Experimental investigation on the hydraulic jump In sloping rectangular closed conduits

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An experimental study was carried out to investigate the characteristics of the hydraulic jump occurring in a sloping closed conduit with a pressurized flow downstream from the jump and a submerged conduit outlet. Experiments were conducted on a closed conduit with relatively small slopes to study the variation of the relative tailwater depth with the main parameters affecting the jump in sloping conduits. These parameters include the channel bottom slope, the initial Froude number, and the ratio of the initial depth to conduit height. Non-dimensional design curves are provided to relate the jump characteristics. Also, empirical equations are provided to determine the relative tailwater depth for different conduit slopes, initial Froude numbers, and ratios of initial depth to conduit height. The results agreed well with the developed equation and with the results of other authors for horizontal conduits.

يهدف هذا البحث الى دراسه معمليه لخصائص القفزه الهيدروليكيه التى تتكون داخل القنوات المستطيله المغلقه المائله والتى يكون فيها السريان خلف القفزه تحت ضغط ومخرج القناه المستطيله المغلقه المائله مغمور. ولهذا الغرض أجريت مجموعه من التجارب المعمليه لدراسة التغير فى العمق النسبى عند المخرج مع التغير فى العوامل الأخرى المؤثره فى القفزه الهيدروليكيه داخل القناه المغلقه. وتشمل هذه العوامل الميول الطوليه للمجرى ورقم فرود الابتدائى وأيضا نسبة العمق الابتدائى الى ارتفاع القناه. وقد تم أعداد مجموعة منحنيات تصميميه لابعديه توضح طبيعة وشكل التغير بين خصائص القفزه. وتم أيضا أستنباط معادله تقريبيه لايجاد عمق المخرج النسبى كداله فى الميل الطولى للقناه ورقم فرود الابتدائى ونسبة العمق الابتدائى الى عمق القناه المغلقه وذلك باستخدام البيانات المعمليه. وقد وجد أن المعادله التقريبيه تمثل العلاقه بين خصائص القفزه تمثيلا جيدا. وتم أيضا مقارنة المعادله التقريبيه مع معادلات مستنتجه من دراسات سابقه.

Keywords: Hydraulic jump, Pressurized flow, Hydraulic structures.

1. Introduction

The hydraulic jump formed in closed conduits below control gates is a phenomenon which has been frequently observed [1]. In open channels, the hydraulic jump provides a natural transition from initial supercritical flow to downstream subcritical free surface flow. In closed conduits, the initial free surface supercritical flow changes to a pressurized flow downstream from the jump and the conjugate depth is confined by the conduit height. The tailwater depth in that case provides the downstream subcritical free surface flow. The jump location in the conduit is very sensitive to any slight variation in the initial depth, conduit height, tailwater depth, or conduit slope. Then, it is extremely important to investigate the interdependency of such variables. The case of horizontal conduits has been earlier studied by Ezzeldin [2]. Earlier researches carried out by Lane and Kindsvater [3] for the case of horizontal conduits followed by Kalinske and Robertson [4] for the case of sloping conduits concentrated on the air pumping capacity of the jump. The jump formation in closed conduits was studied by Haindl [5] for horizontal rectangular conduits and by Rajaratnam [6] for horizontal exponential and circular conduits. A practical case of the hydraulic jump formation in closed conduits includes the occurrence of the hydraulic jump in the barrel of a siphon inlet. Smith and Haid [7] studied the jump characteristics for the case of circular pipes. Later, Smith and Chen [8] investigated the relative height of the hydraulic jump formed in a steeply sloping square conduit without considering the tailwater depth conditions. They derived the theoretically momentum-based equation for the relative height of the hydraulic jump formed in sloping square conduits, but they could not solve it because it contained too many unknowns. Hence they provided set of empirical equations of the form $H_j / D = a F_1^{1.4} + b$, (H_j being the height of jump, D is the conduit height, F_1 is the initial Froude number, and **a** & **b** are coefficients that depend upon the values of the conduit slope and the ratio of the initial depth to conduit height).

In the present paper, an experimental study is carried out and the hydraulic jump is allowed to be formed in sloping closed conduits of different heights. The relevant parameters were measured and non-dimensional design curves were prepared to determine the variation in the tailwater depth with the change in slopes, initial Froude numbers, and ratios of initial depth to conduit heights. Also, empirical equations are provided to determine the relative tailwater depth in terms of the relevant parameters.

2. Theoretical considerations

Figure 1 shows a definition sketch for the hydraulic jump formed in sloping closed conduit. Although the momentum equation along with the energy equation can be written to theoretically express the relationship among the different variables describing the phenomenon, the direct solution for such equations will be somewhat difficult as there are too many unknowns [8]. These unknowns include the weight component of the jump in the direction of flow, the boundary frictional resistance, the exit losses, and the air water ratio at the end of the jump. Therefore, the theoretical solution for the jump in sloping conduits is avoided in the present study.

The relative tailwater depth D_t/d_1 can be expressed in non-dimensional form to be a function of the initial Froude number F_1, the ratio of the initial depth to conduit height d_1/D, and the conduit slope S_o as:

$$D_t/d_1 = f\,(F_1, d_1/D, S_o), \qquad (1)$$

where D_t is the depth just downstream the outlet of the conduit and is termed in this paper as the tailwater depth, d_1 is the initial depth of the jump, S_o is the slope of the conduit, and F_1 is the initial Froude number ($=Q/\ b d_1 \sqrt{g d_1}$) with **Q** being the discharge, **b** is the conduit width, and **g** is the gravitational acceleration. Eq. (1) could be defined and evaluated using the experimental data. The experimental data can be plotted on several planes to completely understand the phenomenon. Such planes may include $[D_t/d_1, F_1]$, $[D_t/d_1, S_o]$, or $[D_t/d_1, d_1/D]$. In each plane, for a constant value of one of the two remaining parameters, a family of curves is drawn for different values of the other one.

