# Shear strength of high strength concrete beams with web reinforcement- An experimental investigation

M. Nasser Darwish and M. Taher Hashem
*Structural Engineering Department, Faculty of Engineering, Alexandria University*
M. Tarek Moustafa
*Engineer, Ministry of Housing and New Communities, Alexandria*

An experimental investigation is presented to study the shear strength of high strength concrete beams with web reinforcement. Nine simply supported reinforced high (medium high) strength concrete HSC beams with mean concrete strength of 65 MPa with web reinforcement in the form of vertical stirrups were experimentally tested under two symmetrically concentrated loads, to determine their shear strength. The test variables were: i) the web reinforcement ratio $\rho_v$=0.178% & 0.237%; ii) longitudinal steel ratio, $\rho$=2.3% and 3.1%; and iii) shear-span-to-depth ratio, namely a/d=1.5, 2.5 & 3.5. Test results were compared with strength predictions using several codes especially ACI-99, ECP-95 & BS-8110 to check the validity of codes when applied to such HSC beams with web reinforcement, noting that the used stirrups' spacing in some of the tested specimens did not fulfill some codes' provisions in that regard. ECP predictions seem always conservative whether its limitations on concrete strength were waived or not. ACI was conservative for beams with a/d=1.5, however for beams with a/d=2.5 and 3.5 its margin of safety was small. Fulfilling ACI provisions of stirrups spacing may render conservative results. BS predicts the shear capacity well for beams with a/d=1.5 but may overestimate that for beams with a/d>2. Zsutty's equation was conservative for short beams with a/d=1.5, however for beams with a/d=2.5 and 3.5 it seemed unconservative. These findings are in good agreement with other reported results. The restrictions set by codes for stirrups spacing would render more conservative results. Other conclusions are drawn.

يتناول هذا البحث دراسة مختبريه لمقاومة القص للكمرات ذات الخرسانة عالية المقاومة والمحتوية علي تسليح جذعي. وتم اختبار تسع كمرات بسيطة الارتكاز من خرسانة عالية المقاومة بمتوسط مقاومة ٦٥ ميجاباسكال وذات تسليح جذعي علي هيئة كانات رأسية تحت تأثير حملين مركزين متماثلين لتعيين مقاومة القص. وقد تم دراسة المتغيرات الآتية: ا) نسبة التسليح الجذعي وكانت ٠,١٧٨ ٪ و ٠,٢٣٧ ٪ ب) نسبة التسليح الطولي ٢,٣ ٪ و ٣,١ ٪ ح) ونسبة بحر القص إلى العمق الفعال a/d = ١,٥ و ٢,٥ و ٣,٥ . وتم مقارنة نتائج الاختبارات مع القيم المقدرة بتطبيق معادلات العديد من الكودات وخاصة الكود المصري وكود معهد الخرسانة الأمريكي الدولي والمواصفات البريطانية لتقييم هذه الكودات عند تطبيقها علي مثل هذ الكمرات من الخرسانة عالية المقاومة والمحتوية علي تسليح جذعي علما بأن التقسيط بين الكانات في بعض العينات المختبرة أكبر مما هو مسموح به في بعض الأكواد. وقد اتضح من النتائج أن الكود المصري في جانب التحفظ دائما سواء تم تطبيق الحد الأقصى الذي يفرضه علي مقاومة الخرسانة أم لا. كما ظهر أن كود معهد الخرسانة الأمريكي في جانب التحفظ للكمرات ذات a/d =١,٥ إلا أن معاملات الأمان تنخفض بالنسبة للكمرات ذات a/d = ٢,٥ و ٣,٥ والوفاء بمتطلبات التقسيط بين الكانات يمكن ان يحقق نتائج أكثر تحفظا . وظهر أن المواصفات البريطانية تحسن تقدير مقاومة القص للكمرات ذات a/d = ١,٥ ولكنها قد تزيد من تقدير المقاومة للكمرات ذات a/d أكبر من ٢,٠ كما أظهرت المقارنات أن معادلة زسوتسي Zsutty للقص في جانب التحفظ بالنسبة للكمرات ذات a/d = ١,٥ ولكنها في غير جانب التحفظ بالنسبة للكمرات ذات a/d = ٢,٥ و ٣,٥ وتتفق هذه النتائج مع تلك المتوصل إليها بمعرفة آخرين. تطبيق اشتراطات الأكواد من حيث التقسيط بين الكانات يمكن أن يؤدي الي نتائج أكثر تحفظاً. كما تم التوصل الى العديد من النتائج الأخرى.

**Keywords**: Beams, Building codes, High strength concrete, Shear strength, Web reinforcement

## 1. Introduction

In shear, beams mainly fail abruptly without sufficient advanced warning and the developed diagonal cracks are considerably wider than flexural cracks [1]. To avoid such abrupt shear failure, adequate amounts of shear reinforcement are required. The shear strength of reinforced concrete beams is dependent on several factors [2-6] including: concrete compressive strength; ratio of longitudinal steel, ρ; shear-span-to-depth ratio, a/d; size effect [7]; residual tensile stresses transmitted directly across cracks [6]; and provisions of web reinforcement. The concrete strength contribution to shear resistance in beams is the sum of three components: compressive zone of the still uncracked concrete above the top of diagonal crack; aggregate interlock along the diagonal crack; and dowel action provided by longitudinal reinforcement [3, 4, 8].

One feature of high strength concrete (HSC) (say with strength >55 MPa) is the tendency of cracks to pass through instead of around the aggregates [9]. This

Alexandria Engineering Journal Vol. 39 (2000). No. 4, 621-634

creates smoother crack surfaces, reducing the aggregate interlock and hence, reducing the shear carried by concrete $V_c$. Because of the reduced aggregate interlock, higher dowel forces occur in the longitudinal reinforcing bars. These higher dowel forces, together with the highly concentrated bond stresses in higher strength concrete beams, result in higher bond splitting stresses where the shear cracks cross the longitudinal tension bars. This effect can lead to brittle shear failures. The inclusion of an appropriate amount of minimum shear reinforcement can control these horizontal splitting cracks and results in improved shear response.

Shear reinforcement increases the ductility of beams and considerably reduces the likelihood of sudden and catastrophic failures that often occur in beams without shear reinforcement [8]. This was true for HSC beams as found earlier for normal strength concrete, NSC [10]. Stirrups not only carry shear themselves but also enhance the strength of other shear transfer mechanisms [3]. They provide support for the longitudinal steel and prevent the bars from splitting the surrounding concrete, hence they greatly increase the strength of dowel action. At the same time, stirrups help to contain the crack, limiting its propagation and keeping its width small. These effects increase both shear carried by aggregate interlock and shear contribution of the uncracked compression zone. Although stirrups do not affect the diagonal cracking load, they enhance the capacity of the different shear transfer mechanisms. Before diagonal cracking, the external shear force produces practically no stress in the web reinforcement [8]. When the diagonal crack forms, any web reinforcement that intercepts the diagonal crack would suddenly carry a portion of the shear force. Minimum shear reinforcement must prevent sudden shear failure on the formation of the first diagonal tension cracking and, in addition, it must adequately control the diagonal tension cracks at service load levels. To control crack widths at service load levels, not only a minimum amount of shear reinforcement must be provided, but the maximum stirrup spacing must also be limited [9]. The effectiveness of web reinforcement in increasing shear strength may be greater in cases of diagonal tension failures than in cases of shear compression failures [3].

