Experimental investigation on the hydraulic jump in adversely sloping rectangular closed conduits

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An experimental study was carried out to investigate the characteristics of the hydraulic jump formed in adversely sloping rectangular closed conduits with a pressurized flow downstream from the jump and a submerged conduit outlet. Experiments were conducted on a closed conduit with relatively small adverse slopes to study the variation of the relative tailwater depth with the main parameters affecting the jump in adversely sloping conduits. These parameters include the channel bottom adverse slope, the initial Froude number, and the ratio of the initial depth to conduit height. Non-dimensional design curves are provided to relate the jump characteristics. Also, empirical equations are provided to determine the relative tailwater depth for different conduit adverse slopes, initial Froude numbers, and ratios of initial depth to conduit height. The results agreed well with the developed equations and with the results of other authors for horizontal conduits.

يهدف هذا البحث الى دراسه معمليه لخصائص القفزه الهيدروليكيه التى تتكون داخل القنوات المستطيله المغلقه ذات الميول العكسيه والتى يكون فيها السريان خلف القفزه تحت ضغط ومخرج القناه المستطيله المغلقه ذات الميول العكسيه مغمور. ولهذا الغرض أجريت مجموعه من التجارب المعمليه لدراسة التغير فى العمق النسبى عند المخرج مع التغير فى العوامل الأخرى المؤثره فى القفزه الهيدروليكيه داخل القناه المغلقه. وتشمل هذه العوامل الميول الطوليه العكسيه للقناه ورقم فرود الابتدائى وأيضا نسبة العمق الأبتدائى الى ارتفاع القناه. وقد تم اعداد مجموعة منحنيات تصميميه لابعديه توضح طبيعة وشكل التغير بين خصائص القفزه. وتم أيضا استنباط معادله تقريبيه لايجاد عمق المخرج النسبى كداله فى الميل الطولى العكسى للقناه ورقم فرود الابتدائى ونسبة العمق الابتدائى الى عمق القناه المغلقه وذلك باستخدام البيانات المعمليه. وقد وجد أن المعادله التقريبيه تمثل العلاقه بين خصائص القفزه تمثيلا جيدا. وتم أيضا مقارنة المعادله التقريبيه مع معادلات مستنتجه من دراسات سابقه.

Keywords: Hydraulic jump, Pressurized flow, Hydraulic structures.

1. Introduction

The hydraulic jump formed in closed conduits below control gates is a phenomenon which has been frequently observed [1]. In open channels, the hydraulic jump provides a natural transition from initial supercritical flow to downstream subcritical free surface flow. In closed conduits, the initial free surface supercritical flow changes to a pressurized flow downstream from the jump and the conjugate depth is confined by the conduit height. The tailwater depth in that case provides the downstream subcritical free surface flow. The jump location in the conduit is very sensitive to any slight variation in the initial depth, conduit height, tailwater depth, or conduit adverse slope. Then, it is extremely important to investigate the interdependency of such variables. The cases of horizontal and sloping closed conduits have been earlier studied by Ezzeldin [2] and Ezzeldin et al. [3], respectively. Earlier researches carried out by Lane and Kindsvater [4] for the case of horizontal conduits followed by Kalinske and Robertson [5] for the case of sloping conduits concentrated on the air pumping capacity of the jump. The jump formation in closed conduits was studied by Haindl [6] for horizontal rectangular conduits and by Rajaratnam [7] for horizontal exponential and circular conduits. A practical case of the hydraulic jump formation in closed conduits includes the occurrence of the hydraulic jump in the barrel of a syphon outlet Smith and Haid [8] studied the jump characteristics for the case of circular pipes Later. Smith and Chen [9] investigated the relative height of the hydraulic jump formed in a steeply sloping square conduits without considering the tailwater depth conditions. They derived the theoretically momentum-based equation for the relative height of the hydraulic jump formed in sloping square conduits. but they could not solve it because it contained too many unknowns. Hence. they provided set of empirical equations of the form $H_j / D = aF_1^{-1} + b$ (**Hj** being the height of jump, **D** is the conduit height. **F₁** is the initial Froude number and a & **b** are coefficients that depend upon the values of the conduit slope and the ratio of the initial depth to conduit height).

In the present paper. an experimental study is carried out and the hydraulic jump is allowed to be formed in adversely sloping closed conduits of different heights. The relevant parameters were measured and non-dimensional design curves were prepared to determine the variation in the tailwater depth with the change in the adverse slopes. the initial Froude numbers, and the ratios of initial depth to conduit height. Also, empirical equations are provided to determine the relative tailwater depth in terms of the relevant parameters.

2. Theoretical considerations

Figure 1 shows a definition sketch for the hydraulic jump formed in sloping closed conduit with adverse slope. Although the momentum equation along with the energy equation can be written to theoretically express the relationship among the different variables describing the phenomenon. the direct solution for such equations will be somewhat difficult as there are too many unknowns [9]. These unknowns include the weight component of the jump against the direction of flow. the boundary frictional resistance, the exit losses, and the air water ratio at the end of the jump. Therefore, the theoretical solution for the jump in adversely sloping conduits is avoided in the present study.

