Initialization techniques for hopfield neural network algorithm for cellular radio channel assignment

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Since the frequency spectrum of the mobile radio communications is limited, the channel assignment problem deserves more attention in order to use the available frequency spectrum with optimum efficiency. A new channel assignment algorithm using a modified Hopfield neural network is proposed in [4]. In this paper, we propose various initialization techniques based on multilevel rearrangement of the channels before applying the algorithm of [4] to decrease the number of iteration and improve the convergence rate. These techniques will guarantee that the neural network will skip the local minimum, and in all cases will converge to optimum arrangement of the channels. The specific characteristics of the channel assignment problem in cellular radio network such as Co-Site Constraints, Adjacent Channel Constraints, and Co-Channel Constraints are considered with the implementation of the preassignment techniques. The results of the proposed techniques are compared with other prior reported techniques for the same eight benchmark problems. The comparison shows the merits of the proposed initialization techniques.

مشكلة تخصيص الخطوط النيناميكية في شبكات الاتصال بالرانيو الخلوية من المشاكل التي تستحق الانتباه و ذلك لأنه ف مثّل هذا النوع من التخطيط يته التعامل مع جميع الخطوط المتاحة للشبكة في أن واحد لتخصيص خط خالي لأي مكالم مطلوبة . و أيضا لزيادة الطلب على خدمة الاتصال بالراديو مع ثبات المدى الترددي المخصص لهذا الغــــرض. تطبيــق تقنوات الشبكة العصبية لحل مثل هذه المشكلة يعتبر اتجاها جديداً أثبت فاعلية و قدم حلولا مشجعة. في السسنوات الأخسيرة ظهرت بعض الأبحاث التي تطرقت لمثل هذه المشكلة. البحث الأول من هذه الأبحاث فرض أربعة شروط أولية يساعد علَّى الوصول للحل الأمثل للشبكات العصبية ذات التغذية الخلفية للطبقة الواحدة. البحث الثاني استخدم نفس الطريقة الأولى مسمع فرض طرق تمهينية ثابتة و أخرى عشوائية بالإضافة إلى ثلاث طرق تحنيث و كانت نتائجه أحسن قليلا من مثيلتـــها فـــى البحث الأول مع التطبيق على نفس مجموعة المشاكل في كلتا الحالتين. في هذا البحث ته فرض طرق تمهيئية جديدة تطبـــق على طريقة البحث الأول لحل نفس مجموعة المشاكل إلى تعرض لها كل من هنين البحثين. هذه الطرق تتلخص في تطبيق أربعة مستويات لعملية التخصيص النيناميكية لآي مكالمة قبل تطبيق نموذج الشبكة العصبية.المستوى الأول: يقسوم بتحديث الخلية ذات أكبر عند من المكالمات و توفير أقل عند من خطوط الخنمة يفي باحتياجات هذه المكالمات مع مراعاة تحقيسق جميع القيود التي تحد من عملية التخصيص لهذه الخلية. هذه القيود هي (أ) قيد المواقع المتماثلة (ب) قيد الخطوط المتماثلــة و (ج) قيد الخطوط المتجاورة. المستوى الثاني: يقوم بتحديد عند المكالمات المطلوبة لباقي الخلايا و تخصيص أقل عند مسن الخطوط لخدمة هذه الخلايا مع مراعاة تحقيق قيد المواقع المتماثلة فقط. الخلايا مرتبة بشكل عشوائي. المستوى الثالث: ويقوم بحل المشاكل المتبقية من المستوى الثاني بعد ترتيب الخلايا ترتيبا تنازليا من حيث عند المكالمات المطلوبة لكل خلية. المستوى الرابع: و يقود بتحقيق باقي القيود علَى عملية التخصيص لكل الخطوط في كل الخلايا.تُـــم تطبيـــق هـــــــد الطــــرق التمهينية على نفس المشاكل التي أخنت في الاعتبار في البحثين السابقين و النتائج التي حصلنا عليها كانت أفضل كثيرا مسي حيث تقليل متوسط عند المحاولات و زيادة معدل التقارب و ضمان إيجاد حل لجميع المشاكل بنسبة ٠٠٠%.

