Lyapunov stability of large-scale power systems considering the voltage regulator and speed governor

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It is developed a new stability approach used to carry out transient stability studies of an N-machine power system. The system mathematical model is derived considering the generator flux decay effect, non-uniform mechanical damping, electromagnetic damping, the voltage regulator effect and the speed governor action. Using the decomposition-aggregation method, the system is decomposed into (N- 1)/2, "three-machine" subsystems, and the system aggregation is performed by using a constructed vector Lyapunov function. An aggregation (square) matrix of the order (N-1)/2 is obtained, stability of this matrix implies asymptotic stability of the system equilibrium state. The developed approach is applied to a 7-machine, 14-bus power system and an estimate for the system asymptotic stability domain is determined. A 3-phase short circuit fault (with successful re-closure) is assumed to be occurred on one of the system lines at point near a generator bus, or near a load bus. The faulted line is isolated for clearing the considered fault, and the critical time for reconnecting the open line is directly determined. It is found that the developed approach is suitable and can be simply used to carry out practical stability studies of power systems.

تم تقديم معيار أتزان جديد أستخدم لأنجاز دراسات الأتزان الأنتقالى لنظام قدره يشتمل على "ن" آله. تم استنتاج النظام الرياضي للنظام مع الأخذ فى الأعتبار الأتي: تأثير تضاؤل مجال المولد - الأخماد الميكانيكى الغير متماثل - الأخماد الألكترومغناطيسى - تأثير منظم الجهد - تأثير متحكم السرعه. بأستخدام طريقة الفك والتراكب تم فك النظام الى عدد (ن-١) ٢ تحت أنظمه كل منها يشتمل على ثلاثة مولدات. تم فك النظام الرياضى للنظام الى عدد (ن -١) /٢ تحت أنظمه مرتبطه كل منها من الدرجة الرابعة عشر. كل نظام مرتبط تم فكه الى تحت نظام حر(يشتمل على ثلاثة دوال غير خطيه) وارتباطات. لكل تحت نظام حر أختيرت دالة ليابونوف مكونه من "صوره مربعه + مجموع تكاملات ثلاثة دوال غير خطيه" . تم تكوين دالة ليابونوف متجهه وأستخد مت هذه الدالة فى أجواء التراكب للنظام. تم الحصول على مصفوفه تراكب للنظام من الدرجة (ن -١)/٢ , أتزان هذه المصفوفه يتضمن أتزان النظام. طبق معيار الأتزان المقدم على نظام قدره مكون من سبعة مولدات و يشتمل على أربعة عشر قضيب وأمكن الحصول على تقدير لحيز الأتزان لهذا النظام. أفترض حدوث قصر ثلاثى الأوجه عند نقطة على أحد خطوط النظام (هذه النقطة تكون ملاصقه لقضيب متصل به مولد , أو لقضيب متصل به حمل) و يتم فصل هذا الخط لعزل منطقة الخطأ. تم بطريقة مباشرة تعيين الزمن الحرج لأرجاع الخط المفصول . وجد أن الزمن المحسوب يكون مساويا تقريبا الزمن الحقيقى الذى تم حسابه بطريقة الخطوه خطوه. وجد أيضا أن معيار الأتزان المقدم مناسبا ويمكن أستخدامه بسهوله لأجراء دراسات الأتزان العمليه لأنظمة القدره كبيرة المقياس

keyword: Lyapunov method, Transient stability, One-axis model, Voltage regulator, Speed governor

1. Introduction

An important step in power system planning is the examination of dynamic and transient stability characteristics of alternative system design. This examination generally involves the time simulation of the behavior of many generators and their controls using a digital computer stability program. However, the computation cost of this process is a function of the complexity with which the power system elements are modeled.

Recently, the power system stability problem has drawn the attention of many, and various analytical techniques have been developed for the study of this problem. Among these techniques the Lyapunov's direct method is a worthwhile contribution. This method can be used to determine the stability of the behavior of higher order power systems and it is useful and practical in many cases [1]. Furthermore, the Lyapunov direct method has proved to be a promising tool of analysis for off-line and on-line studies [2].

Owing to the continuous increase in the sizes and complexities of real power systems, the scalar (function) Lyapunov method did not seem suitable in particular when the problem of the system stability domain estimate is attacked [3]. In addition, it is very difficult to derive a valid Lyapunov function for a power system when the automatic voltage regulator (AVR) is considered [4].

The decomposition-aggregation via vector Lyapunov function method has been appeared more suitable than the scalar function method for application to power systems. Applying the vector Lyapunov function method to power systems, it can be considered a more sophisticated mathematical models of generators and transmission, in addition exact estimates of the overall system stability domain may be defined [5,6].

In the last two decades, the vector Lyapunov function method was used to perform transient stability analysis of an N-machine power system considering the generator classical model (the internal voltage $E^`$ is constant) [5,7-14]. Also, the one-axis model ( the voltage component $E^`_q$ is changing with time), and the two-axis model (the voltage components $E^`_q$ and $E^`_d$ are changing with time) were considered in the papers [15,16]. A 3-machine, 4-bus and A10-machine, 11-bus power systems were used as illustrative numerical examples.

In the present work, transient stability analysis of an N-machine power system is performed using a new Lyapunov stability approach. A more realistic model (the one-axis model) represents each machine, non-uniform mechanical damping and the electromagnetic damping are considered, in addition the machine control systems (the voltage regulator and speed governor) are taken into consideration. The system mathematical model is decomposed into (N-1)/2, 14th-order interconnected subsystems, and an aggregation matrix of the order (N-1)/2 is obtained. As an illustrative example, the developed approach is applied to a 7-machine, 14-bus power system.

2. Power system model

Consider an N-machine power system (the transfer conductance are included) with mechanical and electromagnetic damping in addition to the first-order proportional speed governor [13] and the automatic voltage regulator (AVR), which is approximated by simple first-order lag [17,18]. Representing each of the system machines by the one-axis model, the absolute motion of the i th machine is described by the equations (see Notation)

$$M_i \ddot{\delta}_i + D_i \dot{\delta}_i + \Sigma^* D_{ij} (\dot{\delta}_i - \dot{\delta}_j) = P_{mi} + P_{gi} - P_{ei}$$
$$T'_{doi} \dot{E}^`_{qi} = E_{FDi} - E^`_{qi} + (X_{di} - X^`_{di}) I_{di}$$
$$T_{Ei} \dot{E}_{FDi} = - (E_{FDi} - \hat{E}_{FDi} + K_{Ei} V_{Ti})$$
$$\dot{P}_{gi} = - \mu_i P_{gi} - \alpha_i \dot{\delta}_i, \ i = 1,2...,N, \qquad (1)$$

where P_{ei} is given under the assumption $X'_d = X'_q$ (solid cylindrical rotors are considered) as,

$$P_{ei} = \sum_{j=1}^{N} Y_{ij} \{E'_{qi} [E'_{qj} \cos (\theta_{ij} - \delta_{ij}) - E'_{dj} \sin (\theta_{ij} - \delta_{ij})] + E'_{di} [E'_{dj} \cos (\theta_{ij} - \delta_{ij}) + + E'_{qj} \sin (\theta_{ij} - \delta_{ij})]\}, \ i = 1,2,\dots,N. \qquad (2)$$

In Eq.1, the terminal voltage variation V_{Ti} is given as ($E^`_{di}$ is assumed constant),

$$V_{Ti} = E^`_{qi} - \hat{E}_{qi} + (I_{di} - \overset{o}{I}_{di}) X^`_{di}, i = 1,2,\dots,N. \qquad (3)$$

