Preliminary hull surface generation using finite element method

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This paper discusses the application of the Finite Element Method (FEM) for the preliminary generation of hull surface. Two elements are applied for generation of the ship hull surface. Firstly, the beam element is applied to generate longitudinal curves of the ship such as the sectional area curve and the load waterline. Secondly, the rectangular plate element is applied to generate the body plan and waterlines on the basis of the generated sectional area and load waterline curves. A computer program is developed and some examples are executed to demonstrate the validity of the method. The generated hull surfaces after fairing are compared with actual hull surfaces generated by other methods. The comparison shows good agreement.

من الصعوبات التى تواجه مهندسي تصميم السفن فى المراحل الأولية من خطوات لتصميم هو تحديد خطوط بدن السفينة لذلك كان الغرض من هذا البحث هو تطبيق طريقة العنصر المحدد لاستنتاج خطوط بدن السفينة. استخدام عنصرين هما عنصر الكمرة وعنصر اللوح. أستخدم عنصر الكمرة لاستنتاج المنحنيات الطولية مثل منحنى مقاطع السفينة ومنحنى خط الماء عند التحميل الكامل. بالنسبة لعنصر اللوح فاستخدام لاستنتاج خطوط البدن الكاملة للسفينة معتمدا على خواص منحنى مقاطع السفينة ومنحنى خط الماء عند التحميل الكامل. تم عمل برنامج على الحاسب وأجريت بعض الأمثلة على سفن ذات قاع مرتفع مثل سفن الصيد والقاطرات. قورنت النتائج بأخرى موجودة لهذه السفن وكانت المقارنة مرضية ومحققة الغرض من البحث.

Keywords: Ship hull surface generation, Longitudinal curves, Body plan, Semi displacement ships, Finite element method, Fairness criterion, Plate element and Beam element.

1. Introduction

One of the problems which faces naval architect at an early stage of the ship design is to generate the ship hull form in order to be able to proceed with various preliminary design office calculations. These calculations include hydrostatics, wetted surface, capacities, resistance and propulsive coefficients.

Some of these calculations are based on the below-water hull form such as those for resistance and propulsive coefficients, whereas most of the calculations are based on the below and above-water hull form of the ship. Hence, it is essential to produce the preliminary lines up to the level of the upper deck in order to be able to develop a realistic general arrangement. All calculations involved in the preliminary design stage can be performed more or less exactly once the hull surface are defined, however, resistance and propulsion coefficients are usually estimated by approximate methods. The reliability of the powering estimation depends entirely on the accuracy of the powering method used [1].

There are several methods for generation hull form such as, methodical series interpolation, parent ship, mathematical representation using mathematical functions, FEM and Fuzzy modeling method. The applications of the methodical series and parent ship methods have been widely used due to their simplicity [2]. However they are applied only for a limited range of hull parameters such as length/breadth ratio and coefficient of variation of ship forms to avoid degenerated hull shapes. Recently, great progress has been made in the development of mathematical ship hull form [3,4]. It is a powerful tool to apply to the automation of shipbuilding. It helps engineers to avoid hand lofting and gives preliminary estimation of the power and resistance.

The hull form is generated by representing longitudinal curves using mathematical functions fitted to the required form [5]. However, discontinuities usually show up and sometimes the form is distorted.

Fuzzy model requires a lot of input data and information about the hull form to give the best fit for the ship hull generation [6].

In this paper, mathematical modeling based on the FEM and a fairing criterion is applied to generate the ship hull surface [7]. Two elements are used in the mathematical model. Firstly, the beam element is applied to generate the sectional area curve and the load waterline of the ship. Secondly, the plate element is applied to generate the hull surface (body plan and waterlines) on the basis of the generated sectional area and load waterline curves. Boundary conditions and ship design constrains such as coefficient of variation of ship forms, the contour of bow, stern and the base line are imposed to solve the FEM model.

A computer program based on the two elements is developed to generate the hull form and to calculate the approximate hydrostatics and intact stability of the ship.