Keywords: Neural networks, Mobile radio networks.

1. Introduction

Since the usable frequency spectrum is limited, the channel assignment problem has

become an important issue to be solved in mobile telephone communications. At this time, the mobile management system should find a way to arrange the available channels

utilization. have maximum The implementation of neural network techniques to the channel assignment problem is a new which presents an encouraging problem of channel the solution to assignment [1-8]. In [2], four heuristics were used to improve the convergence rate of channel assignment. They also fixed some frequencies in one or more certain cells in order to accelerate the convergence time. This process has been applied on several problems, the results were favorable in some cases but not in others. In [9], Chan et al. used the feed forward neural network which had a learning process prior to the actual channel assignment. The performance of the algorithm is totally dependent on the training data used. They also considered only co-Kim et al. [10] channel constraints. proposed an algorithm based on various initialization and updating techniques. The initialization methods are: the fixed interval initialization method and the random interval initialization method, along with three updating methods. The results of Kim for the convergence rate and iteration number were better than those of Funabiki [2].

This paper presents a preassignment algorithm which composes of a multilevel initialization technique to be applied on the Funabiki's parallel algorithm neural network model. The new technique guarantees 100 % convergence rate and minimum iteration for the same eight benchmark problems proposed by Funabiki [2] and Kim [4]. In section 2, the channel assignment problem is discussed in view of neural representation considering the network different channel assignment constraints. Section 3 presents the mathematical of the parallel algorithm representation neural network model. In section 4, the steps to be followed on applying the parallel algorithm are discussed. In section 5, the proposed preassignment initialization levels discussed. The results of the proposed algorithm in comparison with prior reported results are given in section 6. The conclusion is drawn in section 7.

2. Channel assignment constraints

On dealing with dynamic channel assignment [13], some constraints have to be considered on assigning a certain frequency to a call in a cell. These constraints are [4-6]:

- [1]The Co-Site Constraints (CSC) which means, any pair of frequencies (channels) assigned to a radio cell must have certain distance in the frequency domain.
- [2]Co-Channel Constraints (CCC), which means the same frequency can not be assigned to certain pairs of radio cells simultaneously.
- [3] The Adjacent Channel Constraints (ACC), which means frequencies adjacent in the frequency domain can not be assigned to adjacent radio cells simultaneously.

The above constraints have to be satisfied before assigning any radio frequency to a call in any cell. In [2] the compatibility matrix C = (cij) is defined as n*n symmetric matrix, where n is the number of cells in the mobile radio network. Each nondiagonal element cii represents the minimum frequency separation between a frequency in cell #i another in cell # j. The CCC is represented by $c_{ij} = 1$, and ACC represented by $c_{ij} = 2$. $c_{ij} = 0$ indicates that cell # i and cell # j are allowed to use the same frequency. Each diagonal element cii represents the minimum separation distance between any two frequencies assigned to cell # i, which is the CSC, where c_{ii} > 1 is always satisfied. The required channel number for each cell # i is represented by the demand matrix D = (d_i) , where $1 \le i \le n$. Let f_{ik} indicates the assigned frequency for the kth call in cell # i, where $1 \le i \le n$ and $1 \le k \le n$ The condition imposed by compatibility constraints between fik and fil is given by $\left|f_{ik} - f_{jl}\right| \ge c_{ij}$, where $1 \le i,j \le n$ and $1 \le k, 1 \le d_i$ except i = j, k = 1.

The goal of the channel assignment problem is to solve for f_{ik} which satisfies the constraint conditions when the number of cells n in the mobile network are given along with the compatibility matrix C and the

number of required channels given by matrix D.