American Concrete Institute (ACI) code provisions [11] may overestimate the shear strength term $V_c$ for HSC beams containing the minimum required amount of web reinforcement that should be related to the concrete compressive strength [12]. Johnson et al. [13] showed that ACI code equations become more conservative as the amount of web reinforcement increases. They also reported that for high strength concrete beams the number of inclined cracks increased with increasing the amount of web reinforcement, indicating an enhanced redistribution of internal forces in such beams. The crack width and hence stirrup strains, tended to increase with higher concrete strengths at failure for beams with minimum amount of web reinforcement. Mphonde et al. [14] reported that the ratio of test to predicted shear strength decreased with the increase of concrete strength. ACI-99 Code [11] requires a minimum amount of shear reinforcement for nonprestressed members, as follows:

$$A_v = 0.33\,\frac{b_w * s}{f_{yv}} \qquad \text{(N. mm units)} \qquad (1)$$

where: $A_v$ = the web reinforcement area; $b_w$= breadth of beam; $s$ = longitudinal spacing of stirrups; and $f_{yv}$= yield stress of web reinforcement. The equation is independent of the concrete strength used and the code limits the square root of the compressive strength to 8.3 MPa when calculating the concrete contribution to shear $V_c$ in equation:

$$V_c = 0.167\sqrt{f'_c}\,(b_w * d) \qquad \text{(N. mm units)} \qquad (2)$$

i.e., ACI does not account for concrete strength greater than 69 MPa in calculating the concrete contribution to shear $V_c$. Note that $f'_c$ is the concrete cylinder compressive strength. To take advantage of concrete strength greater than 69 MPa in calculating $V_c$, ACI 318-99 code requires a minimum amount of shear reinforcement of:

$$A_v = \left(\frac{f'_c}{35}\right)\left(0.33\,\frac{b_w * s}{f_{yv}}\right) \ \text{(N. mm units)} \ < \frac{b_w * s}{f_{yv}} \qquad (3)$$

The Canadian Standard Association, CSA 1994 (cited in [6]), introduced an equation for calculating the minimum amount of shear reinforcement as a function of $\sqrt{f'_c}$, permitting the use of the specified concrete strength $f'_c$ in calculating $V_c$ as follows:

$$A_v = 0.06\sqrt{f'_c}\,\frac{b_w * s}{f_{yv}} \qquad \text{(N. mm units)} \qquad (4)$$

The AASHTO (1994) specifications (cited in [6]), relate the minimum reinforcement required to the concrete strength and require a larger amount of stirrups for high strength concrete and require that:

$$A_v = 0.083\sqrt{f'_c}\,\frac{b_w * s}{f_{yv}} \quad \text{(MPa)} \qquad (5)$$

However, there is some concern that the equation may not be conservative enough for large reinforced concrete members that contain low percentage of longitudinal reinforcement [6]. Ahmed et al. [15] investigated through finite element analysis, beams of normal and HSC to study the effects of web reinforcement position (in the shear span and in the flexure span) on the load carrying capacity. They showed that there was a considerable increase in shear resistance for HSC, especially for a/d<2.0, and that the British Standard (BS 8110) equation may be valid for concretes with strengths higher than 40 MPa. Kong et al. [16] showed that generally for HSC, the shear strength of beams increased with increasing the shear reinforcement ratio and with increasing the longitudinal tensile reinforcement ratio. This might have been due to increased dowel action from the bundling of the longitudinal tensile bars. They also concluded that a/d did not have a significant effect on the shear strength for beams with a/d ≥2.5. However, when a/d < 2.5 the shear strength increased because of arch action. The concrete cover to shear reinforcement cage neither spalled at the time of failure nor affected the shear strength of beams [16]. The nominal stress at failure decreased with increasing the overall beam depth. Besides, the loss of shear strength with increasing beam depth may be attributed to a decrease in aggregate interlock and dowel action for deeper slender beams. Sarasam et al. [10] concluded that the size effect was insignificant on the shear strength of beams with web reinforcement. The effect of concrete strength $f'_c$ on shear strength of beams with web reinforcement, was studied by many researchers [10,12,13,15].

There are several models to represent shear in beams with web reinforcement. These include the truss model and the modified compression field theory. The truss model is based on plasticity theory [6] and uses the lower-bound approach of limit analysis to design reinforced concrete beams [16]. It does not include components of the shear failure mechanism such as aggregate interlock and friction, dowel action of the longitudinal steel, and shear carried across uncracked concrete [8]. In addition, the model completely ignores the favorable interaction between these factors and web reinforcement; to this extent it tends to give conservative results, though the conservatism reduces as the amount of web steel increases [8]. The truss analogy assumes that the failure of the beam is initiated by the yielding or excessive deformation of the web reinforcement. Higher concrete compressive strengths allow further redistribution of internal forces by strengthening the concrete components of the truss model [13]. This redistribution permits increased mobilization of the stirrups and may lead to larger shear strengths if an adequate amount and detailing of the longitudinal and web reinforcement is provided. Increasing the concrete compressive strength increases the diagonal tension cracking load, which results in larger shear stress to be carried by the combination of aggregate interlock, dowel action of the longitudinal reinforcement, uncracked concrete, and web reinforcement. These larger shear stresses induce larger crack widths, which in combination with the smoother surfaces typical of higher strength concrete results in a diminished aggregate interlock contribution. The reduced aggregate interlock and the larger stirrup strains could make the stirrups the weak link in the load carrying system in beams with minimum amount of web reinforcement [13]. In higher strength concrete beams with small amount of web reinforcement, because of the increased shear force to be transferred at the onset of diagonal tension cracking and the reduced aggregate interlock contribution, this transfer of forces may cause the first mobilized stirrups to yield and rupture. Stirrup rupture would stop any further redistribution of forces and could result in diminished reserve capacity [13]. However, in some cases the stress in shear reinforcement did not reach the yield strength [1]. Watanabe et al. [1] used an incremental analytical approach based on the truss mechanism to predict shear strength and shear failure modes of RC beams. The Modified Compression Field Theory (MCFT) [18] is a refined version of the truss model in which the cracked concrete is treated as a new material with its own stress-strain characteristic [18].

cracked concrete is treated as a new material with its own stress-strain characteristic [18]. Equilibrium, compatibility, and constitutive relationships are formed in terms of average stresses and average strains [6,18]. For high strength concrete, it was stated [9] that shear predictions using the MCFT agreed well with experimental results.

Kong et al [16] tested high performance concrete beams with different shear reinforcement experimentally and analytically and developed a theory based on stress analysis of strut and tie model to calculate the shear strength. The stirrups effectiveness in reinforced concrete beams under flexure and shear were studied [19] and a mechanical model was proposed to provide mean ultimate shear stress of beams with stirrups.

In summary current codes provide simple superposition of stirrups and concrete capacities, however ignoring any dependence of stirrups action on the failure mode and beam and arch action interactions [19]. Web reinforcement increases the shear strength and ductility of beams. ACI code equations seem to be more conservative as the amount of web reinforcement increases and relates the minimum amount of shear reinforcement required for beams with $f'_c >$ 69 MPa to the concrete compressive strength. The CSA 1994 and the AASHTO specifications (1994) (cited in [6]) relate the minimum reinforcement required to the concrete strength and require a larger quantity of stirrups for high strength concrete. BS 8110 may be valid for concretes with strengths higher than 40 MPa [9]. Shear span to depth ratio, a/d, seems not to have significant effect on the shear strength when $a/d \geq 2.5$, while for $a/d < 2.5$ the shear strength increased because of arch action [16]. The size effect may be insignificant on the shear strength of beams with web reinforcement, however the nominal stress at failure may decrease with increasing the overall beam depth [10,16].

