

# Adaptive imperfect column with piezoelectric actuators

Abdulmalik A. A. Alghamdi

Department of Mechanical Engineering, College of Engineering  
King Abdulaziz University, PO Box 9027, Jeddah 21413, Saudi Arabia

This paper discusses the stability of long imperfect adaptive (or smart) columns. The mechanic of a smart column having piezoelectric elements as active elements is introduced first. Then, the governing differential equation of the adaptive column is presented. A numerical example is given to show the capability of active piezo-ceramic elements in enhancing structural stability of long imperfect column. Obtained numerical results show improved stability performance of the adaptive column in comparison with the passive one. It is possible to straighten initially imperfected columns using surface attached active elements.

تقدم هذه الورقة استقرار الأعمدة الانضباطية (أو الذكية) الناقصة الطويلة. تم تقديم ميكانيكا العمود الذكي المحتوى على عناصر إجهادية كيربانية كعناصر فعالة. ثم قدمت المعادلات التفاضلية الشاملة للعمود المنضبط. تلى ذلك طرح مثال عندى لتوضيح قدرة العناصر الإجهادية الكيربانية لتحسين الاتزان الهيكلى للأعمدة الطويلة الناقصة. إن النتائج العددية المستقاة تبين تحسن أداء الاستقرار للعمود المنضبط مقارنة مع العمود السلبي. من الممكن استبدال الأعمدة الناقصة ابتدائياً باستخدام عناصر نشطة ملتصقة بالسطح.

**Keywords:** Structural stability, Smart structures, Piezoelectric materials, Adaptive column.

## 1. Introduction

Adaptive or smart structures are defined as structures that adapt themselves to environmental changes by using appropriate sensors, actuators, and control systems integrated into the structure [1]. Although, there exist some differences between smart, adaptive and intelligent structures, these terms are used in this paper interchangeably to denote one concept.

There has been a tremendous amount of research in the area of adaptive control of intelligent or smart structures in the past decade. The first paper in introducing this new concept of adaptive structures was written by Bailey and Hubbard [2]. Since then, researchers were modeling the adaptive structures, experimenting them and finding new applications for them. Active elements integrated in adaptive structures include piezoelectric materials (where the coupling is electromechanical), shape memory alloys (thermomechanical), magnetostrictive materials (magnetomechanical), electrorheological fluids, electrorelaxors, and

so on. Details of advances in adaptive structures can be found in a paper by Crawley [3].

Burke and Hubbard [4] modeled the adaptive structure by treating it as one dimensional simple beam model. Since then, several models have been proposed. Some of these models include uniform strain model [5], one-dimensional Bernoulli-Euler strain model for beams [6], one-dimensional Bernoulli-Euler strain model for plates [7], strain energy model [8], finite element model [9-11], and eigenstrain analysis [12]. Piezoelectric materials have been used in adaptive vibration control by many investigators, see for example Hagood et al. [13], Leo and Inman [14] and Tzou [15].

Baz and Tampe [16] utilized externally attached shape memory alloys in the form of helical spring to enhance the elastic stability of beams by counterbalancing the applied compressive load. Baz, et al. [17] controlled buckling behavior using shape memory alloy wires embedded in the beam. Controlled buckling load increases to three times the uncontrolled one by activating the shape

memory alloy. Krishna and Mei [18] studied buckling and post-buckling of thin rectangular laminated plates with piezoelectric materials using finite element analysis. Their finite element analysis was based on Von-Karman large deflection theory, and capability of piezoelectric layer to buckle the laminated plate was examined, and the critical buckling voltage was predicted. Chandrashekhara and Bhatia [19] investigated the behavior of laminated composite plates with piezoelectric materials using finite element model based on first-order shear deformation plate theory. Obtained results show reduction in the deflection of the plate center due to increase in applied voltage.

In this paper, the stability analysis of long imperfect smart column with piezoelectric materials is introduced. The active elements utilized in this paper are piezoelectric materials bonded to the outer surfaces of the adaptive column. The mechanics of interaction between the bonded active elements and the passive column are solved using simplified pin-force model assuming perfect bond condition. The differential equation for pivoted ends adaptive column is given along with its solution. A numerical example is proposed for a column made of Aluminum material with two externally bonded layers of piezoceramic materials extending along the adaptive column.

