Modeling and simulation of the power transformer faults

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The modeling of power transformer faults and its application to performance evaluation of digital protection algorithms are the objective of this study. This paper presents the algorithm that can be used to simulate an internal turn-to-earth fault occurring on either the primary side or the secondary side of the transformer. The method of simulating external faults occurring at the transformer terminals is also explained. Simulation results showing the validity of the model are presented.

يقدم البحث التفاصيل الكامله لطريقة رياضية جديدة لتمثيل المحولات أثناء الأعطال الداخلية والتى تحدث بسبب إنهيار العزل للملفات ، وأثناء الأعطال الخارجية. وتستخدم هذه الطريقة لحساب النوعيات المختلفة من تيارات القصر التى قد تحدث بسبب الأعطال السابق الإشارة اليها. يبدأ البحث بدراسة للمصفوفه الحثية للمحول عند حدوث عطل داخلى به حيث إن هذه المصفوفه يحدث بها بعض التغيرات نتيجة لهذا العطل. والبحث يشرح أسباب التغير فى المصفوفه وكيفية حسابها . بعد ذلك يتم شرح المعادلات التى تحتاجها طريقه الحساب المقترحة لتمثيل الأعطال الداخلية والخارجية . وينتهى البحث بتقديم النتائج فى هيئة رسومات لتيارات القصر أثناء الأعطال المختلفة. ويخدم هذا العمل الأبحاث التى تجرى لتطوير وتحديث أجهزة الوقاية الرقمية باستخدام الميكروبروسيسور.

Keywords: Power transformer modeling, Electromagnetic transients program, Transformer fault analysis.

1. Introduction

The development and the validation of algorithms for transformer protection require the preliminary determination of a power transformer model [1]. This model must simulate all the situations that will be chosen to study the behavior of the protection algorithm. In particular it must allow for the simulation of internal and external faults. Most of the Electromagnetic transient programs (EMTP) available are able to accurately simulate other phenomena occurring in the transformer like inrush magnetizing current, exciting current and transformer saturation [2]. Developing a model to simulate internal and external faults of a power transformer, and applying it to test a transformer protection algorithm, is a true validation for it.

This paper describes a model that can be used to simulate internal and external faults of a transformer. The model is based on the hysical concept of representing windings as mutually coupled coils. The impedance matrix can easily be derived from commonly available test data. In this paper the analysis presented in [3,4] is used to build the model. The algorithms presented in [5,6] have also been of guidance to the work presented in this paper.

2. Model Outline

The algorithm presented in this paper can be divided into three parts as indicated below.

Part A:

In this part the excitation and short circuit tests in positive and zero sequences are used to compute two matrices $[R_1]$ and $[L_1]$ modeling a healthy transformer [3]. In the case of a three-phase transformer, with two windings for each phase, these matrices are of order 6 as shown in Eqs. (1,2).

$$R_1 = \begin{pmatrix} R_{ap} & 0 & 0 & 0 & 0 & 0 \\ 0 & R_{as} & 0 & 0 & 0 & 0 \\ 0 & 0 & R_{bp} & 0 & 0 & 0 \\ 0 & 0 & 0 & R_{bs} & 0 & 0 \\ 0 & 0 & 0 & 0 & R_{cp} & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{cs} \end{pmatrix} \tag{1}$$

$$L_1 = \begin{pmatrix} L_{ap} & M_{apas} & M_{apbp} & M_{apbs} & M_{apcp} & M_{apcs} \\ M_{asap} & L_{as} & M_{asbp} & M_{asbs} & M_{ascp} & M_{ascs} \\ M_{bpap} & M_{bpas} & L_{bp} & M_{bpbs} & M_{bpcp} & M_{bpcs} \\ M_{bsap} & M_{bsas} & M_{bsbp} & L_{bs} & M_{bscp} & M_{bscs} \\ M_{cpap} & M_{cpas} & M_{cpbp} & M_{cpbs} & L_{cp} & M_{cpcs} \\ M_{csap} & M_{csas} & M_{csbp} & M_{csbs} & M_{cscp} & L_{cs} \end{pmatrix} \tag{2}$$

Where R_i and L_i are the resistance and the self-inductance of coil *i*, and M_{ij} is the mutual inductance between coils *i* and *j*, see Fig. 1. In this manner the transformer will be handled as mutually coupled branches.