Choosing the Nth- machine as a comparison machine, and introducing the following (5N-1) state variables,

$$\sigma_{iN} = \delta_{iN} - \overset{o}{\delta}_{iN} \qquad , i \neq N$$
$$\omega_i = \dot{\delta}_i ; E_{Qi} = E^`_{qi} - \hat{E}_{qi} ; \quad E_{fi} = E_{FDi} - \hat{E}_{FDi} ;$$
$$P_i = P_{gi} - P^o_{gi} , i = 1,2 ,\dots,N \qquad (4)$$

we can derive the whole system motion in the form

$$\dot{\sigma}_{iN} = \omega_i - \omega_N \quad , i \neq N$$
$$\dot{\omega}_i = -\lambda_i \omega_i - \Sigma^* \lambda_{ij} (\omega_i - \omega_j) + (P_i/M_i) - (1/M_i)[G_{ii} (E^2_{Qi} + 2E_{Qi}\hat{E}_{qi}) +$$
$$\Sigma^* Y_{ij}\{A_{ij} f_{ij}(\sigma_{ij}) + \tilde{A}_{ij} g_{ij}(\sigma_{ij}) + [\hat{E}_{qi}E_{Qj} + E_{Qi}(E_{Qj} + \hat{E}_{qj})]\cos(\theta_{ij} - \delta_{ij}) +$$
$$+ [\hat{E}_{di}E_{Qj} + \hat{E}_{dj}E_{Qi}]\sin(\theta_{ij} - \delta_{ij})\}]$$
$$\dot{E}_{Qi} = -\Gamma_i E_{Qi} + \chi_i E_{fi} + K_i \Sigma^* Y_{ij}[\hat{E}_{dj} f_{ij}(\sigma_{ij}) + \hat{E}_{qj} g_{ij}(\sigma_{ij}) + E_{Qj}\sin(\theta_{ij} - \delta_{ij})]$$
$$\dot{E}_{fi} = -\eta_i E_{Qi} - \nu_i E_{fi} - L_i \Sigma^* Y_{ij}[\hat{E}_{dj} f_{ij}(\sigma_{ij}) + \hat{E}_{qj} g_{ij}(\sigma_{ij}) + E_{Qj}\sin(\theta_{ij} - \delta_{ij})]$$
$$\dot{P}_i = -\mu_i P_i - \alpha_i \omega_i \quad ,i = 1,2,\ldots,N \qquad (5)$$

where the nonlinear functions f_{ij} and g_{ij} are given as

$$f_{ij}(\sigma_{ij}) = \cos(\sigma_{ij} + \delta^o_{ij} - \theta_{ij}) - \cos(\delta^o_{ij} - \theta_{ij})$$
$$g_{ij}(\sigma_{ij}) = \sin(\sigma_{ij} + \delta^o_{ij} - \theta_{ij}) - \sin(\delta^o_{ij} - \theta_{ij}). \qquad (6)$$

The functions of Eq.6, satisfy the following conditions:

$$f_{ij}(0) = g_{ij}(0) = 0; \quad 0 < f_{ij}(\sigma_{ij})/\sigma_{ij} \leq \xi_{ij}$$
$$0 < |g_{ij}(\sigma_{ij})/\sigma_{ij}| \leq \zeta_{ij}, i \neq j, i, j = 1,2,\ldots,N, \qquad (7)$$

where ξ_{ij} and ζ_{ij} ,are positive numbers and may be determined as,

$$\xi_{ij} = (\partial f_{ij}(\sigma_{ij})/\partial\sigma_{ij})\big|_{\sigma_{ij}=0},$$
and $\zeta_{ij} = |\partial g_{ij}(\sigma_{ij})/\partial\sigma_{ij}|_{\sigma_{ij}=0}$.

3. Power system decomposition

As a first step for the system decomposition, the system Nth–order reduced admittance matrix Y is determined (the system loads are represented by constant shunt impedance, then all the system nodes, except the machines internal nodes, are eliminated). Applying the triple-wise decomposition [11,12], the system is decomposed into (N–1)/2 interconnected subsystems. Finally, defining the state vector X_I in the form

$$X_I = [\sigma_{iI,N}, \sigma_{iI+1,N}, \omega_{iI}, \omega_{iI+1}, \omega_N, E_{QiI}, E_{QiI+1},$$
$$E_{QN}, E_{fiI}, E_{fiI+1}, E_{fN}, P_{iI}, P_{iI+1}, P_N]^T$$
$$= [X_{I1}, X_{I2}, X_{I3}, \ldots, X_{I14}]^T, \qquad (8)$$

the system mathematical model (Eq.5) can be decomposed into S =(N –1)/2, 14th-order interconnected subsystems. Each subsystem can be written in the general form

$$\dot{X}_I = P_I X_I + B_I F_I(\sigma_I) + h_I(X) \quad , \sigma_I = C^T_I X_I,$$
$$I = 1,2,\ldots, S \qquad (9)$$

and it may be decomposed into the free (disconnected) subsystem

$$\dot{X}_I = P_I X_I + B_I F_I(\sigma_I), \sigma_I = C^T_I X_I, \quad I = 1,2,\ldots,S, \qquad (10)$$

and the interconnections $h_I(X)$.

Now, expanding the 12 (the largest number) nonlinear functions included in each free subsystem, it is found that there are, at most, the following three non-linearity

$$f_{I1}(\sigma_{I1}) = \sin(\sigma_{iI.N} + \delta^o_{iI.N}) - \sin\delta^o_{iI.N}$$
$$f_{I2}(\sigma_{I2}) = \sin(\sigma_{iI+1.N} + \delta^o_{iI+1.N}) - \sin\delta^o_{iI+1.N}$$
$$f_{I3}(\sigma_{I3}) = \sin(\sigma_{iI.iI-1} + \delta^o_{iI.iI-1}) - \sin\delta^o_{iI.iI+1.} \qquad (11)$$

which satisfy the conditions:

$$\sigma_{Ik} f_{Ik}(\sigma_{Ik}) \geq \varepsilon_{Ik}\sigma^2_{Ik} \quad , k = 1,2,3, \qquad (12)$$

where $\varepsilon_{Ik} \in (0, \xi_{Ik})$ and ξ_{Ik} is determined from Eq.(7).

Referring to Eqs.(5), (8), and (10), it is obtained the I-th free subsystem matrices P_I, B_I, F_I and C_I as,

$$P_I = \begin{pmatrix} & \mathbf{I}_2 \quad -\mathbf{a} & \mathbf{O}_{2\times 9} & \\ & -\mathbf{P}_{I1} & -\mathbf{P}_{I2} \quad \mathbf{O}_{3\times 3} \quad \mathbf{P}_{I3} & \\ \mathbf{O}_{14\times 2} & & -\mathbf{P}_{I4} \quad \mathbf{P}_{I5} & \\ & \mathbf{O}_{6\times 3} & & \mathbf{O}_{6\times 3} \\ & & -\mathbf{P}_{I6} \quad -\mathbf{P}_{I7} & \\ & -\mathbf{P}_{I8} & \mathbf{O}_{3\times 6} \quad -\mathbf{P}_{I9} & \end{pmatrix} \tag{13}$$

where **O** and I are zero and identity (square) matrices, respectively, of the indicated dimensions, and **a** is a second-order unit vector and where,

$$\mathbf{P}_{I1} = \begin{bmatrix} -\Lambda_{iI} & \lambda_{iI,iI+1} & \lambda_{iI,N} \\ \lambda_{iI+1,iI} & -\Lambda_{iI+1} & \lambda_{iI+1,N} \\ \lambda_{N,iI} & \lambda_{N,iI+1} & -\Lambda_N \end{bmatrix}$$

$\mathbf{P}_{I2} = \text{diag}\,[\Theta_{iI}, \Theta_{iI+1}, \Theta_N]$

$\mathbf{P}_{I3} = \text{diag}\,[1/M_{iI};\ 1/M_{iI+1};\ 1/M_N]$

$\mathbf{P}_{I4} = \text{diag}\,[\Gamma_{iI}, \Gamma_{iI+1}, \Gamma_N]$

$\mathbf{P}_{I5} = \text{diag}\,[\chi_{iI}, \chi_{iI+1}, \chi_N]$

$\mathbf{P}_{I6} = \text{diag}\,[\eta_{iI}, \eta_{iI+1}, \eta_N]$

$\mathbf{P}_{I7} = \text{diag}\,[\nu_{iI}, \nu_{iI+1}, \nu_N]$

$\mathbf{P}_{I8} = \text{diag}\,[\alpha_{iI}, \alpha_{iI+1}, \alpha_N]$

$\mathbf{P}_{I9} = \text{diag}\,[\mu_{iI}, \mu_{iI+1}, \mu_N]$

$$B_I = \begin{bmatrix} & o_{203} & \\ BI2 & BI2 & BI3 \\ & o_{303} & \end{bmatrix}, \tag{14}$$

where

$BI1 = [-d_{iI\,N}, 0, d_{N,iI}, q_{iI.N}, 0, -q_{N,iI}, -b_{iI,N}, 0, b_{N,iI}]^T$

