Natural convection from an infinite isothermal horizontal circular cylinder

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The natural convection heat transfer from an infinite isothermal horizontal circular cylinder is presented. The governing mass, momentum and energy equations were solved numerically using the Patankar-Spalding technique. A single heat transfer correlation was obtained in the Rayleigh number range from 10^2 to 10^7 and the Prandtl number range from 0.1 to 100. Streamlines and isotherms were developed for different combinations of the Rayleigh and Prandtl numbers to explain the flow around the cylinder. A comparison of the present correlation with previous results from other authors revealed that the present correlation represents the natural convection from the horizontal cylinder better than the previous correlation and gives values for Nusselt numbers which are almost the average of all previous results.

البحث يعرض إنتقال الحرارة بالحمل الحر من إسطوانة دائرية أفقية لانهائية الطول عند درجة حرارة ثابتة . و قد تم حل معادلات الكتلة و كمية الحركة و الطاقة عدديا باستخدام طريقة باتنكر سبلدنج . و قد تم وضع النتائج فى معادلة واحدة لحساب انتقال الحرارة فى مدى رقم رالى من ١٠^٢ إلى ١٠^٧ و رقم براندل من ٠,١ إلى ١٠٠ . و قد تم الحصول على أشكال توزيع خطوط السريان و درجة الحرارة لمجموعات مختلفة من أرقام رالى و براندل لشرح السريان حول الإسطوانة . و قد أوضحت المقارنة بين المعادلة الحالية و النتائج السابقة من باحثين آخرين أن المعادلات الحالية تمثل الحمل الحر من إسطوانة أفقية بطريقة أفضل من المعادلات السابقة و تعطى قيم لأرقام نوسلت تمثل متوسط قيم النتائج السابقة .

Key words: Natural convection, Horizontal cylinder, Numerical solution.

1. Introduction

The problem of natural convection heat transfer from long horizontal circular cylinders to fluids of different Prandtl numbers is very important and has many applications in industry. The applications include hot wire anemometry, boiler design and the rating of electrical conductors. Several workers have given an insight into the problem where only free convection is predominant. Experimental studies on laminar free convection around horizontal cylinders which have been published between 1892 and 1973 were critically reviewed by Morgan [1], who recommended an improved correlation in the form of $Nu=aRa^n$ with different values of the constants "a" and "n" in the range $10^{-10} < Ra < 10^{12}$. Hermann [2] presented the first theoretical work on the problem where he applied the thin boundary layer theory to the analysis about the horizontal cylinder and obtained an approximate solution valid for $Pr \rightarrow \infty$. Theoretical solutions for laminar regime are not very accurate due to the formation of wake at the rear of the cylinder. Nakai and Okazaki [3] analyzed the problem of pure free convection at small Grashof numbers which means thick boundary layers. The temperature field was divided into two regions, one is the "near field" around the cylinder where conduction is dominant in comparison with convection and the other is the "far field" at a large distance above the cylinder in the plume where convection is controlling. Fujii et al. [4] carried out a numerical analysis for the thick boundary layer of steady laminar free convection around the horizontal cylinder and provided a correlation for $10^{-10} < Ra < 10^7$ and $0.7 < Pr < 100$. They used the same function for the effect of Prandtl number which was given by Churchill and Usagi [5] for a vertical isothermal flat plate. Chiang and Kaye [6] used a Blasius type series expansion to solve the equations for a thin boundary layer. Saville and Churchill [7] improved the technique by employing a series expansion of the Goertler type that converged faster. Holster and Hale [8] presented a finite element analysis for transient solution at Gr = 5×10^4 and steady state solution for 2×10^3 < Gr< 7.5×10^4. Raithby and Hollands [9] analyzed the heat transfer from isothermal elliptic ducts from a conduction layer point of view and reported solutions for the geometric limiting cases of the flat plate and circular cylinders.

