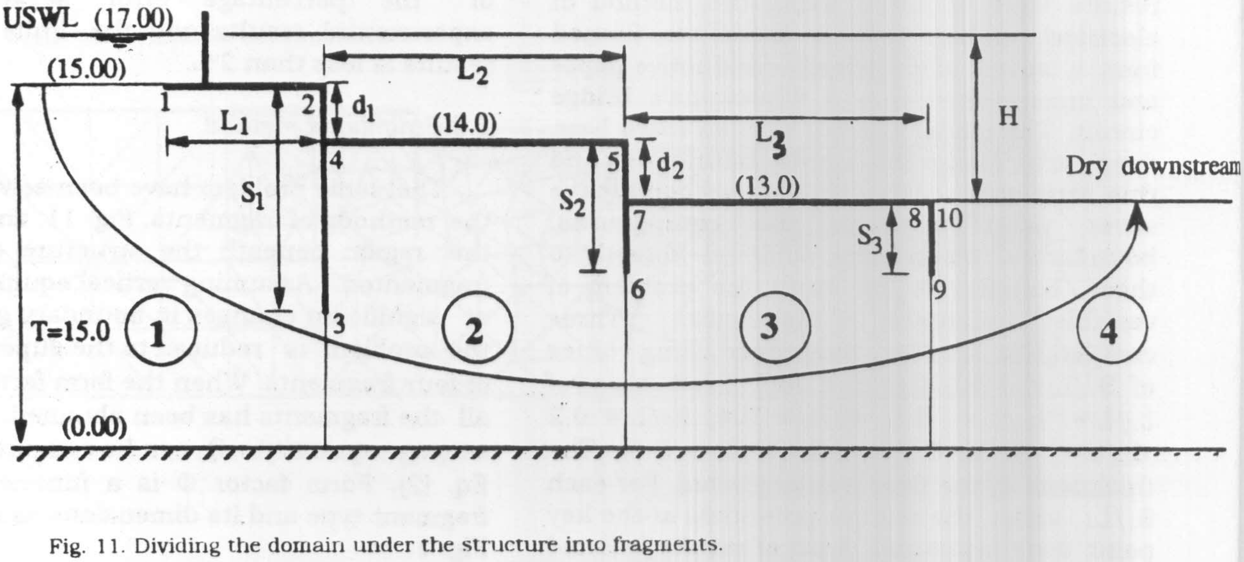


Fig. 11. Dividing the domain under the structure into fragments.

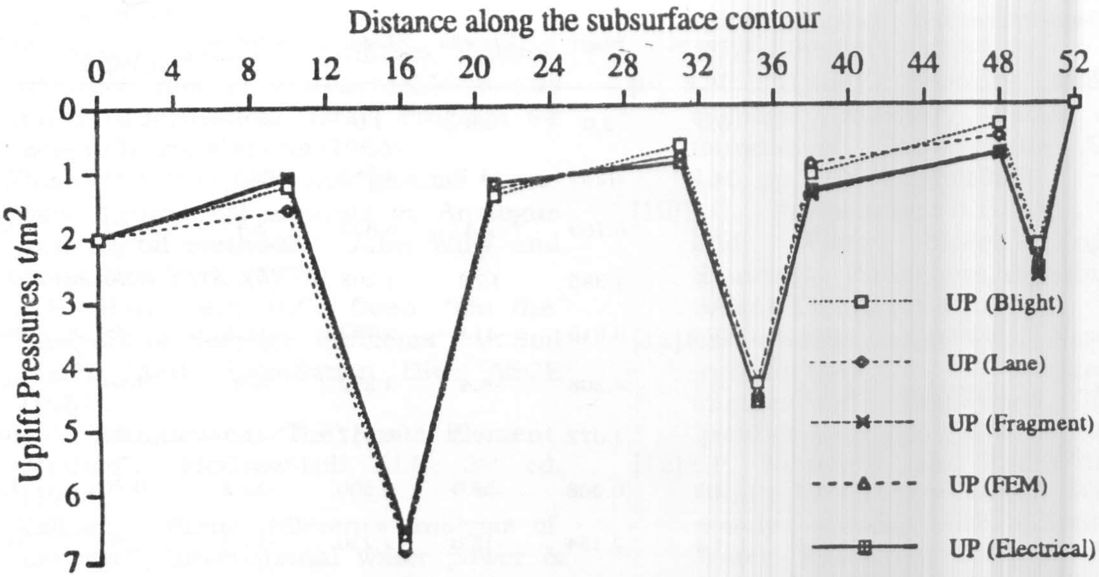


Fig. 12. Comparisaon between the computed uplift pressures (S_1/L_1= 0.6, S_2/L_1=0.4, S_3/L_1=0.2).

the d/s direction of the middle sheet pile. Bligh and Lane computations differ from the finite element one with about 59% and 43%, respectively, for the studied case.

For comparison, the uplift pressures as determined by the above five mentioned methods for the dropped floor are shown in Fig. 12. It is evident that, the uplift pressure calculated using the method of Bligh and the method of Lane, are on the unsafe side, whereas the fragment method agrees reasonably well with the actual uplift pressure(FEM). The figure also shows an excellent agreement between the finite element results and the experimental results.