$BI2 = [0, -d_{iI+1,N}, d_{N,iI+1}, 0, q_{iI+1.N}, -q_{N,iI+1}, 0, -b_{iI+1,N}, b_{N.iI+1}]^T$

$BI3 = [-d_{iI,iI+1}, d_{iI+1,iI}, 0, q_{iI,iI+1}, -q_{iI+1,iI}, 0, -b_{iI,iI+1}, b_{iI+1,iI}, 0]^T$

$$F_I(\sigma_I) = [\mathbf{f}_{I1}(\sigma_{I1}), \mathbf{f}_{I2}(\sigma_{I2}), \mathbf{f}_{I3}(\sigma_{I3})]^T \tag{15}$$

$$C^T{}_I = \left[\begin{array}{cc|c} 1.0 & 0 & \\ 0 & 1.0 & O_{3x12} \\ 1.0 & -1.0 & \end{array}\right] \tag{16}$$

Now, we get the interconnection (vector) matrix $h_I(x)$ as,

$$h_I(X) = [0, 0, h_{I3}(X), h_{I4}(X), \ldots, h_{I11}(X), 0, 0, 0]^T, \tag{17}$$

where

$\mathbf{h}_{I3}(X) = -(1/M_{iI})[G_{iI,iI} X^2_{I6} + \mathbf{C}_{iI,N} f'_{I1}(\sigma_{I1}) + \mathbf{C}_{iI,iI+1} f'_{I3}(\sigma_{I3}) + \Sigma(S_{iI,j} + \widetilde{S}_{iI,j}) + \Sigma^*\{\overline{D}_{iI,j} + X_{I6} L_{iI,j}\}]$

$\mathbf{h}_{I4}(X) = -(1/M_{iI+1})[G_{iI+1,iI+1} X^2_{I7} + \mathbf{C}_{iI+1,N} f'_{I2}(\sigma_{I2}) + \mathbf{C}_{iI+1,iI} f'_{I3}(\sigma_{I3}) + \Sigma(S_{iI+1,j} + \widetilde{S}_{iI+1,j}) + \Sigma^*\{\overline{D}_{iI+1,j} + X_{I7} L_{iI+1,j}\}]$

$\mathbf{h}_{I5}(X) = -(1/M_N)[G_{N.N} X^2_{I8} + \mathbf{C}_{N,iI} f'_{I1}(\sigma_{I1}) + \mathbf{C}_{N,iI+1} f'_{I2}(\sigma_{I2}) + \Sigma(S_{N,j} + \widetilde{S}_{N,j}) + \Sigma^*\{\overline{D}_{N,j} + X_{I8} L_{N,j}\}]$

$\mathbf{h}_{I6}(X) = K_{iI}[\overline{C}_{iI,N} f_{I1}(\sigma_{I1}) + \overline{C}_{iI,iI+1} f'_{I3}(\sigma_{I3}) + \Sigma\,\hat{S}_{iI,j} - \Sigma^*\overline{L}_{iI,j}]$

$\mathbf{h}_{I7}(X) = K_{iI+1}[\overline{C}_{iI+1,N} f_{I2}(\sigma_{I2}) + \overline{C}_{iI+1,iI} f'_{I3}(\sigma_{I3}) + \Sigma\,\hat{S}_{iI+1,j} - \Sigma^*\overline{L}_{iI-1,j}]$

$\mathbf{h}_{I8}(X) = K_N[\overline{C}_{N,iI} f_{I1}(\sigma_{I1}) + \overline{C}_{N,iI+1} f'_{I2}(\sigma_{I2}) + \Sigma\,\hat{S}_{N,j} - \Sigma^*\overline{L}_{N,j}]$

$\mathbf{h}_{I9}(X) = -\mathbf{h}_{I6}(X)[L_{iI}/K_{iI}]$

$\mathbf{h}_{I10}(X) = -\mathbf{h}_{I7}(X)[L_{iI+1}/K_{iI+1}]$

$\mathbf{h}_{I11}(X) = -\mathbf{h}_{I8}(X)[L_N/K_N].$ (18)

In Eq.(13), (14) and (18), we define the following:

$\Gamma_j = [1.0 - (X_{dj} - X'_{dj})B_{jj}]/T'_{doj} \qquad \chi_j = 1/T'_{doj}$

$\Theta_j = 2\hat{E}_{qj} G_{jj}/M_j \qquad ; \nu_j = 1/T_{Ej}$

$\eta_j = [1.0 + X'_{dj} B_{jj}]/T_{Ej} \qquad , j \in \mathbf{J}_{IN}$

$$d_{kj} = (A_{kj} B_{kj} + \widetilde{A}_{kj} G_{kj}) / M_k$$
$$C_{kj} = (A_{kj} G_{kj} - \widetilde{A}_{kj} B_{kj})$$
$$\overline{C}_{kj} = (\hat{E}_{dj} G_{kj} + \hat{E}_{qj} B_{kj})$$
$$q_{kj} = K_k (\hat{E}_{dj} B_{kj} - \hat{E}_{qj} G_{kj})$$
$$b_{kj} = q_{kj} (L_k / K_k) \quad ,k \neq j,\ k, j \in J_{IN}$$
$$f_{I1}(\sigma_{I1}) = \cos(\sigma_{il,N} + \delta^o_{il,N}) - \cos \delta^o_{il,N}$$
$$f_{I2}(\sigma_{I2}) = \cos(\sigma_{il+1,N} + \delta^o_{il+1,N}) - \cos \delta^o_{il+1,N}$$
$$f_{I3}(\sigma_{I3}) = \cos(\sigma_{il,il+1} + \delta^o_{il,il+1}) - \cos \delta^o_{il,il+1}$$
$$L_j = \{(E_{Qi} + \hat{E}_{qi}) f^*(\sigma_{ij}) + \hat{E}_d g^*(\sigma_j)\} Y_j E_{Qj}$$
$$\overline{L}_j = E_{Qj} Y_j g^*(\sigma_j) \quad , \widetilde{S}_j = \lambda_j \omega_j$$
$$\overline{D}_j = \{\hat{E}_{qj} f^*(\sigma_j) - \hat{E}_d g^*(\sigma_j)\} Y_j E_{Qi}$$
$$S_j = \{A_j f_j(\sigma_j) + \widetilde{A}_j g_j(\sigma_j)\} Y_j$$
$$\hat{S}_j = \{\hat{E}_{dj} f_j(\sigma_j) - \hat{E}_{qj} g_j(\sigma_j)\} Y_j$$
$$f^*(\sigma_j) = \cos(\sigma_j + \delta^o_j - \theta_j)$$
$$g^*(\sigma_j) = \sin(\sigma_j + \delta^o_j - \theta_j) \quad , i \in J_{IN} \quad , j \notin J_{IN} \qquad (19)$$

and the nonlinear functions $f_{ij}(\sigma_{ij})$ and $g_{ij}(\sigma_{ij})$ are given by Eq. (6).

4. Power system aggregation

It is constructed [19] an aggregation (square) matrix, $A = [\alpha_{IJ}]$, whose elements (real numbers) obey the inequality

$$\dot{V}_I(X_I) \leq \sum_{J=1}^{S} \alpha_{IJ} U_I(X_I) U_J(X_J),\ I=1,2,\ldots,S, \qquad (20)$$

where $V_I(X_I)$ is a Lyapunov function for the Ith free subsystem, U_I and U_J are comparison functions and they are chosen in the form [5],

$$U_I(X_I) = \|X_I\| = (X_I^T X_I)^{1/2} \quad , I = 1,2,\ldots S. \qquad (21)$$

For each free subsystem, it is accepted a Lyapunov function in the form [5,7,8,11-16]

$$V_I(X_I) = X_I^T H_I X_I + \sum_{m=1}^{3} \gamma_{Im} \int_0^{\sigma I} f_{Im}(\sigma_{Im})\, d\sigma_{Im},$$
$$I = 1,2,\ldots,S, \qquad (22)$$

where, H_I is an 14th-order symmetric positive-definite matrix, the functions f_{Im} are given by eqn 11, and γ_{Im} are arbitrary positive numbers.