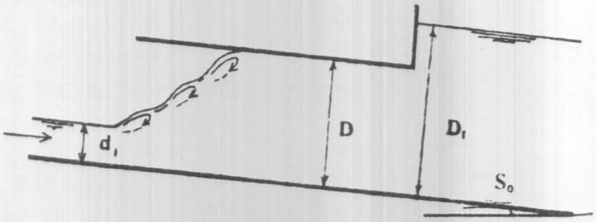


Fig. 1. Definition sketch.

3. Experimental setup and procedure

The experiments were conducted in the hydraulics laboratory of Zagazig university in a tilting glass sided flume 3.0 m long, 10 cm wide, and 31 cm deep as shown in Fig. 2. The discharge was measured using a pre-calibrated orifice meter. An in-line valve fitted into the main supplying pipeline was used to regulate the flow rate. Depth measurements were taken using a needle point gauge with a reading accuracy of ± 0.1 mm. Uniform flow conditions were reached using a carefully designed inlet tank. The slope was adjusted using a screw jack located at the upstream end of the flume while at the downstream end, the flume is allowed to rotate freely about a hinged pivot. The slope was directly determined using a slope indicator. A downstream adjustable gate was used to regulate the tailwater surface elevation.

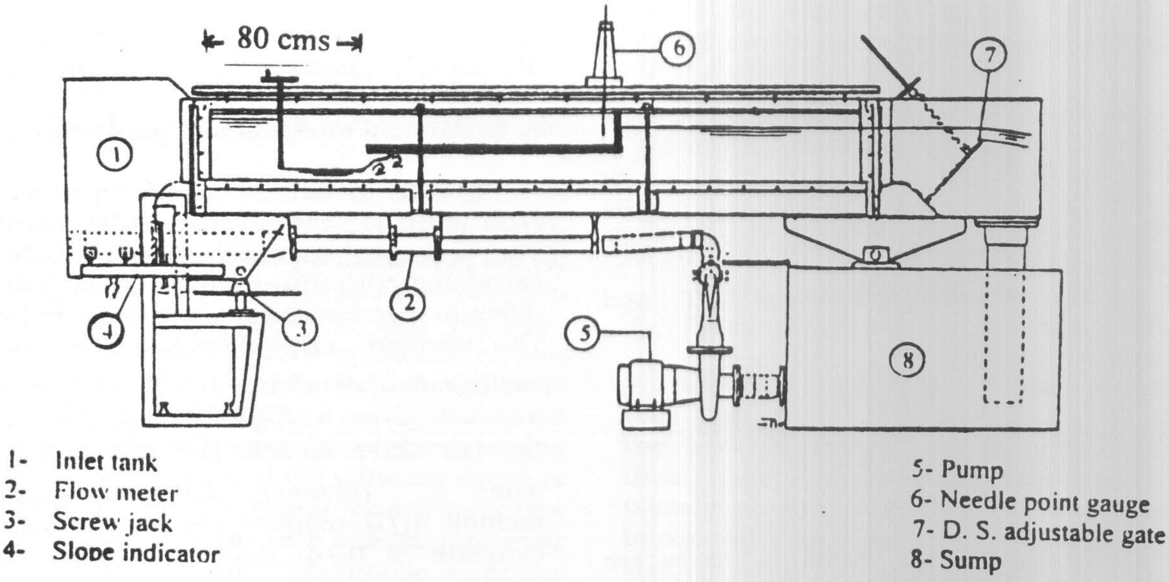


Fig. 2. Experimental apparatus.

The experiments were carried out using five different conduit heights, **D** , of 6,7,8,9, and 10 cm. Seven different conduit slopes, S_o, of 0.002, 0.004, 0.005, 0.0067, 0.008, 0.01, and 0.02 were used to illustrate the effect of conduit slope on the jump formed in sloping conduits. The slopes were selected based on the flume facilities.

For each combination of conduit slope and height, five different flow rates ranging from 342 L/min to 234 L/min were used. The initial Froude number ranges from 4 to 6 For each conduit height. The upstream control gate was so adjusted to produce an initial supercritical depth, d_1. The downstream adjustable gate was adjusted to control the tailwater depth, D_t That enabled the jump to be formed at a certain location in the conduit such that the jump toe is always located at the beginning of the conduit roof in order to make measurement of the initial supercritical depth, d_1, using the needle point gauge possible. The jump location was kept fixed throughout the course of the experiments. For each combination of slope and conduit height, the flow rate and the tailwater depth just downstream the conduit outlet were measured.

4. Experimental results

The variation of D_t/d_1 with F_1 for different tested slopes are presented in Figs. 3 to 7 for d_1/D = 0.21, 0.233, 0.2625, 0.3, and 0.35, respectively. From these figures, it can be observed that for a fixed d_1/D, the trend of variation between D_t/d_1 and F_1 is increasing with a nonlinear trend according to Eq. (2). Also, at a particular F_1, D_t/d_1 increases as the conduit slope increases.

$$D_t/d_1 = a_o + a_1 F_1 + a_2 F_1^2 , \qquad (2)$$

where a_o , a_1, and a_2 are regression coefficients that depend upon the conduit slope and the ratio d_1/D. The values of the coefficients of Eq. (2) are given in Appendix A, Table A1. Shown also in the figures the predicted values for S_o=0.015 using Eq. (5) (solid circles).