The method can be applied for ships with a parallel middle body or with flat of side, however, discontinuities of curves have shown up when ships of flat bottoms are generated. Therefore this research work is applied with semi displacement ships which have a slight rise of floor such as fishing boats and tugboats.

Some examples are executed on different boats with different sizes to verify the capability of the method. The obtained results by FEM are compared with those generated by other methods. It is found that hull surfaces generated by using FEM method requir good fairing in order to perform preliminary design office calculations accurately.

2. FEM descritization

The mathematical representation of a ship form is treated in the literature. The longitudinal curves were dealt with individually. The objective of the mathematical representation could be used for generation, fairing and hydrostatics and hydrodynamic calculations of the longitudinal curves. Each curve is represented by a polynomial function whose accuracy depends on the degree of the polynomial.

However, high degree of the polynomial dictates the number of the design parameters that needs a lot of computing work.

Examples of this representation are given in Ref. [5].

In this paper, the mathematical representation of the longitudinal curves and the body plan sections are dealt with simultaneously. The Finite Element Method (FEM) generates the hull form in two stages.

In the first stage, a beam element is used to generate the main longitudinal curves such as sectional area curve and load waterline.

In this element a simple polynomial function is used to satisfy the continuity of the curve at any point.

In the second stage, the hull surface is generated using the plate element while taking into consideration the generated sectional area and load waterline curves as design constrains. The procedure of FEM can be explained as follows, A simple equation of the hull surface, Y, is defined as follows:

$$Y = g(x, z), \tag{1}$$

The domain of analysis is the vertical central plane of symmetry of the ship bounded by bow, stern, deck shear and base line as shown in Fig. 1. The compatibility equation of the problem is the variational fairness criterion [8]. The fairing criterion is to measure the unfairness in the surface (a fair surface, Y, is attained by minimizing this function).

It is defined as the integral of the sum of squares of the curvature in x, y and z directions. The general form of this criterion is written as follows;

$$F = 1/2\iint_A w_1 k_{xx}^2 + w_2\, k_{xz}^2 + w_3\, k_{zz}^2\, dA, \tag{2}$$

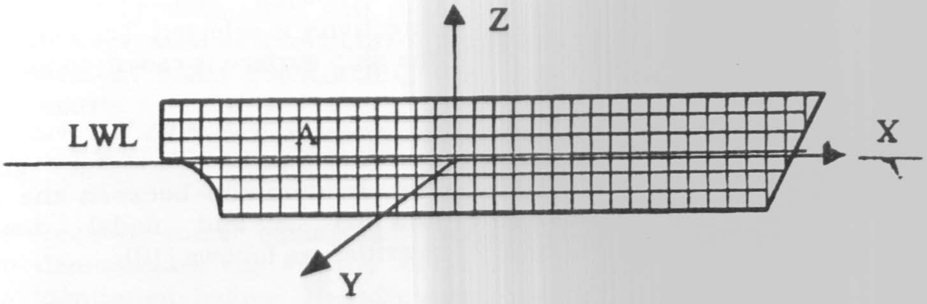

Fig. 1. The domain of analysis (FEM mesh division).

Where;

F is the fairing criterion,

k_{xx} is the curvature in x-direction $= Y_{xx}/(1+Y_{xx}^2)^{3/2}$,

k_{xz} is the curvature in xz- direction $= Y_{xz}/(1+Y_{xz}^2)^{3/2}$,

k_{zz} is the curvature in z- direction $= Y_{zz}/(1+Y_{zz}^2)^{3/2}$,

w_i is the weight function in i- direction,

x , z are the coordinates in x and z directions, and

A is the domain of analysis.

The above equation is linearized and simplified as follows,

$$F = 1/2 \iint_A w_1 Y_{xx}^2 + w_2 Y_{xz}^2 + w_3 Y_{zz}^2 dA, \quad (3)$$

where, $Y_{xx} = \partial^2 Y/\partial x^2, Y_{xz} = \partial^2 Y/\partial x \partial z$ and $Y_{zz} = \partial^2 Y/\partial z^2$.

Then, the procedure of FEM is processed to formulate the governing stiffness matrix [k] and to solve Eq. (3). In the following sections, the governing stiffness matrix [k] is formulated for two different elements, the beam element and the plate element [8].