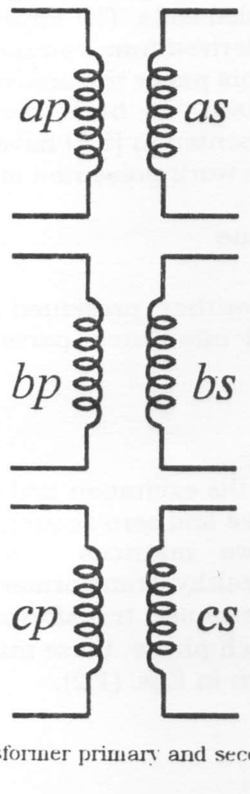

Fig. 1. Transformer primary and secondary windings.

Part B:

In this part the winding of a transformer suffering from a turn-to-earth fault is divided into two coils, and the transformer is then described with two 7x7 matrices [R_2] and [L_2]. In [4] a method is described for the determination of the 7x7 matrices, this method can be applied for a faulty coil on either the primary side or the secondary side of the transformer.

Part C:

It is the objective of this paper to describe Part **C** of the algorithm. Part **C** utilizes the analysis presented in Parts **A** and **B** and develops complete equations that can be used to simulate an internal turn-to-earth fault occurring on either the primary side or the secondary side of the transformer. It is also used to simulate external faults occurring at the transformer terminals.

3. Turn-To-Earth Faults

3.1. Matrices calculations

When a turn-to-earth fault occurs in phase *bp*, its coil will be divided into two sub-coils *bp1* and *bp2*, as shown in Fig. 2. Each sub-coil will have its own resistance, self-inductance and mutual inductance with the other sub-coil and all remaining coils of the transformer. This leads to the formation of new 7x7-[R_2] and [L_2] matrices [4].

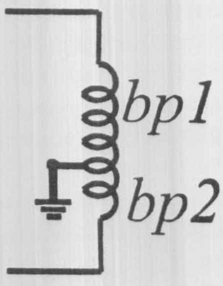

Fig. 2. Turn-to-earth fault in winding bp.

The 7x7-matrix $[R_2]$ will be determined with the help of the following relations:

$$R_{bp1} = \frac{n_{bp1}}{n_{bp}} R_{bp}$$
$$R_{bp2} = \frac{n_{bp2}}{n_{bp}} R_{bp} \qquad (3)$$

where;

n_{bp} is the Number of turns of coil bp,
n_{bp1} is the Number of turns of coil bp1,
n_{bp2} is the Number of turns of coil bp.

The main difficulty is to determine the new 7x7-matrix $[L_2]$. The elements that related to the faulty sub-coils *bp1* and *bp2* in Eq. (4) are unknown. The other ones are determined in a straightforward manner by the tests described in [3] and can thus be considered as known data.

$$L_2 = \begin{pmatrix} L_{ap} & M_{apas} & M_{apbp1} & M_{apbp2} & M_{apbs} & M_{apcp} & M_{apcs} \\ M_{asap} & L_{as} & M_{asbp1} & M_{asbp2} & M_{asbs} & M_{ascp} & M_{ascs} \\ M_{bp1ap} & M_{bp1as} & L_{bp1} & M_{bp1bp2} & M_{bp1bs} & M_{bp1cp} & M_{bp1cs} \\ M_{bp2ap} & M_{bp2as} & M_{bp2bp1} & L_{bp2} & M_{bp2bs} & M_{bp2cp} & M_{bp2cs} \\ M_{bsap} & M_{bsas} & M_{bsbp1} & M_{bsbp2} & L_{bs} & M_{bscp} & M_{bscs} \\ M_{cpap} & M_{cpas} & M_{cpbp1} & M_{cpbp2} & M_{cpbs} & L_{cp} & M_{cpcs} \\ M_{csap} & M_{csas} & M_{csbp1} & M_{csbp2} & M_{csbs} & M_{cscp} & L_{cs} \end{pmatrix} \qquad (4)$$

The first step is to determine the self and mutual inductance of the faulty sub-coils: L_{bp1}, L_{bp2} and M_{bp1bp2}. This purpose will be achieved according to three rules: consistency, leakage and proportionality.

Consistency

If, when using the 7x7-matrix, coils *bp1* and *bp2* are supplied in series without any fault, the same results must be found as when the 6x6 matrix is used. If it is assumed that coils *bp1* and *bp2* are supplied in series and that a current *i* flows through them. All other coils are on no-load. Hence

$$\varphi_{bp1} = \left(L_{bp1} + M_{bp1bp2}\right) i \qquad (5)$$
$$\varphi_{bp2} = \left(L_{bp2} + M_{bp1bp2}\right) i$$

In this way

$$\varphi_{bp1} + \varphi_{bp2} = \left(L_{bp1} + 2M_{bp1bp2} + L_{bp2}\right) i \qquad (6)$$
$$\varphi_{bp} = \left(L_{bp}\right) i$$

These relations lead to the well-known expression of L_{bp}, considered as two inductance in series, as follows:

$$L_{bp} = L_{bp1} + 2M_{bp1bp2} + L_{bp2} \qquad (7)$$

L_{bp1}, L_{bp2} and M_{bp1bp2} are elements of the 7x7 matrix.
L_{bp} is an element of the 6x6 matrix.