## 2. Mechanics of the adaptive column

Figure 1 shows an imperfect long, homogenous and isotropic column with two outside layers of piezoceramic materials. Amplitude of the imperfection is  $a$ , length of the column is  $L$ , width of the column is  $w$ , thickness of the column is  $t$ . The length of the piezoceramic elements is  $l_a$ , width is  $w_a$ , and thickness is  $t_a$ . Length of the piezoelements is the same as the length of the column (i.e.,  $l_p=L$ ). Thus,  $L$  is used to denote the length of the adaptive column. Also, as shown in Fig. 1, the piezoelements and the column have the same width (i.e.,  $w_a=w$ ). But, the thickness of the piezoelements is less than that of the column ( $t_a < t$ ).

Now, assume an equal and opposite electrical potential ( $V$ ) is applied to the left and

right piezoelements, as shown in Fig. 2. The left piezo will expand causing the left surface of the column to be in tension, while the right piezo will contract causing the right surface of the column to be in compression. Thus, the applied voltage causes the column to be straight by minimization of initial imperfection.

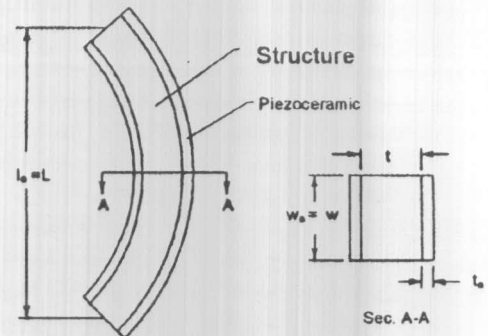


Fig. 1. Imperfect column.

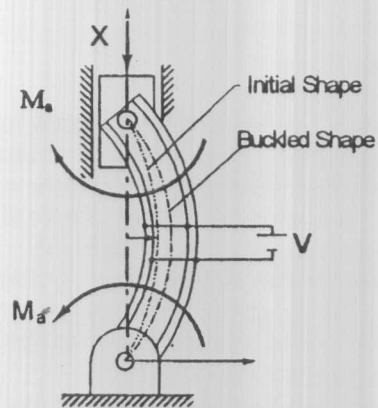


Fig. 2. System of adaptive column.

The free uniform strain in the piezoelements is given by [20],

$$\varepsilon = d_{31} \left( \frac{V}{t_a} \right) = \Lambda, \quad (1)$$

where  $d_{31}$  is the piezoelectric strain coupling constant,  $V$  is the applied potential at the outside electrodes of the piezoelements. The

resulting active force ( $F_a$ ) in the piezo is the product of the stress by the area,

$$F_a = d_{31} V w E_a, \quad (2)$$

where  $E_a$  is the modulus of elasticity of the piezoelements. The two equal and opposite forces will produce an active moment  $M_a$ ,

$$M_a = F_a (t_a + t). \quad (3)$$

The induced active moment ( $M_a$ ) will cause strain in the outer fibers of the column given as,

$$\varepsilon^s = \frac{M_a t}{2 EI}, \quad (4)$$

where  $EI$  is the weighted flexural rigidity given by,

$$EI = \frac{W}{12} [E t^3 + E_a t_a (8 t_a^2 + 6 t^2 + 12 t t_a)]. \quad (5)$$

Assuming perfect bond condition between the piezoelements and the structure, the total uniform strain in the piezoelement is,

$$\varepsilon_a = \Lambda - \frac{F_a}{E_a w t_a}, \quad (6)$$

where the first part in Eq. (6) is the free strain given by Eq. (1), while the second part is the mechanical strain due to the active force ( $F_a$ ). Minus sign means that the resistance of the structure to the expansion (or contraction) of the piezoelement. Now, applying the displacement compatibility conditions at the interface surface and assuming uniform strain in the piezoceramic, one can get [5],

$$F_a = \frac{2}{\alpha} EI E_a w t_a \Lambda, \quad (7)$$

where  $\alpha$  is given by,

$$\alpha = E_a w t_a (t + t_a) t + 2 EI. \quad (8)$$

The induced electro-mechanical moment is,

$$M_a = \frac{2}{\alpha} EI E_a w t_a (t + t_a) \Lambda, \quad (9)$$

The induced moment is proportional to the applied voltage and the piezoelectric strain coupling constant.

For the given approximations and simplifications, the accuracy of the proposed pin-force model is acceptable. However, more tedious analysis can be obtained using analysis methods such as adaptive classical laminate theory [7], strain energy model [8] and eigenstrain techniques [12].