3. Neural network model

The neural network model [12] is composed of a large number of massively connected simple processing elements (neurons). The processing element has an input and an output. The input of a processing element is connected with outputs of several processing elements including the processing element itself as shown in Fig. 1. The parallel algorithm [2, 13, 14] is based on a two dimensional neural network model. A total of n*m processing elements is required for solving n-cell m-frequency problem, where n is the number of radio cells and m is the total number of frequencies (channels). The output of ijth processing element (Vij) indicates that whether or not frequency # j is assigned to cell # i. The non zero output (Vii = 1) indicates that frequency # j is assigned to cell # i. The zero output $(V_{ij} = 0)$ indicates that frequency # j is not assigned to cell # i. The output and the input of a processing element is given by:

$$V_i$$
 = 1, if U_i > UTP (upper trip point)
= 0, if U_i < LTP (lower trip point)
unchanged otherwise. (1)

The channel assignment constraints are represented as:

3.1. Channel requirements

For a required number of channels d_i for cell # i, which has non zero output, the following value will be zero

$$\sum_{q=1}^{m} V_{iq} - d_{i} . {2}$$

3.2. Co-site constraint

If the frequency # q within distance c_{ii} from frequency # j ($|j-q| < c_{ii}$) is assigned to

cell # i, then frequency # j must not be assigned to cell # i and for the following term:

$$\sum_{\substack{q=j-(c_{ii}-1)\\q\neq j\\1\leq q\leq m}}^{j+(c_{ii}-1)}V_{iq}.$$
(3)

The output of the above equation will have nonzero value if the co-site constraint is violated.

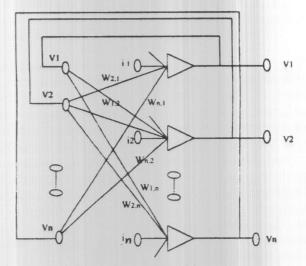


Fig. 1. Single layer feedback neural network.

3.3. Co-channel and adjacent channel constraints

If frequency # q within distance c_{ip} from frequency # j ($|j-q| < c_{ip}$) is assigned to cell # p for $c_{ip} > 0$ and p \neq i, frequency # j must not be assigned to cell # i and the following term will be nonzero if the assignment of frequency # j to cell # i violates co-channel and /or adjacent channel constraints.

$$\sum_{\substack{p=1\\p\neq i\\ p\neq i\\ c_{ip}\geq 0}}^{n}\sum_{\substack{j+(c_{ip}-1)\\ v_{pq}\\ l\leq q\leq m}}^{j+(c_{ip}-1)}V_{pq}. \tag{4}$$

The above equation will have nonzero output if co-channel and/or adjacent channel constraints are violated.

The motion equation of ijth processing element in the n-cell-m-frequency problem is given by:

$$\frac{dU_{ij}}{dt} = -A \left(\sum_{q=1}^{m} V_{iq} - d_{i} \right) - B \left(\sum_{\substack{q=1-(c_{ij}-1)\\q=j}}^{j-(c_{ij}-1)} V_{iq} + \sum_{\substack{p=1\\q=j-(c_{ij}-1)\\q=1\\1\leq q\leq m}} \sum_{\substack{1-(c_{ij}-1)\\q=j-(c_{ij}-1)\\q=1\\1\leq q\leq m}}^{j-(c_{ij}-1)} V_{pq} \right).$$
(5)

The motion equation (energy function) represents the distance between the current state of the neural network system and the solution state. The motion equation is determined by considering all constraints in the problem. The goal of the neural network model for solving combinatorial optimization problems is to minimize the motion equation. Only local minimum is guaranteed by neural network. In order to increase the frequency of the global minimum convergence, the following four heuristics have been introduced empirically:

3.3.1. A-term saturation heuristic

By applying bounding function to A-term, we have:

$$-\operatorname{Af}\left(\sum_{q=1}^{m} V_{iq} - d_{i}\right), \tag{6}$$

where $f(x)=A_{max}$ if $x > A_{max}$, $f(x)=A_{min}$ if $x < A_{min}$ and x otherwise.

