Performance evaluation of frequency hopping spread spectrum due to co-channel interference

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An investigation into the performance analysis of frequency-hopped spread spectrum systems is presented. The study is instigated by the increased interest in the design and performance analysis of these systems, which are inherently capable of anti-interference. A mathematical expression for P_e , which represents a quantified measure for system performance under various operating configurations, has been introduced. The accuracy of the approximation is checked by computer simulation. It exhibits the quantified reduction of co-channel interference effect.

يعرض البحث تحليل الأداء للقفز الترددي في منظومات الطيف المنتشر نظراً للأهمية المتزايدة لتحليل التصميم والأداء لهذه المنظومات ذات القابلية الذاتية على تفادي التداخل. لقدتم التوصل إلى تعبير رياضي لاحتمالية الخطأ والذي يمثل مقياساً كمياً لجودة الأداء لمختلف تشكيلات التشغيل. كما جرت محاكاة المنظومة بالحاسب للتأكد من سلامة التقريب للدقة المطلوبة حيث تبين بوضوح الانخفاض الكمي لتأثير التداخل بين القنوات التي تعمل على نفس التردد.

Keywords: Mobile communications, Channel Interference, Digital communications.

1. Introduction

Within both the commercial and military arenas, there has been an increased interest in the design and performance analysis of spread spectrum communication systems in general, and frequency-hopped systems in particular, because of their inherent anti-interference and low probability of intercept characteristics. Direct sequence (DS), frequency hopping (FH) and hybrid, DS/FH, schemes have been proposed for such applications [1-8].

Most interference encountered in digital communications systems is introduced in the transmission of the information-bearing signal through the channel. The characteristics of the interference depend to a large extent on its origin. It may be categorized as being either broad band or narrow band relative to the bandwidth of the information bearing signal, and either continuous in time or pulsed in time. If it is broad band, it may be characterized statistically as an equivalent additive white Gaussian noise with power spectral density J_0[9].

The anti-interference capabilities of various fast frequency-hopped (FFH) M-array frequency shift keying (MFSK) systems have been subject for quite a few publications [10,11] and the references therein.

The purpose of this study is to conduct performance analysis for a fast frequency-hopped spread spectrum subject to the threat of wide band interference due to other users generated by the network (i.e., Co-channel and adjacent channel interference) and wide band Gaussian noise.

A typical approach of implementing a FHSS system is to use frequency shift keying (FSK) for a base band modulation. For BFSK and BFSK are not defined, the symbol rate R_s and the bit rate R_b are assumed equal. The Interval during which the carrier stays in a specific frequency is known as dwell interval. Further, let s be the number of symbols transmitted in a dwell interval, then R_s=s.R_h ,where R_h is the hopping rate, and hop dwell time $T_h=1/R_h$ with received energy per hop E_h=s T_h. For arbitrary base band data modulation, the FH carrier partitions W_{ss} into N_c contiguous, non-overlapping channels, each with bandwidth $W_m=W_{ss}/N_c$, such that the entire signal energy in any hop lies within a single channel. The chip rate R_c is given by the maximum of R_h and R_s. Now suppose that s(t) is, an FH with SS bandwidth W_{ss} for an M-array FSK system, can be divided into $N_b=W_{ss}/M/R_c$ channels. For BFSK each of these channels is further divided into two sub-channels to transmit 1 and 0 respectively.

In a FHSS system, in addition to channel noise, the detection of a specific bit is also affected by another signal which is simultaneously transmitted at the same frequency, i.e., co-channel. This phenomenon is known as a hit.

2. Analytical channel model

A few years ago, propagation measurements were performed to determine UHF propagation characteristics in and around office building [12] and in factories [13]. These measurements show that the indoor channel in buildings is a Rayleigh channel, whereas the propagation in factories is Rician.

In this study concentration only is on the consideration of Rayleigh channel model. The relation between the transmitted signal s(t) and received signal r(t) for N-path interference channel can be established as follows:

$$r(t) = \sum_{j=0}^{N-1} \beta_j s(t - t_j) + n(t) , \qquad (1)$$

where

$n(t)$ represents the channel noise, and is assumed to be additive white Gaussian noise (AWGN) with two-sided power spectral density $N_o/2$,

β_j is the strength of the j-th path, and

t_j is the relative delay between the j-th path and the zero path, i.e., direct path.

$$s(t) = \sin\left(2\pi f_{i,n} t\right), \quad i = 0,1 \text{ and } n = 1, \ldots, N_b . \qquad (2)$$

Furthermore, β_j, t_j are assumed independent random variables. $\beta_0, \beta_1, \ldots \beta_{N-1}$ are independent and uniformly distributed random variables.

For analytical simplicity, it is assumed that the receiver observation interval (0,T) contains an integer number $N_h=T/T_h$ of complete hops. Also, fading is assumed slow in the sense B_j doesn't change significantly within a bit duration T_b. Finally, each user hops among the N_b channels in a random fashion, whereas in practice, hopping can be controlled by a pseudo-random number generator.