Now computing the total time derivative of the function V_I along the motion of the interconnected subsystem of Eq.9, the left-hand side of inequality 20 is determined. Then, a number of majorizations are introduced and used for majorizing $\dot{V}_I$. Finally, referring to Eq.20, elements of the aggregation (square) matrix $A = [\alpha_{IK}]$ of order (N−1) / 2, are obtained and defined as

$$\alpha_{IK} = \begin{cases} \lambda^*_I & , K = I \\ 2 Z_{IK}, & K \neq I,\ K, I = 1,2,\ldots, S = (N-1)/2 \end{cases} \qquad (23)$$

In Eq. (23), λ* is the minimal (positive) eigenvalue of the 15th-order symmetric matrix R_I, whose elements are given by Eq. (A-1), and Z_{IK} is defined by Eq. (A-2). It is of importance to note that, stability of the aggregation matrix A, implies asymptotic stability of the system equilibrium [19].

5. Numerical example

The developed approach is applied, in this example, to the 7-machine, 14-bus power system shown in Fig.1. The system loads are represented by equivalent shunt admittances, and the reactance X'_d of each generator is inserted. Then the system 7th-order (symmetric) admittance matrix Y is computed (the generators internal nodes are kept) and its elements are given in Table 1 (see Appendix). Selecting machine 7 as the comparison machine and applying the triple-wise decomposition, the system is decomposed into three "3-machine" interconnected subsystems. The following parameters are chosen:

$h^k_{13} = h^k_{24} = h^k_{33} = h^k_{44} = 1.0$; $h^k_{66} = h^k_{77} = 400$, k = 1,2 ,3 ; $h^1_{55} = 3.9$, $h^2_{55} = 4.5$, $h^3_{55} = 4.1$ $h^1_{88} = h^3_{88} = 38.0$, $h^2_{88} = 40.0$; $\lambda_1 = \lambda_2 = 2.7$, $\lambda_3 = \lambda_4 = 2.6$, $\lambda_5 = \lambda_6 = 2.8$, $\lambda_7 = 9.9$ $T'_{doi} = 5.0$, $K_{Ei} = 20.0$,

$T_{Ei} = 0.30$, $i = 1, 2, 3 ,...,6$; $T'_{do7} = 4.5$, $K_{E7} = 15.0$, $T_{E7} = 0.20$ $\alpha_i = 0.5$, $\mu_i = 15.0$, $i=1,2,....,7$; $\lambda_{ij} = 0.010$, $i \neq j$, $i,j =1,2,..,6$, $\lambda_{Nj} = 0.001$ $j = 1,2,...,6$

$\varepsilon_{11} = 0.58$, $\varepsilon_{12} = 0.54$; $\varepsilon_{21} = 0.59$, $\varepsilon_{22} = 0.55$; $\varepsilon_{31} = 0.61$, $\varepsilon_{32} = 0.59$

Then, using expression (23), it is computed the aggregation matrix

$$A = \begin{bmatrix} -0.508502 & 0.313846 & 0.316635 \\ 0.318380 & -0.753105 & 0.354680 \\ 0.317915 & 0.323244 & -0.725602 \end{bmatrix}$$

which satisfies the Hick's conditions[23], and is thus a stable matrix . This implies the asymptotic stability of the system equilibrium. Finally, for the system asymptotic stability domain it is determined [12] the estimate $\tilde{E}_1$ given as

$$\tilde{E}_1 = \{X: [V_1(X_1)+V_2(X_2)+V_3(X_3)] \leq 8.9229\} \quad (28)$$

where V_1 , V_2 and V_3 are the free subsystem Lyapunov functions, given by Eq. (22).

Now, the system transient stability computations are carried out assuming two severe cases for occurrence of a3-phase short circuit fault (with successful reclosure) as follows:

1. The fault is assumed to be occurred close to generator bus 1, at 5% length of the line connecting buses 1 and 7. For isolating the faulted bus it is assumed that the two lines 1-7 and 1-14 are simultaneously opened using 5-cycle circuit breakers. It is assumed also that, due to operation of the under-voltage relay, the load connected to bus 1 is removed after 0.16 sec from the fault instant. Applying the developed approach, it is found directly that the critical time for reclosing the open two lines with reconnecting the removed load is equal to 0.199 sec (computed from the fault instant). Note that, the exact critical time, by using the step-by-step method, equals 0.247 sec. Figs. 2-a, b and c, show variations of the first subsystem (includes machines 1, 2 and 7) states just after reconnecting the open lines and the removed load.
2. It is assumed that the fault is occurred close to load bus 11, at 5% length of the line connecting buses 11 and 14. Using 10-cycle C.Bs., the faulted line is opened for clearing the fault. A pulsating load of the value (1.2+j0.80) is assumed to be added, after 0.30 sec from the fault instant, to the load of bus 6. The critical time for reclosing the open line with removing the added load is found, by applying the developed approach, to be 0.363 sec (the exact time equals 0.418 sec) from the fault instant. Figs. 3-a, b and c, show variations of the third subsystem (includes machines 5, 6 and 7) states just after reclosing the open line with removing the added load. It is to be noted that, variations of the states of machine "7" for the considered two fault cases are very small and hence they are not shown in Figs. 2 and 3.

6. Conclusions

1. The transfer conductances are considered in the developed approach, hence for stability studies of a considered power system, resistance of the lines can be taken into consideration and the system network can be greatly simplified by eliminating all load nodes.
2. Order of the developed aggregation matrix is (N – 1)/2, where N is number of system machines, instead of number of system buses as usual. Hence, for a considered real power system (number of machines is, in general, greatly smaller than number of the system buses) computations of the aggregation matrix and its stability conditions are simpler.
3. For transient stability studies of a real power system the developed approach is more suitable than the decomposition aggregation approaches developed so far [7-16]. Note that, in the developed approach a more realistic generator model (that is, the one-axis model) is considered, and the action of both the voltage regulator and the speed governor is taken into consideration.
4. The developed approach is simple and suitable to use for practical stability

studies of power systems. Note that, in the given numerical example, the computed critical times for the assumed two cases of a 3- phase short circuit fault are nearly equal (about 87%) the exact times computed by the step-by-step method.

5. In the developed approach the parameters of voltage regulator and speed governor systems are included, hence new horizons to sensitivity analysis problem of power systems can be opened. Furthermore, a comparison between the subsystem, or the whole system, stability domain estimates can be easily obtained for assumed different values of the voltage regulator and speed governor parameters.

List of symbols

P_{mi}, P_{ei} = mechanical electrical power of ith machine

P_{gi}= variation of ith generator mechanical power

P^o_g= steady-state (pre-transient) variation of generator mechanical power

δ = rotor angle, or position of the rotor q-axis from the reference

X_d,X_q=d-axis, q-axis synchronous reactances

$X^`_d$, $X^`_q$= d-axis, q-axis transient reactances

E_{FD}=exciter voltage referred to the armature circuit

$E^`$ = voltage behind d-axis transient reactance

$E^`_d$, $E^`_q$= d-axis, q-axis components of the voltage $E^`$

E_q=armature emf corresponding to the field current

δ^o, $\hat{E}_{FD}$, $\hat{E}_q$, $\hat{E}_d$ = steady-state values of δ, E_{FD}, $E^`_q$ and $E^`_d$

V_t, V_T= terminal voltage, terminal voltage variation

V_{tqi} , V_{tdi} = q-axis and d-axis components of the voltage V_t

V^o_t , V^o_{tq} = steady-state values of V_t and V_{tq}

I^o_q , I^o_d= steady-state q-axis and d-axis current components

K_E, T_E = the exciter gain and time constant

ω = rotor speed with respect to the synchronous speed

Y_{ij} = Y_{ji} = modulus of transfer admittance between internal nodes of ith and jth generators

