# Evaluation of accuracy in curve ranging by linear measurements

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The methods used in horizontal curve setting-out are so diverse regarding instrumentation and geometrical configuration. Methods involving linear measurements, usually used in alignment of roadwork of normal accuracy, pipelines and navigation channels, are introduced and examined. A comparative study is carried out for the attainable accuracy of these methods by varying such parameters as degree of curve, and observational a priori standard deviations. The mathematical models necessary for analysis are derived. The criteria used are based on spatial distribution of error ellipses for the methods under investigation.

تختلف الطرق المستخدمة فى توقيع المنحنيات الأفقية من حيث التقنية والأجهزة التي تستخدم في الرصد ، كمـــا تتوقـــف دقـــة الطريقة على الغرض الذي من اجله صممت هذه المنحنيات ، لذا فقد بات من الضروري تقييم الدقة المتوقع الحصول عليها عند توقيع منحنى ما بالطرق المختلفة حتى يتسنى اختيار الطريقة التي تناسب درجة الدقة المطلوبة.وقد تم في هذا البحث دراســـة ثلاث طرق تعتمد على القياسات الخطية. وقد تعرض هذا البحث أيضا باستفاضة لدراسة أساسيات ما يسمى بانتشـــار الخطـــأ العشوائي وأيضا منحنى الخطأ لكي يتسنى تصميم النماذج الرياضية التي تحاكى طرق التوقيع ومنها نحصل على كمية الخطـــأ المتوقعة عند توقيع نقاط هذه المنحنيات . أخيرا تم عمل دراسة مقارنة للطرق الثلاثة المستخدمة من خلال إنشاء منحنى القطع الناقص للأخطاء عند مختلف النقط على طول المنحنى المراد توقيعه.

**Keywords**: Horizontal curves, Setting out, Accuracy, Error, Ellipse

## 1. Introduction

The main task of this paper is to analyze and quantify the amount of error in curve setting-out resulting from the errors in survey observations. Three methods are investigated; viz: offsets from the long chord, offsets from tangent, and prolongation of chords. Whenever necessary, these methods are referred to as Method I, Method II and Method III, respectively. Firstly, a brief study of fundamentals of random error propagation, and error- ellipse technique is introduced; then the mathematical models, which quantify the amount of error represented by variances and covariances, are developed (7). Comparative analysis between different methods is carried out through construction of the error-ellipses of point position determined along the curve.

## 2. Preview of methods

### *2.1. Offsets from the long chord (Method I)*

Fig. 1. shows the geometry of this method. A point P on the curve is determined by the formula (8);

$$y_p = [R^2 - (z - x_p)^2]^{1/2} - (R^2 - z^2)^{1/2}.$$

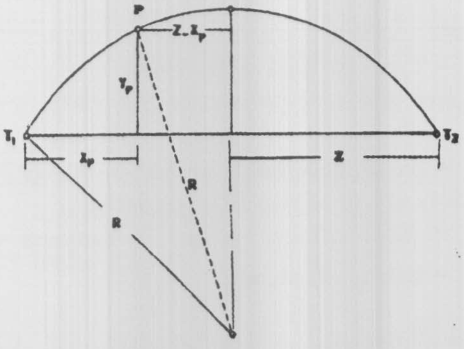


Fig. 1. Setting out by offsets from the long chord.

Where: $x_p, y_p$ are horizontal coordinates of point p according to the shown system, and z is half the long-chord-length.

*2. 2. Offsets from the tangent (Method II)*

Let $y_p$ is the perpendicular offset $D_p$ at any distance $x_p$ along the tangent,
$T_1 I$ (see Fig. 2.),
$T_1 D = x_p$
$y_p$ is obtained from the following equation

$$y_p = R - (R^2 - X_p^2)^{1/2}$$

the distance $x_p$ is measured along the tangent, then point p $(x_p, y_p)$ is fixed by erecting the offset $y_p$.