The analysis of natural convection from a horizontal cylinder is very much more complicated, therefore, extensive experimental data have been obtained over a wide range of conditions from which empirical correlation have been developed.
The general form of the correlation in most of the published work is give as:

$$Nu = a + bRa^n + cRa^m, \tag{1}$$

and different authors reported different values of the exponents and constants. Gebhart et al. [10] reported experimental results of natural, forced and mixed convection for fine horizontal wires in air and in two silicone liquids of Pr = 6.3 and 63, for a single wire of L/D = 8000. Collis and Williams [11] found, for air, that the effect of L/D can be neglected if its value is higher than 10^4. Churchill and Chu [12] developed a correlation for free convection from a horizontal cylinder by joining the asymptotic solutions at Prandtl equals 0 and ∞ and adjusting the equation constants in view of the experimental values of Ra at 0 and ∞. Their correlation is given by:

$$Nu = \left[0.6 + 0.387\left(\frac{Ra}{\left[1+\left(0.559/Pr\right)^{9/16}\right]^{16/9}}\right)^{1/6}\right]^2 \tag{2}$$

The above correlation is then used to represent the experimental data for all Pr and Ra. Honda and Yamashita [13] provided experimental data for water and freon-22 (2<Pr < 7) for 0.5 < Ra < 10^5. Kuehn and Goldstein [14] obtained a correlation for natural convection from a single horizontal cylinder valid at all Ra and Pr. Clemes et al.. [15] reported a new set of measurements in air from isothermal long horizontal cylinders of circular and non-circular cross sections at various orientations, covering the range of Rayleigh number from 10^3 to 10^9. Their data agreed with the Kuehn and Goldstein [14] correlation as well as the correlation given by Raithby and Hollands [9] if the exponent m=3.337 was modified to be 15.

2. Mathematical analysis

The present study deals with the laminar free convection heat transfer from an infinite isothermal horizontal circular cylinder.
The differential equations governing the conservation of mass, momentum and thermal energy together with their boundary conditions will be solved numerically.
The fluid around the cylinder will be considered as a Newtonian constant property fluid except for the density in the buoyancy force components in the momentum equations. The Boussinesq approximation will relate the variable density to the local temperature. The set of steady state two dimensional governing equations caste in a dimensionless form are as follows:

$$\left(\frac{\partial V_r}{\partial R} + \frac{V_r}{R} + \frac{\partial V_\phi}{R\partial\phi}\right) = 0, \tag{3-a}$$

$$\left(V_r\frac{\partial V_r}{\partial R} + V_\phi\frac{\partial V_r}{R\partial\phi} - \frac{V_\phi^2}{R}\right) = -\frac{Ra}{8Pr}\theta\cos\phi + \frac{\partial^2 V_r}{\partial R^2} + \frac{1}{R}\frac{\partial V_r}{\partial R} - \frac{V_r}{R^2} + \frac{\partial^2 V_r}{R^2\partial\phi^2} - \frac{2}{R^2}\frac{\partial V_\phi}{\partial\phi}, \tag{3-b}$$

$$\left(V_r\frac{\partial V_\phi}{\partial R}+V_\phi\frac{\partial V_\phi}{R\partial\phi}+\frac{V_rV_\phi}{R}\right)=\frac{Ra}{8Pr}\theta\sin\phi+$$

$$\left(\frac{\partial^2V_\phi}{\partial R^2}+\frac{1}{R}\frac{\partial V_\phi}{\partial R}-\frac{V_\phi}{R^2}+\frac{\partial^2V_\phi}{R^2\partial\phi^2}+\frac{2}{R^2}\frac{\partial V_r}{\partial\phi}\right), \quad (3\text{-c})$$

$$\left(V_r\frac{\partial\theta}{\partial R}+V_\phi\frac{\partial\theta}{R\partial\phi}\right)=\frac{1}{Pr}\left[\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\theta}{\partial R}\right)+\frac{\partial^2\theta}{R^2\partial\phi^2}\right]. \quad (3\text{-d})$$