### 3. Stability of the adaptive column

Figure 2 illustrates an initially bent or imperfect long adaptive column subjected to some external force ( $P$ ) causing the column to buckle in the direction of initial imperfection. The amplitude of the buckling depends on the initial imperfection amplitude ( $a$ ) and a magnification factor [21]. Now, by applying a potential voltage to the piezoelements, an induced moment ( $M_a$ ) will work to compensate for buckling by deforming the column in the direction opposite to its initial bent direction.

The differential equation for initially bent column subjected to axial force ( $P$ ) and having pivoted ends free of any applied moments ( $M_a=0$ ) is a classical problem in structural stability and can be written as [22],

$$EI y'' + P (y_0 + y) = 0, \quad (10)$$

where  $y''$  is the second derivative of the lateral displacement  $y$ , and  $y_0$  is the initial shape of the column. Solution for Eq. (10) is given by,

$$y = \frac{a}{1 - \beta} \sin\left(\frac{\pi x}{L}\right), \quad (11)$$

where  $L$  is the length of the column,  $a$  is the amplitude of imperfection,  $x$  is the coordinate along column length and  $\beta$  is the amplification factor given by

$$\beta = \frac{PL^2}{\pi^2 EI} \quad (12)$$

The differential equation for a column subjected to a bending moment ( $M_a$ ) at its end and no axial load ( $P=0$ ) is,

$$y'' = \frac{M_a}{EI} \quad (13)$$

Solution of Eq.(13) is written as,

$$y = \frac{M_a}{2EI} x(x-L) \quad (14)$$

By superposition [21], the total deflection due to the axial force and the end moments is obtained by adding Eq. (11) and (14),

$$y = \frac{a}{1-\beta} \sin \frac{\pi x}{L} + \frac{M_a}{2EI} x(x-L) \quad (15)$$

The first term in Eq. (15) depends on the value of the applied axial force ( $P$ ), while the second term depends on the magnitude of the active ends moment. Note that the sign of the applied active moment has to be negative to minimize the buckling deformation.

Assuming long elastic column, the critical buckling load can be written as,

$$P_{cr} = \frac{\pi^2 EI}{L^2} + \frac{M_a}{y_{max}} \quad (16)$$

where  $y_{max}$  is given as,

$$y_{max} = \frac{M_a L^2}{8EI} + \frac{a}{a-\beta} \quad (17)$$

#### 4. Results and discussion

Consider the following example of long

imperfect column with pivoted ends. The column is made of aluminum where,  $E_a=70$  GPa,  $L=2$  m,  $w=50$  mm,  $t=10$  mm, initial imperfection amplitude  $a=5$  mm. The column undergoes a constant 500 N axial force ( $P$ ). The active material is piezoceramic Lead-Zirconate-Titanate type 5H (PZT-5H). The thickness of the actuator is 1mm, the strain coupling constant ( $d_{31}$ ) is  $-274E-12$  m/V, and modulus of elasticity is 64GPa [23].

Figure 3 shows the lateral deflections of the adaptive column. The solid line represents the initial shape (imperfect shape) of the column under zero axial force ( $P=0$ ). The dashed line shows the passive buckling shape of the column when subjected to a 500 N axial force. The critical load ( $P_{cr}$ ) for this configurations of long column is 1200N, based on Euler column theory. The increase in the lateral deflection is in the direction of the initial imperfect shape.

The amplitude of the deflected shape is normalized with respect to the initial imperfection amplitude ( $a=5$ mm). The normalized amplitude (imperfection amplitude) increases from 1 for zero axial force to about 1.75 for 500N. Now, the piezoelements are activated by applying a 1000V across their electrodes causing an electric field of 1kV/mm. The corresponding applied active moment ( $M_a$ ) is positive (counterclockwise) at the lower end of the column and negative (clockwise) at the upper end. Thus, the resultant adaptive deflection, shown by dashed-dotted line, is obtained in the direction opposite to the passive deformed shape. The value of the applied moment is 7.08 Nm. The normalized amplitude of the deflection decreases from 1.75 at zero voltage to 0.27 at 1000V. In the absence of the external axial load, the active elements can be used to minimize the initial imperfection amplitude, i.e. straightening of imperfected structures.