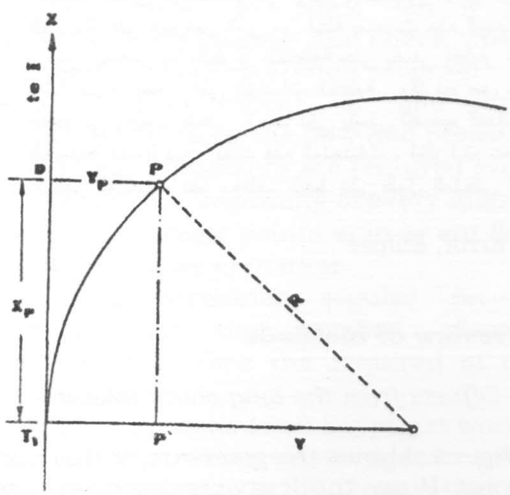

Fig. 2. Setting out by offsets from the tangent.

*2.3. Prolongation of chords (Method III)*

i. Case of unequal chords (Fig. 3):

Assume;
$T_1p_1 = T_1p_1'$ (initial sub-chord) $= \ell_1$,
$p_1, p_2, p_3$, etc. are points on the curve,
$T_1p_1 = \ell_1$, $p_1p_2 = \ell_2$, $p_2p_3 = \ell_3$, etc.,
$T_1I$ = rear tangent, angle $p_1 T_1 p'_1 = \gamma$ = deflection angle of the first chord,
$p'_1p_1$ = first offset, $p'_2p_2$ = second offset, $p'_3p_3$ = third offset, etc.

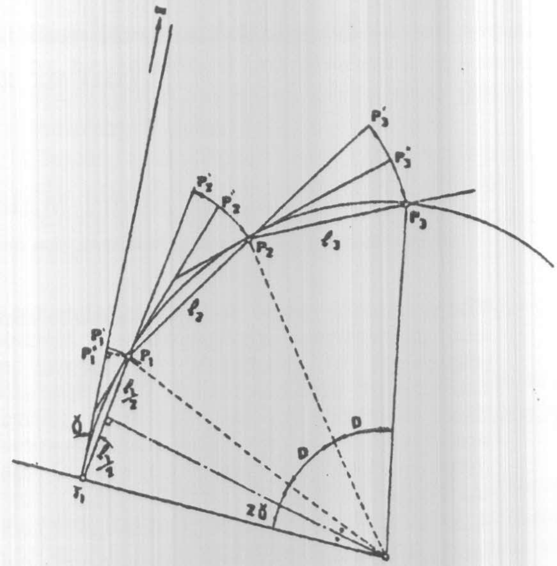

Fig. 3. Setting out the curve by deflection distances (prolongation of chords).

In case of small angle ($\gamma$):

$$p_1p_1'' = p_1p_1' = \ell_1 \sin\gamma = \ell_1^2/2R,$$

first offset/

$$h_1 = \ell_1^2/2R.$$

Offset from the first full chord is

$$h_2 = p_2''p_2' + p_2''p_2 = h_1 \frac{\ell_2}{\ell_1} + \frac{\ell_2^2}{2R}$$

$$= \frac{\ell_1\ell_2}{2R} + \frac{\ell_2^2}{2R} = \ell_2(\ell_1 + \ell_2)/2R.$$

Offset from the second full chord is

$$h_3 = p_3''p_3' + p_3p_3'' = \ell_3(\ell_2 + \ell_3)/2R.$$

Generally, the equation becomes

$$h_n = \ell_n(\ell_n + \ell_{n-1})/2R.$$

*ii. Case of equal chords (No subchord)*

$$\ell_1 = \ell_2 = \ell_3 = \dots = \ell_n = \ell$$

then the first offset $h_1 = \ell^2/2R$ and subsequent offsets are ,

$h_2 = h_3 = h_4 = \ldots = h_n = \ell^2/R$.