$\theta_{ij} = \theta_{ji}$ = phase angle of transfer admittance Y_{ij}

G_{ij}, B_{ij} = transfer conductance, transfer susceptance

T'_{do} = direct-axis transient open-circuit time constant

λ_i = (D_i/M_i)= mechanical damping coefficient

λ_{ij} = (D_{ij}/M_i)= electromagnetic damping coefficient

(1 / μ)= time constant of first-order speed governor

(α / μ)= gain of first-order speed governor

J_{IN} ={ iI , iI+1 , N } = set introduced to denote the Ith subsystem three machines

$J_I \subset J_{IN}$ = { iI , iI+1}

$\delta_{ij} = \delta_i - \delta_j = \delta_{iN} - \delta_{jN}$; $\sigma_{ij} = \delta_{ij} - \delta^o_{ij} = \sigma_{iN} - \sigma_{jN}$,

$\sigma_{kN} = \delta_{kN} - \delta^o_{kN}$, $k \varepsilon J_I$

$A_{ij} = A_{ji} = \hat{E}_{qi}\, \hat{E}_{qj} + \hat{E}_{di}\, \hat{E}_{dj}$;

$\tilde{A}_{ij} = - \tilde{A}_{ji} = \hat{E}_{qi}\hat{E}_{dj} - \hat{E}_{di}\, \hat{E}_{qj}$

$K_j = (X_{dj} - X'_{dj}) / T'_{doj}$; $L_j = K_{Ej}\, X'_{dj} / T_{Ej}$;

$\Lambda j = \lambda j + \Sigma^* \lambda_{ji}$,

$j \varepsilon J_{IN}$

Σ^* and Σ are defined and as $\sum_{j\neq i}^{N}$ and $\sum_{j\notin JI}^{N-1}$,respectively

Z_2, Z_3 = two functions, defined as follows:

$Z_2(\alpha , \phi) = \min \{ \sqrt{2} \max (| \alpha | , | \phi |) ; (| \alpha | + | \phi |) \}$

$Z_3(\alpha, \phi, \gamma) = \min \{2 \max (|\alpha |, | \phi |,| \gamma |) ; (|\alpha| + |\phi |+| \gamma |)$;

$Z_2 [Z_2 (\alpha, \phi) , \gamma]$;

; $Z_2 [Z_2(\phi , \gamma), \alpha] ; Z_2 [Z_2 (\gamma , \alpha) , \phi] \}$

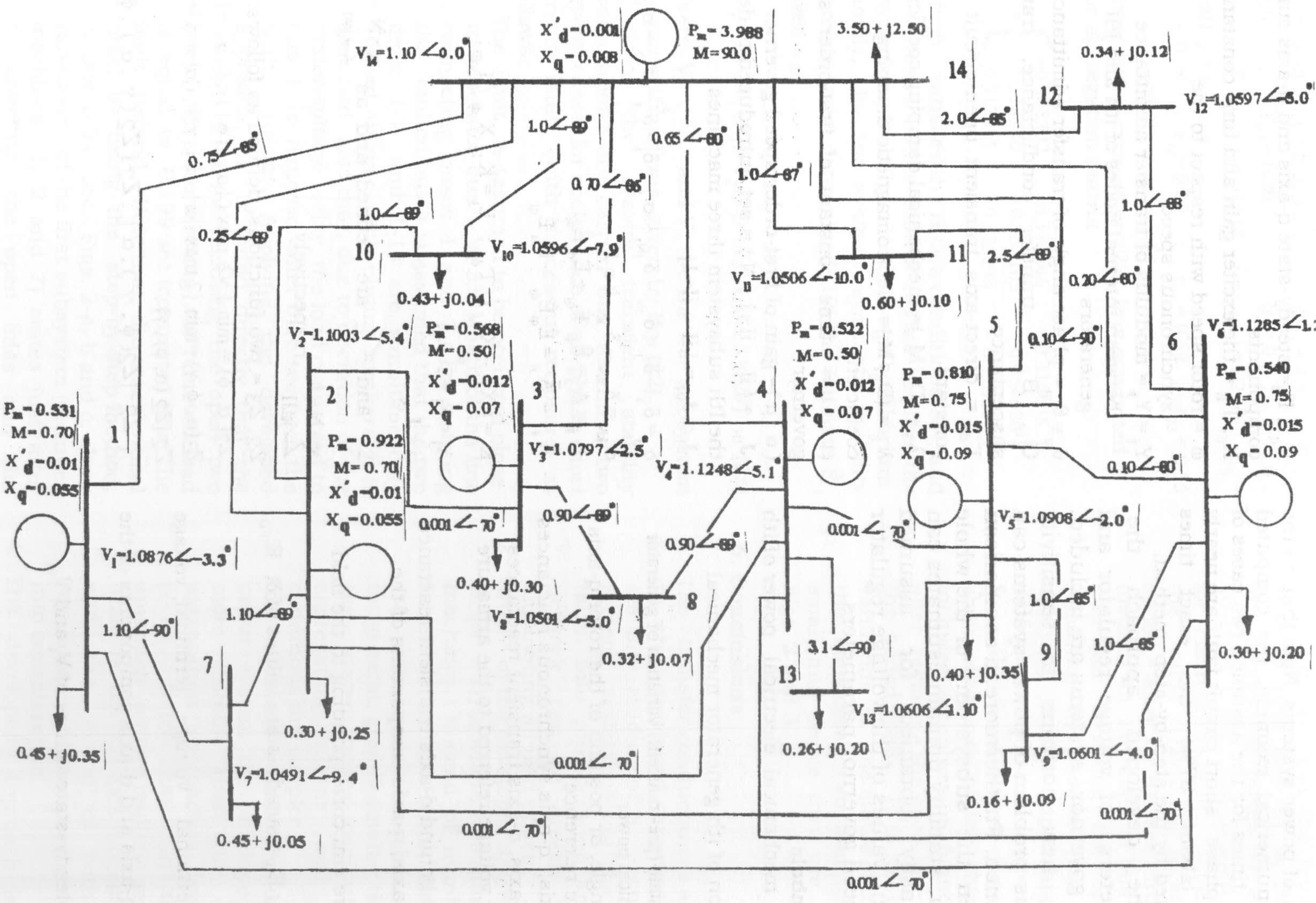

Fig. 1. Seven-machine, 14-bus power system.