Similarly, the variation of D_t/d_1 with S_o for different F_1 is shown in Figs. 8 to 12 for d_1/D = 0.21, 0.233, 0.2625, 0.3, and 0.35 respectively. From these figures, it can be observed that for a fixed d_1/D, the trend of variation between D_t/d_1 and S_o is increasing with a nonlinear trend according to Eq. (3). Also, at a particular S_o, D_t/d_1 increases as F_1 increases.

$$\mathbf{D_t/d_1 = b_o + b_1S_o + b_2S_o^2}, \qquad (3)$$

where $\mathbf{b_o}$, $\mathbf{b_1}$, and $\mathbf{b_2}$ are regression coefficients that depend upon $\mathbf{F_1}$ and $\mathbf{d_1/D}$. The values of the coefficients of Eq.(3) are given in Appendix A, Table A2. Shown also in the figures the predicted values for $\mathbf{S_o}$ = 0.015 using Eq. (5) (solid circles).

It is also possible to combine Eq. (2) and Eq. (3) together to determine $\mathbf{D_t/d_1}$ as a function of both $\mathbf{F_1}$ and $\mathbf{S_o}$ in the form:

$$\mathbf{D_t/d_1=(c_o+c_1S_o+c_2S_o^2)+(c_3+c_4S_o+c_5S_o^2)F_1} \\ \mathbf{+(c_6+c_7S_o+c_8S_o^2)F_1^2}, \qquad (4)$$

where, the coefficients from $\mathbf{c_o}$ to $\mathbf{c_8}$ are functions of $\mathbf{d_1/D}$ only. In order to deduce a general equation in the form of Eq. (1), it is necessary to relate the coefficients ($\mathbf{c_o}$ to $\mathbf{c_8}$) to $\mathbf{d_1/D}$ and back substitution in Eq. (4) yields the required equation. However, such equation will be too long and will have too many coefficients. A simpler and more practical equation may be obtained via the use of statistical analysis.

5. Statistical analysis and predictions

In order to derive a general equation in the form of Eq. (1), the statistical analysis is used. The relative tailwater depth, $\mathbf{D_t/d_1}$, is correlated with the parameters of Eq. (1) in different combinations as shown in Table 1. The standard error of estimate, SEE, and the coefficient of determination R^2, are calculated and are given in Table 1. It is observed that the last three models give approximately the same R^2. However, the last model that include $\mathbf{d_1/D}$ reduces the standard error of estimate of $\mathbf{D_t/d_1}$ more than other models. Testing the residuals of the last three models, it is observed that they provide an error of less than ± 5%. The last model has the merit that it contains all the variables controlling the hydraulic jump in sloping closed conduit and hence enables the analysis and studying the effect of each of these parameters on the phenomenon being under consideration.

Table 1. R^2 and SEE for the statistically tested models.

Model	Variables	SEE	R^2	No. of Variables
1	d_1/D	1.105404	0.0006	1
2	S_o	1.030187	0.1320	1
3	F_1	0.437344	0.8436	1
4	$F_1^{1.5}$	0.431800	0.8475	1
5	F_1^2	0.428100	0.8501	1
6	S_o^2	1.038990	0.1156	1
7	Model 3+4	0.427192	0.8516	2
8	Model 2+7	0.142375	0.9836	3
9	Model 8+6	0.12749	0.9869	4
10	Model 9+1	0.124751	0.9876	5

The variation of R^2 with SEE for the different tested regression models is shown in figure (13).

* The results of the last regression model is given as follows:

Regression Output for Model No. 10

Constant 7.018489 Std. Err. Of $\mathbf{D_t/d_1}$ Est. 0.124751 R squared 0.987566 No. of
Observations 175 Degree of Freedom 169

	$\mathbf{F_1}$	$\mathbf{F_1^{1.5}}$	$\mathbf{S_o}$	$\mathbf{S_o^2}$	$\mathbf{d_1/D}$
Coefficients	-3.78181	1.573178	121.169	-2119.53	0.555433
Std. Err. Of Coeff.	0.506447	0.150482	7.396823	315.3282	0.18998

Thus model No. 10 has the following form:

$$\mathbf{D_t/d_1 = 7.018 - 3.782F_1 + 1.5732F_1^{1.5} + 121.169S_o - 2119.53S_o^2 + 0.5554(d_1/D)}. \quad (5)$$

The prediction of model No. 10 , Eq.(5), is presented against the measured values of $\mathbf{D_t/d_1}$ in Fig. 14. Also, the predictions of Eq. (5) for $\mathbf{S_o}$ = 0.015 are presented in Figs. from 3 to (12. From these Figs. 3-12 and 14, good agreement is observed between measured and predicted values of $\mathbf{D_t/d_1}$ for different $\mathbf{d_1/D}$ and conduit slopes ranging from 0.002 to 0.02 for each $\mathbf{d_1/D}$. The variation of errors versus the measured values of the tailwater depth is presented in Fig. 15 which clearly indicates the prediction of $\mathbf{D_t/d_1}$ with a maximum error of ±5% which is an acceptable error for practical design purposes.

6. Sensitivity analysis

Model No. 10, Eq. (5) is used to study the effect of different parameters on $\mathbf{D_t/d_1}$. Fig. 16 shows the typical effect of $\mathbf{d_1/D}$ at different $\mathbf{F_1}$ of 4, 4.5, 5 and 5.5 at fixed conduit slope of 0.015. It is clear that for the investigated range of slope, the ratio of the initial depth to conduit height has insignificant effect on $\mathbf{D_t/d_1}$ as $\mathbf{D_t/d_1}$ is increasing very slightly with the increase of $\mathbf{d_1/D}$. The figure indicates also that $\mathbf{D_t/d_1}$ increases as $\mathbf{F_1}$ increases.