3. The beam element

An important role of the beam element is the mathematical representation of the longitudinal ship curves such as sectional area curve and waterlines. A beam element with two degree of freedom at each node such as the translation displacement and the rotation displacement is selected. In this case, the fairness criterion is rearranged as follows [9],

$$F = 1/2 \iint_A w_x Y_{xx}^2 dA. \quad (4)$$

The above equation may be formulated in a relationship between the displacement function Y and nodal displacement as follows:

$$\{Y\} = [\phi]\{d\}, \quad (5)$$

where:

$[\phi]$ is the shape function given as[10],

$[\phi]$ is the $[\phi_1 \quad \phi_2 \quad \phi_3 \quad \phi_4]$,

where;

$\phi_1 = 1 - 3(x/l)^2 + 2(x/l)^3$,

$\phi_2 = x - (2x^2/l) + (x^3/l^2)$,

$\phi_3 = 3(x/l)^2 + 2(x/l)^3$,

$\phi_4 = -(x^2/l) + (x^3/l^2)$,

l is the length of the beam element,

A is the area of the beam element, and

{d} is the vector of the nodal displacement.

Eq. (5) is rewritten as follows:

$$\{Y\} = [k]\{d\}. \quad (6)$$

The governing stiffness matrix [k] is calculated as:

$[k] = 1/2\iint w_x [\phi_{xx}]^T [\phi_{xx}] dA$, and
$[\phi_{xx}] = \partial^2\phi/\partial x^2$
^{T} =the transpose of the matrix $[\phi_{xx}]$,
w_x = the weight function, given as $1/(1+x^n)$, and
n = the power of the weight function.

Here, the vector $\{d\}$ includes nodal displacement, the boundary conditions and the design constrains of the longitudinal curves.

4. Boundary conditions and design constrains

In the first stage, the solution of the stiffness matrix of the longitudinal curve is achieved by minimizing the unfairness of this curve. This means; to find nodal displacements that minimize the unfairness function and to satisfy the boundary conditions and design constrains. The Lagrangian multipliers or the Penalty function is applied to solve this problem [11].

The boundary conditions are selected to satisfy the geometrical properties such as the maximum value of the longitudinal curve (C_m or half breadth) and values of tangent at different points on the curve such as value of the tangent at the entrance of the waterline and at the middle point of the curve. The design constrains are selected to satisfy coefficient of variations of the curve. The prismatic coefficient, C_p and the longitudinal center of buoyancy, LCB are related to the sectional area curve while, the waterline area coefficient, C_w, and the longitudinal center of floatation, LCF are related to load waterline. The boundary conditions and design constrains of the longitudinal curves are imposed on Eq. (6) as a function of nodal displacements.

5. The plate element

The basis of the plate element is the mathematical representation of the hull surface (body plan and waterlines).

A rectangular plate element with three degrees of freedoms at each node such as the translation displacement in y-direction and the rotation displacement in x and z directions is selected. The fairness criterion of the ship surface is rewritten as follows:

$$F = 1/2\iint_A w_1 Y_{xx}^2 + w_2 Y_{xz}^2 + w_3 Y_{zz}^2 dA. \qquad (7)$$

The relationship between the displacement function Y and nodal displacement is rewritten as follows [10],

$$\{Y\} = [\phi]\{d\}, \text{ and} \qquad (8)$$

$$[\phi] = [\phi_1 \;\; \phi_2 \;\; \phi_3 \text{ ------------- } \phi_{11} \quad \phi_{12}]$$

where; ϕ_i is the shape function at each node, and
$\{d\}$ is the vector of the nodal displacement

This equation is rearranged as follows:

$$\{Y\} = [k]\{d\}. \qquad (9)$$

Where, the governing stiffness matrix is calculated as,

$$[k] = 1/2(\iint w_x[\phi_{xx}]^T [\phi_{xx}] + w_{xz}[\phi_{xz}]^T [\phi_{xz}] + w_z[\phi_{zz}]^T [\phi_{zz}]), \qquad (10)$$

where;

$[\phi_{xx}] = \partial^2\phi/\partial x^2$.
$[\phi_{xz}] = \partial^2\phi/\partial xz$, and
$[\phi_{zz}] = \partial^2\phi/\partial z^2$.