The relative tailwater depth. D_t/d_1 . can be expressed in non-dimensional form to be a function of the initial Froude number. F_1 . the ratio of the initial depth to conduit height. d_1/D and the conduit adverse slope S_o as

$$D_t/d_1 = f\,(F_1,\ d_1/D,\ S_o). \tag{1}$$

Where D_t is the depth just downstream the outlet of the conduit and is termed in this paper as the tailwater depth. d_1 is the initial depth of the jump. S_o is the conduit adverse slope. and F_1 is the initial Froude number ($= Q / bd \sqrt{gd}$) with **Q** being the discharge. **b** is the conduit width. and **g** is the gravitational acceleration. Eq. (1) could be defined and evaluated using the experimental data. The experimental data can be plotted on several planes to completely understand the phenomenon. Such planes may include [D_t/d_1. F_1]. [D_t/d_1. S_o]. or [D_t/d_1. d_1/D]. In each plane. for a constant value of one of the two remaining parameters. a family of curves is drawn for different values of the other one.

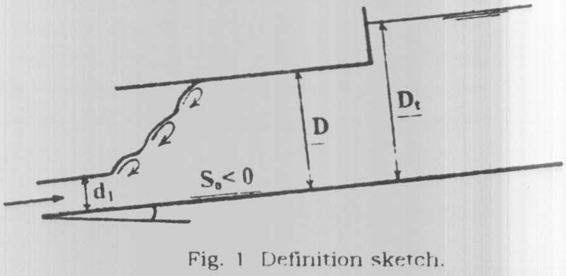

Fig. 1 Definition sketch.

3. Experimental setup and procedure

The experiments were conducted in the hydraulics laboratory of Zagazig university in a tilting glass sided flume 3.0 m long, 10 cm wide, and 31 cm deep as shown in Fig. 2. The discharge was measured using a pre-calibrated orifice meter. An in-line valve fitted into the main supplying pipeline is used to regulate the flow rate. Depth measurements were taken using a needle point gauge with a reading accuracy of ± 0.1 mm. Uniform flow conditions were reached using a carefully designed inlet tank. The adverse slope was adjusted using a screw jack located at the upstream end of the flume while at the

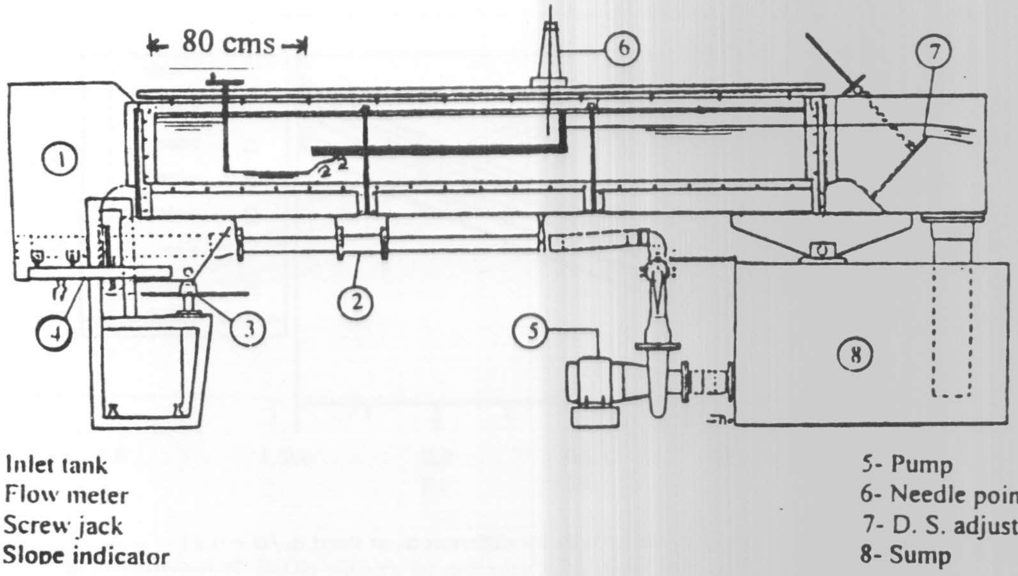

Fig. 2. Experimental apparatus.

downstream end, the flume is allowed to rotate freely about a hinged pivot. The slope was directly determined using a slope indicator. A downstream adjustable gate was used to regulate the tailwater surface elevation.

The experiments were carried out using five different conduit heights, D, of 6, 7, 8, 9, and 10 cm. Six different channel bottom adverse slopes, S_o, of 0.002, 0.004, 0.005, 0.0067, 0.01, and 0.02 were used to illustrate the effect of conduit adverse slope on the jump formed in adversely sloping conduit. The adverse slopes were selected based on the flume facilities.

For each combination of adverse slope and conduit height, five different flow rates ranging from 342 L/min to 234 L/min were used. The initial Froude number ranges from 4 to 6. For each conduit height, the upstream control gate was so adjusted to produce an initial supercritical depth, $\mathbf{d_1}$, and the downstream adjustable gate was adjusted to control the tailwater depth, $\mathbf{D_t}$, and which enabled the jump to be formed at a certain location in the conduit such that the jump toe is always located at the beginning of the conduit roof in order to make measurement of the initial supercritical depth, $\mathbf{d_1}$, using the needle point gauge possible. The jump location was kept fixed throughout the course of the experiments. For each combination of adverse slope and conduit height, the flow rate and the tailwater depth just downstream the conduit outlet were measured.