where the following dimensionless variables are used

$$V_r=\frac{v_r}{(\nu/a)},\ V_\phi=\frac{v_\phi}{(\nu/a)},\ R=\frac{r}{a} \quad \text{and}$$

$$\theta=\left(\frac{T-T_\infty}{T_w-T_\infty}\right).$$

3. Boundary conditions

The calculation domain for the natural convection problem is shown in Fig. 1. Since the flow and heat transfer is symmetrical about the vertical axis, it suffices to consider only one half of the flow field on either side of the vertical axis. The inside boundary is determined by the solid surface of the infinite isothermal horizontal circular cylinder .On this surface , the no slip condition of the fluid requires that the radial and circumferential fluid velocity components to be annihilated and the temperature of the fluid equals the uniform temperature of the cylinder T_w. The calculation domain has no solid surface to confine it on the external side as the hot cylinder does on the inside boundary Therefore, we have to lay the external boundary at a fictitious circle in the fluid far away from the cylinder.

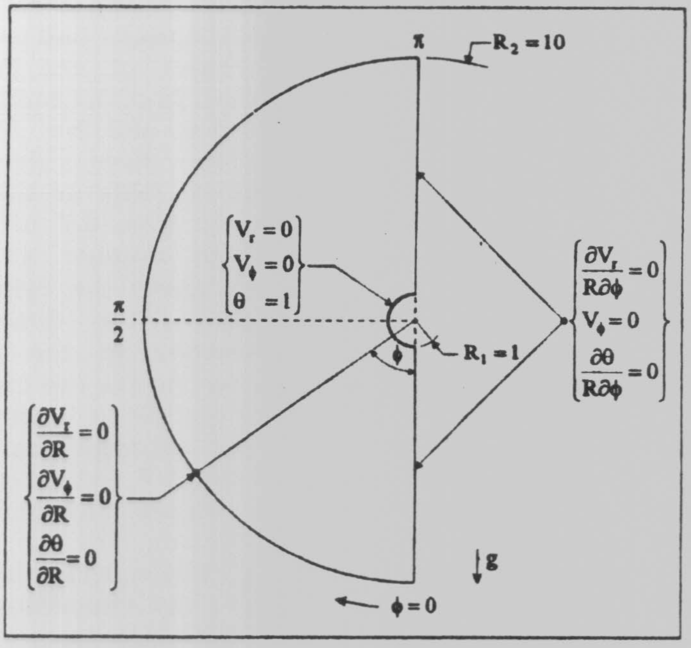

Fig. 1. Boundary conditions of the natural convection problem.

We chose to locate this circle far enough such that the radial variation of the independent variables at its location can be neglected. We assumed that at ten tube diameters this condition is justified as shown by Holster and Hale [8]. On the symmetry line, the circumferential gradients of the independent variables equal zero. Expressing the boundary conditions in terms of dimensionless variables, we get:

at $R = 1$ and $0 \le \phi \le \pi$

$$V_r = V_\phi = 0 \ , \ \theta = 1 \quad \text{(4-a)}$$

, at $R = 10$ and $0 \le \phi \le \pi$

$$\frac{\partial V_r}{\partial R} = \frac{\partial V_\phi}{\partial R} = \frac{\partial \theta}{\partial R} = 0 \ , \quad \text{(4-b)}$$

and at $1 < R < 10$, $\phi = 0$ or π

$$\frac{\partial V_r}{R\partial \phi} = V_\phi = \frac{\partial \theta}{R\partial \phi} = 0 \ . \quad \text{(4-c)}$$

The local Nusselt number , Nu_ϕ is given as:

$$Nu_\phi = -2\frac{\partial \theta}{\partial R} \ , \quad (5)$$

and the average Nusselt number, Nu around the cylinder is given as:

$$Nu = \frac{h(2a)}{k} = \frac{1}{\pi}\sum_{0}^{\pi}\left(-2\frac{\partial \theta}{\partial R}\right)\Delta\phi \ . \quad (6)$$