Leakage

Taking into account a leakage factor between coils *bp1* and *bp2* is essential since the fault current will largely depend on the leakage [4,5,6]. The leakage factor is given by:

$$\sigma_{bp1bp2} = 1 - \frac{M^2_{bp1bp2}}{L_{bp1} L_{bp2}} \qquad (8)$$

A complete analysis for calculating the leakage factors for faulty coils is presented in [4] and is used in the model described in this paper. Hence, the leakage factor will be considered as a known parameter.

Proportionality

To determine the three unknowns L_{bp1}, L_{bp2} and M_{bp1bp2} a third equation must be added:

$$\frac{L_{bp1}}{L_{bp2}} = k^2. \qquad (9)$$

where,

$$k = \frac{n_{bp1}}{n_{bp2}}.$$

Eq. (9) is approximate. It expresses that k is the voltage ratio between coils *bp1* and *bp2*. It is strictly true if there is no leakage (σ_{bp1bp2}=0). However, it represents an excellent numerical approximation when σ_{bp1bp2} is close to zero and is very widely used. Solving Eqs. (7-9) for three unknowns L_{bp1}, L_{bp2} and M_{bp1bp2} will yield:

$$L_{bp1} = \frac{L_{bp}}{\frac{1}{k^2} + \frac{2\sqrt{1-\sigma_{bp1bp2}}}{k} + 1}, \quad (10)$$

$$L_{bp2} = \frac{L_{bp}}{k^2 + 2k\sqrt{1-\sigma_{bp1bp2}} + 1}, \quad (11)$$

$$M_{bp1bp2} = \frac{L_{bp}\sqrt{1-\sigma_{bp1bp2}}}{k + \frac{1}{k} + 2\sqrt{1-\sigma_{bp1bp2}}} . \quad (12)$$

The second step is to determine the mutual inductance between the coil *bp1* and any other coil *j* except for coil *bp2* (as well as those between the coil *bp2* and any other coil *j* except for *bp1*). The consistency principle leads to [4,5]:

$$M_{bpj} = M_{bp1j} + M_{bp2j} \quad (13)$$

The proportionality relation leads to:

$$\frac{M_{bp1j}}{M_{bp2j}} = \frac{n_{bp1}}{n_{bp2}} = k \quad (14)$$

Eqs. (13) and (14) lead to:

$$M_{bp1j} = \frac{k}{1+k} M_{bpj}, \quad M_{bp2j} = \frac{1}{1+k} M_{bpj} \quad (15)$$

It should be noted that M_{bpj} is considered as known data [3].

3.2. Internal faults algorithm

Fig. 3 shows the model used to simulate single phase-to-earth faults. The model consists of a 3-phase supply that is connected to the primary side of the transformer via a short transmission line represented by its resistance R_{TL} and inductance L_{TL}. The secondary side of the transformer is connected to a variable load (R_{Lo}, L_{Lo}). The faulty winding *bp* is divided into two sub-windings *bp1* and *bp2*, and their resistance and inductance are calculated as indicated in the previous section. In practice, ground resistance are used to limit the ground faults to acceptable values, hence three ground resistance R_{g1}, R_{g2} and R_{g3} are added to the model.

With the aid of Fig. 3 the machine terminal voltages, during an internal single phase to ground fault, can be expressed as follows:

$$\begin{aligned}
e_{ap} &= V_m \sin(\omega t) - R_{TL} i_{ap} - L_{TL} p i_{ap} - e_{g1} - e_{g2} , \\
e_{as} &= -R_{Lo} i_{as} - L_{Lo} p i_{as} - e_{g3} , \\
e_{bp1} &= V_m \sin(\omega t - 120) - R_{TL} i_{bp1} - L_{TL} p i_{bp1} - e_{g1} , \\
e_{bp2} &= -e_{g2} , \\
e_{bs} &= -R_{Lo} i_{bs} - L_{Lo} p i_{bs} - e_{g3} , \\
e_{cp} &= V_m \sin(\omega t + 120) - R_{TL} i_{cp} - L_{TL} p i_{cp} - e_{g1} - e_{g2} , \\
e_{cs} &= -R_{Lo} i_{cs} - L_{Lo} p i_{cs} - e_{g3} .
\end{aligned} \quad (16)$$

where,

V_m is the maximum value of supply voltage,

ω is the supply frequency in rad/sec,

$p = \frac{d}{dt}$,

$e_{g1} = (i_{ap} + i_{bp1} + i_{cp}) R_{g1}$,

$e_{g2} = (i_{ap} + i_{bp2} + i_{cp}) R_{g2}$, and

$e_{g3} = (i_{as} + i_{bs} + i_{cs}) R_{g3}$.