Figure 17 presents the effect of the initial Froude number $\mathbf{F_1}$ at fixed $\mathbf{d_1/D}$ and different conduit slopes. It is observed that the effect of $\mathbf{F_1}$ on $\mathbf{D_t/d_1}$ is significant. The $\mathbf{D_t/d_1}$ increases non-linearly with the increase of $\mathbf{F_1}$. Also, the higher the slope, the greater the ratio $\mathbf{D_t/d_1}$ which proves that the slope has an increasing effect on $\mathbf{D_t/d_1}$.

Also, Fig. 18 shows that the slope has a major effect on $\mathbf{D_t/d_1}$ which is comparable with the effect of $\mathbf{F_1}$ on $\mathbf{D_t/d_1}$ where $\mathbf{D_t/d_1}$ increases non-linearly with the increase of conduit slope. Also, confirmed the increase of $\mathbf{D_t/d_1}$ with the increase of $\mathbf{F_1}$ at fixed $\mathbf{d_1/D}$.

7. Comparisons

Although Smith and Chen [8] analyzed only the height of jump ratio and no data on the relative tailwater depth are available in their paper, it is possible to compare the present results with their results for horizontal conduits. Assuming that $\mathbf{d_2}$ is the sequent depth of jump and $\mathbf{H_j}$ is the height of jump in horizontal conduit. The empirically developed equations for horizontal conduit are used to generate $\mathbf{d_2/D}$. It is known that $\mathbf{H_j/D=(d_2-d_1)/D}$ and $\mathbf{d_1}$ is calculated from $\mathbf{d_1/D}$ knowing the conduit height and hence $\mathbf{d_2/D}$ can be obtained. It is also assumed that $\mathbf{d_2}$ is approximately equal to $\mathbf{D_t}$ if the exit and other conduit losses are neglected. Figs 19a and 19b present the comparison between $\mathbf{D_t/D}$ and $\mathbf{d_2/D}$ for $\mathbf{d_1/D}$= 0.2 and $\mathbf{d_1/D}$ = 0.3, respectively. $\mathbf{D_t/D}$ is obtained by computing $\mathbf{D_t/d_1}$ using Eq. (5) and then multiplied by $\mathbf{d_1/D}$ assuming the effect of small slope can be neglected. The deviation between the present results and Smith and Chen [8] results can be attributed to be mainly due to the absence of the effect of small slope when Eq. (5) is used and also due to the exit and other losses which are contained in $\mathbf{D_t/D}$.

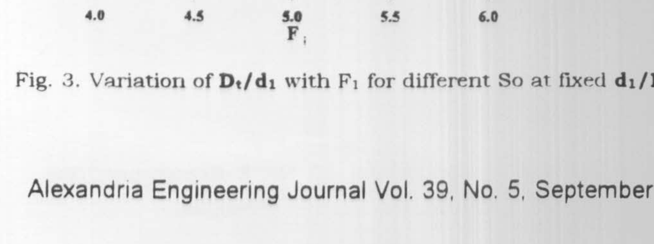


Fig. 3. Variation of $\mathbf{D_t/d_1}$ with F_1 for different So at fixed $\mathbf{d_1/D}$ = 0.21.

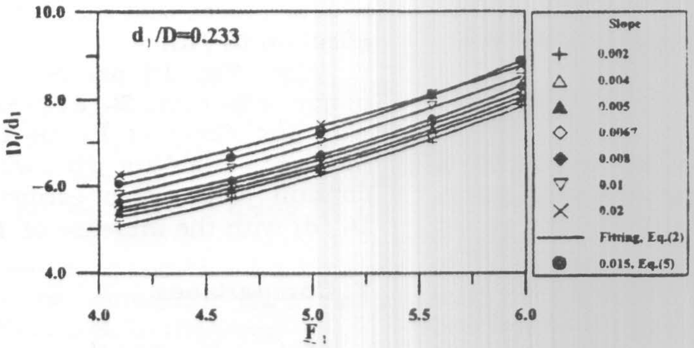


Fig. 4. Variation of D_t/d_1 with F_1 for different So at fixed d_1/D = 0.233.