3.3.2. Omega function heuristic

There are two forms of the B term, which are:

If
$$(t \mod T) < \omega$$

$$-\mathbf{B}\left(\begin{array}{c} \sum_{\substack{j=1\\q=j-1\\1\leq q\leq m}}^{j-1} V_{iq} + \sum_{\substack{p=1\\p=1\\c_{\phi}\neq 0}}^{n} \sum_{\substack{j=1\\q\leq q\leq m}}^{j-1} V_{pq} \\ V_{ij}, \end{array}\right) V_{ij}, \tag{7}$$

else

where t is the number of iteration steps, and T and ω are constant parameters.

3.3.4. The hill climbing heuristic

$$+ ch \left(\sum_{q=1}^{m} V_{1q} - d_{1} \right) (1 - V_{1j}),$$
 (8)

where h(x) = 1 if x < 0, h(x) = 0 if $x \ge 0$ and c is a random variable. The c term encourages ijth processing element to have nonzero output if fewer than d_i processing element for cell # i have nonzero output and $v_{ij} = 0$.

3.3.5. The input saturation heuristic

$$\begin{array}{ll} U_{ij} = U_{\max} & \text{if } U_{ij} > U_{\max} \\ = U_{\min} & \text{if } U_{ij} < U_{\min} \end{array} \tag{9}$$

where Umax and Umm are constant parameters.

Procedural steps of the parallel algorithm

The following procedure describes the proposed parallel algorithm based on the motion equation with the four heuristics.

- 1- Set t = 0, A = B = 1, C = 3, 4, or 5, T = 10, ω = 5, U_{max} = 30, U_{min} = -30, A_{max} = 5, A_{min} = -5, UTP = 5, LTP = -5 and T_{max} = 500.
- 2-The initial values of $U_{ij}(t)$ is a random variable netween 0 and U_{min} . Assign the initial values of $V_{ii}(t)$ to zero.
- 3-Compute the change of input $\Delta U_{ij}(t)$ based on the motion equation as:

$$\begin{split} & \text{If } \big(t \text{ mod } T \big) < \omega \\ & \Delta U_{ij}(t) = - \text{Af} \Bigg(\sum_{q=1}^{m} V_{iq}(t) - d_i \Bigg) - \text{B} \\ & \Bigg(\sum_{\substack{j=1\\ q \neq j\\ 1: j \neq m}}^{j=(c_z-1)} V_{iq}(t) + \sum_{\substack{p=1\\ p \neq i\\ 1: j \neq m}}^{n} \sum_{\substack{j=1\\ c_{g+1} = 1\\ j \neq j}}^{j=(c_z-1)} V_{pq}(t) \Bigg) \\ & V_{ij}(t) + ch \Bigg(\sum_{q=1}^{m} V_{iq}(t) - d_i \Bigg) \Big(l - V_{ij}(t) \Big), \\ & \text{else} \\ & \Delta U_{ij}(t) = - \text{Af} \Bigg(\sum_{\substack{q=1\\ q \neq j\\ 1: j \neq m}}^{m} V_{iq}(t) + \sum_{\substack{p=1\\ p \neq i\\ c_{g+1}}}^{j=(c_g-1)} V_{pq}(t) \Bigg) \\ & + ch \Bigg(\sum_{q=1}^{m} V_{iq}(t) - d_i \Bigg) \Big(l - V_{ij}(t) \Big). \end{split}$$

4- Update input
$$U_{ij}(t+1)$$
 as:
 $U_{ij}(t+1) = U_{ij}(t) + \Delta U_{ij}(t)$ (12)

5- Use the input saturation heuristic:

$$U_{ij}(t+1) = U_{max}$$
 if $U_{ij}(t+1) > U_{max}$
 $U_{ij}(t+1) = U_{min}$ if $U_{ij}(t+1) < U_{min}$, (13)