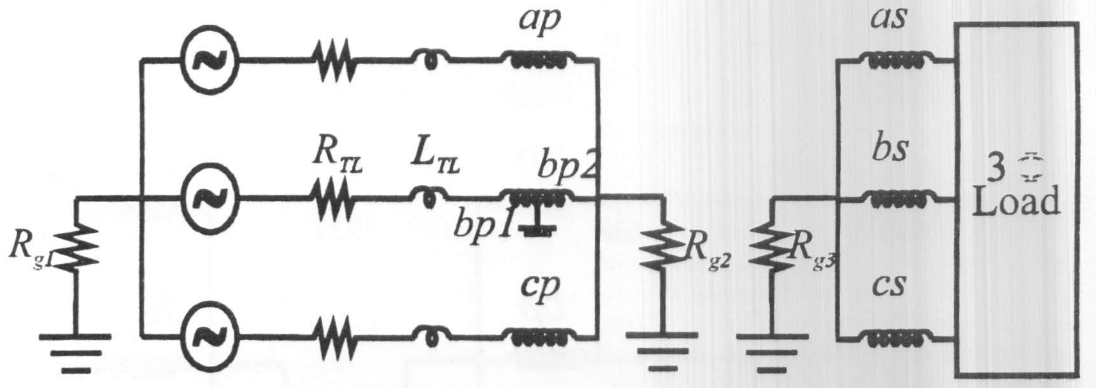

Fig. 3. System model for a turn-to-earth fault in winding bp.

The transformer voltage vector $e=[e_{ap}, e_{as}, e_{bp1}, e_{bp2}, e_{bs}, e_{cp}, e_{cs}]^t$ is related to the flux linkage vector $\psi=[\psi_{ap}, \psi_{as}, \psi_{bp1}, \psi_{bp2}, \psi_{bs}, \psi_{cp}, \psi_{cs}]^t$, winding resistance matrix $[R_2]$ and current vector $i=[i_{ap}, i_{as}, i_{bp1}, i_{bp2}, i_{bs}, i_{cp}, i_{cs}]^t$ by:

$$e = p\psi + R_2\, i, \tag{17}$$

The flux linkage vector ψ is also related to the current vector i by:

$$\psi = L_2\, i, \tag{18}$$

The algorithm utilizes Eqs. (16-18) to simulate a transformer suffering from a single phase-to-earth fault. The technique provided above can simulate an internal fault occurring on either the primary side or the secondary side of the transformer.

4. External faults

As the model is intended to be used to validate protection algorithms, simulation of external faults is as important as the simulation of internal faults. External faults can occur either on the primary side of the transformer or the secondary side. The simulation model should be able to simulate faults on either side of the transformer. The sections presented below describe the simulation of an external single phase to ground fault on the primary side of the transformer and an external two phase to ground fault on the secondary side of the transformer.

4.1. Single phase to ground fault

Fig. 4 shows an external single phase to ground fault occurring on the primary side of the transformer. The following equations describe the transformer terminal voltages during the fault.

$$\begin{aligned}
e_{ap} &= -e_{g2}, \\
0 &= V_m \sin(\omega t) - R_{TL} i_{so} - L_{TL} p i_{so} - e_{g1}, \\
e_{as} &= -R_{Lo} i_{as} - L_{Lo} p i_{as} - e_{g3}, \\
e_{bp} &= V_m \sin(\omega t - 120) - R_{TL} i_{bp} - L_{TL} p i_{bp} - e_{g1} - e_{g2}, \\
e_{bs} &= -R_{Lo} i_{bs} - L_{Lo} p i_{bs} - e_{g3}, \\
e_{cp} &= V_m \sin(\omega t + 120) - R_{TL} i_{cp} - L_{TL} p i_{cp} - e_{g1} - e_{g2}, \text{ and} \\
e_{cs} &= -R_{Lo} i_{cs} - L_{Lo} p i_{cs} - e_{g3},
\end{aligned} \tag{19}$$

where,

$$\begin{aligned}
e_{g1} &= (i_{so} + i_{bp} + i_{cp}) R_{g1}, \\
e_{g2} &= (i_{ap} + i_{bp} + i_{cp}) R_{g2}, \text{ and} \\
e_{g3} &= (i_{as} + i_{bs} + i_{cs}) R_{g3},
\end{aligned}$$