The Patankar-Spalding technique [16] was used to solve the above set of equations subjected to its boundary conditions. The calculation grid has fifty nodes in each of the radial and circumferential directions. Equal spacing is used in the circumferential direction and a non-uniform spacing is used in the radial direction, with smaller spacing near the cylinder surface and coarser away from it. The distribution density near the cylinder was varied for all runs depending on the values of Ra and Pr until the percentage variation in the calculated average Nusselt number is less than 0.2 %. The Rayleigh and Prandtl number ranges are $0.1 < Pr < 100$ and $10^2 < Ra < 10^7$. Divergence in the numerical solution was encountered for $Ra < 100$ and $Pr < 0.1$ The number of iterations needed for convergence of all nodal values of velocity and temperature in the calculation domain for each run was about 2000 iterations.

4. Results and discussion

The streamlines of the flow around the hot isothermal horizontal cylinder due to the natural convection are plotted in Fig. 2. The results in this figure are parametrized by the Rayleigh and Prandtl numbers. The stream lines for Ra = 100 and Pr = 0.7 indicate that the fluid at the bottom and sides of the cylinder and far from it , is entrained towards the cylinder and forced to flow vertically upward inside a plume above the cylinder. Increasing the Rayleigh number from 10^2 to 10^7 while holding the Prandtl number at 0.7, speeds up the radially inward flow towards the cylinder and attracts further ambient fluid from the upper half towards the rising plume. The effect of the Prandtl number can be deduced from the comparison of the results in Figs. 2-a. and 2-c. The values of the stream function have decreased approximately hundred folds as the Prandtl number was changed from 0.7 to 100 while keeping the Rayleigh number at 10^2. Also, the rising plume above the cylinder became thinner. A sample of the dimensionless temperature distribution in the flow field around the cylinder for the two fluids of Prandtl numbers of 0.7 and 100 is illustrated in Fig. 3.

The Rayleigh number was varied between 10^2 and 10^7, which corresponds to the lower and upper extremes of the present calculations. For the same Prandtl number, as the Rayleigh number is increased from 10^2 to 10^7, the temperature field shrinks around the cylinder. Also, for the same Rayleigh number, as the Prandtl number is increased from 0.7 to 100, a similar behaviour is noticed.