4. Experimental results

The variation of $\mathbf{D_t/d_1}$ with $\mathbf{F_1}$ for different tested adverse slopes are presented in Figs. 3 to 7 for $\mathbf{d_1/D}$=0.21, 0.233, 0.2625, 0.30, and 0.35, respectively. From these figures it can be observed that for a fixed $\mathbf{d_1/D}$, the trend of variation between $\mathbf{D_t/d_1}$ and $\mathbf{F_1}$ is increasing with a nonlinear trend according to Eq. (2). Also, at a particular $\mathbf{F_1}$, $\mathbf{D_t/d_1}$ decreases as the adverse slope of the conduit increases.

$$\mathbf{D_t / d_1 = a_o + a_1 F_1 + a_2 F_1^2} . \tag{2}$$

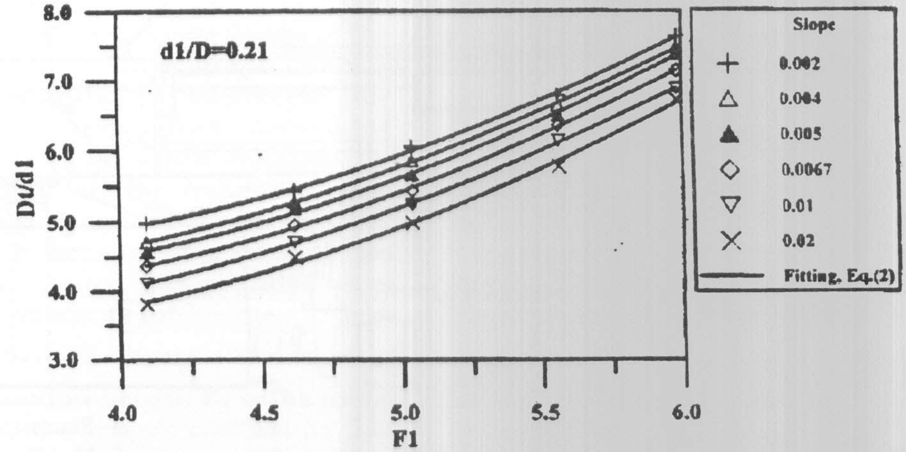

Fig. 3. Variation of D_t/d_1 with F_1 for different S_o at fixed $d_1/D = 0.21$.

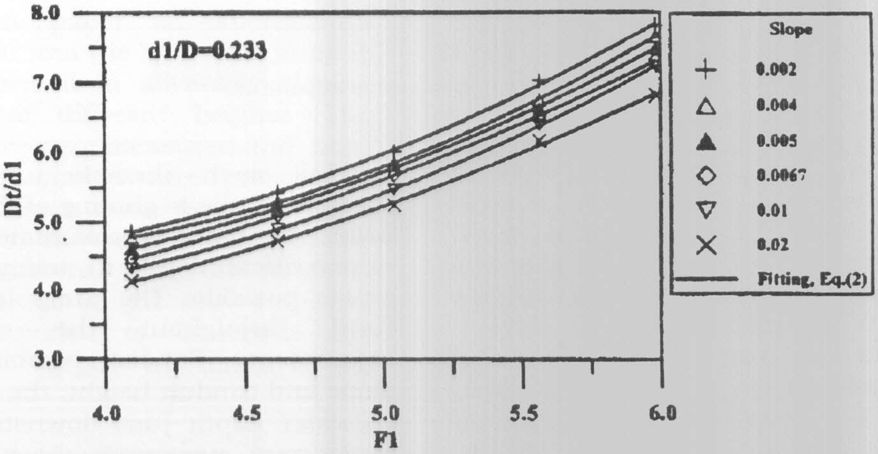

Fig. 4. Variation of D_t/d_1 with F_1 for different S_o at fixed $d_1/D = 0.233$.

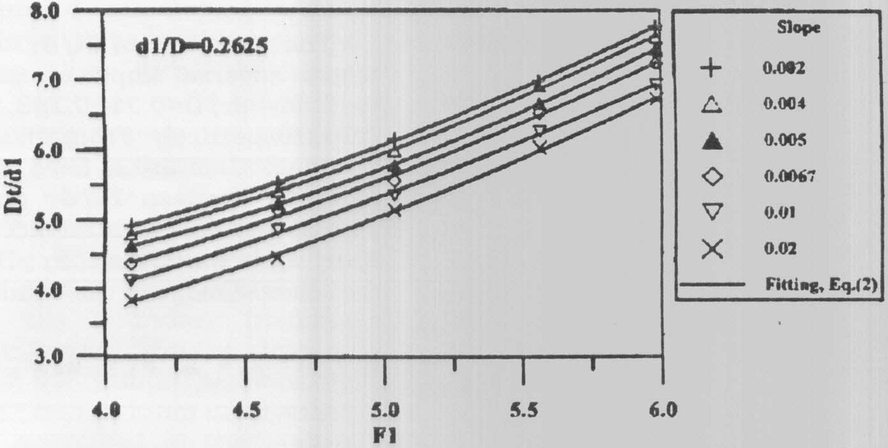

Fig. 5. Variation of D_t/d_1 with F_1 for different S_o at fixed $d_1/D = 0.2625$.