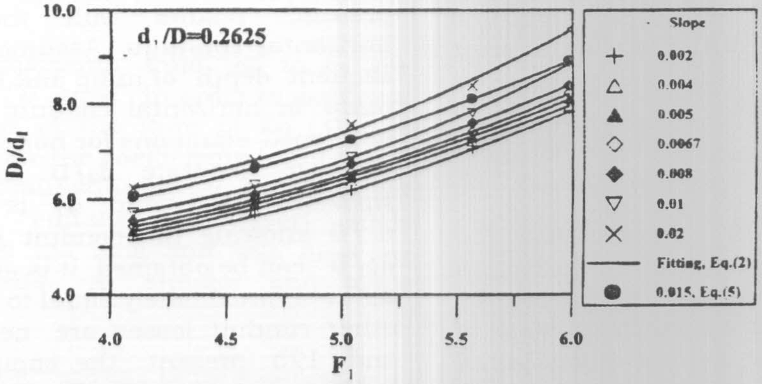


Fig. 5. Variation of D_t/d_1 with F_1 for different So at fixed d_1/D = 0.2625

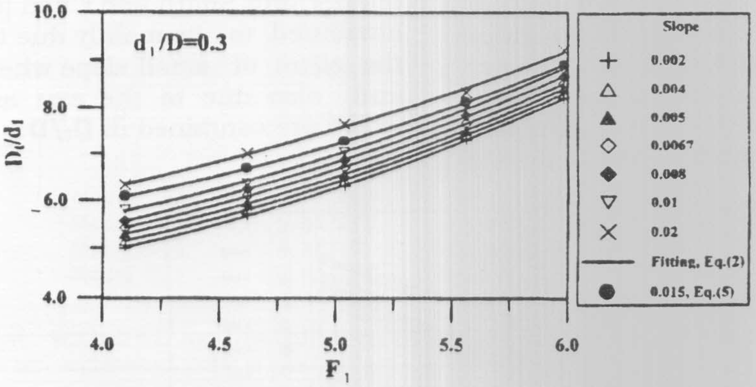


Fig. 6. Variation of D_t/d_1 with F_1 for different So at fixed d_1/D = 0.30

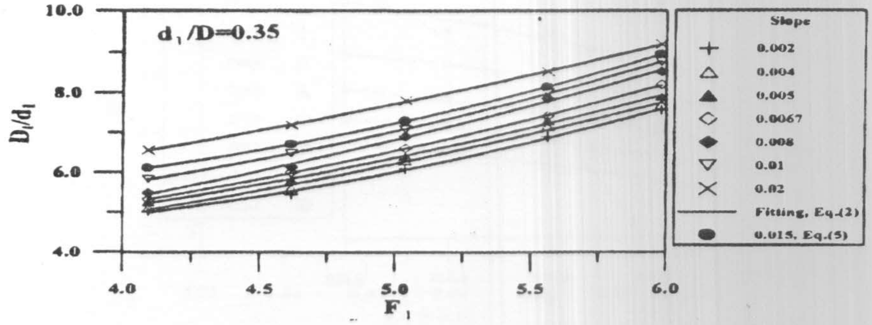


Fig.7. Variation of $\mathbf{D_t/d_1}$ with $\mathbf{F_1}$ for different $\mathbf{S_o}$ at fixed $\mathbf{d_1/D}$ = 0.35.

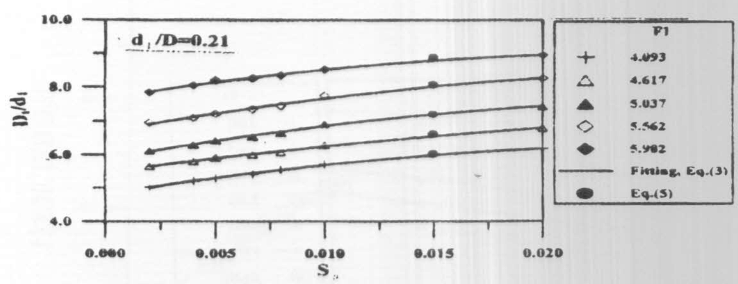


Fig.8. Variation of $\mathbf{D_t/d_1}$ with $\mathbf{S_o}$ for different $\mathbf{F_1}$ at fixed $\mathbf{d_1/D}$ = 0.21.

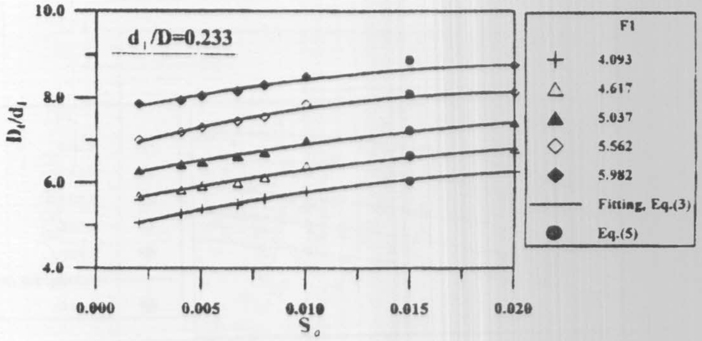


Fig. 9. Variation of $\mathbf{D_t/d_1}$ with $\mathbf{S_o}$ for different $\mathbf{F_1}$ at fixed $\mathbf{d_1/D}$ = 0.233.

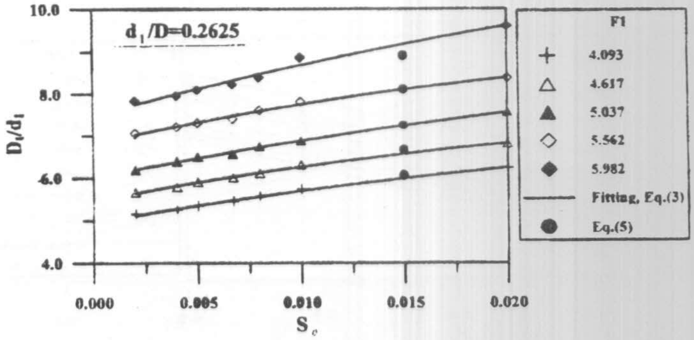


Fig. 10. Variation of $\mathbf{D_t/d_1}$ with $\mathbf{S_o}$ for different $\mathbf{F_1}$ at fixed $\mathbf{d_1/D}$ = 0.2625.

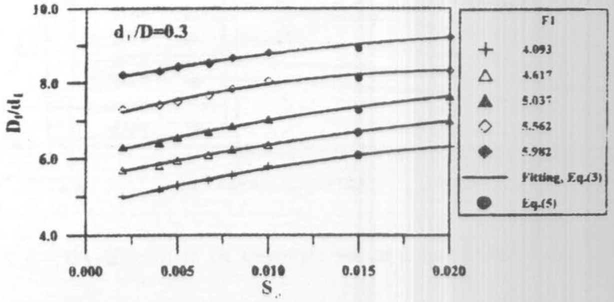


Fig. 11. Variation of $\mathbf{D_t/d_1}$ with $\mathbf{S_o}$ for different $\mathbf{F_1}$ at fixed $\mathbf{d_1/D}$ = 0.30.