## 2. Research significance

Although there are numerous studies on shear strength of HSC beams with web reinforcement, however, results are still inconclusive and shear in HSC beams remains not fully understood. Codes shear equations are based on empirical equations that are derived from experimental tests on beams with normal strength concrete and their extrapolation to HSC beams should be verified. Hence there is a need for further research in such area. In this paper the shear strength of HSC beams with web reinforcement is experimentally investigated. Nine HSC beams (with concrete strength 65 MPa) with web reinforcement in the form of vertical stirrups were tested. The main variables include the amount of stirrups; the tensile steel ratio; and the shear-span-to-depth ratio. In addition the validity of several codes' equations, e.g. ACI, Egyptian Code of Practice ECP [20], and BS 8110, for shear is assessed when applied to such HSC beams with web reinforcement. Within the scope of the limited number of tests and variables and noting that the stirrups' spacings in some of the tested specimens (s=150 & 200 mm.) did not fulfill the provisions set by some codes, especially with the relatively small height of specimens tested (h=260mm), it was observed that ECP predictions seem always conservative whether code limitations on concrete strength were waived or not. In addition, ACI was conservative for beams with a/d=1.5, however for beams with a/d=2.5 and 3.5 its margin of safety was small. Fulfilling ACI provisions of stirrups' spacing is supposed to render more conservative results. The BS predicts the shear capacity well for beams with a/d=1.5 but may overestimate that for beams with a/d>2. Zsutty's equation [21] was conservative for short beams with a/d=1.5, however for beams with a/d=2.5 and 3.5 it seemed unconservative. This may be because the equation was obtained from test results of beams with different conditions including stirrups maximum spacing and concrete strength. The findings are in agreement with other reported results [4,9,10]. The restrictions set by codes for stirrups' spacing may render more conservative results. Other conclusions are drawn.

## 3. Experimental program

Nine simply supported reinforced high (medium high) strength concrete HSC beams were tested under two symmetrically concentrated loads, Fig. 1, to study the effects of several variables on the structural response and shear capacity of beams with web reinforcement. The loads were applied on the beams' top surface through a hydraulic jack of 500 kN capacity (connected to a load cell and a strain indicator). A spreader steel box section was used to transmit the load to two cylindrical bars one at each loading point. Beams were supported on two cylindrical bars one at each support point, Fig. 1. All beams were rectangular with breadth, b=160mm; height, h=260mm; and effective depth, d=230 mm. The mean concrete cube strength was 65 MPa based materials (sand, pink limestone and Ordinary Portland Cement, type 1) and reducing the water cement ratio to 0.29 and using a commercial type F super plasticizer to enhance workability. Other details regarding the used concrete mix, loading, supports and instrumentation are found in Ref. [22]. The tested beams had web reinforcement consisting of two branch vertical stirrups each 6mm in diameter made of ordinary mild steel with measured average yield stress $f_{yv}$=300 MPa, and ultimate strength $f_{ult}$ =430 MPa. All beams had compression steel consisting of 2 plain bars, 10 mm in diameter with measured average yield stress = 310 MPa, and ultimate strength =490 MPa. The variables studied were: i) the effect and amount of web reinforcement; ii) longitudinal steel ratio, $\rho = A_s/(bd)$, namely $\rho$=2.3% and 3.1%; where: $A_s$ the area of the tensile reinforcement consisting of deformed bars (Table 1) with average yield stress, $f_y$ =390 MPa and ultimate strength, $f_{ult}$ =640 MPa; and iii) shear-span-to-depth ratio a/d (namely a/d=1.5, 2.5 and 3.5). The web steel ratio, $\rho_v = A_v/(bs)$, where: $A_v$= cross sectional area of stirrups; and s= longitudinal spacing between stirrups. The considered web steel ratios are: i) 2 branches ø 6 mm. @ 200mm matching the practical construction minimum stirrups specified in the current Egyptian Code of Practice (ECP) [20] (ECP clause 4-2-2-1-6-a), i.e., $\rho_v$=0.178%; and ii) 2 branches ø 6mm @ 150mm, i.e. $\rho_v$=0.237%. The used amount of stirrups and spacing as compared to several codes' provisions will be discussed later in detail. Mid-span deflections were measured by dial gauges. Cracks were visually traced and marked throughout all loading stages. A tested beam specimen in the test setup is shown in Photo. 1.

## 4. Test results and discussion

The effects of the previous variables, $\rho_v$, $\rho$ and a/d, on the shear strength of HSC beams with web reinforcement are provided in Figs. 3, 4, and 5 and Tables 1 and 2. In addition test results are discussed hereafter and comparisons are made with some relevant codes' equations, e.g., ACI, ECP & BS. These are presented in Table 2 and Fig. 5.

### *4.1. Cracking patterns and load deflection curves*

As the load increased, flexural cracks formed in the beam central region. With further load increase new flexural cracks formed in the shear span between load point and support. Then diagonal shear cracks were observed in the shear span at the same loading levels as those in beams without web reinforcement, comparing beams with web reinforcement to similar ones without web reinforcement in Ref. [22]. Hence, web reinforcement had no effect on the cracking shear loads, and had no effect prior to cracking. However the role of the web reinforcement was obvious after inclined shear cracking, as will be discussed hereafter. In addition, there were no horizontal splitting cracks at the level of the longitudinal reinforcement, as opposed to what generally happens in beams without web reinforcement. The beams had more cracks with smaller width than cracks in beams without web reinforcement. In addition, the load-mid span deflection curves are presented in Fig. 2.

### *4.2. Effect of web reinforcement*

The effect of including web reinforcement on the shear strength is depicted in Table 1. In addition, Fig. 5 illustrates the effect of web reinforcement index, $\rho_v f_{yv}$, on the shear strength.

#### *4.2.1. Beams with a/d =1.5*

The ultimate shear loads, $V_u$ of the tested beams with a/d=1.5 sharply increased over their cracking shear loads, $V_{cr}$ as expected (Table 1). The ratio between the ultimate shear load $V_u$ and the cracking shear load, $V_{cr}$; (i.e. $V_u/V_{cr}$), for beams S1, S2 & S3 was 213%, 208% & 236%; respectively. Strangely the increase in web reinforcement from $\rho_v$= 0.178% to 0.237% did not increase the shear strength, comparing beams S3 and S2, but adversely the results were about 4% lower. This may be partially attributed to the fact that beams with a/d=1.5 (short beams) showed relatively high shear strength ($v_u$= 3.8 to 4.4 MPa) due to arch action where part of the load was transmitted directly by diagonal compression to the support and thereby reduced the demand on the other types of load transfer, e.g. web reinforcement. This may have overshadowed the effect of the web reinforcement. However, this may be due to concrete variability and some experimental discrepancies. Variations in the position of the first stirrup near the support may have affected the results. Nevertheless a clearer explanation may still be required. To recall, other reported results [23] on HSC beams with a/d=1.69 and vertical web reinforcement showed that as the web reinforcement index, $\rho_v f_{yv}$, increased the ultimate shear strength increased. However, the increase in strength was small and not proportional to the total amount of the stirrups. This may be because the ultimate shear resistance was only in part due to the stirrups. In any case the beam due to arch action can still take considerably more loads after the stirrups have yielded [24]. Moreover, a redistribution of internal forces may have taken place, and a further increase in the load carrying capacity depends on the strength of the compression zone of the beam.