Procedure for setting out the curve:

- Locate the tangent points $T_1$ (P.C.) and $T_2$ (P.T.).
- Calculate the length $\ell_1$ of the first sub-chord.
- With zero mark at $T_1$, spread the tape along the first tangent to point $p_1$ on it such that $T_1p_1 = \ell_1$ length of the first sub-chord.
- With $T_1$ as center and $T_1p_1$ as radius swing the tape such that the arc $p_1p_1' =$ calculated offset $h_1$, fix the point $p_1$ on the curve.
- Spread the chain along $T_1p_1$ and pull it straight in this direction to a point $p_2'$ such that the distance $p_1p_2' = \ell_2$ = length of normal chord.
- With zero of the chain centered at $p_1$ and $p_1p_2$ as radius swing the chain to a point $p_2$ such that $p_2p_2'$ = length of the second offset, fix the point $p_2$ on the curve.
- Repeat the above steps till the last point is reached.
- The last point must coincide with the point of tangency $T_2$.

## 3. Error and correlation

### 3.1. Covariance and correlation

An observation may be two-dimensional. It consists of pair of observations, one for variate x and the other for variate y. These variates may not be independent but may be affected in a similar manner by some outside factor, which causes some sort of dependency. Such variates are said to be correlated. A measure of correlation is the so called covariance. It is designated by the symbol cov (x,y), or by $\sigma_{xy}$, and is defined by the following expectation:

$$\text{cov}(x,y) = \sigma_{xy} = E[(x-\mu_x)(y-\mu_y)]. \tag{1}$$

Where;

$\mu_x$ is the separate mean of x

$\mu_y$ is the separate mean of y

Correlation may be expressed by the correlation coefficient, designated by $\rho_{xy}$ independent of the units of measurement:

$$\rho_{xy} = E\left[\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right)\right], \tag{2}$$

which can be simplified to

$$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x\sigma_y}. \tag{3}$$

### 3. 2. Propagation of variances

Suppose a variate $y_1$, is linear Function of the variates $x_1, x_2, x_3, \ldots, x_n$ ; and taking n = 3 for convenience :

$$y_1 = a_1 x_1 + a_2 x_2 + a_3 x_3 \tag{4}$$

the variance of $y_1$ is given by

$$\sigma^2_{y1} = a_1^2\sigma^2_{x_1} + 2a_1a_2\sigma_{x_1x_2} + a_2^2\sigma^2_{x_2} + 2a_1a_3\sigma_{x_1x_3} + 2a_2a_3\sigma_{x_2x_3} + a_3^2\sigma^2_{x_3}, \tag{5}$$

the variance of another function of the same variates such as,

$$y_2 = b_1x_1 + b_2x_2 + b_3x_3, \tag{6}$$

is

$$\sigma_{y2} = b_1^2\sigma^2_{x_1} + 2b_1b_2\sigma_{x_1x_2} + b_2^2\sigma^2_{x_2} + 2b_1b_3\sigma_{x_1x_3} + 2b_2b_3\sigma_{x_2x_3} + b_3^2\sigma^2_{x_3}. \tag{7}$$

Further, the covariance of $y_1$ and $y_2$ is given by

$$\sigma_{y_1y_2} = a_1b_1\sigma^2_{x_1} + (a_1b_2 + a_2b_1)\sigma_{x_1x_2} + a_2b_2\sigma^2_{x2} + (a_1b_3 + a_3b_1)\sigma_{x_1x_3} + (a_2b_3 + a_3b_2)\sigma_{x_2x_3} + a_3b_3\sigma^2_{x_3} \tag{8}$$

It will be noticed that the coefficients in formulae (5), (7) and (8) are the same as the

coefficients of the square or cross product of the Eq. (4) and (6), respectively.
If a function F of $y_1$ and $y_2$ is given by,

$$F = cy_1 + dy_2. \tag{9}$$

And if the variance of the function may be found by the same rule,

$$\sigma_F^2 = c^2 \sigma_{y_1}^2 + 2cd\,\sigma_{y_1 y_2} + d^2 \sigma_{y_2}^2. \tag{10}$$

The values obtained for $\sigma^2_{y_1}$, $\sigma^2_{y_2}$, $\sigma_{y_1y_2}$ may then replaced by the variances with respect to $x_1$, $x_2$ and $x_3$ from the previous equations. Should the variates $x_1$,$x_2$ and $x_3$ be uncorrelated, then all cross products in (5), (7) and (8) are canceled since,

$$\sigma_{x_1x_2} = \sigma_{x_1x_3} = \sigma_{x_2x_3} = 0.0,$$

the term $\sigma_{x_1y_2}$ remains. It is easy to realize that $y_1$ and $y_2$ are correlated since both have been derived from the same variates. If the relationship,

$$y_1 = f_1\,(x_1,x_2,x_3)\,, \tag{11}$$

is not linear it can be expanded as a series according to Taylor, so that linear functions are obtained (second and higher terms being omitted).