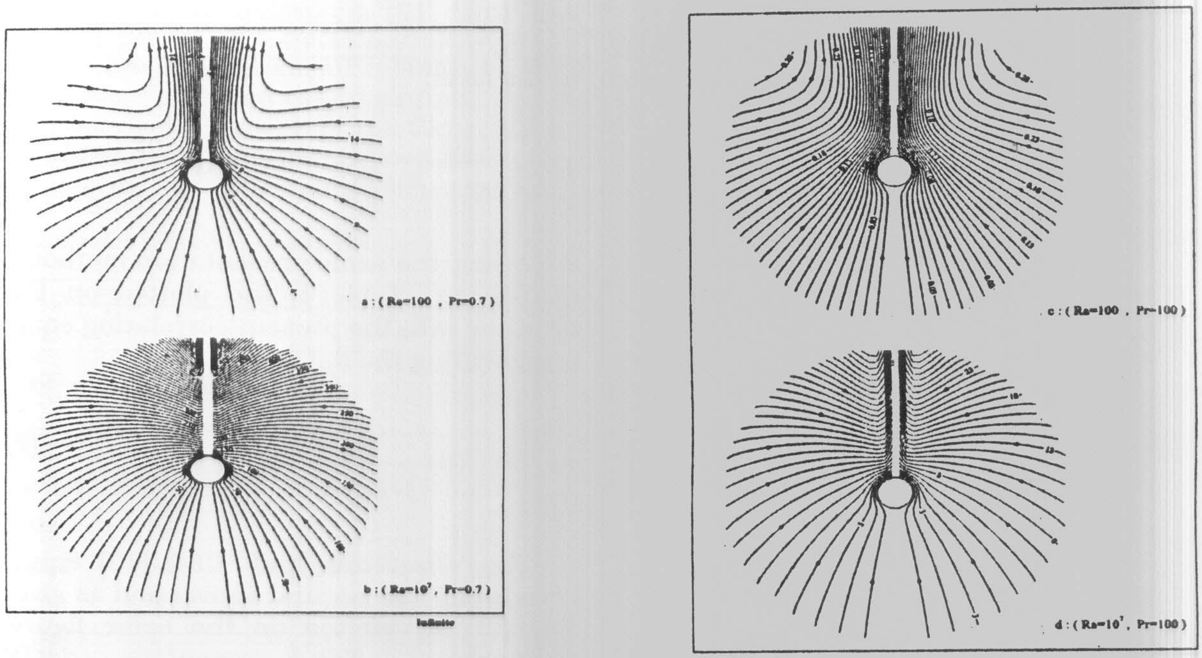

Fig. 2. The streamline patterns of natural convection around an infinite isothermal horizontal circular cylinder.

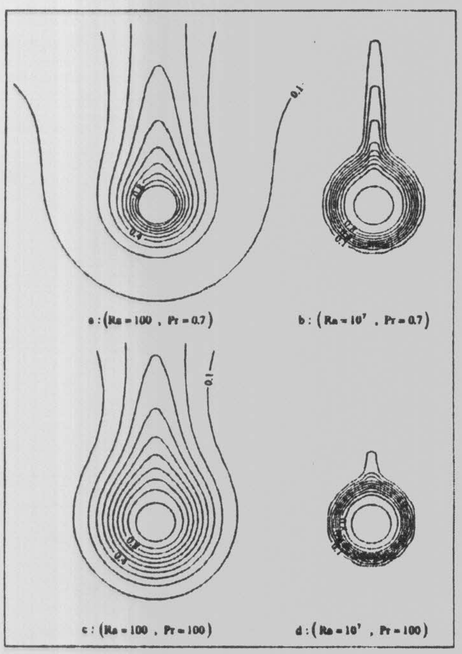

Fig. 3. The dimensionless temperature distribution around an infinite isothermal horizontal circurlar cylinder.

This is quite expected since the increase of the Prandtl number implies the decrease of the fluid thermal diffusivity, or in other words, the spread of the temperature field inside the flow decreases.

The Nusselt number values in the Rayleigh number range 10^2 to 10^7, and for four Prandtl numbers 0.1, 0.7, 10 and 100 are compiled in Table 1.

Table 1. Dependence of the nusselt number on the rayleigh and prandtl numbers in natural convection from an infinite isothermal horizontal circular cylinder.

Ra	Nu			
	Pr=0.1	Pr=0.7	Pr=10	Pr=100
10^2	-	1.630	2.024	2.099
10^3	2.153	2.827	3.403	3.630
10^4	3.565	4.610	5.561	5.987
10^5	5.915	7.724	9.323	9.991
10^6	10.085	13.258	15.993	17.038
10^7	17.488	23.149	27.781	29.465

The first case in the table corresponding to Ra = 10^2 and Pr = 0.1 could not be determined because the numerical solution did not converge to a reasonable solution. Churchill and Chu [12] suggested the presentation of the natural convection heat transfer results in the form:

$$\frac{Ra}{\left[1+(0.559/Pr)^{9/16}\right]^{16/9}} \text{ versus } \left(Nu^{1/2}-0.6\right).$$

Following the same presentation method , the results in Table 1. are plotted on Fig. 4. together with the present correlating equation representing the data:

$$Nu = \left[0.6+0.454\left(\frac{Ra}{\left[1+(0.559/Pr)^{9/16}\right]^{16/9}}\right)^{0.15}\right]^2 . \quad (7)$$

The Churchill and Chu [12] equation. correlating the natural convection as given by Eq. (2) is plotted on the same figure for comparison. The percentage deviation between the correlating Eq. (7) and the numerical data in Table 1. (excluding the first point in the table) is within 6.8 % .