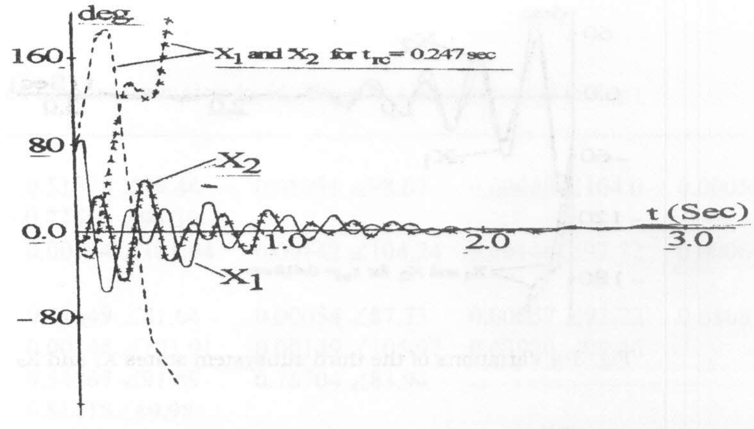

Fig. 2-a Variations of the first subsystem states X_1 and X_2

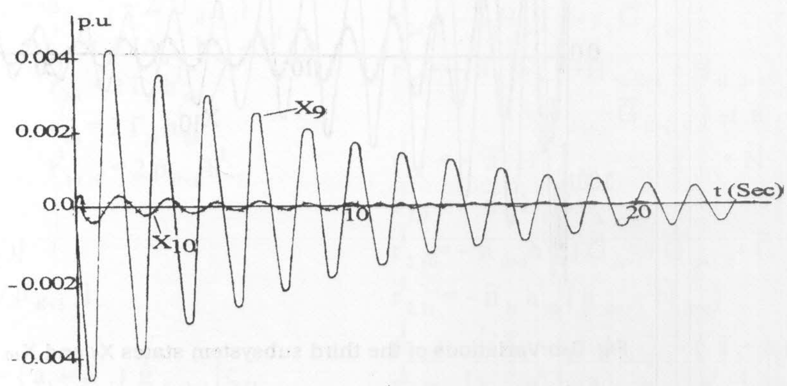

Fig. 2-b Variations of the first subsystem states X_9 and X_{10}

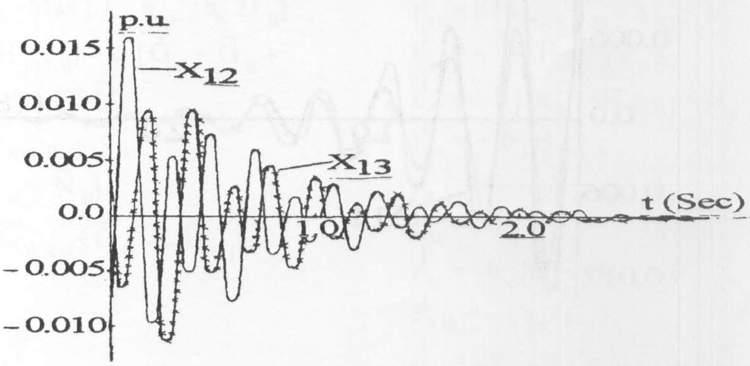

Fig. 2-c Variations of the first subsystem states X_{12} and X_{13}

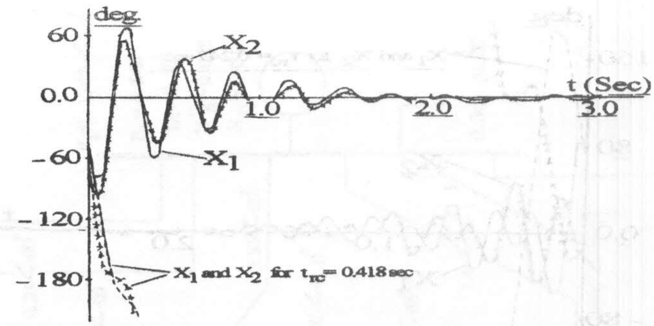

Fig. 3-a Variations of the third subsystem states X_1 and X_2.

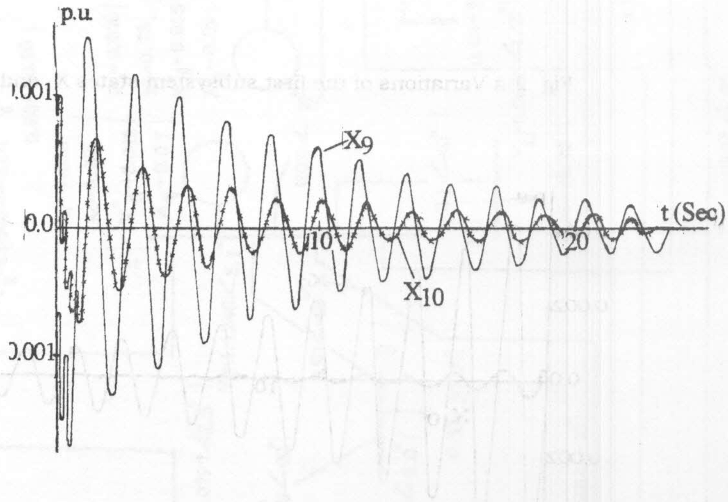

Fig. 3-b Variations of the third subsystem states X_9 and X_{10}.

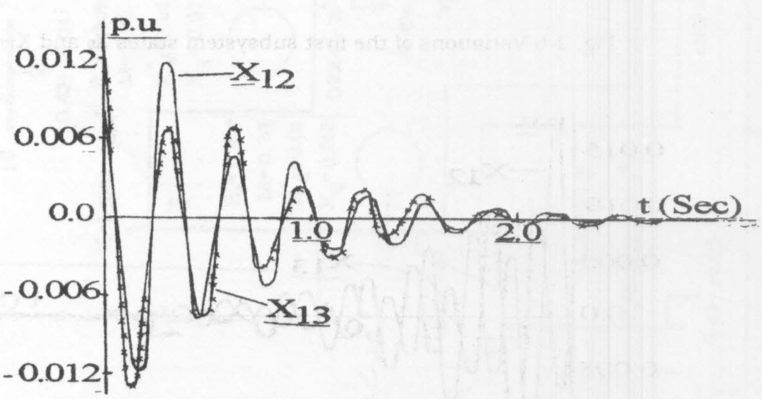

Fig. 3-c Variations of the third subsystem states X_{12} and X_{13}.

Appendix

Table 1 The system reduced admittance matrix (moduli in p.u. and arguments in deg.)

1	1.68799 ∠–71.68	0.51323 ∠79.44	0.00051 ∠98.07	0.00048 ∠104.0	0.00058 ∠88.92
	0.00155 ∠103.64	0.73660 ∠94.39			
2	1.60395 ∠–74.06	0.00144 ∠101.94	0.00142 ∠104.24	0.00146 ∠97.72	0.00060 ∠84.22
	0.71713 ∠83.55				
3	1.47795 ∠–72.23	0.41549 ∠81.64	0.00054 ∠87.73	0.00057 ∠93.72	0.68687 ∠93.48
4	1.33515 ∠–73.44	0.00145 ∠103.91	0.00149 ∠105.97	0.63920 ∠99.46	
5	1.89280 ∠–69.0	0.54867 ∠91.89	0.76704 ∠83.94		
6.	1.66321 ∠–75.80	0.81418∠89.98			
7.	7.43937 ∠–62.43				

Definition of the elements of 15-th order matrix R

$r^{I}_{11} = 2\, a_I \{D_{iI}\, \varepsilon_{11} - \widetilde{D}_{iI} - m_{iI,\, iI+1} - \Sigma\, U_{iI,\, j}\}$

$r^{I}_{22} = 2\, \bar{a}_I \{D_{iI+1}\, \varepsilon_{12} - \widetilde{D}_{iI+1} - m_{iI+1.\, iI} - \Sigma\, U_{iI+1,\, j}\}$

$r^{I}_{33} = 2\, (\Lambda_{iI}\, h^{I}_{33} - h^{I}_{13})$, $r^{I}_{44} = 2\, (\Lambda_{iI+1}\, h^{I}_{44} - h^{I}_{24})$

$r^{I}_{55} = 2\, \Lambda_N\, h^{I}_{55}$, $r^{I}_{66} = 2\, \Gamma_{iI}\, h^{I}_{66}$

$r^{I}_{77} = 2\, \Gamma_{iI+1}\, h^{I}_{77}$, $r^{I}_{88} = 2\, \Gamma_N\, h^{I}_{88}$

$r^{I}_{99} = 2\, \rho_{iI}\, h^{I}_{66}$, $r^{I}_{10,10} = 2\, \rho_{iI+1}\, h^{I}_{77}$

$r^{I}_{11,11} = 2\, h^{I}_{88}\, \rho_N$

$r^{I}_{12,12} = 2\, \rho^*_{iI}\, [b_I + (a_I / \mu_{iI})]$

$r^{I}_{13,13} = 2\, \rho^*_{iI+1}\, [\bar{b}_I + (\bar{a}_I / \mu_{iI+1})]$

$r^{I}_{14,14} = 2\, \rho^*_N\, b_N$

$r^{I}_{12} = -|(a_I - \bar{a}_I)\, g_{iI,\, iI+1} - (a_I + \bar{a}_I)\, \hat{g}_{iI,\, iI+1}|\, \zeta_{13}$

$r^{I}_{13} = -b_I [\widetilde{D}_{iI} + m_{iI,\, iI+1} + \Sigma\, \overline{U}_{iI,\, j}]$

$r^{I}_{14} = -\bar{b}_I\, m_{iI+1,\, iI} - \lambda_{iI.iI+1}\, h^{I}_{13}$

$r^{I}_{15} = -b_I\, D_{iI}\, \xi_{iI} - b_N\, \bar{g}_{iI} - \text{Max}\, [\widetilde{T}_{iI}\, h^{I}_{13}\, ;\, b_N\, \hat{D}_{iI}]$

$r^{I}_{16} = -a_I\, [n_{iI} + H_{N,\, iI} + H_{iI+1,\, iI}] - c_I\, [\breve{G}_{iI} + \widetilde{G}_{iI} + \overline{G}_{iI,\, iI+1}] - (a_I + c_I)\, \Sigma\, \widetilde{U}_{iI.\, j}$