Assuming that the normal distribution of the variate derived from the linear function would be a sufficient approximation of non-normal distribution of $y_1$ in Eq. (11) the variance can be calculated by the following formula,

$$\sigma_{y_1}^2 = \left(\frac{\partial f_1}{\partial x_1}\right)^2 \sigma_{x_1}^2 + \left(\frac{\partial f_1}{\partial x_2}\right)^2 \sigma_{x_2}^2 + \left(\frac{\partial f_1}{\partial x_3}\right)^2 \sigma_{x_3}^2$$

$$+2\frac{\partial f_1}{\partial x_1}\frac{\partial f_1}{\partial x_2}\sigma_{x_1x_2} + 2\frac{\partial f_1}{\partial x_1}\frac{\partial f_1}{\partial x_3}\sigma_{x_1x_3} + 2\frac{\partial f_1}{\partial x_3}\frac{\partial f_1}{\partial x_3}\sigma_{x_2x_3}. \tag{12}$$

If there are two relationships, viz.

$$y_1 = f_1\,(x_1, x_2, x_3), \text{ and}$$

$$y_2 = f_2\,(x_1, x_2, x_3)\,. \tag{13}$$

Then $\sigma y_2{}^2$ would be give by the same formula (12) by replacing $f_1$ with $f_2$.
The covariance becomes

$$\sigma_{y1y2} = \left(\frac{\partial f_1}{\partial x_1}\frac{\partial f_2}{\partial x_1}\right)\sigma^2_{x_1} + \left(\frac{\partial f_1}{\partial x_2}\frac{\partial f_1}{\partial x_2}\right)\sigma^2_{x_2} + \left(\frac{\partial f_1}{\partial x_3}\frac{\partial f_2}{\partial x_3}\right)\sigma^2_{x_3}$$

$$+\left(\frac{\partial f_1}{\partial x_1}\frac{\partial f_2}{\partial x_2} + \frac{\partial f_1}{\partial x_2}\frac{\partial f_2}{\partial x_1}\right)\sigma_{x_1x_2}$$

$$+\left(\frac{\partial f_1}{\partial x_1}\frac{\partial f_2}{\partial x_3} + \frac{\partial f_1}{\partial x_3}\frac{\partial f_2}{\partial x_1}\right)\sigma_{x_1x_3}$$

$$+\left(\frac{\partial f_1}{\partial x_2}\frac{\partial f_2}{\partial x_3} + \frac{\partial f_1}{\partial x_3}\frac{\partial f_2}{\partial x_1}\right)\sigma_{x_2x_3}. \tag{14}$$

If only estimates of $\sigma_{x_1x_2}$, $\sigma^2_{x_2}$, $\sigma^2_{x_1}$ are known, then according to the same rules, only estimates of $\sigma^2_{y_2}$, $\sigma^2_{y_1}$ and $\sigma_{y_1y_2}$ can be calculated.

*3. 3. Error ellipse technique*

The error ellipse technique is the most common method of determining positioning accuracy. The area inside the ellipse is described as a confidence region, i.e. there exists a certain probability or confidence that the true position falls within the ellipse [6].
Let the position variances and covariance of a point P referred to chosen axes x, y, be $\sigma_x^2$, $\sigma_y^2$, $\sigma_{xy}$.