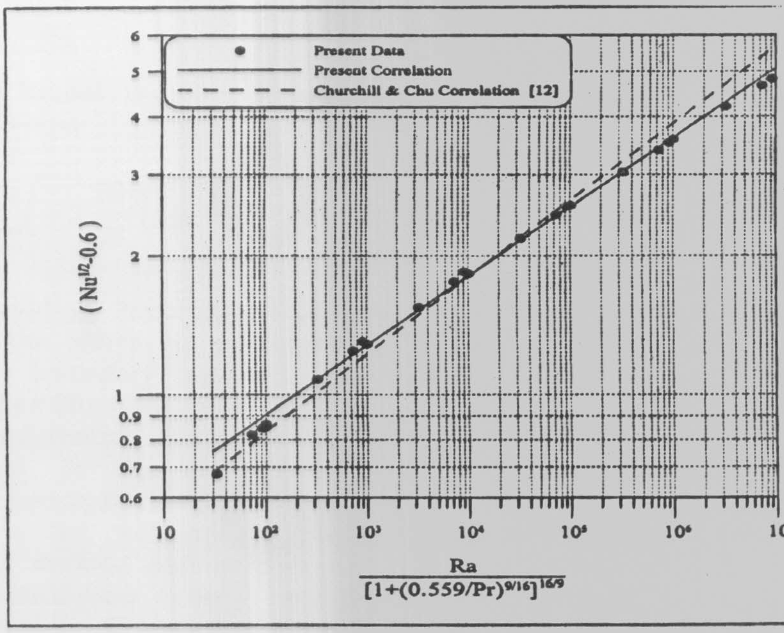

Fig. 4. Natural convection nusselt number from an infinite isothermal horizontal circular cylinder.

Table 2. contains the heat transfer results of six authors for air together with the present data, calculated from Eq. (7), In the Rayleigh number range 35 to 10^7. Some of these results were obtained experimentally and the rest numerically, except for the Churchill and Chu correlation [12] which was determined from joining the asymptotic solutions at the limiting cases of Pr = 0 and ∞. Inspection of Fig. 5. reveals that the present correlation is very close to all other results in the Rayleigh number range 35 to 10^6 except for the Churchill and Chu correlation [12], which is much lower in the Rayleigh number range 35 to 10^5.

Table 2. Recent data of natural convection from an infinite isothermal horizontal circular cylinder to air (Pr = 0.7).

	Nu (for air Pr=0.7)						
Ra $\times 10^{-3}$	Present Correlation	Clemes et al. 1994	Kuehn-Goldstein 1976	Raithby-Hollands 1985	Churchill-Chu 1975	Morgan 1975	Raithby-Hollands Modified
0.0346	1.57	1.71	1.79	1.69	1.39	1.72	1.69
0.0532	1.68	1.80	1.91	1.81	1.49	1.84	1.81
0.116	1.91	2.12	2.16	2.06	1.71	2.08	2.05
0.215	2.13	2.30	2.39	2.30	1.92	2.33	2.29
0.33	2.30	2.53	2.57	2.48	2.09	2.53	2.47
0.57	2.54	2.76	2.83	2.75	2.32	2.80	2.73
0.939	2.79	3.00	3.10	3.02	2.58	3.08	3.00
1.45	3.04	3.25	3.37	3.29	2.82	3.34	3.26
2.36	3.35	3.60	3.69	3.63	3.14	3.66	3.58
3.86	3.70	4.01	4.06	4.02	3.50	4.01	3.95
5.90	4.04	4.33	4.42	4.39	3.86	4.35	4.31
9.50	4.48	4.80	4.87	4.87	4.32	4.76	4.76
15.1	4.95	5.28	5.36	5.39	4.82	5.32	5.25
23.3	5.45	5.73	5.87	5.94	5.36	5.93	5.76
37	6.04	6.52	6.49	6.61	6.02	6.66	6.37
58.8	6.72	7.24	7.17	7.36	6.77	7.47	7.05
90.2	7.42	7.82	7.88	8.15	7.57	8.32	7.75
143	8.27	8.77	8.73	9.12	8.55	9.34	8.61
226	9.23	9.79	9.67	10.20	9.65	10.46	9.54
351	10.27	10.72	10.69	11.39	10.87	11.68	10.55
557	11.50	11.86	11.89	12.82	12.34	13.11	11.75
885	12.90	13.27	13.24	14.46	14.04	14.72	13.09
1370	14.40	14.52	14.67	16.22	15.87	16.42	14.51
2200	16.24	16.50	16.44	18.43	18.18	18.49	16.26
3530	18.33	18.53	18.45	20.96	20.83	20.80	18.25
5390	20.46	20.47	20.51	23.56	23.56	23.12	20.30
8540	23.07	23.15	23.12	26.81	26.99	25.95	22.39