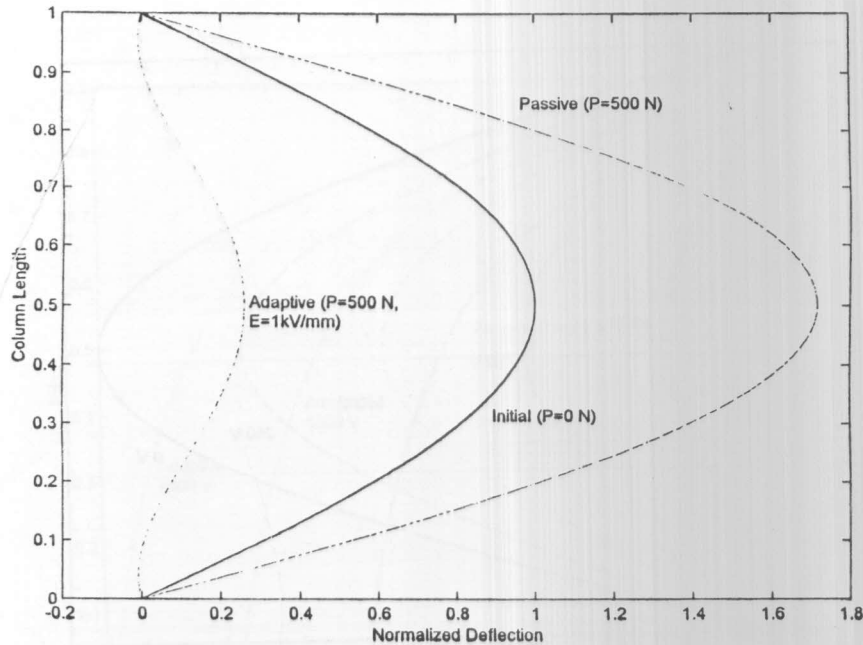


Figure 3: Deflections of the Passive and Adaptive Columns.

Figure 4 illustrates the effect of increasing the applied voltage on the adaptive deflection. The solid line represents the deformed shape of the column due to 500N applied axial force and zero voltage. The dashed, dashed-dotted, dotted, "star" and "cross" lines show the behavior of the column when the active elements are activated to 250V, 500V, 750V, 1000V and 1250V, respectively.

The corresponding applied active moments are 1.77, 3.54, 5.31, 7.08, 8.85 Nm, respectively. The reduction in the amplitude of the deflection is proportional to the applied voltage. Negative deflection appears in the (\*) and (+) curves due to the large applied active moment resulting from the high applied voltage, 1000V and 1250V. The PZT-5H can be subjected up to 2000V/mm without losing its piezoelectricity effect [23].

Another way of showing the effect of increasing the applied voltage on the deflection of the column is represented in Fig. 5. The contour surface depicts the deflection of the column vs. the applied voltages.

The effect of adaptivity on the relation between midpoint deflection and the load ratio  $P/P_{cr}$  is shown in Fig. 6. The solid line depicts the passive behavior of the column showing the increase in the midpoint deflection as a function of the applied load ratio. The deflection increases slowly at first then increases rapidly as the applied load reaches the critical load. However, by activating the piezoelements, the deflection decreases as if the column had less imperfection amplitude. The figure shows the adaptive response for two different values of the applied voltage. At 1000 and 2000 V the normalized midpoint deflection starts by a negative value meaning that, in the absence of the axial load ( $P=0$ ) the active elements will deflect the column to the other side of the initial imperfect shape. The adaptive column needs larger applied load to produce the same amount of point deflection of the passive column. The amount of increase in the carrying capacity of the column is proportional to the applied voltage.