Semi-major and semi-minor axes $\lambda_1$ and $\lambda_2$ respectively are obtained from the following expressions:

$$\left.\begin{aligned}\lambda_1 &= \frac{\sigma_x^2 + \sigma_y^2}{2} + [\frac{(\sigma_x^2 - \sigma_y^2)^2}{4} + \sigma_{xy}^2]^{1/2}\\ \lambda_2 &= \frac{\sigma_x^2 + \sigma_y^2}{2} - [\frac{(\sigma_x^2 - \sigma_y^2)^2}{4} + \sigma_{xy}^2]^{1/2}\end{aligned}\right\} \tag{15}$$

while rotation angle $\beta$ can be obtained from the following expression (3,4):

$$\tan 2\beta = \frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2} . \qquad (16)$$

The quadrant of $2\beta$ is determined in the usual way from the sign of numerator $2\sigma_{xy}$ and denominator $(\sigma_x^2 - \sigma_y^2)$.

## 4. Mathematical models for variance propagation

### *4. 1. Offsets from long chord and offsets from tangent (method i & ii):*

Referring to Fig. 1 and 2, it is clear that in these methods, the two variates x, y are independent (correlation - free). Consequently the covariance $\sigma_{xy}$ is zero.
Positioning errors in x and y directions, represented by $\sigma_x$ and $\sigma y$, respectively, for the considered point can be obtained directly as, (1, 5)

$$\sigma_x = \sigma_\ell \sqrt{\frac{x}{\ell}}, \text{ and} \qquad (17)$$

$$\sigma_y = \sigma_\ell \sqrt{\frac{x}{\ell}},$$

where:

$\sigma_x$ is the standard deviation of distance x,
$\sigma_y$ is the standard deviation of distance y,
$\sigma_\ell$ is the standard deviation of measuring one tape set up,
$\ell$ is the length of the measuring tape.

### *4. 2. Prolongation of chords (method III)*

Referring to Fig. 4, the Cartesian coordinate system is chosen such that the origin o coincides with the point $T_1$, x and y axes are the tangent and the perpendicular direction at $T_1$ respectively. For such model, all chords of the curve (peg Intervals) are assumed to be equal, of length, say $\ell$ , this implies that all of the offsets except the first one are equal, of length say $h_2$, while the first offset is of length $h_1$.

From the geometry of Fig. 4, let the arcs $p_1p'_1$, $p_2p'_2$ , $p_3p'_3$, ... ect. be equal to their chords (arc-chords approximation), then,
$p_1p'_1 = h_1$, and

$p_1p_2 = p_3p'_3 = p_4p'_4 = ... = h_2$ .

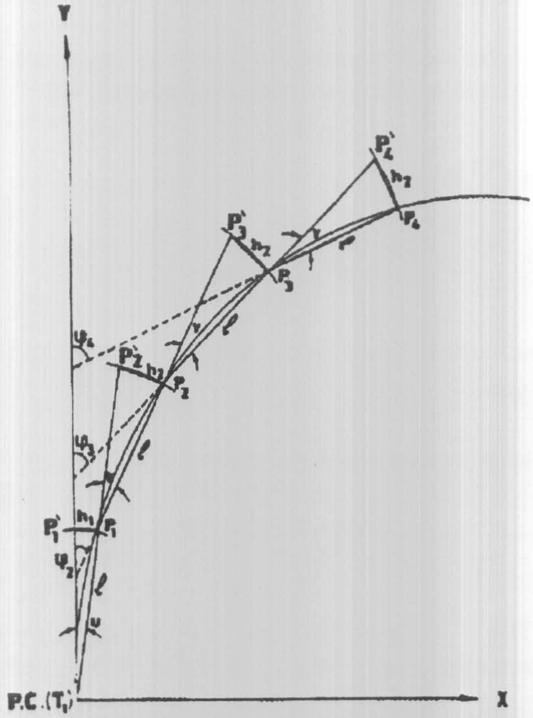


Fig. 4 Prolongation of chords method (coordinates system)

Let $\psi_1$, $\psi_2$, $\psi_3$,..., etc. denote the angles between chords $T_1p$, $p_1p_2$, $p_2p_3$,... etc. and the y axes, respectively, hence.

$$\left.\begin{aligned} \Psi_1 &= u \\ \Psi_2 &= u + v \\ \Psi_3 &= u + 2v \\ &\dots \\ \Psi_i &= u + (i-1)v \end{aligned}\right\}, \qquad (18)$$

where u is the first tangential angle and v is the deflection angle between the prolongation of the preceding chord and the considered chord.