$r^{I}_{17} = -a_I H_{iI,\, iI+1} - \overline{C}_I\, \overline{G}_{iI+1,\, iI}$

$r^{I}_{18} = -a_I H_{iI,\, N} - c_N [\breve{g}_{iI} + \widetilde{N}_{iI}]$

$r^{I}_{19} = -\widetilde{n}_{iI}\, h^{I}_{66} [\breve{G}_{iI} + \overline{G}_{iI,\, iI+1} + \widetilde{G}_{iI} + \Sigma\, \widetilde{U}_{iI,\, j}]$

$r^{I}_{1,10} = -\widetilde{n}_{iI+1}\, h^{I}_{77}\, \overline{G}_{iI,\, iI+1}$

$r^{I}_{1,11} = -\widetilde{n}_N\, h^{I}_{88} [\breve{g}_{iI} + \widetilde{N}_{iI}]$

$r^{I}_{23} = -b_I\, m_{iI.\, iI+1} - \lambda_{iI+1,iI}\, h^{I}_{24}$

$r^{I}_{24} = -\bar{b}_I [\widetilde{D}_{iI+1} + m_{iI+1.\, iI} + \Sigma\, \overline{U}_{iI+1,\, j}]$

$r^{I}_{25} = -\bar{b}_I D_{iI+1}\, \xi_{iI+1} - b_N\, \bar{g}_{iI+1} - \text{Max}\, [\widetilde{T}_{iI+1}\, h^{I}_{24}\, ;\, +b_N\, \hat{D}_{iI+1}]$

$r^{I}_{26} = -\bar{a}_I H_{iI+1,iI} - c_I\, \overline{G}_{iI.\, iI+1}$

$r^{I}_{27} = -\bar{a}_I\, [n_{iI+1} + H_{N,\, iI+1} + H_{iI.\, iI+1}] - \overline{C}_I [\breve{G}_{iI+1} + \widetilde{G}_{iI+1} + \overline{G}_{iI+1.\, iI}] - (\bar{a}_I + \overline{C}_I)\, \Sigma\, \widetilde{U}_{iI+1,\, j}$

$r^{I}_{28} = -\bar{a}_I\, H_{iI+1,\, N} - c_N [\breve{g}_{iI+1} + \widetilde{N}_{iI+1}]$

$r^{I}_{2,9} = -\widetilde{n}_{iI}\, h^{I}_{66}\, \overline{G}_{iI.\, iI+1}$

$r^{I}_{2,10} = -\widetilde{n}_{iI+1} h^{I}_{77} [\breve{G}_{iI+1} + \overline{G}_{iI+1.iI} + \widetilde{G}_{iI+1} + \Sigma\, \widetilde{U}_{iI+1,\, j}]$

$r^{I}_{2,11} = -\widetilde{n}_N\, h^{I}_{88} [\breve{g}_{iI+1} + \widetilde{N}_{iI+1}]$

$r^{I}_{2,15} = -|(\bar{a}_I - a_I)\, \hat{d}_{iI,\, iI+1} - (\bar{a}_I + a_I)\, \bar{e}_{iI.\, iI+1}|$

$r^{I}_{35} = -|\lambda_{iI,N}\, h^{I}_{33} + \lambda_{N,iI}\, h^{I}_{55} - h^{I}_{13}|$

$r^{I}_{36} = -b_I\, [n_{iI} + H_{N.\, iI} + H_{iI+1,\, iI} + \Sigma\, \widetilde{U}_{iI,\, j}]$

$r^{I}_{37} = -b_I\, H_{iI.\, iI+1}$, $r^{I}_{38} = -b_I\, H_{iI,\, N}$

$r^{I}_{45} = -|\lambda_{iI+1,N}\, h^{I}_{44} + \lambda_{N,iI+1}\, h^{I}_{55} - h^{I}_{24}|$

$r^{I}_{46} = -\bar{b}_I\, H_{iI+1,\, iI}$

$r^{I}_{47} = -\bar{b}_I\, [n_{iI+1} + H_{N.\, iI+1} + H_{iI.\, iI+1} + \Sigma\, \widetilde{U}_{iI+1.\, j}]$

$r^{I}_{48} = -\bar{b}_I\, H_{iI+1,\, N}$

$r^{I}_{4,15} = -|(\bar{b}_I - b_I)\, \hat{d}_{iI,\, iI+1} - (\bar{b}_I + b_I)\, \bar{e}_{iI,\, iI+1}|$

$r^{I}_{56} = -b_N\, H_{N,iI}$, $r^{I}_{57} = -b_N\, H_{N,\, iI+1}$

$r^{I}_{58} = -b_N\, [n_N + H_{iI,\, N} + H_{iI+1,\, N} + \Sigma\, \widetilde{U}_{N,\, j}]$

$r^{I}_{5,12} = -(a_I / \mu_{iI})$, $r^{I}_{5,13} = -(\bar{a}_I / \mu_{iI+1})$

$r^{I}_{67} = -Y_{iI,\, iI+1} \sqrt{\ c_I^{2} + \overline{C}_I^{2} - 2\, \overline{C}_I\, c_I \cos(2\, \theta_{iI.\, iI+1})}$

$r^{I}_{68} = -Y_{iI,\, N} \sqrt{\ \{c_I^{2} + c_N^{2} - 2\, c_I\, c_N \cos(2\, \theta_{iI.\, N})\}}$

$r^{I}_{6,10} = -Y_{iI,\, iI+1}\, \widetilde{n}_{iI+1}\, h^{I}_{77}$

$r^I_{6,11} = -Y_{iI,N}\, \tilde{n}_N\, h^I_{88}$, $r^I_{6,15} = -c_I\, \breve{e}_{iI+1,iI}$

$r^I_{78} = -Y_{iI+1,N} \sqrt{\{\overline{C}_I^{\,2} + c_N^{\,2} - 2\,\overline{C}_I\, c_N \cos(2\theta_{iI+1,N})\}}$

$r^I_{79} = -Y_{iI,iI+1}\, \tilde{n}_{iI}\, h^I_{66}$

$r^I_{7,11} = -Y_{iI+1,N}\, \tilde{n}_N\, h^I_{88}$, $r^I_{7,15} = -\overline{C}_I\, \breve{e}_{iI,iI+1}$

$r^I_{89} = -Y_{iI,N}\, \tilde{n}_{iI}\, h^I_{66}$, $r^I_{8,10} = -Y_{iI+1,N}\, \tilde{n}_{iI+1}\, h^I_{77}$

$r^I_{9,15} = -\tilde{n}_{iI}\, h^I_{66}\, \breve{e}_{iI+1,iI}$, $r^I_{10,15} = -\tilde{n}_{iI+1} h^I_{77}\, \breve{e}_{iI,iI+1}$

$$r^I_{15,15} = 2\, a_I\, (\hat{d}_{iI,iI+1} + \overline{e}_{iI,iI+1})\, /\, \xi_{I3} \qquad \text{(A-1)}$$

where the other elements of this matrix are zero.