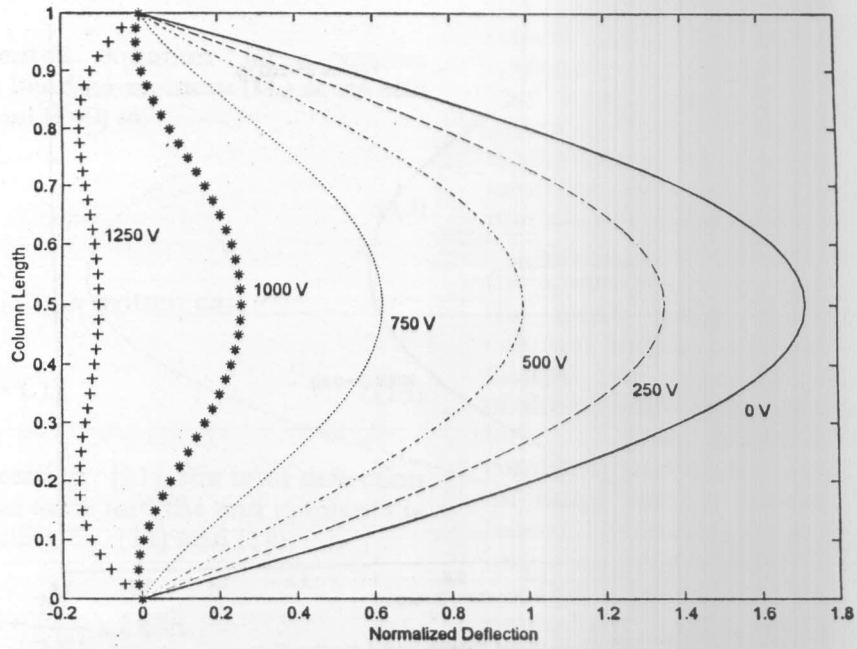


Figure 4: Effect of the Applied Voltage on the Deformed Shape of the Column.

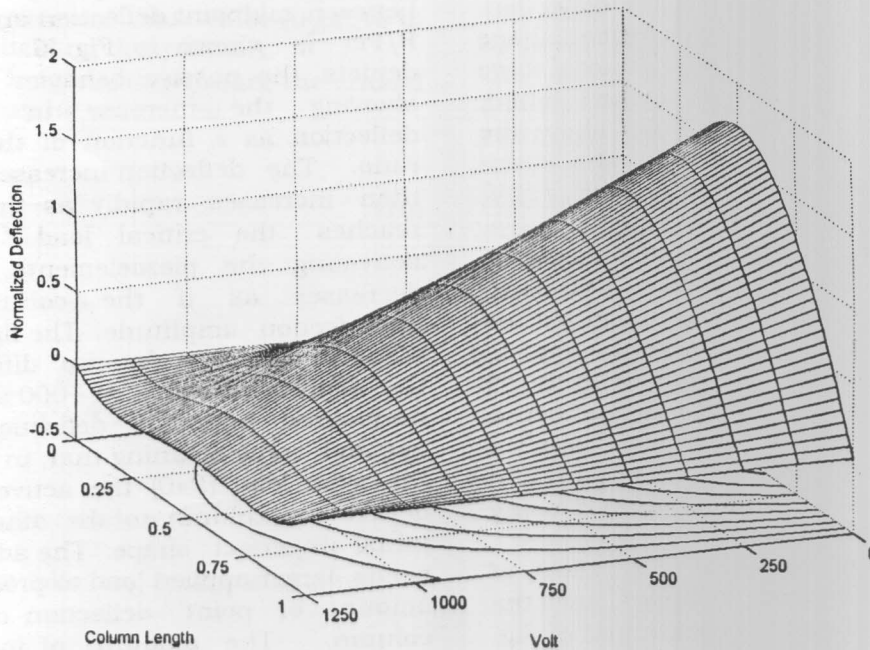


Figure 5: Effect of the Applied Voltage on the Deformed Shape of the Adaptive

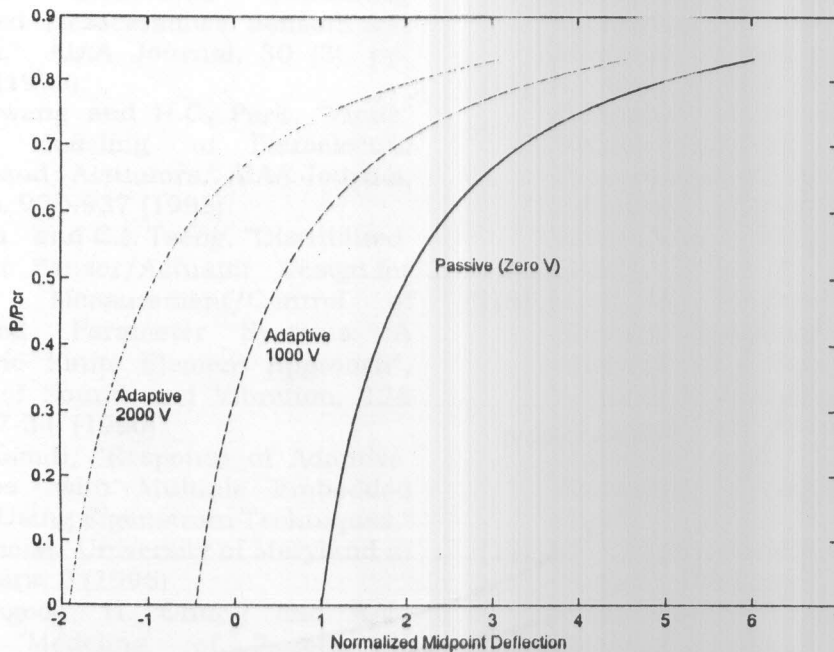


Fig. 6. Load deflection curves of the adaptive column.