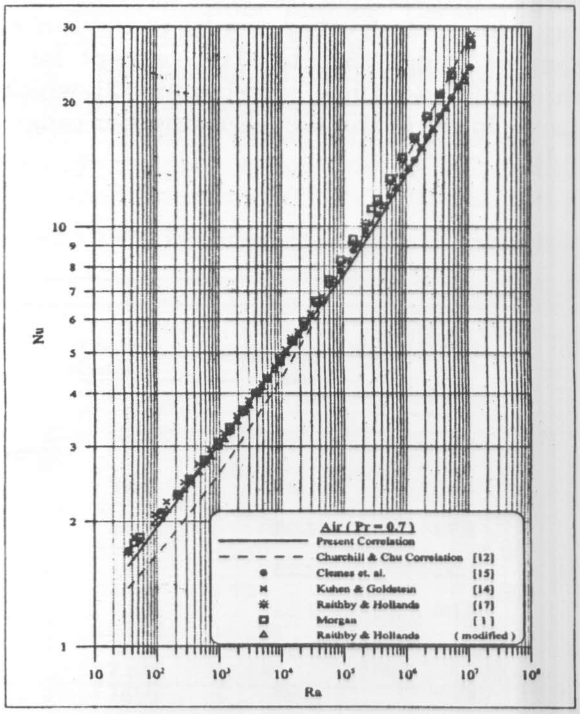

Fig.5. Comparison of the present correlation with the recent published data.

For a Rayleigh number between 10^6 and 10^7, the present correlation is in complete coincidence with the results of Clemes et al. 1994 [15], Kuehn and Goldstein [14] and Raithby and Hollands "modified", but it is lower than those of Morgan [1] and Raithby and Hollands [17]. If we exclude the work by Raithby and Hollands [17], in favor of the revised work by the same authors (Raithby and Hollands "Modified"), the work of Morgan [1] will be the only one which lies above the present correlation. On the other hand, the Churchill and Chu correlation [12] is below all the results up to Ra $\cong 10^5$ and above them beyond that up to $\cong 10^7$. It runs out clearly that the correlation in Eq. (7) represents the available natural convection heat transfer data from a horizontal cylinder to air better than the Churchill and Chu correlation [12], Eq. (2). The effect of Pr on Nu is shown on Fig. 6. for different values of Prandtl numbers (0.1, 10 and 100) along with results from other authors for comparison with the present results.

It is seen that the present study gives values between all previous results. The highest deviation is 30 % which is that from the Churchill and Chu correlation [12]. Thus the present correlation gives the average effect of the Prandtl number on natural heat transfer from a horizontal cylinder.