### *4.2.2. Beams with a/d =2.5*

The ultimate shear loads of beams M1, M2 and M3 with a/d=2.5 increased over their shear cracking loads (Table 1). The ratio between the ultimate shear load $V_u$ and the cracking shear load, $V_{cr}$; (i.e. $V_u/V_{cr}$), for beams M1, M2 & M3 was 128%, 129% and 189%; respectively, and the beams did not fail soon after the formation of the first diagonal shear cracks. This is because stirrups not only carry a part of applied shear load by themselves, but also they enhance the strength of other shear transfer mechanisms. Increasing the amount of web reinforcement from $\rho_v$=0.178% in beam M1 to $\rho_v$=0.237% in beam M3, i.e. a 33% increase, led to increasing the ultimate shear load of beam M3 by 57%. Besides, it led to increasing the number of inclined cracks.

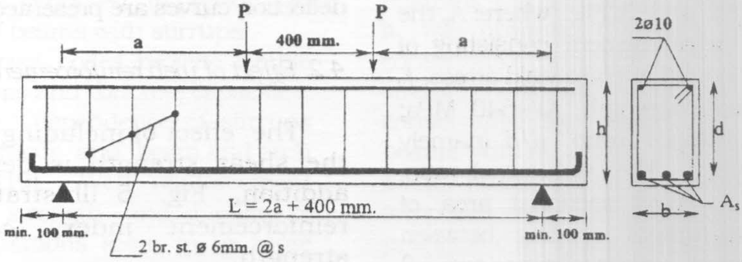

Fig.1. Details of tested beams.

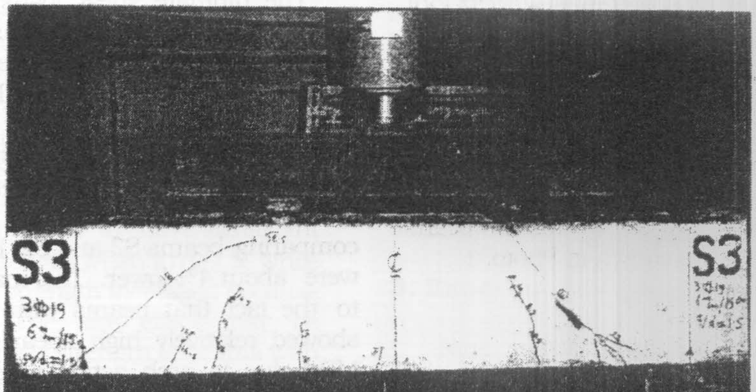

Photo. 1. Tested specimen in test setup

Table 1. Details and test results of tested beams.

| Beam | a/d | $A_s$ mm² | Main Steel $\rho$=$A_s$/bd % | Stirrups $\rho_v$ = $A_v$/(bs) % | $f'_c$ MPa | $f_{cu}$ MPa | $V_{cr}$ kN | $V_u$ kN | $v_{cr}$ MPa | $v_u$ MPa |
|---|---|---|---|---|---|---|---|---|---|---|
| S1 | 1.5 | 3 Φ 19<br>( 851 ) | 2.3 % | 2 ø 6mm @ 20cm<br>(0.178 %) | 60 | 63 | 75 | 160 | 2.03 | 4.34 |
| S2 | 1.5 | 3 Φ 22<br>( 1140 ) | 3.1 % | 2 ø 6mm @ 20cm<br>(0.178 %) | 60 | 63 | 65 | 135 | 1.76 | 3.66 |
| S3 | 1.5 | 3 Φ 19<br>( 851 ) | 2.3 % | 2 ø 6mm @ 15cm<br>(0.237 %) | 60 | 63 | 55 | 130 | 1.49 | 3.53 |
| M1 | 2.5 | 3 Φ 19<br>( 851 ) | 2.3 % | 2 ø 6mm @ 20cm<br>(0.178 %) | 56 | 62.5 | 45 | 57.5 | 1.22 | 1.56 |
| M2 | 2.5 | 3 Φ 22<br>( 1140 ) | 3.1 % | 2 ø 6mm @ 20cm<br>(0.178 %) | 56 | 62.5 | 52.5 | 67.5 | 1.42 | 1.83 |
| M3 | 2.5 | 3 Φ 19<br>( 851 ) | 2.3 % | 2 ø 6mm @ 15cm<br>(0.237 %) | 56 | 62.5 | 45 | 85 | 1.22 | 2.3 |
| N1 | 3.5 | 3 Φ 19<br>( 851 ) | 2.3 % | 2 ø 6mm @ 20cm<br>(0.178 %) | 60 | 69 | 50 | 67.5 | 1.35 | 1.83 |
| N2 | 3.5 | 3 Φ 22<br>( 1140 ) | 3.1 % | 2 ø 6mm @ 20cm<br>(0.178 %) | 60 | 69 | 55 | 63.7 | 1.49 | 1.73 |
| N3 | 3.5 | 3 Φ 19<br>( 851 ) | 2.3 % | 2 ø 6mm @ 15cm<br>(0.237 %) | 55.7 | 57 | 50 | 72.5 | 1.35 | 1.97 |

a/d= shear span to depth ratio
$f'_c$ = average cylinder compressive strength of concrete, MPa.
$f_{cu}$ = average cube compressive strength of concrete, MPa.
$\rho$ = longitudinal steel ratio = $A_s$ /(bd)%
$\rho_v$ = web steel ratio $A_v$/(bs)%
Φ = bar diameter , High Tensile Deformed bar, mm.
ø = bar diameter , Ordinary Mild Steel, mm.
V = half the applied load = 2P/2, kN.
$V_{cr}$ = cracking shear load, kN.
$V_u$ = ultimate shear load, kN.
$v_{cr}$ = cracking shear stress, MPa.
$v_u$ = ultimate shear stress, MPa.

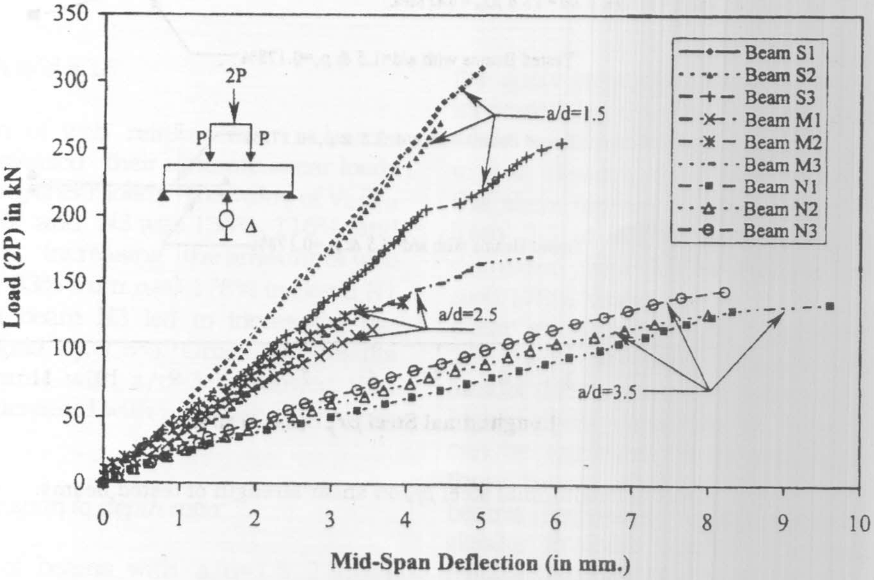

Fig. 2. Load-deflection curves for tested beams.