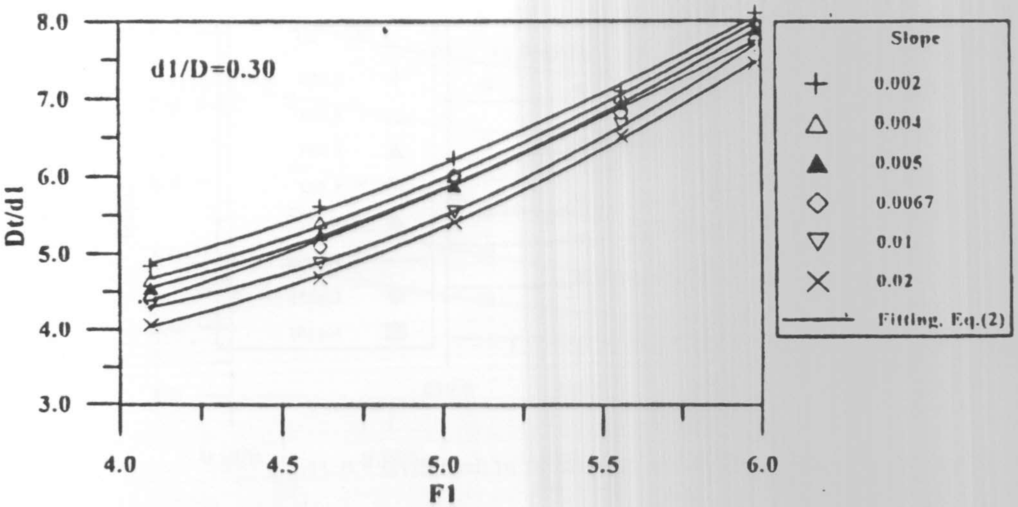

Fig. 6. Variation of D_t/D_1 with F_1 for different S_o at fixed d_1/D = 0.30.

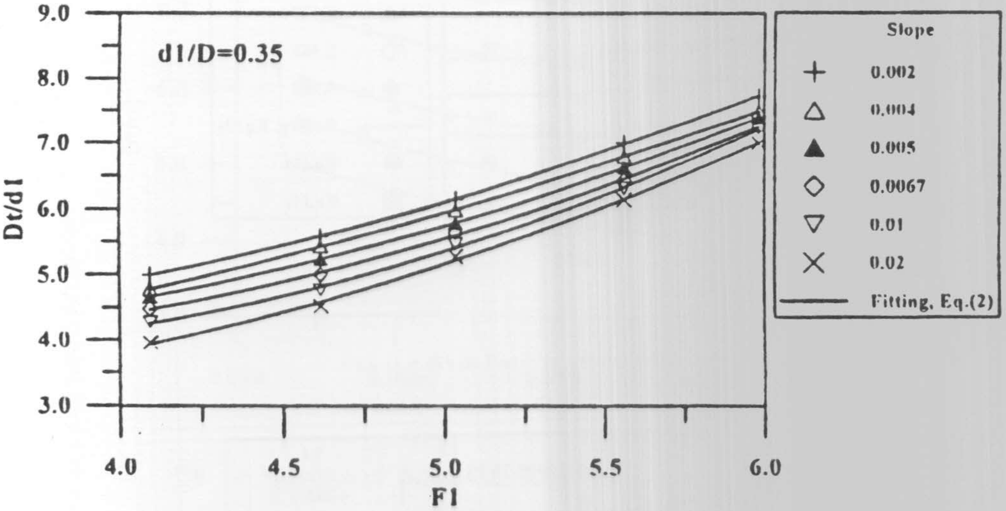

Fig. 7. Variation of D_t/D_1 with F_1 for different S_o at fixed d_1/D = 0.35.

where a_o, a_1, and a_2 are regression coefficients that depend upon the conduit adverse slope and the ratio d_1/D. Similarly, the variations of D_t/d_1 with S_o for different F_1 are shown in Figs. 8 to 12 for d_1/D=0.21, 0.233, 0.2625, 0.30, and 0.35, respectively. From these figures, it can be observed that for a fixed d_1/D, the trend of variation between D_t/d_1 and S_o is decreasing with a nonlinear trend according to Eq. (3). Also, at a particular S_o, D_t/d_1 increases as F_1 increases.

$$D_t/d_1 = b_o + b_1 S_o + b_2 S_o^2 , \tag{3}$$

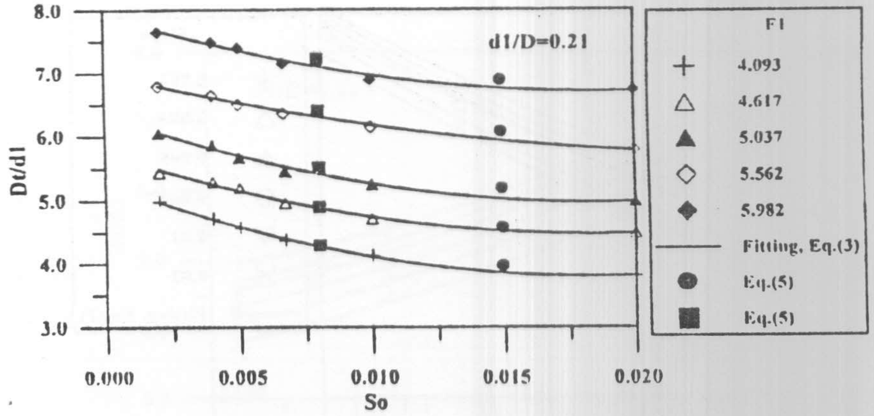

Fig. 8. Variation of D_t/D_1 with S_o for different F_1 at fixed d_1/D = 0.21.