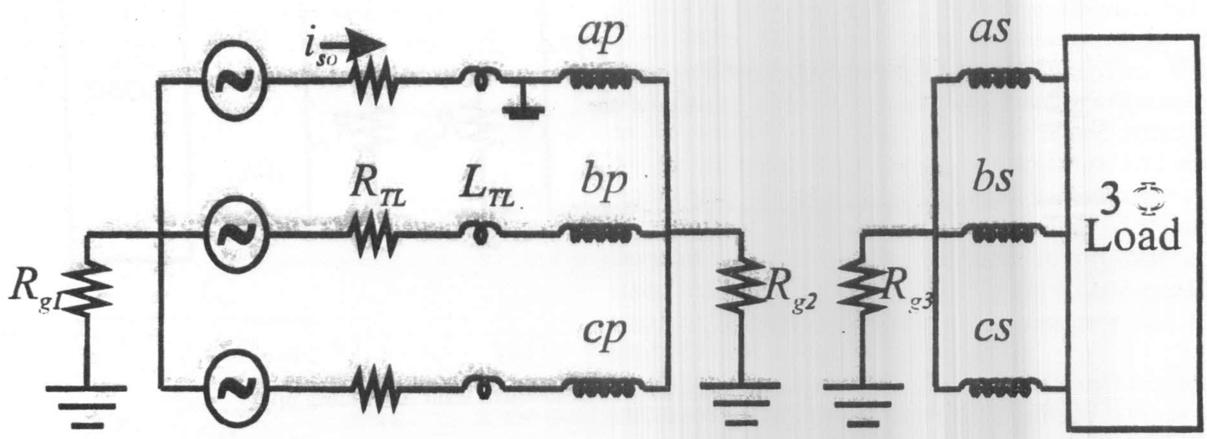

Fig. 4. System model for an external single phase to ground fault.

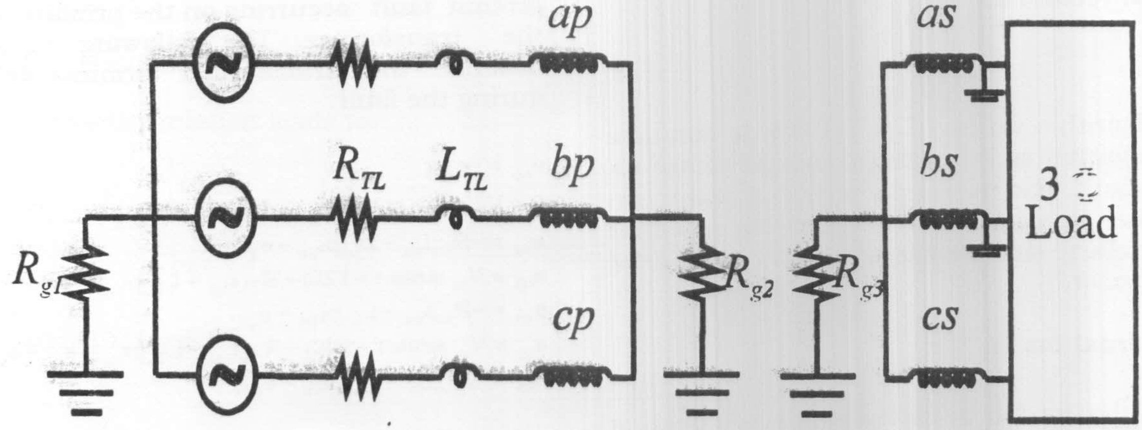

Fig. 5. System model for an external two phase to ground fault.

As in the case of internal single phase to ground faults the remaining equations that constitute the external fault model are:

$$e = p\psi + R_1 i, \quad (20)$$

$$\psi = L_1 i, \quad (21)$$

where,

$e = [e_{ap}, e_{as}, e_{bp}, e_{bs}, e_{cp}, e_{cs}]$,
$\psi = [\psi_{ap}, \psi_{as}, \psi_{bp}, \psi_{bs}, \psi_{cp}, \psi_{cs}]$, and
$i = [i_{ap}, i_{as}, i_{bp}, i_{bs}, i_{cp}, i_{cs}]$.