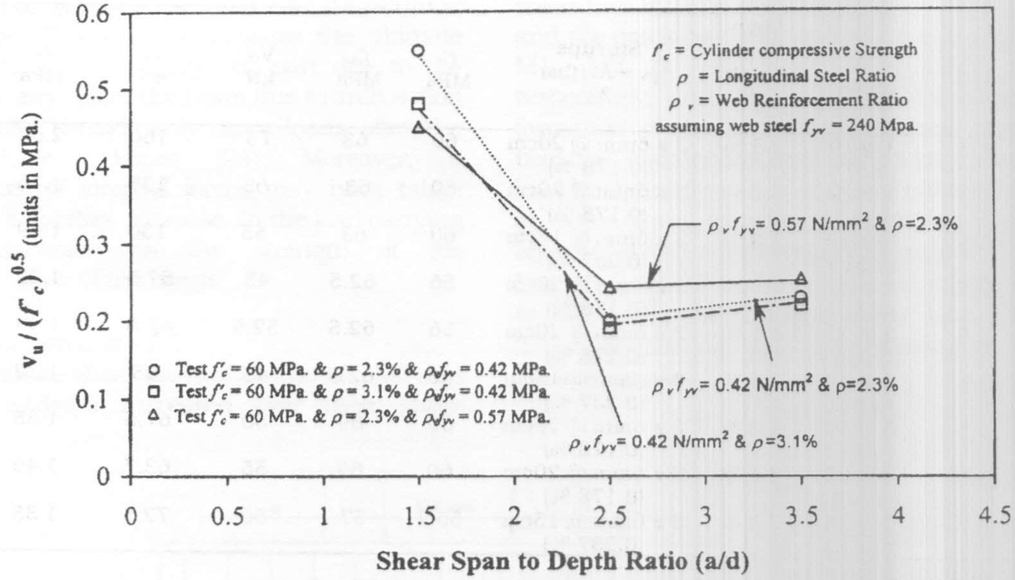

Fig. 3. Effect of a/d on shear strength of tested beams.

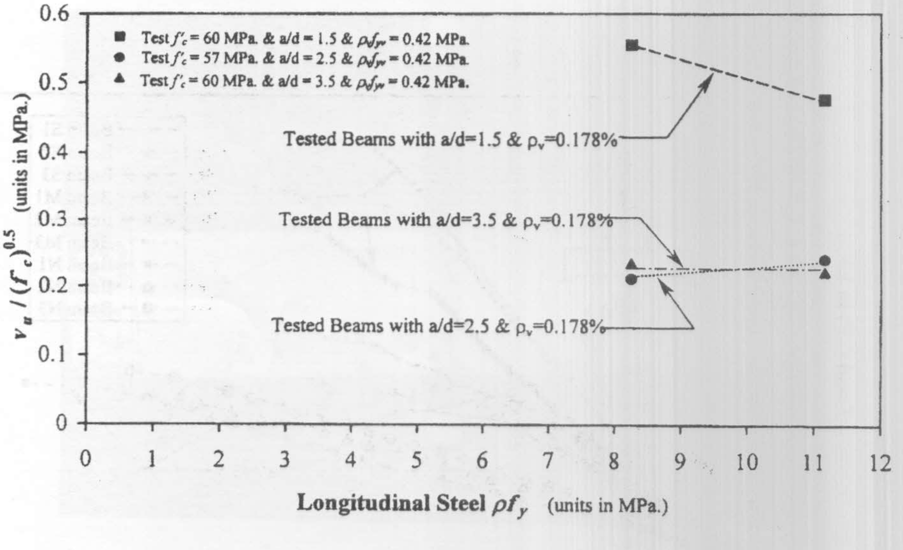

Fig. 4. Effect of longitudinal steel $\rho f_y$ on shear strength of tested beams.

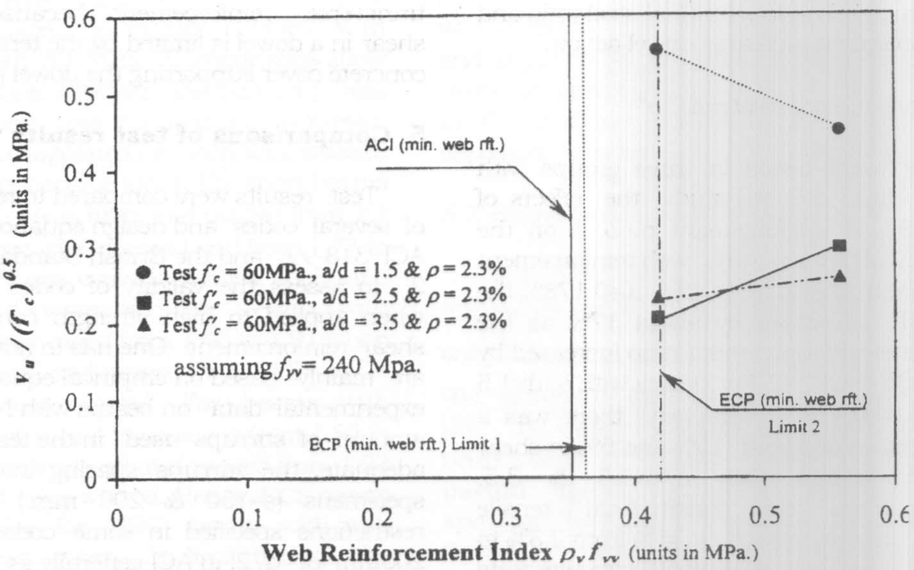

Fig. 5. Effect of web reinforcement index on shear strength of tested beams.

ECP (minimum web reinforcement):
$A_v$ = 0.15 bs % for steel 24/35, for $f_{yv}$ =240 MPa, $\rho_v f_{yv}$ = 0.36 MPa. (Limit 1)
= 0.10 bs % for steel 36/52
however not less than 2 br. st. ϕ6mm. @ 200mm.
for b= 160 mm. & $f_{yv}$ = 240 MPa min. $\rho_v f_{yv}$ = 0.42 MPa (Limit 2)

ACI (minimum web reinforcement):
$A_v = 0.33\ (b\ s)/f_{yv}$ eq. (11-13)

### 4.2.3. Beams with a/d =3.5

The inclusion of web reinforcement in beams with a/d=3.5 increased their ultimate shear loads over their cracking shear loads. The value of $V_u/V_{cr}$ for beams N1, N2 and N3 was 135%, 116%, and 145%; respectively. Increasing the amount of web reinforcement by 33% from $\rho_v$=0.178% in beam N1 to $\rho_v$=0.237% in beam N3 led to increasing the ultimate shear load by 7.5%. Other test results [13,16,25] on beams with a/d=3~5 showed that shear strength increased with increasing the shear reinforcement.