The weight function, w_i, is assumed as the following polynomial function;

$w_x = 1/(1+x+ x^n)$,

$w_{xz} = 1/(1+xz+ zx^n)$, and

$w_z = 1/(1+z+ z^n)$.

In the second stage of solution, the Lagrangian multipliers or the Penalty function is reapplied to minimize the unfairness in the hull surface. The solution of Eq. (9) must satisfy boundary conditions and design constrains of the ship form. Half breadths at the midship section, stem, stern and base line are considered as boundary conditions. The properties of the generated sectional area curve and the load waterline are considered as the design constrains. It is noted that values of boundary conditions and design constrains are expressed as a function of the nodal displacement when constructing the hull stiffness matrix.

6. Examples

In this section some examples are executed to demonstrate the validity of the method. As mentioned before, the following examples are concerned with semi displacement ships which have rise of floors such as trawlers (I and II) and tugboat. A computer program based on the FEM procedure explained in the previous sections is developed. The flow chart of this computer program is shown in Fig.2.

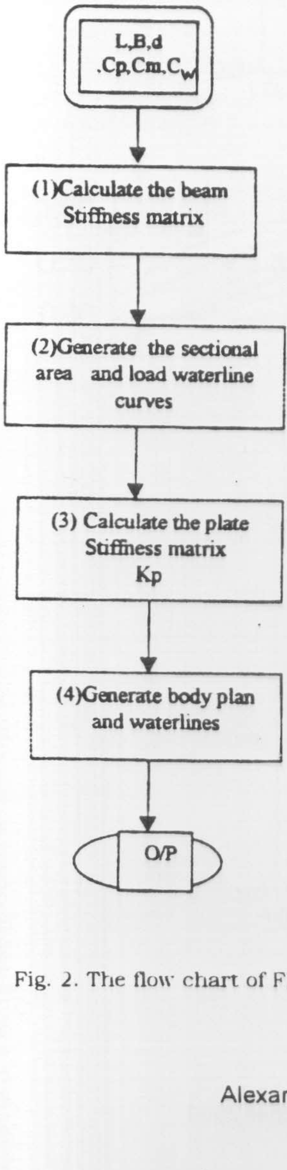

Fig. 2. The flow chart of FEM program

The domain of the analysis is the longitudinal vertical section of the ship hull bounded by the contours of the stem, the stern and the base line. The mesh division consists of 160 plate elements, 189 nodes and 20 beam elements. The sectional area curve and load waterline of the ship are firstly developed. Secondly the body plan and different waterlines are generated based on the developed sectional area and load waterline curves.

6.1. Generation of sectional area and load waterline curves

A trawlerI of a length 43.4m, breadth 9.5m, draft 4.16m and a prismatic coefficient of 0.57 is considered. Twenty beam elements and twenty one nodes are used here to generate the sectional area curve and the load waterline of the ship. It is more appropriate to normalize values of x and y between 0 and 1. Design constrains such as values of prismatic coefficient (0.57), longitudinal center of buoyancy (-o.51L aft), and sectional area curve coefficient, (o.8) were used to generate the sectional area curve. While maximum half breadth, longitudinal center of flotation, (-0.46L aft), waterline area coefficient, (0.69) and half angle of entrance (16°) were used to generate load waterline. Fig. 3. shows the generated sectional area curve and the load waterline of the ship based on these input data.

6.2. Generation of hull surface

The plate mesh division of the vertical central plane of the ship is as shown in Fig.1. The half breadths of the contour of the stern, base line, stem and shear deck are considered as boundary conditions. The value of each section area and corresponding half breadth at the load waterline are considered as the design constrains (calculated in the first stage). In order to describe the type of the section area; U or V, half breadths of the waterline below the draft are also considered. Figures 4 and 5 show the generated body plan and the waterlines for the selected ship.