It should be noted that $i_{so} = i_{ap}$ prior to the occurrence of the fault. During the fault i_{so} is obtained by applying a numerical integration technique to solve Eq. (19). The flux linkage vector ψ is also obtained by applying a numerical integration technique to solve Eq. (20), and finally the current vector i is obtained be using the obtained vector ψ and inverting the matrix $[L_1]$.

4.2. Two phase to ground fault

The transformer during a two phase to ground fault is represented as shown in Fig. 5. The model used for this simulation is very similar to the model described in the previous section. The only difference is in the equations representing the machine terminal voltages, which are given by:

$$\begin{aligned}
e_{ap} &= V_m \sin(\omega t) - R_{TL} i_{ap} - L_{TL} p i_{ap} - e_{g1} - e_{g2} , \\
e_{as} &= -e_{g3} , \\
e_{bp} &= V_m \sin(\omega t - 120) - R_{TL} i_{bp} - L_{TL} p i_{bp} - e_{g1} - e_{g2} , \\
e_{bs} &= -e_{g3} , \\
e_{cp} &= V_m \sin(\omega t + 120) - R_{TL} i_{cp} - L_{TL} p i_{cp} - e_{g1} - e_{g2} , \\
e_{cs} &= -R_{Lo} i_{cs} - L_{Lo} p i_{cs} - e_{g3} ,
\end{aligned} \quad (22)$$

where,

$$e_{g1} = (i_{ap} + i_{bp} + i_{cp}) R_{g1}.$$

The voltages e_{g2} and e_{g3} are obtained in the same manner described in the previous section. Eq. (20,21) are also used in this model.

5. Simulation results

Prior to the occurrence of a fault, the transformer is assumed to be operating normally at its full load. The model presented in [3] is used to simulate normal operating conditions. The transformer parameters used for internal and external fault simulations are the same parameters used in [3]. The model described above can work equally well using physical quantities or per-unit values. However in this paper the physical quantities are used. Eqs. (16-18) have been used to simulate an internal single phase to ground fault at 60% of coil *bp* (i.e. n_{bp1}=0.6n_{bp}). The ground resistance are R_{g1}=42 ohm, R_{g2}=0 and R_{g3}=10 ohm. These values were chosen to limit the fault currents to acceptable values. The simulation results are shown in Figs. 6,7. It should be noted that fault inception time is 0.045s. Figs. 8,9 show an external two phase to ground fault occurring in phases *as* and *bs*. It should be noted that the ground resistance are not effective in limiting the fault currents in the second case.

6. Conclusions

It has been shown that internal single phase to ground faults and external terminal faults could be modeled in a power transformer. In the case of internal faults a new inductance matrix has to be computed. The procedure of calculating the required matrix is explained.

Inclusion of the ground resistance to limit the fault currents to acceptable values is necessary when validating different protection schemes. Ground resistance has been included in the model, and its effect in limiting single phase to ground fault currents is shown in the simulation results.