From the geometry of Fig. 4, u and v can be given as:

$$\left.\begin{aligned} u &= \cos^{-1}(m) \\ v &= \cos^{-1}(m) \end{aligned}\right\}. \qquad (19)$$

Where $m = \left(\frac{\ell^2 - h_1^2}{\ell^2}\right)^{\frac{1}{2}}$, and $n = \left(\frac{\ell^2 - h_2^2}{\ell^2}\right)^{\frac{1}{2}}$,

then the coordinates of point $p_n$ ($x_{pn}$, $y_{pn}$), on the curve, may be written, in general, as:

$$\left.\begin{aligned} x_{pn} &= \ell \sum_{i=1}^{n} \sin \Psi_i \\ y_{pn} &= \ell \sum_{i=1}^{n} \cos \Psi_i \end{aligned}\right\}. \qquad (20)$$

Substituting from (18) and (19) into Eq. (20) one gets:

$$\left.\begin{aligned} x_{pn} &= \ell \sum_{i=1}^{n} \sin \{ \cos^{-1}(m) + (i-1)\cos^{-1}(n)\} \\ y_{pn} &= \ell \sum_{i=1}^{n} \cos \{ \cos^{-1}(m) + (i-1)\cos^{-1}(n)\} \end{aligned}\right\} \qquad (21)$$

The procedure for determining the variances $\sigma_{x_{pn}^2}$, $\sigma_{y_{pn}^2}$ and covariance $\sigma_{xy_{pn}}$ could be carried out in steps by using the laws of variance-covariance propagation. Since u and v are, often, small angles then we can reasonably assume that, $h_2 = 2h_1$ (very nearly). Differentiating Eqs. (21) partially with respect to $\ell$, $h_1$ and $h_2$, for the $n_{th}$ point, taking into account the last assumption, one gets:

$$\frac{\partial y_{pn}}{\partial \ell} = \sum_{i=1}^{n} \sin(\Psi_i) + C_1 \sum_{i=1}^{n} (2i-1)\cos(\Psi_i),$$

$$\frac{\partial y_{pn}}{\partial \ell} = \sum_{i=1}^{n} \cos(\Psi_i) + C_1 \sum_{i=1}^{n} (2i-1)\sin(\Psi_i),$$

$$\frac{\partial x_{pn}}{\partial h1} = C_2 \sum_{i=1}^{n} \cos(\Psi_i), \frac{\partial y_{pn}}{\partial h1} = C_3 \sum_{i=1}^{n} \sin(\Psi_i),$$

$$\frac{\partial x_{pn}}{\partial h2} = C_2 \sum_{i=1}^{n} (i=1)\cos(\Psi_i), \frac{\partial y_{pn}}{\partial h2} = C_3 \sum_{i=1}^{n} (i=1)\sin(\Psi_i), \qquad (23)$$

where;

$C_1 = \frac{\sqrt{2}\, h_2}{\ell} = \frac{h_2}{\sqrt{2}\,\ell}$, and $C_2 = - C_3 = 2$.

Let $\sigma_{h_1}$ and $\sigma_{h_2}$ are the corresponding variances of the measured distances $h_1$ and $h_2$ respectively. $\sigma_{h_1}$ and $\sigma_{h_2}$ may be obtained from the following forms suggested by (1),

$$\left.\begin{aligned} \sigma_{h1}^2 &= \frac{h_1}{\ell}\sigma_\ell^2 \\ \sigma_{h2}^2 &= \frac{h_2}{\ell}\sigma_\ell^2 \end{aligned}\right\}, \qquad (24)$$

where $\sigma_\ell$ and $\ell$ are standard deviation and the length of the tape, respectively. It was assumed previously that all measurement components were independent, consequently, each of the values $\sigma_{hi\,hi+1}$ and $\sigma_{\ell hi}$ are zero values (i=1,2,3,...n).