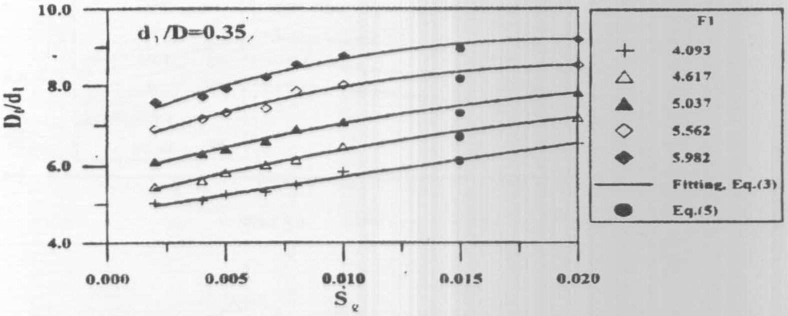


Fig. 12. Variation of $\mathbf{D_t/d_1}$ with $\mathbf{S_o}$ for different $\mathbf{F_1}$ at fixed $\mathbf{d_1/D}$ = 0.35

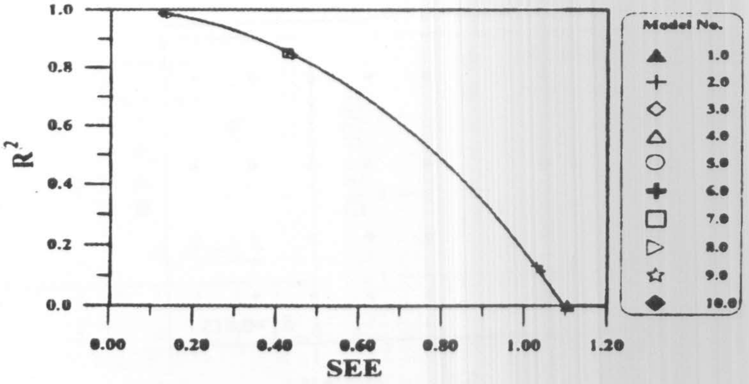


Fig. 13. Variation of R2 with SEE for the tested regression models.

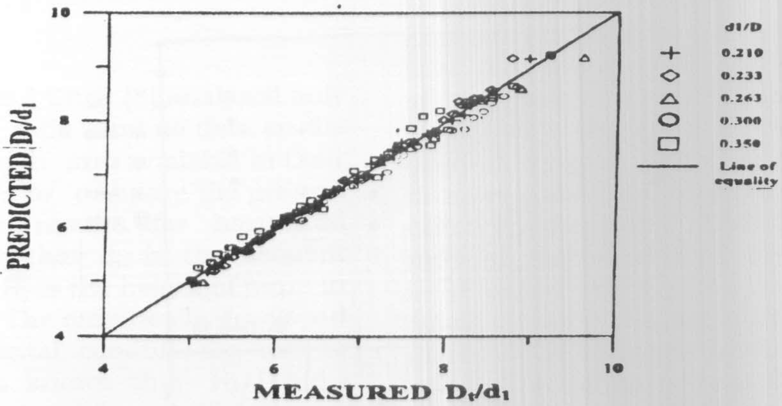


Fig. 14. Measured values of D_t/d_1 versus prediction of (Eq. (5))

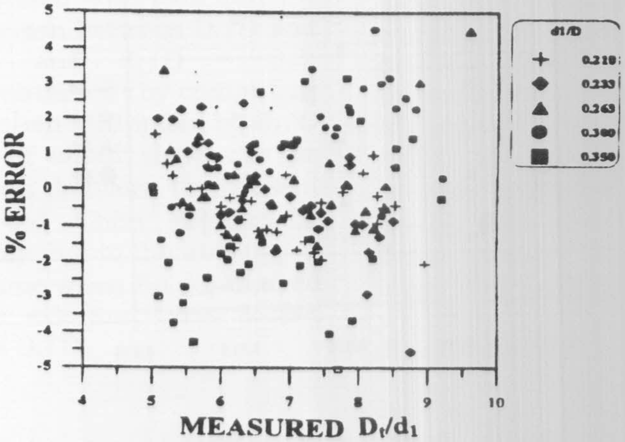


Fig. 15. Variation of % error with D_t/d_1 using Eq. (5).

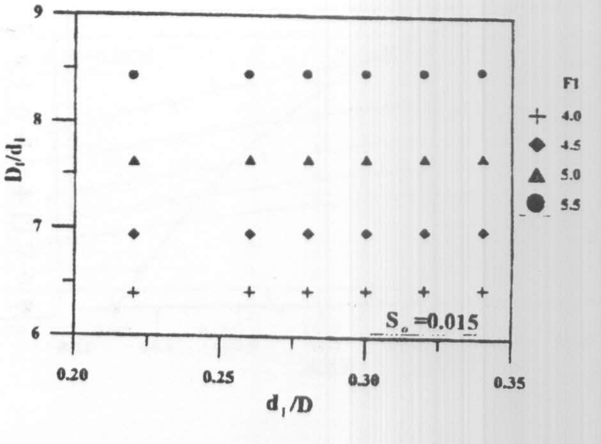


Fig. 16. Effect of $\mathbf{d_1/D}$ **on** $\mathbf{D_t/d_1}$ for different $\mathbf{F_1}$ at fixed slop of