Experimental analysis is time consuming and can not be adapted to account for differences in contact between horizontal and vertical surfaces along the creep path. The utility of linear approximation methods lies in their simplicity. Cutoff depth should always be determined by Lane's weighted creep theory. In the interest of safety, however, uplift should be calculated according to Bligh if there is any doubt about the composition of the foundation. Lane's method may be used where horizontal stratification is known to exist. Fragment method is based on simplifying assumptions. Its accuracy is often good for practical purposes. Finite element method is highly accurate but as the calculations are lengthy, it often requires the use of computer. Nowadays, ready use finite element programs are available. On important structures, all methods, including the electrical analogue, should probably be investigated.

The computed seepage flow using Fragment method is 1.145 (m^3/day) comparing with the finite element result (1.114), with difference 3%.

Table 2. Comparison between uplift pressures for different methods, (S_1/L_1=0.6, S_2/L_1=0.4, S_3/L_1=0.2).

Method / Point No.	F.E.M	Fragment	Error %	Bligh	Error%	Lane	Error %	Electrical analogue	Eror %
1	2.0	2.0	0.0	2.0	0.0	2.0	0.0	2.00	0.0
2	1.1078	1.091	-1.5	1.231	11.1	1.583	42.9	1.10	-0.7
3	6.6112	6.546	-1.0	6.769	2.4	6.833	3.4	6.60	-0.2
4	1.2012	1.290	7.4	1.385	15.3	1.208	0.6	1.20	-0.1
5	0.8921	0.778	-12.8	0.615	-31.0	0.792	-11.3	0.89	-0.2
6	4.6041	4.573	-0.7	4.308	-6.4	4.292	-6.8	4.60	-0.1
7	1.3383	1.387	3.6	1.077	-19.5	0.917	-31.5	1.33	-0.6
8	0.7480	0.765	2.3	0.308	-58.9	0.500	-33.2	0.76	1.6
9	2.5339	2.641	4.2	2.154	-15.0	2.250	-11.2	2.50	-1.3
10	0.0	0.000	0.0	0.0	0.0	0.0	0.0	0.0	0.0

4. Conclusions

Uplift pressures under hydraulic structures with dropped floors are computed using five different methods. In order to clear differences between Bligh, Lane, the method of fragments, and the electrical analogue method, the finite element method is applied to solve an illustrative problem and its results are considered as a reference because of its accuracy.

From the computed values, for the case study, it is observed that:

- All the considered empirical and approximate methods (Bligh, Lane, and fragments) give different uplift pressures from the finite element ones.
- Fragments method is the closest one to the finite element method compared the others with difference of 13 %.
- Bligh's method gives uplift pressure values differ from the finite element with about 59% while Lane differs with about 43%.
- Using finite element method, seepage characteristics are easily computed.
- With the advent of computer process, numerical methods may be used instead of the empirical and approximate methods especially for the more complex problems

Notations

The following symbols are used in this paper:

b is the width of apron;
C is the creep head ratio;
d_1, d_2 is the drop heights;
H is the effective head;
h is the potential head;
K, K' is the complete elliptic integrals of the first kind, with modules m and m', respectively;
K_X is the horizontal permeability;
K_Y is the vertical permeability;
L_1 is the length of upstream apron;
L_2 is the length of middle apron:
L_3 is the length of downstream apron:
L_T is the percolation length;
S,s',s'' is the depth of sheet pile;
S_1,S_2,S_3 is the depth of front face of sheet piles
T is the depth of permeable layer; and
x,y is the cartesian coordinate;

References

[1] V.I Aravin., and S.N. Numerov, "Theory of fluid flow in undeformable porous media", Jerussliem, Israel Program for Scientific translations (1965).

[2] Rushton, "Seepage and ground water flow (Numerical analysis by Analogue and digital methods)", John Willy and Sons, New York (1979).

[3] M.E. Harr, and R.C. Deen "On the analysis of Seepage Problems", J. Soil Mech. And Foundation Div., ASCE (1961).

[4] O.C. Zienkiewicz, "The Finite Element Method", McGraw-Hill, U.K. 3rd ed. (1983).

[5] Kalkani, "Finite difference analysis of seepage", International water power & Dam construction, 21-23, January (1992).

[6] L. Smith, and R.A. Freeze,"Stochastic analysis of steady state groundwater flow in a bounded domain", Water Resources Res. Vol 15(6), pp. 1543-1559 (1979).

[7] D.V. Griffiths, "Rationalized charts for the method of fragments applied to confined seepage", Geotechnique, London, Vol. 34 (2), pp. 229-238 (1984).

[8] L.F. Harza, "Uplift and seepage under Dams on sand", Transactions ASCE, Vol. 100, pp.1352-1406 (1935).

[9] E.W. Lane, "Security from under seepage masonry dams on earth foundation", Transactions ASCE, Vol. 100, pp.1235-1351 (1935).

[10] B.C Punmia, and B.B. LAL., "Irrigation and Water Power Engineering", Standard Publishers distributors, NAI SARAK DELHI (1983).

[11] R.D. Holtz, and W.D. Kovacs, "An introduction to geotechnical engineering", Engiewood Gliffs, N.J., Prentice-Hall, pp. 258-270 (1981).

[12] S.P. Neuman, and P.A. Witherspoon, "Finite Element Method of Analysing steady seepage with a Free Surface", Water Resources Res., Vol. 6 (3), pp. 889-897 (1971).

[13] S.R. Gutti, "Stability of an Overfall founded on Permeable Soils", J. of Hyd. Div., ASCE, Vol. 92 (HY3) (1966).

[14] V.I. Vaidyanathan, "The Application of Electrical Analogy to the design of Hydraulic structures," Publication No. 55, Central Board of Irrigation and Power, New Delhi, India (1955).

Received August 10, 1999
Accepted September 3, 2000