Substitution from Eqs. (23) and (24) into Eqs. (12) and (14)., taking the last assumption into account, variances $\sigma_x^2$, $\sigma_y^2$ and covariance $\sigma_{xy}$ for the $n^{th}$ point established on the curve, are:

$$\sigma_{xpn}^2 = \left\{\sum_{i=1}^{n} \sin(\Psi_i) + C_1 \sum_{i=1}^{n} (2i-1)\sin(\Psi_i)\right\}^2 \sigma_\ell^2 + \left\{C_2 \sum_{i=1}^{n} \cos(\Psi_i)\right\}^2 \sigma_{h_1}^2 + \left\{C_2 \sum_{i=1}^{n} (i-1)\cos(\Psi_i)\right\}^2 \sigma_{h_2}^2$$

$$\sigma_{xpn}^2 = \left\{\sum_{i=1}^{n} \cos(\Psi_i) + C_1 \sum_{i=1}^{n} (2i-1)\sin(\Psi_i)\right\}^2 \sigma_\ell^2 + \left\{C_3 \sum_{i=1}^{n} \sin(\Psi_i)\right\}^2 \sigma_{h_1^2} + \left\{C_3 \sum_{i=1}^{n} (i-1)\sin(\Psi_i)\right\}\sigma_{h_2}^2$$

$$\begin{aligned} \sigma_{xy_{pn}} = &\left\{\sum_{i=1}^{n} \sin(\Psi_i) + C_1 \sum_{i=1}^{n} (2i-1)\cos(\Psi_i)\right\} * \left\{\sum_{i=1}^{n} \cos(\Psi_i) + C_1 \sum_{i=1}^{n} (2i-1)\sin(\Psi_i)\right\}\sigma_\ell^2 \\ &+ C_2 C_3 \left\{\sum_{i=1}^{n} \cos(\Psi_i)\right\} * \left\{\sum_{i=1}^{n} \sin(\Psi_i)\right\}\sigma_{h_1}^2 \\ &+ C_2 C_3 \left\{\sum_{i=1}^{n} (i=1)\cos(\Psi_i)\right\} * \left\{\sum_{i=1}^{n} (i=1)\sin(\Psi_i)\right\}\sigma_{h_2}^2 \end{aligned} \qquad (25)$$

Eqs. (25) are the mathematical model for determining the expected precision and correlation represented by variances and covariance, in setting out points on the curve.

## 5. Analysis of results

According to the previously derived mathematical models, error ellipses were constructed for each of the three methods under study. A wide range of variables including the degree of curve, and linear measurement observational accuracy were dealt with. Only the cases with degree = 4 ° and tape stad. dev. = 0.005 m are introduced here . Fig. 5, 6 and 7 show the resulting error ellipse distribution for the three cases under study .

Error ellipses derived from the mathematical models, introduced in sec. 4, are considered as very suitable criteria for comparison between different methods of layout.

Visual inspection of the distribution and size of error ellipses is very convenient for judging accuracy, giving favor to the method that exhibits smaller sizes of error ellipses. However, in the present case, it is clear that even a large component of positioning error in the direction running along the curve is not significant. On the other hand, the radial component of error, which causes a

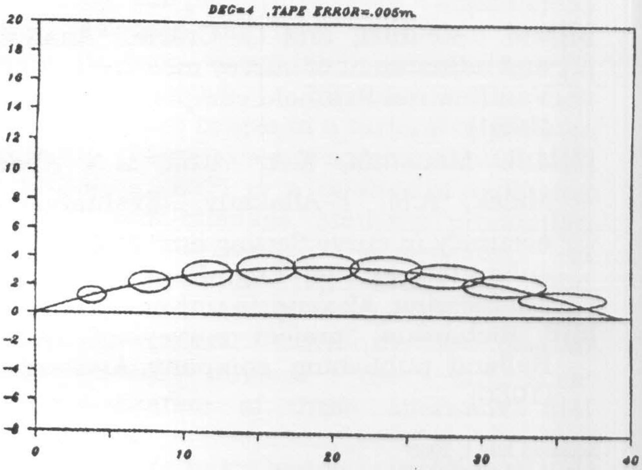


Fig. 5. Error ellipse distribution-method I-by offsets from Chord.