### 4.3 Effect of shear span to depth ratio

Three series of beams with a/d=1.5, 2.5 and 3.5 were tested to study the effect of a/d on the shear strength of beams with web reinforcement. Test results depicted in Table 1 and Fig.3 show that the shear strength of beams with ρ=3.1% & $\rho_v$=0.178% increased by 6% as a/d decreased from 3.5 to 2.5, while it increased by 100% as a/d decreased from 2.5 to 1.5. Besides, for beams with ρ=2.3% & $\rho_v$=0.237% the shear strength increased by 17% as a/d decreased from 3.5 to 2.5, while it increased by 53% as a/d decreased from 2.5 to 1.5. For beams with ρ=2.3% & $\rho_v$=0.178% unexpectedly there was a decrease in the shear strength by 17% as a/d decreased from 3.5 to 2.5. But the shear strength increased sharply by almost 178% when a/d decreased from 2.5 to 1.5. The higher shear capacities of short beams with a/d=1.5 can be attributed to the arch action that developed in those beams. This indicates that the shear strength of beams increased as a/d decreased especially from slender to short beams, in agreement with other test results [10, 15, 16]. In addition, the increase in shear strength of beams with the decrease of a/d (from 3.5 to 2.5) becomes more obvious as the longitudinal steel ratio and the web reinforcement increases. This is because the increase in longitudinal steel ratio and the web reinforcement increases dowel action.

### 4.4. Effect of longitudinal steel ratio "$\rho$"

Six beams were tested in three groups with a/d=1.5, 2.5 and 3.5 to study the effects of longitudinal tensile reinforcement ratio, $\rho$ on the shear strength of beams with web reinforcement (Table 1). For beams with a/d=2.5 & $\rho_v$=0.178% the shear strength increased by about 17% as the longitudinal tensile reinforcement ratio increased by 35% from 2.3% to 3.1%. For beams with a/d=1.5 & 3.5 with $\rho_v$=0.178%, strangely there was a marginal decrease of about 15% and 6% in shear strength for beams with a/d=1.5 & 3.5, respectively, when the longitudinal tensile reinforcement ratio increased by 35% from 2.3% to 3.1%. To recall for normal strength concrete tests on beams with a/d≈1.6 containing web reinforcement [24] proved that the greater the value of $\rho$ the higher the load carrying capacity of the beam.

### 4.5. General discussion

The existence of web reinforcement improved the shear strength of beams by delaying brittle shear failures. Appropriate amounts of shear reinforcement control horizontal splitting cracks and result in improved shear response. Moreover, stirrups help to contain the crack, limiting its propagation and keeping its width small. For the tested beams with a/d=1.5 the results were erratic partially due to the effect of arch action. Besides, for such beams the effects of increasing $\rho$ on enhancing the shear strength seems more pronounced than the effect of increasing vertical web reinforcement. From tests it can be generally observed that as the longitudinal steel ratio increases the shear strength of beams with web reinforcement increases, in agreement with other reported results [3,25]. One notes from other reported studies [6,22] that the effect of longitudinal steel ratio "$\rho$" on the ultimate shear strength of beams is more obvious in beams with web reinforcement than in beams without web reinforcement. This is because the stirrups provide support for the longitudinal steel and prevent the bars from splitting the surrounding concrete, hence increasing the dowel action. The dowel action may not be very significant in members without transverse reinforcement, because the maximum shear in a dowel is limited by the tensile strength of the concrete cover supporting the dowel [6].

## 5. Comparisons of test results with codes

Test results were compared to relevant predictions of several codes and design equations especially ECP; ACI 318-99; and the British Standard BS 8110, Table 2, to assess the validity of codes shear predictions when applied to high strength concrete beams with shear reinforcement. One has to note that these codes are mainly based on empirical equations derived from experimental data on beams with NSC. Although the amount of stirrups used in the tested specimens are adequate, the stirrups' spacing in some of the tested specimens (s=150 & 200 mm.) did not fulfill all restrictions specified in some codes. Note: in ECP s< 200mm or d/2; in ACI generally s< 600mm or d/2; in BS8110 s<0.75d (cited in [6]); in CSA 1994 s< 600mm or 0.7d (cited in [9]). This is because of the relatively small height of the specimens tested (h=260mm), while most real beams in practice have larger heights (generally h>400 mm) for which the used stirrups' spacings may be adequate. Hence this is a more severe test for codes' shear predictions. The effects of such issue on the comparisons will be further discussed.

### 5.1. Egyptian Code of Practice (ECP 1995)

ECP-95 [20] stipulates that the ultimate shear strength $v_u$ can be generally calculated as follows:

$$v_u = V_u/bd = v_s + 0.5\,v_c = v_s + 0.5 * 0.75\sqrt{(f_{cu}/\gamma_c)} < 2.2\sqrt{(f_{cu}/\gamma_c)}\ \&< 30\ \mathrm{kg/cm^2} \tag{6}$$

and $$v_s = \frac{A_v * {}^{f_{yv}}/_{\gamma_s}}{s * b} \tag{7}$$

where: $v_s$ is the shear strength provided by shear reinforcement; $f_{cu}$ is the concrete cube compressive strength; and $\gamma_c$ & $\gamma_s$ are the strength reduction factors for concrete and steel; respectively. For beams with a/d<2, ECP allows the reduction of shear forces by a/2d. For the web reinforcement (stirrups) the code stipulates that the minimum web reinforcement, $\rho_{v(\min.)}=4/f_{yv}$ where: $f_{yv}$ in kg/cm² but not less than 2 branches ø6mm. @ 200mm; and maximum spacing s= d/2. Comparisons with code predictions, $V_{ECP}$ were made twice; once with $f_{cu}$ limited to 30 MPa (ECP*), then with such limit assumed waived, i.e. ECP equations extrapolated to include concrete with strength more than 30 MPa. Comparisons show that ECP predictions whether the limit on $f_{cu}$ was imposed or waived were conservative. However, when ECP was extrapolated i.e., with limit waived, the factor of safety became lower. For short beams (a/d=1.5), $V_{u\,test}/V_{ECP}$ was high, and ranged from 2.2 to 3.05 when $f_{cu}$ limit was imposed and from 1.77 to 2.4 when the limit was waived. Besides, such ratio decreased as a/d increased. For beams with a/d=2.5, $V_{u\,test}/V_{ECP}$ ranged from 1.46 to 1.91 when the limit was imposed and ranged from 1.15 to 1.54 when the limit was waived. For beams with a/d=3.5 the ratio of $V_{u\,test}/V_{ECP}$ ranged from 1.62 to 1.71 when limit was imposed and ranged from 1.23 to 1.3 when limit was waived. Hence, ECP seems always conservative for HSC beams with web reinforcement, even when its limitation on the maximum spacing between stirrups was violated. This is because ECP disregards half of the concrete contribution to shear strength after diagonal cracking, i.e., only 0.5 $v_c$ is considered towards $v_u$.