The preceding procedure are executed on trawlerII of a length 75.8m, breadth 12.8 m, draft 5.72m and a prismatic coefficient of 0.66 to demonstrate the capability of the method when the trawler has a flat of side and small rise of floor. The generated body plan and the waterlines for this ship are shown in Figs. 6 and 7. Figure 8 shows the generated body plan of a tugboat of a length 15.4m, breadth 4.6 m, draft 1.75m and a prismatic coefficient of 0.55.

All results generated by FEM are compared with those originally generated by other method, it is found that good fairing needs to satisfy the objective required.

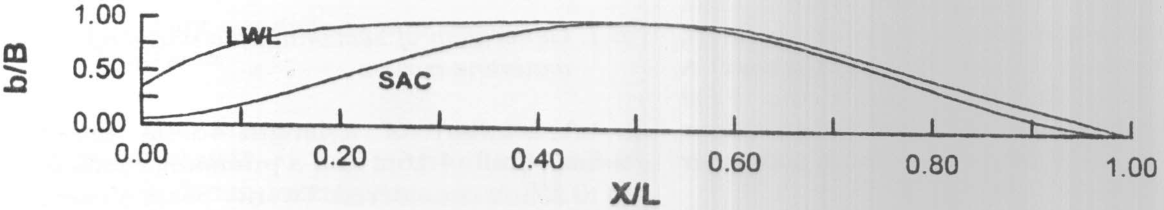

Fig. 3. The sectional area and load waterline curves

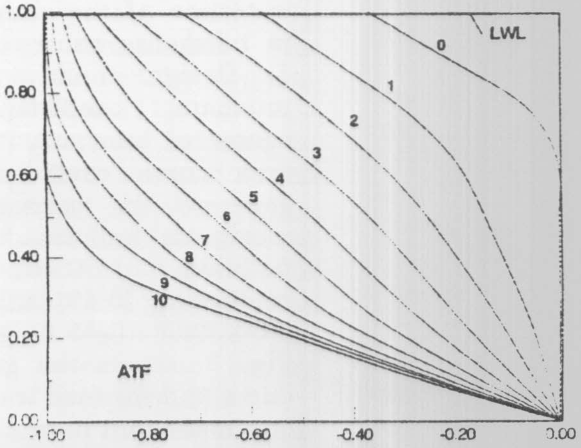

Fig. 4-a. The body plan (aft) of the trawleri

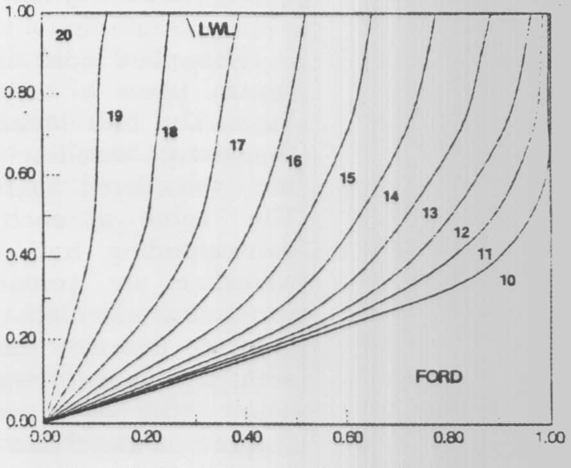

Fig. 4-b. The body plan (Ford) of the trawleri

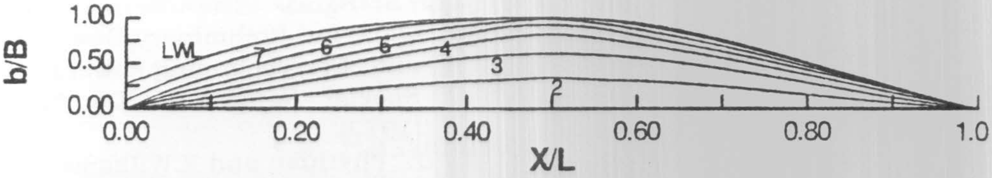

Fig. 5. Waterlines curves of the trawleri.