Definition of the aggregation matrix off-diagonal elements:

In Eq.23, Z_{IK} is given as (see Notation),

$$Z_{IK} = Z_3\, [Z_2(\overline{Z}_{I,K}\, ,\, \overline{Z}_{I,K+1})\, ;\, Z_2(\widetilde{Z}_{I,K}\, ,\, \widetilde{Z}_{I,K+1})\, ;\, Z_2(\hat{Z}_{I,K}\, ,\, \hat{Z}_{I,K+1})] \qquad \text{(A-2)}$$

where

$$\overline{Z}_{I,K} = Z_2\, [\, Z_3(\, \overline{R}_I\, \overline{U}_{iI,iK}\, ;\, \tilde{R}_I\, \overline{U}_{iI+1,iK}\, ;\, b_N\, \overline{U}_{N,iK})\, ;\, Z_2\, \{Z_3(\, c_I\, \tilde{U}_{iI,iK}\, ;\, \overline{C}_I\, \tilde{U}_{iI+1,iK}\, ;\, c_N\, \tilde{U}_{N,iK})\, ;\, Z_3(\tilde{n}_{iI}\, h^I_{66}\, \tilde{U}_{iI,iK}\, ;\, \tilde{n}_{iI+1}\, h^I_{77}\, \tilde{U}_{iI+1,iK}\, ;\, \tilde{n}_N\, h^I_{88}\, \tilde{U}_{N,iK})\}]$$

$$\overline{Z}_{I,K+1} = Z_2 [Z_3(\overline{R}_I\, \overline{U}_{iI,iK+1}\, ;\, \tilde{R}_I\, \overline{U}_{iI+1,iK+1}\, ;\, b_N\, \overline{U}_{N,iK+1})\, ;\, Z_2\{Z_3(c_I\, \tilde{U}_{iI,iK+1}\, ;\, \overline{C}_I\, \tilde{U}_{iI+1,iK+1}\, ;\, c_N\, \tilde{U}_{N,iK+1})\, ;\, Z_3(\tilde{n}_{iI}\, h^I_{66}\, \tilde{U}_{iI,iK+1}\, ;\, \tilde{n}_{iI+1}\, h^I_{77}\, \tilde{U}_{iI+1,iK+1}\, ;\, \tilde{n}_N\, h^I_{88}\, \tilde{U}_{N,iK+1})\}]$$

$$\widetilde{Z}_{I,K} = Z_3\, [\hat{h}_I\, \lambda_{iI,iK}\, ;\, \breve{h}_I\, \lambda_{iI+1,iK}\, ;\, h^I_{55}\, \lambda_{N,iK}]$$

$$\widetilde{Z}_{I,K+1} = Z_3\, [\hat{h}_I\, \lambda_{iI,iK+1}\, ;\, \breve{h}_I\, \lambda_{iI+1,iK+1}\, ;\, h^I_{55}\, \lambda_{N,iK+1}]$$

$$\hat{Z}_{I,K} = Z_3\, [\, Z_3(\overline{R}_I\, H_{iI,iK}\, ;\, \tilde{R}_I\, H_{iI+1,iK}\, ;\, b_N\, H_{N,iK})\, ;\, Z_3(\, c_I\, \tilde{a}_{iI,iK}\, ;\, \overline{C}_I\, \tilde{a}_{iI+1,iK}\, ;\, c_N\, \tilde{a}_{N,iK})\, ;\, Z_3(\tilde{n}_{iI}\, h^I_{66}\, \tilde{a}_{iI,iK}\, ;\, \tilde{n}_{iI+1}\, h^I_{77}\, \tilde{a}_{iI+1,iK}\, ;\, \tilde{n}_N\, h^I_{88}\, \tilde{a}_{N,iK})]$$

$$\hat{Z}_{I,K+1} = Z_3\, [Z_3(\overline{R}_I\, H_{iI,iK+1}\, ;\, \tilde{R}_I\, H_{iI+1,iK+1}\, ;\, b_N\, H_{N,iK+1})\, ;\, Z_3(\, c_I\, \tilde{a}_{iI,iK+1}\, ;\, \overline{C}_I\, \tilde{a}_{iI+1,iK+1}\, ;\, c_N\, \tilde{a}_{N,iK+1})\, ;\, Z_3(\tilde{n}_{iI}\, h^I_{66}\, \tilde{a}_{iI,iK+1}\, ;\, \tilde{n}_{iI+1}\, h^I_{77}\, \tilde{a}_{iI+1,iK+1}\, ;\, \tilde{n}_N\, h^I_{88}\, \tilde{a}_{N,iK+1})] \qquad \text{(A-3)}$$

In Eqs.(A-1) and (A - 3), the following constants are defined,

$a_I = h^I_{13} / M_{iI}$, $\overline{a}_I = h^I_{24} / M_{iI+1}$

$b_I = h^I_{33} / M_{iI}$, $\overline{b}_I = h^I_{44} / M_{iI+1}$

$b_N = h^I_{55} / M_N$

$c_I = K_{iI}\, h^I_{66}$, $\overline{C}_I = K_{iI+1}\, h^I_{77}$

$c_N = K_N\, h^I_{88}$

$\xi_{I3} = \cos\delta^o_{iI,iI+1}$, $\zeta_{I3} = |\sin\delta^o_{iI,iI+1}|$

$\hat{h}_I = Z_2\, (h^I_{13}\, ;\, h^I_{33})$, $\overline{R}_I = \hat{h}_I / M_{iI}$

$\breve{h}_I = Z_2\, (h^I_{24}\, ;\, h^I_{44})$, $\tilde{R}_I = \breve{h}_I / M_{iI+1}$

$D_j = (A_{jN}\, B_{jN} + \tilde{A}_{jN}\, G_{jN})$

$\tilde{D}_j = |A_{jN}\, G_{jN} - \tilde{A}_{jN}\, B_{jN}|\, \zeta_j$

$\tilde{G}_j = |\hat{E}_{qN}\, B_{jN} + \hat{E}_{dN}\, G_{jN}|\, \zeta_j$

$\overline{g}_j = |A_{jN}\, G_{jN} + \tilde{A}_{jN}\, B_{jN}|\, \zeta_j$

$\tilde{N}_j = |\hat{E}_{dj}\, G_{jN} + \hat{E}_{qj}\, B_{jN}|\, \zeta_j$

$\xi_j = \cos\delta^o_{j,N}$, $\zeta_j = |\sin\delta^o_{j,N}|$

$\hat{D}_j = (A_{jN}\, B_{jN} - \tilde{A}_{jN}\, G_{jN})\, \xi_j$

$\breve{G}_j = |\hat{E}_{dN}\, B_{jN} - \hat{E}_{qN}\, G_{jN}|\, \xi_j$

$\breve{g}_j = |\hat{E}_{dj}\, B_{jN} - \hat{E}_{qj}\, G_{jN}|\, \xi_j$, $j \in \mathbf{J}_I$

$n_j = \hat{E}_{qj}\, G_{jj}$, $\rho^*_j = \mu_j / \alpha_j$

$\tilde{n}_j = (K_{Ej}\, X'_{dj} / T_{Ej}\, T'_{doj}\, \eta_j)$

$\rho_j = 1.0 / (\eta_j\, T_{Ej}\, T'_{doj})$, $j \in \mathbf{J}_{IN}$

$m_{k,j} = |A_{k,j}\, G_{k,j} - \tilde{A}_{k,j}\, B_{k,j}|\, \zeta_{I3}$

$\tilde{T}_j = [\Lambda^*_j + (1 / M_j\, \rho^*_j)]$

$g_{k,j} = A_{k,j}\, |G_{kj}|$, $\hat{g}_{k,j} = \tilde{A}_{k,j}\, B_{kj}$

$\hat{d}_{k,j} = A_{k,j}\, B_{k,j}$, $\overline{e}_{k,j} = \tilde{A}_{k,j}\, G_{k,j}$

$e_{k,j} = |\hat{E}_{qj}\, G_{k,j} - \hat{E}_{dj}\, B_{k,j}|\, |\cos\delta^o_{k,j}|$

$\breve{e}_{k,j} = |\hat{E}_{dk}\, B_{k,j} - \hat{E}_{qk}\, G_{k,j}|$

$\overline{G}_{k,j} = |\hat{E}_{dj}\, G_{k,j} + \hat{E}_{qj}\, B_{k,j}|\, \zeta_{I3}$, $k \neq j,\, k, j \in \mathbf{J}_I$

$\tilde{U}_{k,j} = Y_{k,j}\, (\hat{E}_{qj}\, \zeta_{k,j} + |\hat{E}_{dj}|\, \xi_{k,j})$

$U_{k,j} = Y_{k,j}\, \text{Max}\, (\hat{E}_{qk}\, |\hat{E}_{dj}|\, ;\, |\hat{E}_{dk}|\, \hat{E}_{qj})\, \zeta_{k,j}$

$\overline{U}_{k,j} = [\, U_{k,j} + Y_{k,j}\, A_{k,j}\, \xi_{k,j}]$

$\tilde{a}_{k,j} = Y_{k,j}\, \xi_{k,j}$, $H_{k,j} = Y_{k,j}\, \hat{E}_k$, $k \in \mathbf{J}_{IN}$, $j \notin \mathbf{J}_{IN}$

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Received October 26, 1999
Accepted May14, 2000