### 5.2. American Concrete Institute (ACI)

Comparisons of test results with ACI predictions, $V_{ACI}$ show that ACI [11] was conservative for beams with a/d=1.5 even when the spacing of web reinforcement, specified by ACI, was violated. For the tested beams with a/d=2.5 and 3.5 ACI seemed not always conservative and its margin of safety was small. For the tested beams with both a/d=2.5 & 3.5 and $\rho_v$=0.178%, ACI seems unconservative. This may be because the used spacing between web reinforcement violated the maximum spacing specified by ACI (i.e. 120mm). For beams with a/d=2.5 & 3.5 and $\rho_v$= 0.237%, $V_{u\ test}/V_{ACI}$ seemed conservative with a small margin of safety however, the margin of safety in beams with a/d=2.5 was higher than that in beams with a/d=3.5. These results agree with other tests [3,10] on HSC beams that concluded that ACI equation was conservative. Besides, it was reported [13] that for HSC beams with a/d=3.1 and $\rho_v f_{yv}$= 0.35 MPa the overall reserve shear strength after diagonal tension cracking diminished with the increase in $f'_c$. Fulfilling ACI provisions for stirrups spacing is supposed to render more conservative results.

### 5.3. British Standard (BS 8110)

The British Standard BS 8110 (cited in [10]) stipulates that if the shear stress, v= $V/(bd)$ exceeds $0.8\sqrt{f_{cu}}$ or 5 N/mm$^2$, whichever is less, the product $(bd)$ must be increased to reduce v [8]. It also stipulates that the minimum shear reinforcement index $\rho_v f_{yv} \geq 0.4$ N/mm$^2$ with a partial safety factor of 0.87 for $f_y$, and the spacing between stirrups should not exceed 0.75$d$. Comparisons of test results with those of the BS, $V_{BS\ 8110}$ show that BS seems conservative for beams with a/d=1.5. The ratio $V_{u\ test}/V_{BS\ 8110}$ was high for beams with a/d=1.5 and decreased for beams with a/d>2. As $\rho_v f_{yv}$ increased $V_{u\ test}/V_{BS\ 8110}$ increased. This is in agreement with other tested HSC and NSC beams [15] with web reinforcement that concluded that BS equations for shear strength predict the beam shear capacity well for beams with a/d=2.0 but overestimate that for beams with a/d>2.

### 5.4. Zsutty equation

Zsutty's equation [10,21] originally developed for shear in normal strength concrete beams correlates the concrete compressive strength $f'_c$, $\rho$ and $a/d$ through a power of 1/3. In the comparisons the partial factor of safety of 0.85 was waived. Comparisons of the tested beams with Zsutty's equation show that the equation seems conservative for short beams with a/d=1.5 with a safety factor ranging between 30~69%. For beams with a/d=2.5 and 3.5 it seems unconservative in agreement with other results [10].

Table 2 Comparisons of test results with codes' equations for HSC beams with web reinforcement.

| Beam | $V_{cr\ test}$ kN | $V_{u\ test}$ kN | $\frac{V_{u\ test}}{V_{ECP^*}}$ | $\frac{V_{u\ test}}{V_{ECP}}$ | $\frac{V_{u\ test}}{V_{ACI}}$ | $\frac{V_{u\ test}}{V_{BS\ 8110}}$ | $\frac{V_{u\ test}}{V_{Zsutty}}$ |
|---|---|---|---|---|---|---|---|
| S1 | 75 | 160 | 3.05 | 2.4 | 2.25 | 1.72 | 1.69 |
| S2 | 65 | 135 | 2.58 | 2.02 | 1.8 | 1.32 | 1.31 |
| S3 | 55 | 130 | 2.2 | 1.77 | 1.7 | 1.30 | 1.3 |
| M1 | 45 | 57.5 | 1.46 | 1.15 | 0.87 | 0.82 | 0.7 |
| M2 | 52.5 | 67.5 | 1.71 | 1.35 | 0.99 | 0.89 | 0.77 |
| M3 | 45 | 85 | 1.91 | 1.54 | 1.19 | 1.11 | 0.98 |
| N1 | 50 | 67.5 | 1.71 | 1.3 | 1.02 | 0.94 | 0.89 |
| N2 | 55 | 63.75 | 1.62 | 1.23 | 0.95 | 0.81 | 0.77 |
| N3 | 50 | 72.5 | 1.63 | 1.35 | 1.04 | 0.97 | 0.91 |

$V_u$ = shear strength $= V_c + V_s$, kN.

$V_c$ = contribution to shear strength provided by concrete, kN.

$V_s$ = contribution to shear strength provided by shear reinforcement, kN.

$V_u$ = Ultimate shear strength, kN.

**ECP (Egyptian Code of Practice) [20]:**

$$V_{ECP} = \{\frac{1}{2} \cdot [0.237 \sqrt{(f_{cu} / \gamma_c)}]^* (b \cdot d) + \frac{Av * fyv/\gamma s}{s} * d\} \text{ units in (MN, m.)}$$

-For a/d<2, can reduce shear forces by a/2d (code clause 4-2-2-1-1-b), hence multiply equation by 2d/a

-Values reported in the table are calculated for $\gamma_c = \gamma_s = 1.0$

-ECP* = Egyptian Code of Practice (with limit for $f_{cu}$ = 30 MPa. imposed)

- If we neglect reducing shear forces by a/2d for a/d<2 (code clause 4-2-2-1-1-b) for beams S1, S2, & S3:

* $V_{u\ test}/V_{ECP^*}$= 4.06, 3.43, 2.92; respectively.

* $V_{u\ test}/V_{ECP}$ = 3.19, 2.69, 2.35; respectively.

**ACI [11]:** $V_{ACI} = [(\sqrt{f'c} + 120\, \rho\, d/a) \cdot (b*d) / 7] + \frac{Av * fyv * d}{s}$ (MN, m.)

**BS 8110 (British Standard) (cited in [6])**

$$V_{BS\ 8110} = (0.79/\gamma_m) \cdot (100A_s/bd)^{1/3} (400/d)^{1/4} (f_{cu}/25)^{1/3} (b \cdot d) + \frac{Av * fyv * d}{s} \quad \text{MN, m (for } a/d \geq 2)$$

$$V_{BS\ 8110} = (0.79/\gamma_m) (100A_s/bd)^{1/3} (400/d)^{1/4} (f_{cu}/25)^{1/3} (2 \cdot d/a) (b \cdot d) + \frac{Av * fyv * d}{s} \quad \text{MN, m (for } a/d<2)$$

where: $\gamma_m$ = materials partial safety factor, and assumed=1.0 in the comparisons & $400/d \leq 1.0$

**Zsutty's Equation [21]:** $V_{Zsutty} = 2.2\, (f_c \cdot \rho \cdot d/a)^{1/3} \cdot (b \cdot d) + \frac{Av * fyv * d}{s}$ N, m

## 6. Conclusions

An experimental investigation is presented to study the shear strength of high strength concrete beams with web reinforcement. Nine simply supported reinforced high (medium high) strength concrete HSC beams with mean concrete cube strength of 65 MPa with web reinforcement in the form of vertical stirrups were tested, to determine their shear strength. The test variables were the web reinforcement ratio $\rho_v$; the longitudinal steel ratio; and shear-span-to-depth ratio, namely a/d=1.5, 2.5 & 3.5. Test results were compared with strength predictions using several codes especially ACI-99, ECP-95 & BS8110 to assess the validity of codes when applied to HSC beams with web reinforcement, noting that the used stirrups spacing in some beams did not fulfill the restrictions set by some codes. Within the scope of the tested beams and limited variables the following conclusions can be drawn:

1- Web reinforcement enhances the shear strength of beams by delaying brittle shear failures, and improves ductility. Appropriate amounts of shear reinforcement control horizontal splitting cracks and result in improved shear response. Moreover, stirrups help to contain the crack, limiting its propagation and keeping its width small. In addition, it enhances dowel action.