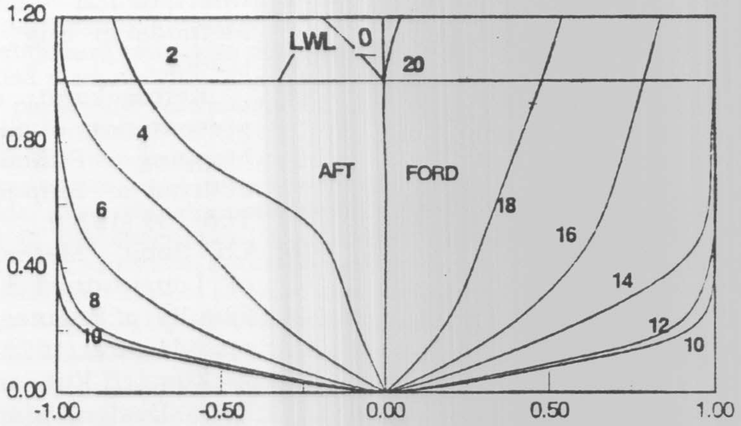

Fig. 6. The body plan of the trawlerl.

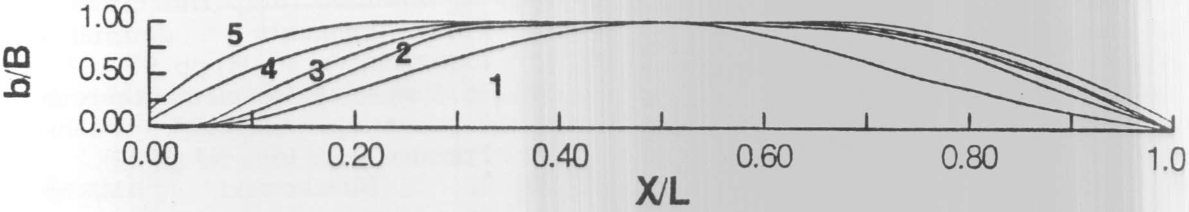

Fig. 7. Waterlines curves of the trawlerII.

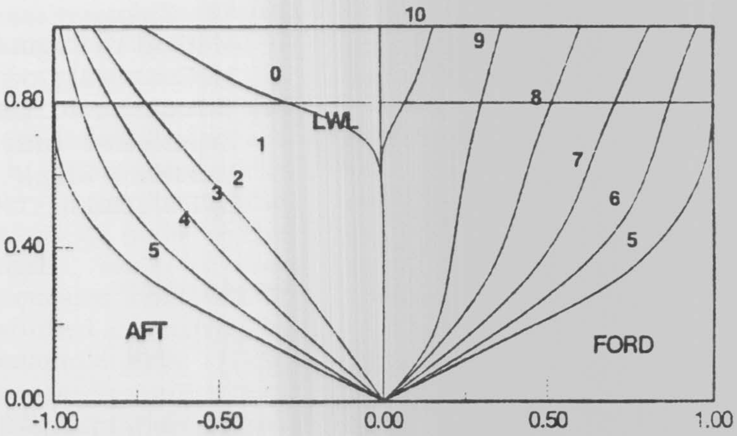

Fig. 8. The body plan of a the tugboat.

6.3. Further research work and applications

The application of FEM for generating hull forms is considered as the first stage of several further research works in other applications of the numerical methods in the ship design. For example the generation of ship surfaces with flat bottom and new hull forms needs more investigation.

7. Conclusions

The FEM is proposed to generate the hull surface, and from this research work the following can be concluded:
1-The method given above can be easily applied to define the preliminary hull form of a ship for the purpose of calculating the hydrostatic curves, approximate intact stability and hydrodynamic properties.
2-The generated hull surface requires good fairing in order to perform preliminary design office calculations accurately.
3-The preliminary hull form is mainly dependent on the given constrains and boundary conditions, therefore several hull forms may be generated. The hull form that satisfies a minimum resistance is then selected.
4-Problems arises if the ship has a flat bottom. This is because the bottom of the ship has an infinite slope in z-direction with respect to the global coordinates This can be overcome if the bottom has a slight rise of floor.

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