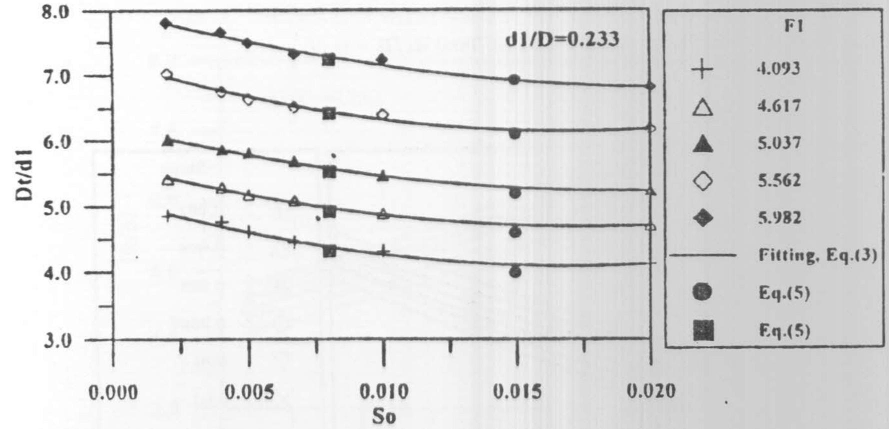

Fig. 9. Variation of D_t/D_1 with S_o for different F_1 at fixed d_1/D = 0.223.

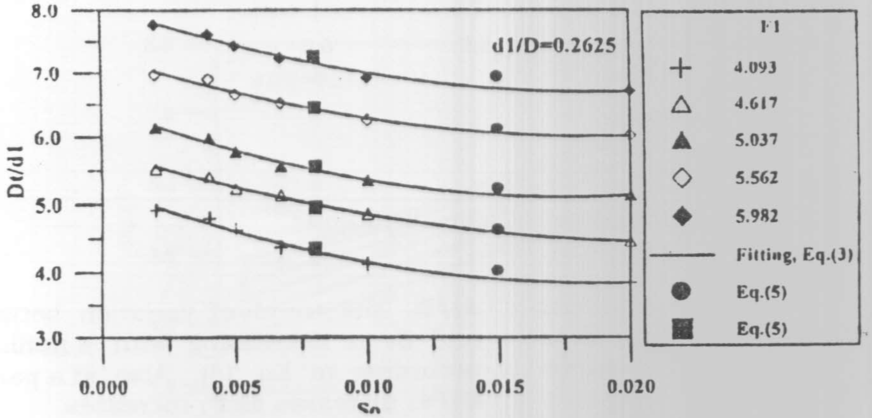

Fig. 10. Variation of D_t/D_1 with S_o for different F_1 at fixed d_1/D = 0.2625.

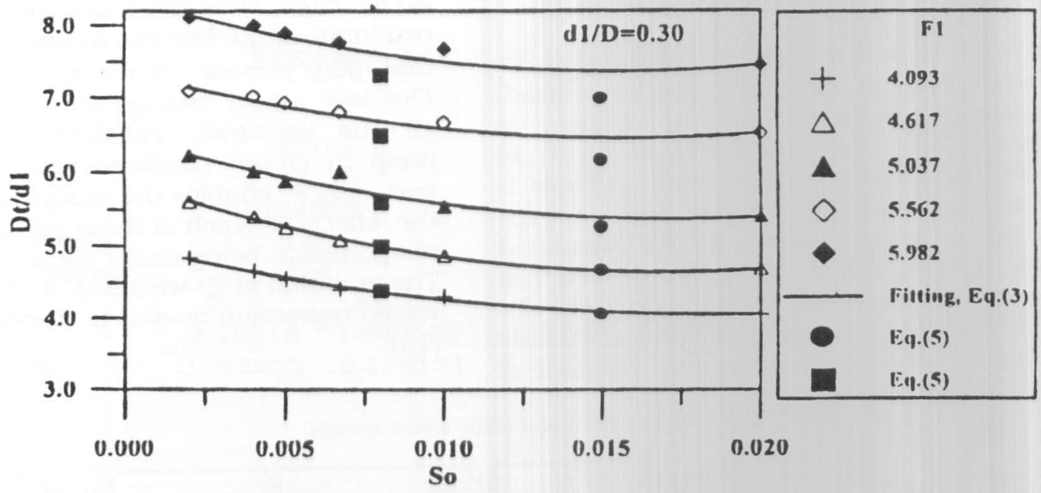

Fig. 11. Variation of D_t/D_1 with S_o for different F_1 at fixed d_1/D = 0.30.