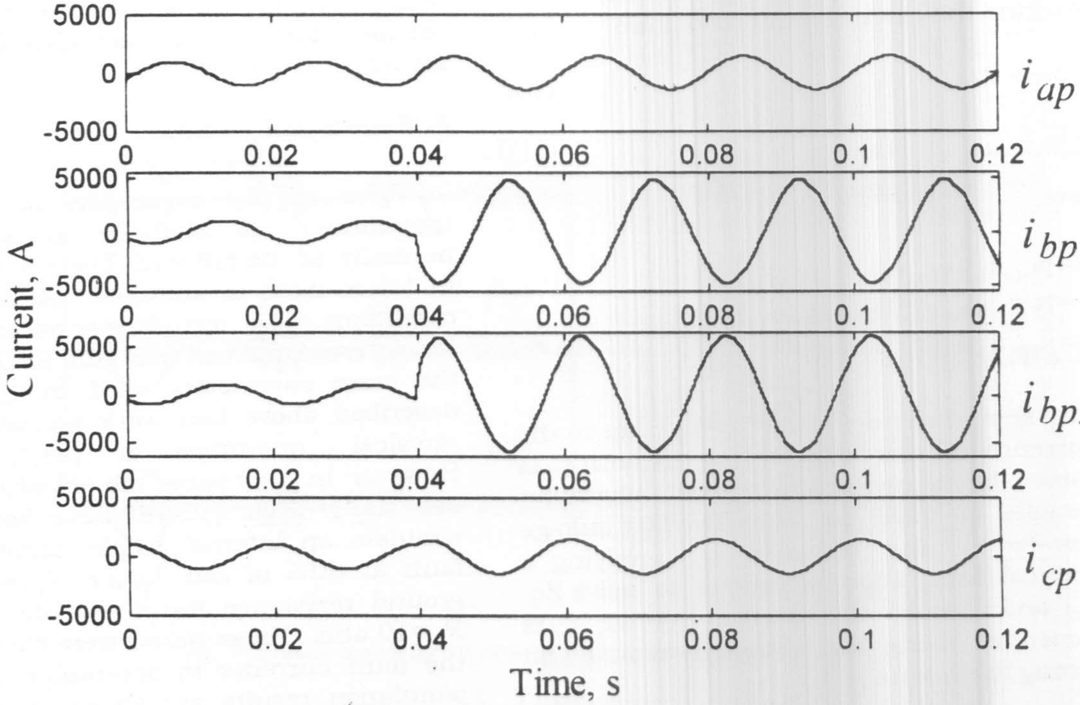

Fig. 6. Primary side currents for an internal single phase to ground fault..

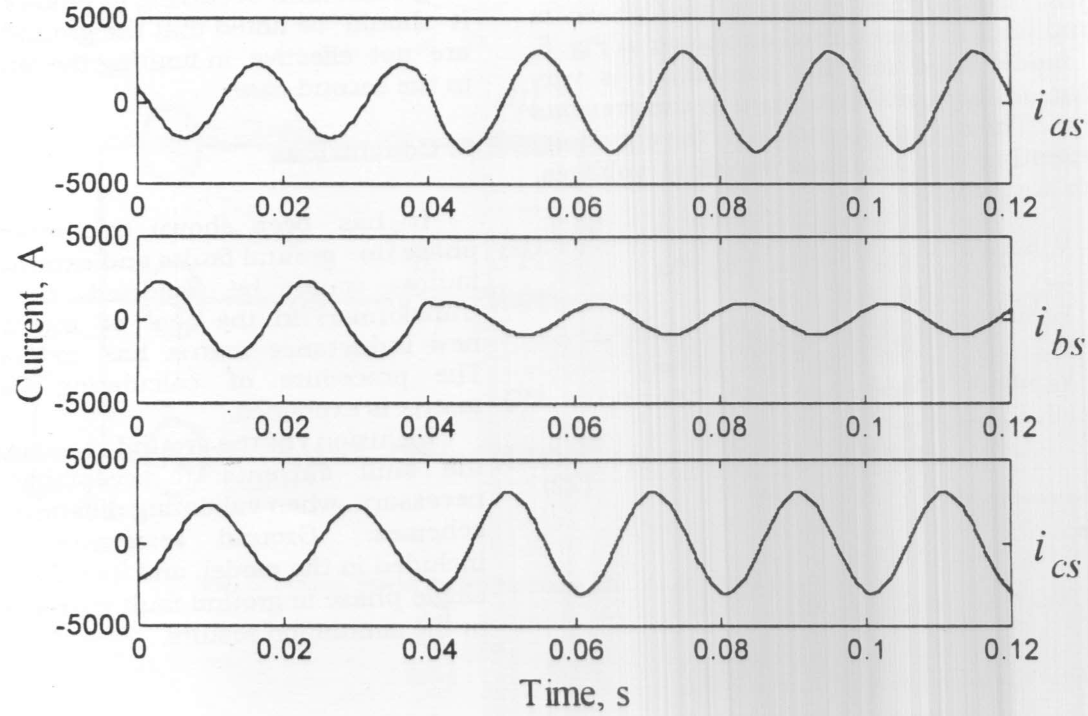

Fig. 7. Secondary side currents for an internal single phase to ground fault.