Figure 7 illustrates the increase in the load carrying capacity ( $P_{cr}$ ) as a function of the column length ( $L$ ). In this figure all parameters are held constant except the column length which is varied from 2 to 12m. The solid line shows the passive response of the column, whereas the dashed line presents the adaptive response of the column at 1000 V. The voltage applied to the piezoceramic elements resulted in increasing the elastic stability of the column by increasing the critical load at certain length. Since the applied voltage is constant, the active end moment is constant, and hence the gap between the passive curve and the adaptive decreases as the column length increases.

## Conclusions

In conclusion, attachment of two layers of piezoelectric elements to an imperfect long column resulted in improving the elastic stability of the column. Using simple pin-force model and linear differential equation it was possible to integrate the active moment of the piezoelement and show its work in increasing the elastic stability of imperfect long column. As the applied voltage increases, the adaptive imperfection decreases. If the applied voltage is increased beyond a certain limit, a negative deflection may appear. Thus, active elements, such as piezoceramics, can be used to straighten imperfectioned structures.

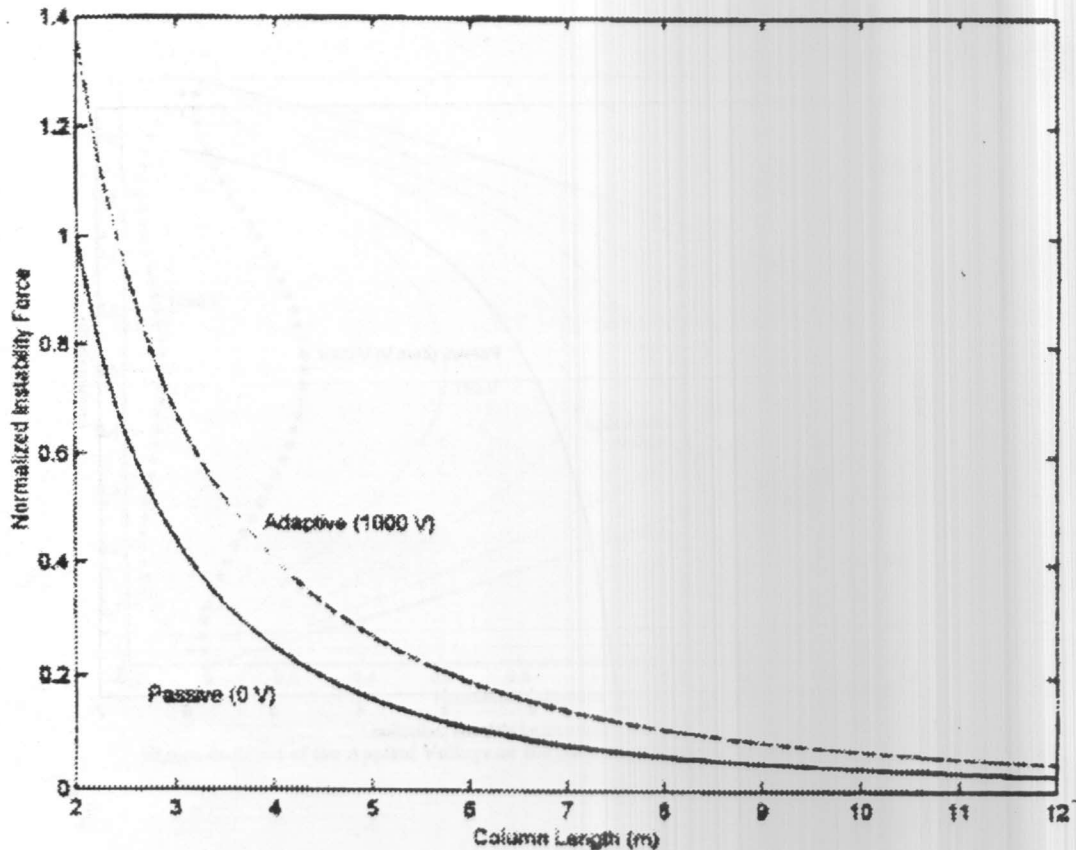


Fig. 7. Effect of the applied voltage on the load carrying capacity of the column.

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