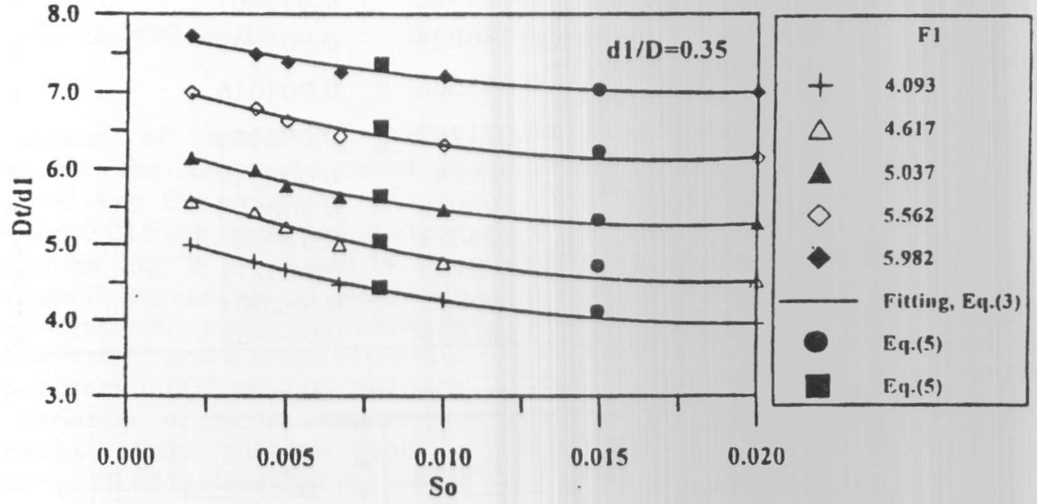

Fig. 12. Variation of D_t/D_1 with S_o for different F_1 at fixed d_1/D = 0.35.

where b_o, b_1, and b_2 are regression coefficients that depend upon the F_1 and the ratio d_1/D. Shown also on the figures the predicted values for S_o=0.008 and 0.015 using Eq. (5) (solid circles and squares). It is also possible to combine Eq. (2) and Eq. (3) together to determine D_t/d_1 as a function of both F_1 and S_o in the form:

$$D_t/d_1 = (c_o + c_1 S_o + c_2 S_o^2) + (c_3 + c_4 S_o + c_5 S_o^2) F_1 + (c_6 + c_7 S_o + c_8 S_o^2) F_1^2 . \qquad (4)$$

Where the coefficients from c_o to c_8 are functions of d_1/D only. In order to deduce a general equation in the form of Eq. (1), it is necessary to relate the coefficients (c_o to c_8) to d_1/D and back substitution in Eq. (4) yields the required equation. However, such equation will be too long and will have too many coefficients. A simpler and more practical equation may be obtained via the use of statistical analysis.

5. Statistical analysis and predictions

In order to derive a general equation in the form of Eq. (1), the statistical analysis is used. The relative tailwater depth, $\mathbf{D_t/d_1}$, is correlated with the parameters of Eq.(1) in different combinations as shown in Table 1. The standard error of estimate, SEE, and the coefficient of determination, R^2, are calculated and are given in Table 1. It is observed that the last two models give approximately the same R^2. However, the last model that include $\mathbf{d_1/D}$ reduces the standard error of estimate of $\mathbf{D_t/d_1}$ more than the other model. Testing the residuals of the last two models, it is observed that they provide an error of less than ± 5%. The last model has the merit that it contains all the variables controlling the hydraulic jump in closed conduits with adverse slopes and hence enables the analysis and studying the effect of each of these parameters on the phenomenon being under consideration.

The variation of R^2 with SEE for the different tested regression models is shown in Figure (13).

Table 1. R^2 and SEE for the statistically tested models.

Model	Variable	SEE	R^2	No. of variables
M1	$\mathbf{d_1/D}$	1.101308	0.002261	1
M2	$\mathbf{S_o}$	1.064491	0.067857	1
M3	$\mathbf{F_1}$	0.354422	0.896667	1
M4	$\mathbf{F_1^{1.5}}$	0.346824	0.901049	1
M5	$\mathbf{F_1^{2}}$	0.341586	0.904016	1
M6	$\mathbf{S_o^{2}}$	1.071787	0.055034	1
M7	M3+M4	0.33934	0.905914	2
M8	M2+M4	0.195079	0.968906	2
M9	M5+M6	0.223871	0.95905	2
M10	M2+M7	0.179783	0.973771	3
M11	M6+M10	0.146929	0.982601	4
M12	M1+M11	0.137525	0.984862	5

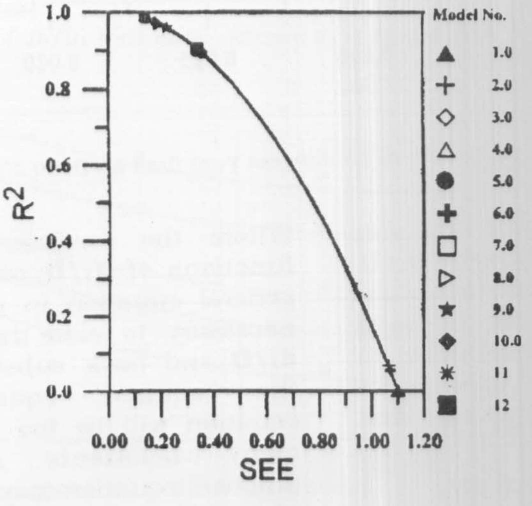

Fig. 13. Variation of R^2 with SEE for the tested regression models.

The results of the last regression model are given as follows:

Regression output for model No. 12

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M12 Regression Output:
   Constant                    7.742767
   Std. Err. of Y Est.         0.137525
   R Squared                   0.984862
   No. of Observations            150
   Degrees of Freedom             144
                          F1        F1^1.5      So        So^2          d1/D
X Coefficient(s)      -4.10218   1.680542  -128.058   3501.501      1.049159
Std. Err. of Coef.    0.603035   0.179181   8.92272   382.0423      0.226219
```

Thus, model No. 12 has the following form:

$$D_t/d_1=7.743-4.1022F_1+1.6805F_1^{1.5}-128.058S_o$$
$$+3501.5\,S_o^2+1.0492\,(d_1/D). \quad (5)$$

The prediction of model No. 12 is presented against the measured values of D_t/d_1 in Fig. 14. Also, the prediction of Eq.(5) for S_o=0.008 and 0.015 are presented in Fig. 8 to 12. From these Fig. 8 to 12 and 14, it is observed that good agreement is observed between measured and predicted values of D_t/d_1 for different d_1/D and a range of conduit adverse slopes from 0.002 to 0.02 for each d_1/D. The variation of errors versus the measured values of the tailwater depth is presented in Fig. 15. It is clear that the model predicts D_t/d_1 with a maximum error of about ±5% which is an acceptable error for practical design purposes.

6. Sensitivity analysis

Model No. 12 is used to study the effect of the different parameters on D_t/d_1. Fig. 16 shows the typical effect of d_1/D for different F_1 of 4, 4.5, 5, and 5.5 at fixed conduit adverse slope of 0.015. It is clear that for the investigated range of the adverse slopes, the ratio of the initial depth to conduit height d_1/D has insignificant effect on D_t/d_1 as D_t/d_1 is increasing very slightly with the increase of d_1/D. The figure indicates also that D_t/d_1 increases as F_1 increases. Fig. 17 presents the effect of the initial Froude number F_1 at fixed d_1/D and different conduit adverse slopes. It is observed that the effect of F_1 on D_t/d_1 is significant. The relative tailwater depth, D_t/d_1, increases non-linearly with the increase of F_1. Also, the higher the adverse slope, the less the ratio D_t/d_1 is, which proves that the adverse slope has a decreasing effect on D_t/d_1. This result is confirmed by observing the sign of the coefficient of S_o in Eq.(5).

Also, Fig. 18 shows that the adverse slope has a major effect on D_t/d_1 which is comparable with the effect of F_1 on D_t/d_1 where D_t/d_1 decreases non-linearly with the increase of the conduit adverse slope. Also, confirmed the increase of D_t/d_1 with the increase of F_1 at a fixed d_1/D.

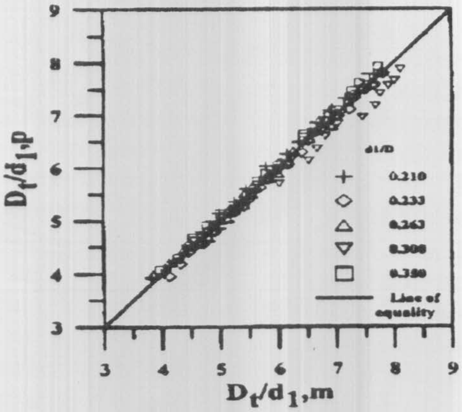


Fig. 14. Measured values of D_t/d_1 versus prediction of Model No. 12 (Eq.(5)).

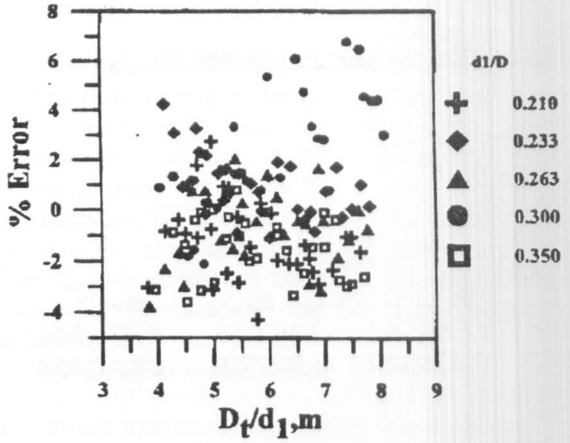


Fig. 15. Variation of % error with D_t/d_1 using model No. 12 (Eq.(5)).

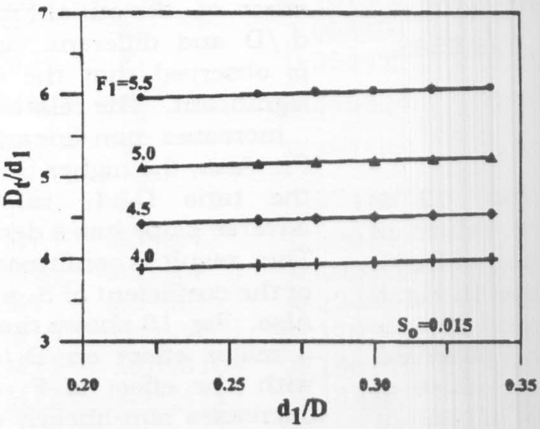


Fig. 16. Effect of d_1/D on D_t/d_1 for different F_1 at a fixed adverse slope of 0.015.

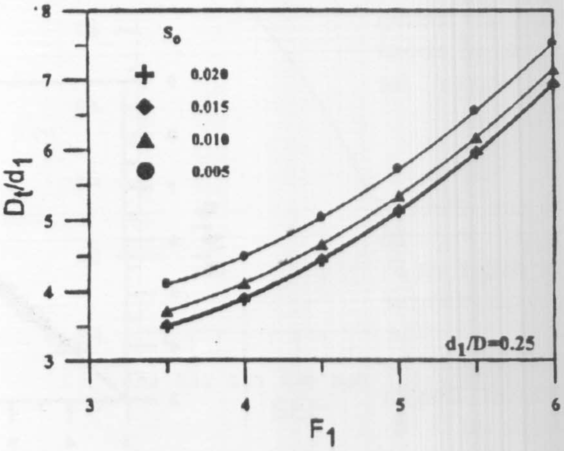


Fig. 17. Effect of F_1 on D_t/d_1 for different adverse slopes at fixed d_1/D.

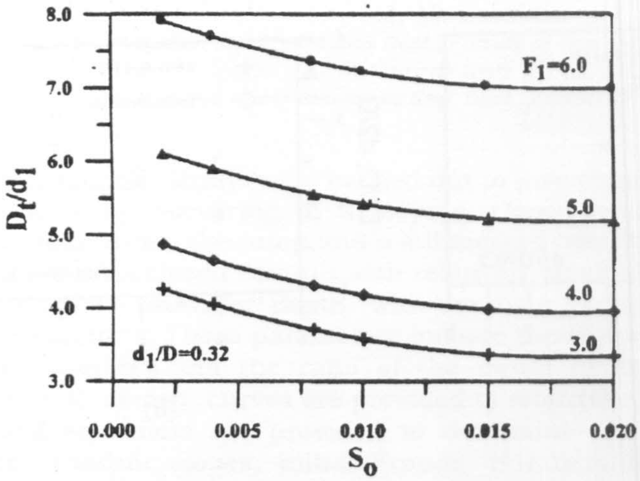


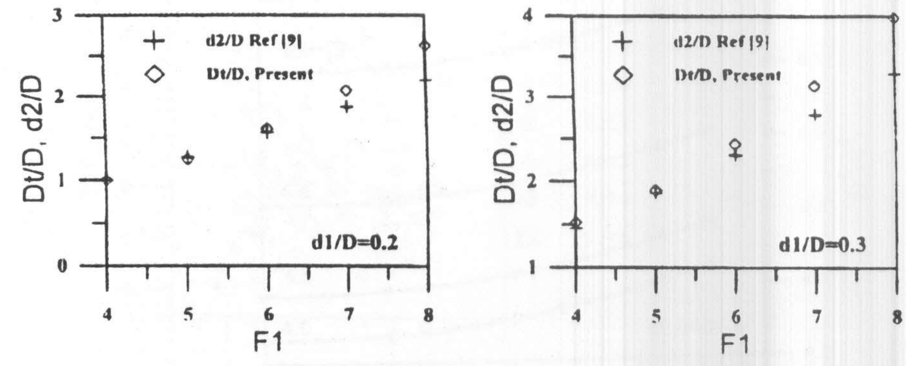