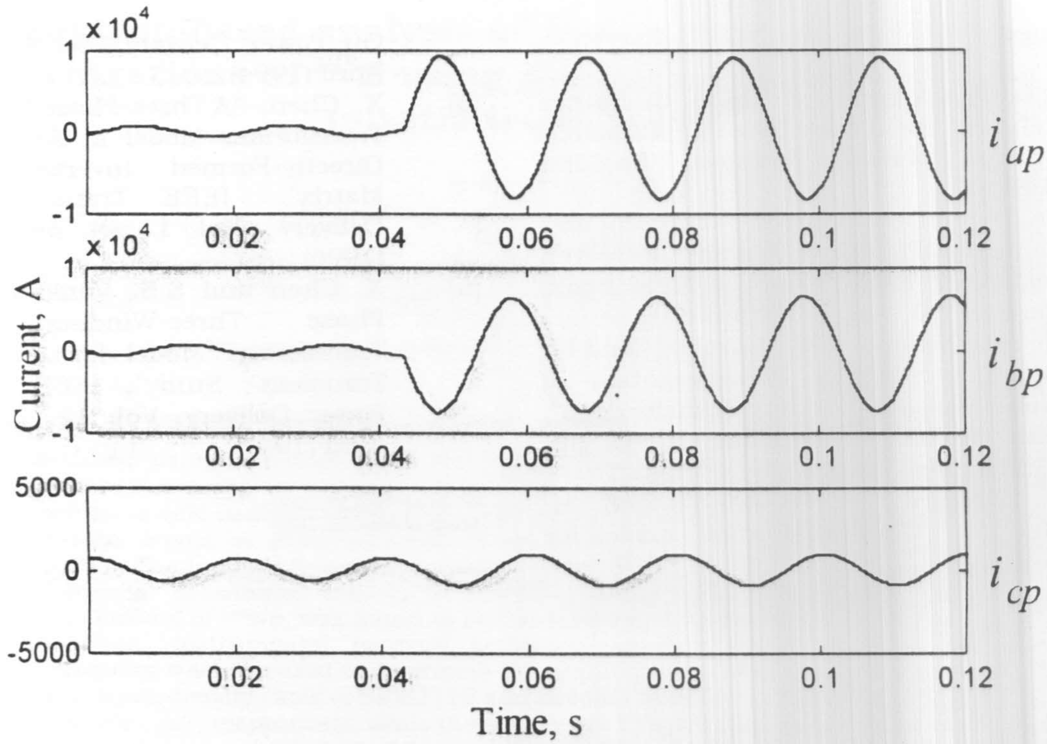

Fig. 8. Primary side currents for an external two phase to ground fault.

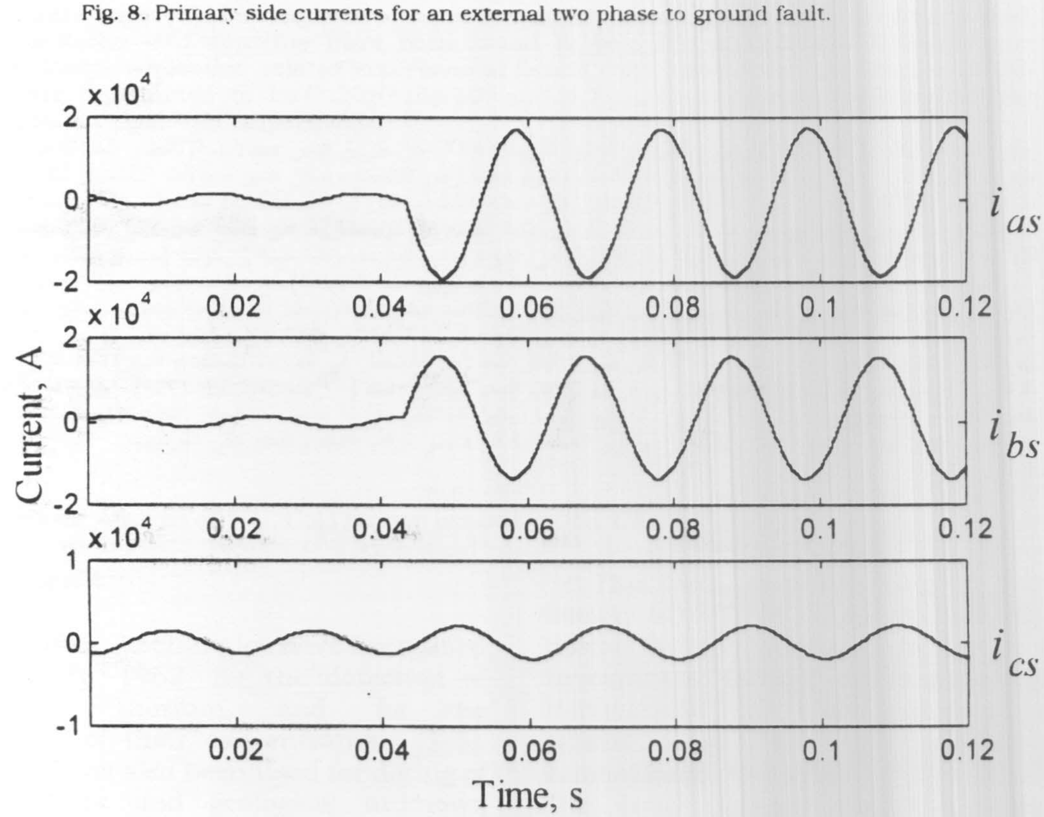

Fig. 9. Secondary side currents for an external two phase to ground fault.

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