6- Update output Vij(t+1) as:

$$V_{ij}(t+1) = 1 \qquad \text{if } U_{ij} (t+1) > \text{UTP}$$

$$= 0 \qquad \text{if } U_{ij} (t+1) < \text{LTP}$$

$$\text{unchanged otherwise} \qquad (14)$$

7-Check the termination condition:

if
$$\left(\left(\sum_{q=1}^{m} V_{iq} - d_{i}\right) = 0\right) \text{ and }$$

$$\left(V_{ij}(t) = land \left(\sum_{\substack{q=1,\cdots,c_{k}-1\\q=j\\1\leq q\leq m}}^{j-c_{k}-1} V_{iq} + \sum_{\substack{p=1\\p=i1\\c_{k}-0}}^{n} \sum_{\substack{q=1,\cdots,c_{k}-1\\q=j\\c_{k}-0}}^{j-c_{k}-1} V_{pq}\right) = 0\right),$$
(15)

for i = 1,...,n and $j \in \{1,...,m\}$ or $t = T_{max}$

then terminate this procedure, otherwise increment t by 1, and go to step 2.

To run the neural network parallel algorithm program, the input values U_{ij} are sequentially updated, while all output values are fixed. Then all output values V_{ij} are sequentially updated, while all input values U_{ij} are fixed.

5. The proposed preaassignment initialization levels

The following levels are proposed to be applied as a prestep of the parallel algorithm in order to decrease the number of iterations and increase the convergence rate.

- (1) Level-1 (Achieve assignment of CellMax)
- Finf the cell with maximum traffic (CellMax). Assign the minimum number of channels needed to serve this cell which is given by (((c_{ii}-1)*d_{max})-1) to meet co-site constraint.
- Assign the demand calls for the other cells, taking into account the achievement of co-site constraint in all cells, plus co-channel and adjacent channel constraints in CellMax. The cells are arranged in random order.
- (2) Level-2 (Check and solve violation between cells)

This level is mainly for solving the violation of the co-site constraint between cells, and this will be done by the following steps:

- If the assignment of neuron (i,j) (i.e. cell #
 i, and channel # j) violates the co-site
 constraints with other cells, try the search
 process for a free channel that makes no
 violation, whether in favor of the left side
 neuron (i, j-1) or right side neuron (i, j+1).
- After the initial direction is decided, (to the right or to the left), the next neurons are tested sequentially until the search is finished by true or false.
- For false search a global trace is followed.
 The global trace will be done by repeating the search process for a free channel in all the other cells that make violations

with the existed one in the same way as the first two steps.

- For false search, return the old assignment with cell # i.
- (3) Level-3 (Check and solve violation in descending order)

This level will be executed if level-2 fails in solving the problem of violation and it will follow the same steps of level-2 but with cells arranged in descending order.

- (4) Level-4 (Consider the other constraints)
 - Consider co-channel and adjacent channel constraints between cells, and count the number of errors.
- If the number of errors is less than two, supply the input and output matrices into the Hopfield network.
- For two errors or more, the system will solve these problems either by the search for free channels for these assignment or for the assignment which cause violations with the existed ones.

6. Simulation results and discussion

Table 1 shows the specification of the examined eight benchmark problems with their compatibility matrix C and demand vector D. For more details see [], [] where

the details of the problems are discussed. The number of radio cells varies between 4 and 25 for number of frequencies ranges from 11 to 533. The CSC is indicated by the value in column c_{ii}.

The initialization steps discussed in the previous section have to be executed in advance of the the neural network parallel algorithm steps. The cell with maximum number of calls and so maximum required channel number must be determined with the aid of the demand vector. In the same way the required channel number for the other cells have to be determined. Fixing the frequency assignment of CellMax accelerates the convergence time.