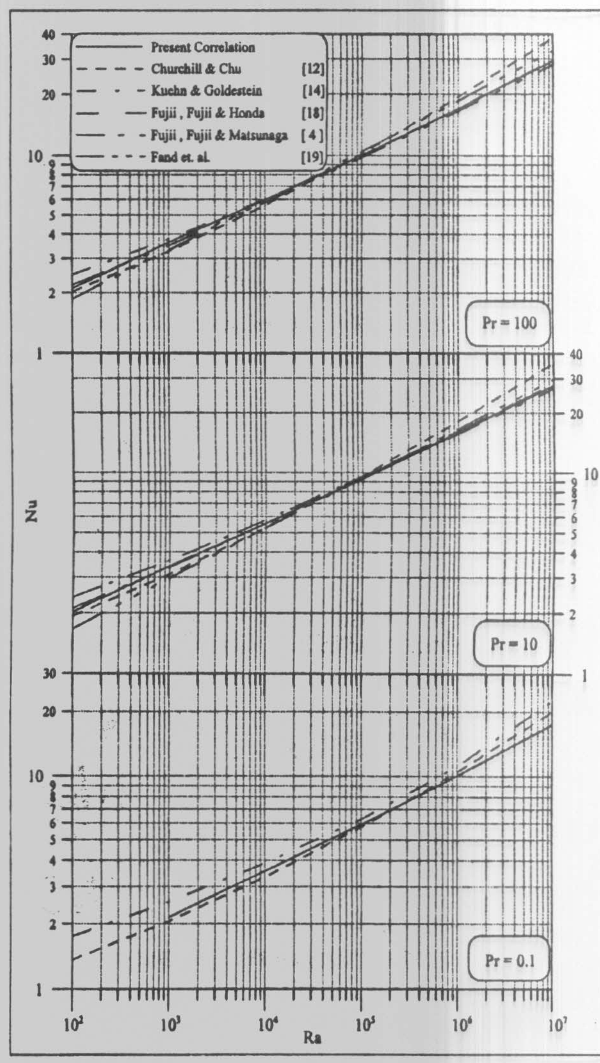

Fig. 6. Comparison of the present correlation with other published data for different prandtl numbers.

5. Conclusions

The problem of natural convection from an infinite isothermal horizontal circular cylinder was analyzed and solved numerically. A correlation between average Nusselt number and the Rayleigh and prandtl numbers in the range $10^2 < Ra < 10^7$ and $0.1 < pr\ 100$ was derived. The percentage difference between the present average Nusselt number for pr= 0.7 and that calculated from Churchill and Chu correlation is - 10.55 % at Ra= 10^2 and 17.26 % at Ra= 10^7.

Nomenclature

A	Cylinder radius, m.
G	Gravitational acceleration, m/s^2.
Gr	Grashof number, $g\beta(T_w - T_\infty)(2a)^3/\nu^2$
H	Average heat transfer coefficient, W/m^2 K.
H_ϕ	Local heat transfer coefficient, W/m^2 K.
K	Fluid thermal conductivity , W/m K.
Nu	Average Nusselt number , $h(2a)/k$.
Nu_ϕ	Local Nusselt number, $h_\phi(2a)/k$.
Pr	Prandtl number, ν/α .
R	Radial coordinate , m .
R	Dimensionless radial coordinate, r/a .
Ra	Rayleigh number, $g\beta(T_w - T_\infty)(2a)^3/\nu\alpha$.
T	Local fluid temperature, K.
V_r	Radial velocity, m/s.
V_r	Dimensionless radial velocity.
V_ϕ	Tangential velocity, m/s.
V_ϕ	dimensionless tangential velocity.

Greek

α	thermal diffusivity , m^2/s .
β	coefficient of volumetric thermal expansion, K^{-1}.
ν	Kinematics viscosity, m^2/s .
ϕ	angular coordinate, rad.
θ	dimensionless temperature, $(T - T_\infty)/(T_w - T_\infty)$.

Subscript

1	inner
2	outer
W	wall
∞	ambient

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Received March 30, 2000
Accepted June 26, 2000