Fig. 18. Effect of adverse slope on $\mathbf{D_t/d_1}$ for different $\mathbf{F_1}$ at fixed $\mathbf{d_1/D}$.

7. Comparisons

Although Smith and Chen [9] analyzed only the height of jump ratio and no data on the relative tailwater depth are available in their paper, it is possible to compare the present results with their results for horizontal conduits. Assuming that d_2 is the sequent depth of jump and H_j is the height of jump in horizontal conduit. The empirically developed equations for horizontal conduit are used to generate d_2/D. It is known that $Hj/D = (d_2-d_1)/D$ and d_1 is calculated from d_1/D knowing the conduit height and hence d_2/D can be obtained. It is also assumed that d_2 is approximately equal to D_t if the exit and other conduit losses are neglected. Fig. 19-a and 19-b present the comparison between D_t/D and d_2/D for $d_1/D=0.2$ and $d_1/D = 0.3$, respectively. D_t/D is obtained by computing D_t/d_1 using Eq (5) and then multiplied by d_1/D assuming the effect of small slope can be neglected. The deviation between the present results and Smith and Chen [9] can be attributed to be mainly due to the absence of the effect of small slope when Eq. (5) is used and also due to the exit and other losses which are contained in D_t/D

8. Conclusions

The hydraulic jump in closed conduits with adverse slopes considering the tailwater depth at the outlet is analyzed with the aid of experimentally collected data. It is concluded that the relative tailwater depth is a function of the initial Froude number, the conduit adverse slope and the ratio of the initial depth to conduit height. Both of the initial Froude number and the conduit adverse slope have major effect on the jump characteristics while the ratio of the initial depth to conduit height is of minor effect when the adverse slope is relatively small. In all cases, the relative tailwater depth increases nonlinearly with the increase of the initial Froude number and/or the decrease of the conduit adverse slope. Set of equations are presented in terms of the initial Froude number and conduit adverse slope. Statistical methods are used to analyze the experimental data and to derive empirical prediction equation. The developed prediction equation provides the calculation of the relative tailwater depth with a maximum percent error of about ±5% and it can be used to study the effect of the different parameters on the relative tailwater depth as indicated in Figs. 16-18. The present results are compared with other authors published results for the same slope and $\mathbf{d_1/D}$ as indicated in Fig. 19.



(a) (b)

Fig. 19. Comparison between results of present study and results of Smith and Chen [9] for horizontal conduit, (a) for $\mathbf{d_1/D}$=0.2 and (b) for $\mathbf{d_1/D}$=0.3.

Nomenclature

a_o-a_2	coefficients of Eq.(2),
b_o-b_2	coefficients of Eq.(3),
c_o-c_8	coefficients of Eq.(4),
d_1	initial depth of supercritical flow = d1 on the figures,
D	conduit height,
D_t	tailwater depth, =Dt on the figures,
S_o	conduit adverse slope,
F_1	initial Froude number,
D_t/d_1	relative tailwater depth=Dt/d1on the figures,
d_1/D	ratio of initial depth to conduit height, and
d_2	sequent depth of jump according to Ref. [9].

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