Table 2 shows the average iteration number and the convergence rate for each problem at the end of each of the initialization levels. The average iteration number is the average number of iterations, which are increased until the energy is equal to zero. Convergence rate is the probability that the network has an energy of zero value before the maximum number of iteration is reached. In the simulation the maximum number of iterations is fixed at 500. To investigate the number of iteration and the convergence rate, 100 simulation runs were performed with different initial seed values.

Table 1. Specifications of the simulated problems [6].

Problem #	Number of Radio cells	Number of Frequencies	Cii	Compatibility Matrix C	Demand vector D	
	N	M				
1	4 .	11	2	C ₁	D ₁	
2	25	73	2	C ₂	D ₂	
3	21	381	5	C ₃	D ₃	
4	21	533	7	C ₄	D ₃	
.5	21	533	7	C ₅	D ₃	
6	21	221	5	C ₃	D ₄	
7	21	309	7	C4	D ₄	
8	21	309	7	C ₅	D ₄	

Table 2. Simulation results of the suggested initialization levels.

Problem #	Level 1		Level 2		Level 3		Level 4	
N PA	Average Iter No.	Convergen ce rate	Average Iter No.	Convergen ce rate	Average Iter No.	Convergen ce rate	Average Iter. No.	Convergen ce rate
1	16	100%	1	100%	1	100%	1	100%
2	43	100%	23.3	100%	1	100%	1	100%
3	200	100%	1	100%	1	100%	1	100%
4	139	100%	1	100%	1	100%	1	100%
5	150	100%	1	100%	1	100%	1	100%
6		=	271	20%	1	100%	1	100%
7	115	25%	1	100%	1	100%	1	100%
8			271	18%	253	20%	16	100%

Table 3. Comparison with prior reported results.

Problem #	Method I ₁ with B ₂ [4]		Funabiki [6]		Results of Level 4	
	Average Iter No.	Conver gence rate	Average Iter. No.	Conver gence rate	Average Iter. N.	Conver gence rate
2	279.9	62%	294.0	9%	1	100%
3	67.4	99%	147.8	93%	1	100%
4	64.4	100%	117.5	100%	1	100%
5	126.8	98%	100.3	100%	1	100%
6	62.4	97%	234.8	79%	1	100%
7	127.7	99%	85.6	100%	1	100%
8	151	52%	305.6	24%	16	100%

From the table we can notice that we can get a solution for problems 1 to 5 with 100 % convergence rate at the end of level 1 but the system could not get a solution for problems 6 and 8 at this level. At the second level, the average iteration number for all the problems has decreased to a great extent with a solution to problems 6 and 8. At the end of level 4, the convergence rate is 100 % for all the problems with one iteration except problem 8 which has an average iteration number of 16.

Table 3 shows a comparison between the achieved result at the end of level 4 with that reported in [2] and [10]. In [2] a couple of frequencies in certain cells are fixed for accelerating the convergence time, while in [4] with its initialization and updating methods had similar or better results than those found in [6] although the frequencies are not fixed in any cell. The proposed initialization levels combine the advantages of both of these methods. In the proposed algorithm the frequencies of CellMax are the only fixed frequencies as a step to accelerate the convergence rate in addition to the

proposed initialization levels to decrease the number of iteration.

7. Conclusions

This paper proposes a preassignment algorithm to the channel assignment problem as a prestep to the implementation of the algorithm. The preassignment parallel algorithm drives itself to the optimum solution by fixing the assignment of the cell with maximum traffic and solving violation of any of the constraints according to its importance and connection weight successively. The interconnection weight to any of the neurons takes into account the required channel number in each cell and the three channel assignment constraints. The three constraints that are considered are the CSC, CCC and ACC. The obtained average iteration number and the convergence rate are compared with prior reported results. The proposed algorithm drastically reduced the average iteration number and improved the convergence rate to 100 % for the considered eight benchmark problems.

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Received April 15, 2000 Accepted August 10, 2000