2- The minimum amount of shear reinforcement required for beams with HSC should be related to the concrete compressive strength as qualitatively specified in ACI-99, CSA 1994 and AASHTO 1994, requiring a larger quantity of stirrups for HSC. This is because there is more demand on stirrups in HSC beams, partially because HSC is deficient in aggregate interlock and the concrete shear contribution to shear strength increases at a slower rate than $\sqrt{f'_c}$.

3- Comparisons with codes' predictions for the shear capacity of the studied beams with web reinforcement were performed, noting that the stirrups' spacings in some of the tested specimens did not fulfill the restrictions specified in some design codes. Comparisons show that:

* ECP-95 seems always conservative whether code limitations on concrete strength were waived or not, regardless of the violation of the maximum stirrups spacing. This is attributed to the fact ECP disregards half of the concrete contribution to shear strength after diagonal cracking.

* ACI-99 was conservative for beams with a/d=1.5, however for beams with a/d=2.5 and 3.5 its margin of safety was small. ACI code equations seem to be more conservative as the amount of web reinforcement increases. If ACI specified amount of minimum web reinforcement or maximum stirrups' spacing are violated, its shear strength predictions may be unconservative for HSC beams with a/d=2.5 & 3.5. Fulfilling ACI provisions of stirrups spacing may render conservative predictions.

* BS 8110 predicts the shear capacity well for beams with a/d=1.5 but may overestimate that of beams with a/d>2.

* Zsutty's equation was conservative for short beams with a/d=1.5, however for beams with a/d=2.5 and 3.5 it seemed unconservative. This may be because the equation was obtained from test results of beams with different conditions including stirrups maximum spacing and concrete strength.

The previous findings are in agreement with other reported results. The restrictions set by codes for stirrups' spacing may render more conservative results. More tests are required on high strength concrete beams with web reinforcement with relatively large practical dimensions to make the comparisons more realistic and feasible.

## References

[1] F. Watanabe and J. Lee, "Theoretical Prediction of Shear Strength and Failure Modes of Reinforced Concrete Beams," ACI Structural Journal, Vol. 95, No. (6), pp.749-757 (1998).

[2] ACI Committee, "State of the Art Report on High Strength Concrete" (ACI 363R-92), (Reapproved 1997) American Concrete Institute, Detroit, p. 55 (1999).

[3] A. Elzanaty, A.Nilson. and F. Slate, "Shear Capacity of Reinforced Concrete Beams Using High Strength Concrete," ACI Journal, Vol. 83, (2), pp.290-296, (1986).

[4] S. Ahmad, A. Khaloo. and A. Poveda., "Shear Capacity of Reinforced High Strength Concrete Beams," ACI Journal, Vol. 83, (2) pp.297-305, (1986).

[5] J. Taub and A. Neville, "Resistance to Shear of Reinforced Concrete Beams. Part 1-Beams without Web Reinforcement," ACI Journal, Proceedings Vol. 57, (8), August, pp. 193-220, (1960).

[6] ASCE – ACI committee 445 on shear and torsion, "Recent Approaches to Shear Design of Structural Concrete," Journal of Structural Engineering, Vol. 124, (12), Dec., pp.1375-1417, (1998).

[7] Z. Bazant and M. Kazemi, "Size Effect on Diagonal Shear Failure of Beams without Stirrups," ACI Structural Journal, Vol. 88, (3), pp.268-276, (1991).

[8] F. Kong and R. Evans, "Reinforced and Prestressed Concrete," Third Edition, Chapman & Hall, Great Britain, p. 508 (1993).

[9] Y. Yoon, W. Cook and D. Mitchell, "Minimum Shear Reinforcement in Normal, Medium, and High Strength Concrete Beams," ACI Structural Journal, Vol. 93, (5), pp. 576-584, (1996).

[10] K. Sarsam. and J. Al- Musawi, "Shear Design of High and Normal Strength Concrete Beams with Web Reinforcement," ACI Structural Journal, Vol. 89, (6), pp. 658-664, (1992).

[11] "ACI Building Code Requirements for Structural Concrete (ACI 318-99) and Commentary (ACI 318R-99)," American Concrete Institute, Detroit, P. 369 (1999).

[12] J. Roller and H. Russell., "Shear Strength of High Strength Concrete Beams With Web Reinforcement," ACI Structural Journal, Vol. 87, (2), pp.191-198 (1990).

[13] M. Johnson and J. Ramirez, "Minimum Shear Reinforcement in Beams With Higher Strength Concrete," ACI Structural Journal, Vol. 86 (4), pp.376-382 (1989).

[14] A. Mphonde and G. Frantz, "Shear Tests of High and Low Strength Concrete Beams Without Stirrups," ACI Journal, Vol. 81, (4), pp.350-357 (1984).

[15] S. Ahmad S., F. Park and K. Dash, "Web Reinforcement Effects On Shear Capacity of Reinforced High Strength Concrete Beams," Magazine of Concrete Research, 47, (172), September, pp.227-233 (1995).

[16] Y. Kong and B. Rangan, "Shear Strength of High–Performance Concrete Beams," ACI Structural Journal, Vol. 95, (6), November - December, pp.677-688 (1998).

[17] K. Al-Nahlawi and J. Wight, "Beam Analysis Using Concrete Tensile Strength in Truss Models," ACI Structural Journal, Vol. 89, (3), pp.284-289 (1992).

[18] F. Vecchio and M. Collins, "Predicting the Response of Reinforced Concrete Beams Subjected to Shear Using Modified Compression Field Theory," ACI Structural Journal, Vol. 85(2), pp.258-268 (1988).

[19] G. Russo and G. Puleri, "Stirrup Effectiveness in Reinforced Concrete Beams under Flexure and Shear," ACI Structural Journal, Vol. 94, (3), pp.227-238 (1990).

[20] "Egyptian Code of Practice For Concrete Structures 1995," Ministry of Housing, Cairo, Egypt, (in Arabic) (1995).

[21] T. Zsutty, "Beam Shear Strength Prediction by Analysis of Existing Data," ACI Journal, Proceedings Vol. 65, (11), pp. 943-951 (1968).

[22] M. Moustafa, "Behaviour of High Strength Concrete Beams," M.Sc. Thesis, Department of Civil Engineering, Alexandria University, Alexandria, 142 p. (1999).

[23] K. Tan, F. Kong, S. Teng and L. Weng, "Effect of Web Reinforcement on High Strength Concrete Deep Beams," ACI Structural Journal, Vol. 94, (5), pp.572-582 (1997)

[24] J. Taub and A. Neville, "Resistance to Shear of Reinforced Concrete Beams. Part 2-Beams with Vertical Stirrups," ACI Journal, Proceedings Vol. 57, (9), September, pp. 315-336, (1960).

[25] G. Ozcebe, U. Ersoy and T. Tankut, "Evaluation of Minimum Shear Reinforcement Requirements for High Strength Concrete," ACI Structural Journal, Vol. 96, (3), pp.361-368, (1999).

Received March 5. 2000
Accepted July 2. 2000