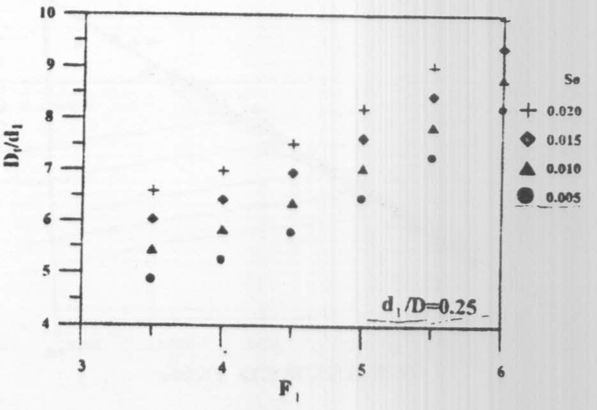


Fig. 17. Effect of $\mathbf{F_1}$ on $\mathbf{D_t/d_1}$ for different slopes at fixed $\mathbf{d_1/D}$

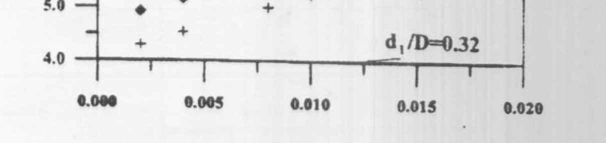
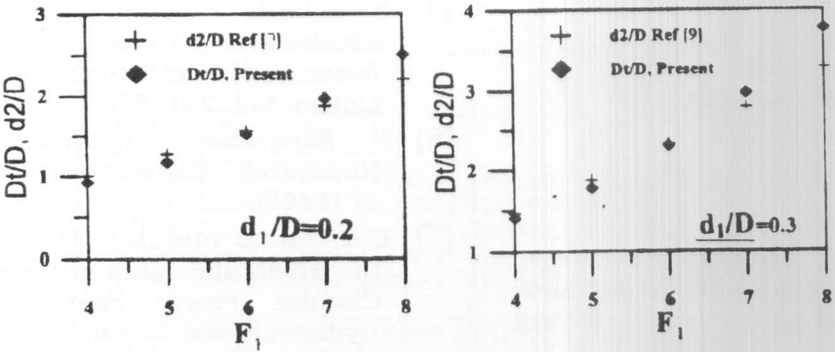


Fig. 18.Effect of slope on $\mathbf{D_t/d_1}$ for different F_1 at fixed $\mathbf{d_1/D}$



(a) (b)

Fig. 19. Comparison between present and Smith and Chen [8] results for horizontal conduit. (a) for $d_1/D = 0.2$ and (b) for $d_1/D=0.3$

7. Comparisons

Although Smith and Chen [8] analyzed only the height of jump ratio and no data on the relative tailwater depth are available in their paper, it is possible to compare the present results with their results for horizontal conduits. Assuming that d_2 is the sequent depth of jump and H_j is the height of jump in horizontal conduit. The empirically developed equations for horizontal conduit are used to generate d_2/D. It is known that $Hj/D = (d_2-d_1)/D$ and d_1 is calculated from d_1/D knowing the conduit height and hence d_2/D can be obtained. It is also assumed that d_2 is approximately equal to D_t if the exit and other conduit losses are neglected. Fig. 19-a and 19-b present the comparison between D_t/D and d_2/D for $d_1/D=0.2$ and $d_1/D = 0.3$, respectively. D_t/D is obtained by computing D_t/d_1 using Eq (5) and then multiplied by d_1/D assuming the effect of small slope can be neglected. The deviation between the present results and Smith and Chen [8] can be attributed to be mainly due to the absence of the effect of small slope when Eq. (5) is used and also due to the exit and other losses which are contained in D_t/D

8. Conclusions

The hydraulic jump in sloping rectangular closed conduits considering the tailwater depth at the outlet is analyzed with the aid of experimentally collected data. It is concluded that the relative tailwater depth is a function of the initial Froude number, the conduit slope, and the ratio of the initial depth to conduit height. Both of the initial Froude number and the conduit slope have major effect on the jump characteristics while the ratio of the initial depth to the conduit height is of minor effect when the slope is relatively small. In all cases, the relative tailwater depth increases nonlinearly with the increase of the initial Froude number and/or the increase of the conduit slope. Set of equations are presented in terms of the initial Froude number and conduit slope. Statistical methods are used to analyze the experimental data and to derive an empirical prediction equation. The developed prediction equation provides the calculation of the relative tailwater depth with a maximum percent error of about ± 5% and it can be used to study the effect of the different parameters on the relative tailwater depth as indicated in Figs. 16-18. The present results are compared with other authors published results for the same slope and $\mathbf{d_1/D}$ as indicated in Fig. 19.

Nomenclature

$\mathbf{a_o}$ - $\mathbf{a_2}$	coefficients of Eq. (2).
$\mathbf{b_o}$ - $\mathbf{b_2}$	coefficients of Eq. (3).
$\mathbf{c_o}$ – $\mathbf{c_8}$	coefficients of Eq. (4).
$\mathbf{d_1}$	initial supercritical depth.
$\mathbf{d_1/D}$	ratio of initial depth to conduit height.