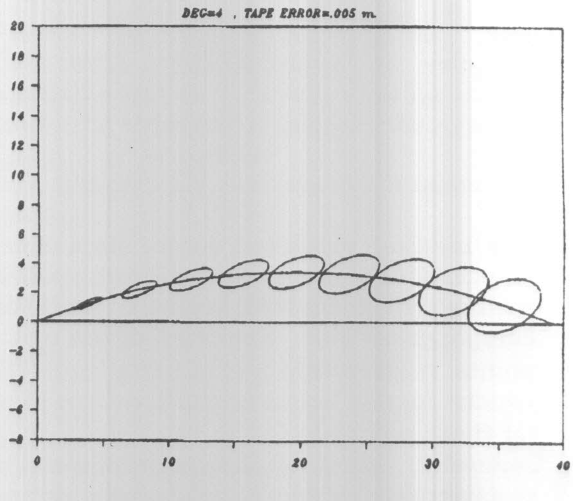


Fig. 6. Error ellipse distribution-method II-by offsets from the tangent.

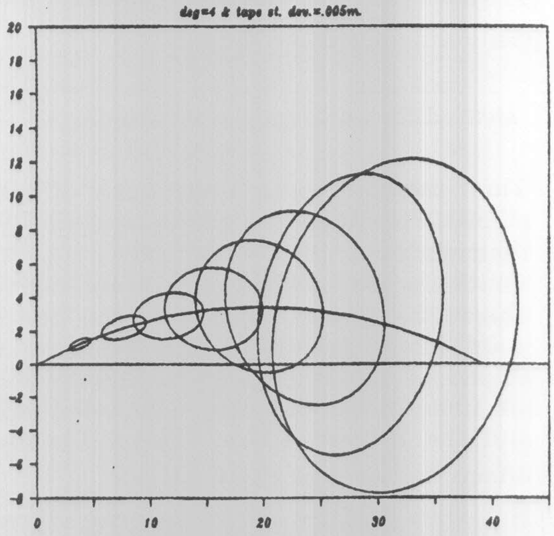


Fig. 7. Error ellipse distribution-method III-by prolongation of chords.

## 6. Conclusions

### 6.1. Method I (offsets from the long chord)

i. Dimensions of major and minor axes of the error ellipses are proportional to the ordinate lengths of considered point.
ii. For curves with large radii, the major axes of the error ellipses goes nearly tangential to the curve path which is a favorable situation.

iii. The radial component of standard deviation is approximately equal for all points on the curve that means existence of homogeneity, again, a favorable situation.

*6. 2. Method II (offsets from the tangent)*

i. This method exhibited larger sizes of error ellipses as compared to the previous method. Nevertheless, smaller radial error components were exhibited in the first few points neighboring the starting point (P.C.) which means better positing determination for these points .

ii. Generally, sizes of the error ellipses are considerably larger in case of greater curve degrees, consequently greater values of ($\sigma_r$) are recorded which means weak positioning accuracy.

*6.3. Method III (prolongation of chords)*

i. This method gives huge sizes of error ellipses, investigation shows irregularity in their shapes.

ii. Quick accumulation of errors can be observed as we move from the starting point along the curve. For example, the ellipse of the fifth point on curve is about six times larger than that of the first point; and the ellipse of the 9th point is about fifteen times larger than the first.

iii. Not only that the size of the error ellipses grows larger as we move away from the starting point; but also their orientation changes in such away that the major axis turns from the tangential to the radial position, which means more deterioration of accuracy.

*6. 4. General*

1. As expected an increase in the precision of measurements used in setting out procedure, increases the positioning accuracy. However, the effect is not always significant depending on the case as will be seen later.
2. For methods depending on Cartesian coordinates. VIZ. Offsets from the long chord and offsets from the tangent, the size of the error ellipse is proportional to the ordinate length of located points. Therefore, such methods are preferable in case of flat and short curves.
3. In case of using method of prolongation of chords, weak positioning accuracy with non-homogenous radial error component is observed. Quick accumulation of error exists since the error ellipses grow quickly in size as we move away from the starting point.

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