D	conduit height,
D_t	tailwater depth,
D_t/d_1	relative tailwater depth,
F_1	initial Froude number, and
S_o	conduit slope,

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APPENDIX A

Table A1. Values of the coefficients of Eq. (2).

d_1/D	S_o	a_o	a_1	a_2	SEE	R^2
0.2100	0.0020	7.1164	-1.8519	0.3292	0.0660	0.9982
0.2100	0.0040	7.9777	-2.1397	0.3586	0.0718	0.9979
0.2100	0.0050	7.6113	-1.9704	0.3443	0.0830	0.9973
0.2100	0.0067	7.4210	-1.8407	0.3305	0.0478	0.9991
0.2100	0.0080	8.0727	-2.0453	0.3492	0.0399	0.9994
0.2100	0.0100	5.2148	-0.8200	0.2297	0.0221	0.9998
0.2100	0.0200	3.8383	-0.0466	0.1510	0.0307	0.9996
0.2330	0.0020	4.4501	-0.7406	0.2181	0.0700	0.9980
0.2330	0.0040	5.1927	-0.9325	0.2328	0.0277	0.9997
0.2330	0.0050	6.2229	-1.3026	0.2689	0.0097	1.0000
0.2330	0.0067	5.5779	-1.0145	0.2420	0.0492	0.9989
0.2330	0.0080	6.9129	-1.5098	0.2915	0.0329	0.9995
0.2330	0.0100	3.8998	-0.2139	0.1649	0.0380	0.9994
0.2330	0.0200	-1.3159	0.0268	-0.0766	0.0243	0.9999
0.2625	0.0020	7.4098	-1.9051	0.3305	0.0121	0.9999
0.2625	0.0040	5.5594	-1.1182	0.2540	0.0289	0.9996
0.2625	0.0050	5.8415	-1.2041	0.2641	0.0204	0.9998
0.2625	0.0067	6.7714	-1.5507	0.2995	0.0115	0.9999
0.2625	0.0080	6.0454	-1.2423	0.2732	0.0305	0.9996
0.2625	0.0100	10.4819	-3.0642	0.4661	0.0497	0.9992
0.2625	0.0200	13.2172	-3.1454	0.5721	0.1705	0.9958
0.3000	0.0020	5.9008	-1.5255	0.3195	0.0287	0.9997
0.3000	0.0040	6.0897	-1.5328	0.3183	0.0247	0.9998
0.3000	0.0050	6.6408	-1.6816	0.3307	0.0240	0.9998
0.3000	0.0067	5.5497	-1.1766	0.2797	0.0211	0.9999
0.3000	0.0080	5.2426	-0.9995	0.2629	0.0158	0.9999
0.3000	0.0100	6.3877	-1.3674	0.2973	0.0698	0.9984
0.3000	0.0200	8.2739	-0.4141	0.3251	0.1110	0.9976
0.3500	0.0020	5.6148	-1.2015	0.2562	0.0734	0.9975
0.3500	0.0040	2.7345	-0.0259	0.1445	0.1023	0.9955
0.3500	0.0050	3.6447	-0.3406	0.1769	0.0710	0.9979
0.3500	0.0067	3.4237	-0.2611	0.1768	0.0231	0.9998
0.3500	0.0080	2.0113	0.2812	0.1361	0.0839	0.9977
0.3500	0.0100	5.0507	-0.7534	0.2300	0.0225	0.9998

APPENDIX A

Table A2. Values of the coefficients of Eq.(3).

d_1/D	F_1	b_0	b_1	b_2	SEE	R^2
0.2100	4.0930	4.8015	107.8545	-1913.6497	0.0220	0.9978
0.2100	4.6170	5.4671	90.2143	-1147.7331	0.0360	0.9941
0.2100	5.0370	5.8666	120.5106	-2052.9033	0.0437	0.9935
0.2100	5.5620	6.6815	118.9939	-1964.1343	0.0586	0.9885
0.2100	5.9820	7.6567	106.9451	-2111.3101	0.0310	0.9949
0.2330	4.0930	4.8373	119.4889	-2441.5212	0.0209	0.9981
0.2330	4.6170	5.4590	102.5672	-1790.4746	0.0643	0.9803
0.2330	5.0370	6.0603	101.5333	-1678.1530	0.0615	0.9827
0.2330	5.5620	6.6939	141.6131	-3495.8521	0.0591	0.9844
0.2330	5.9820	7.5991	107.3333	-2465.4955	0.0595	0.9765
0.2625	4.0930	4.9621	81.2603	-860.7208	0.0316	0.9951
0.2625	4.6170	5.4496	94.7119	-1325.6194	0.0338	0.9949
0.2625	5.0370	6.0126	95.6155	-908.4301	0.0297	0.9970
0.2625	5.5620	6.8006	110.8564	-1573.6904	0.0506	0.9915
0.2625	5.9820	7.4911	131.5241	-1270.4064	0.1195	0.9750
0.3000	4.0930	4.7232	123.3306	-2160.4613	0.0446	0.9933
0.3000	4.6170	5.4772	101.4273	-1261.6805	0.0293	0.9969
0.3000	5.0370	6.0366	114.4775	-1698.0861	0.0386	0.9952
0.3000	5.5620	6.9881	129.0090	-3061.6953	0.0652	0.9791
0.3000	5.9820	7.9934	95.8516	-1728.9882	0.0371	0.9921
0.3500	4.0930	4.7559	98.3004	-403.0253	0.0903	0.9809
0.3500	4.6170	5.0901	154.9137	-2485.6316	0.0654	0.9918
0.3500	5.0370	5.7266	159.4116	-2767.2852	0.0700	0.9903
0.3500	5.5620	6.4682	194.9803	-4597.8551	0.1049	0.9767
0.3500	5.9820	7.0048	226.4995	-5823.4387	0.1177	0.9738