Let $B_{t-N}, \ldots, B_{t-2}, B_{t-1}, B_t, B_{t+1}, B_{t+2}, \ldots, B_{t-N}$ be the transmitted bit sequence over the channel in which B_t denotes the bit currently being received and processed by the receiver.

The interference considered in this analysis is that due to B_{t-1} or that due to a different user operating at the same frequency, i.e. co-channel interference. In other words, is the interference that results from the simultaneous transmission of a bit from a different user at the same frequency. Consequently, the detection of B_t fails if its value is reversed jointly by the channel noise and a signal hit, or if it is hit by at least two interfering signals.

A simplified block diagram of the receiver is shown in Fig. 1. Let B_{ic} (0) to denote the value of B_{ic} of the desired signal which arrives with no interference, then

$$B_{ic}(0) = \int_0^{T_b} \beta_o \sin(\omega_{i,n} t + \theta_o) \times \sqrt{\frac{2}{T_b}} \cos(\omega_{i,n} t)\, dt = \sqrt{\frac{2}{T_b}} \beta_o \sin\theta_o \qquad (3)$$

Similarly,

$$B_{is}(0) = \sqrt{\frac{T_b}{2}}\, \beta_o \cos\theta_o . \qquad (4)$$

In (2) and (3), θ is assumed uniformly distributed over (0, 2π), and t_0 is also assumed equals 0 for simplicity.

Further, let B_{ic} (N) to denote the value of B_{ic} due to channel noise with a zero-mean Gaussian random variable, then

$$B_{ic}(N) = \int_0^{T_b} n(t) \sqrt{\frac{2}{T_b}} \cos\left(\omega_{i,n} t\right) dt . \qquad (5)$$

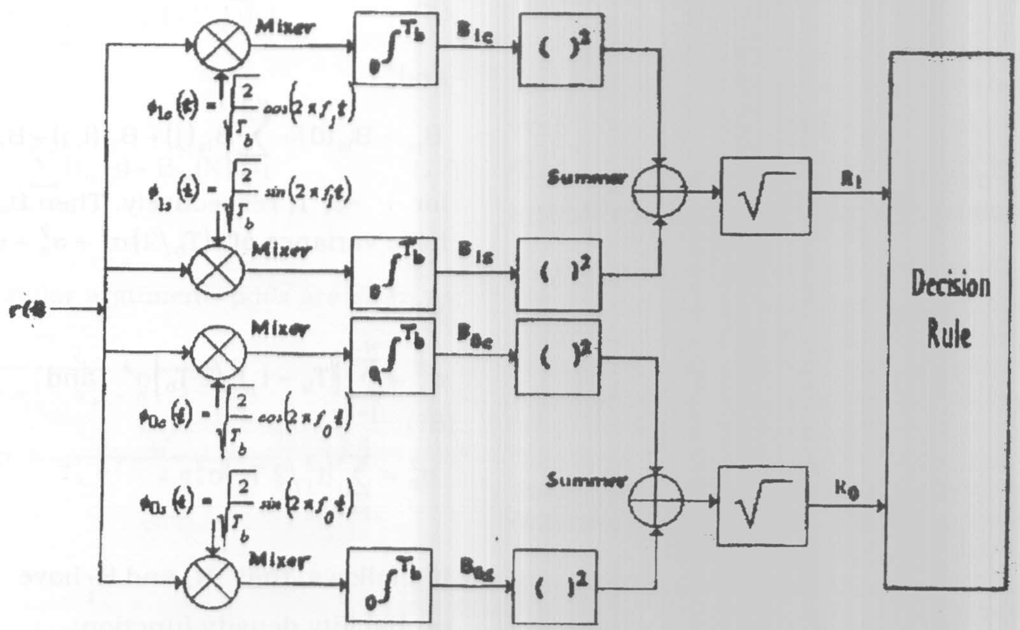

Fig. 1. Receiver block diagram

Similarly, the contribution from the j-th path signal due to interference from other users is

$$B_{ic}(j) = \int_0^{T_b} \beta_j \sin\left(\omega_{i,n} t + \theta_j\right) \times \sqrt{\frac{2}{T_b}} \cos\left(\omega_{i,n} t\right) dt \qquad (6)$$

$$= \sqrt{\frac{\left(T_b - t_j\right)^2}{2\,T_b}}\ \beta_j \sin\ \theta_j .$$

Consequently,

$$B_{is}(j) = \sqrt{\frac{\left(T_b - t_j\right)^2}{2\,T_b}}\ \beta_j \cos\ \theta_j . \qquad (7)$$

From the fundamentals of probability theory, it is know that if β is a Rayleigh random variable with mean $\sqrt{\frac{\pi}{2}}\sigma$, second moment $2\,\sigma^2$ and if θ is a random variable uniformly distributed over $(0$ to $2\pi)$ then $\beta \cos\theta$ and $\beta \sin\theta$ are two independent Gaussian random variables with zero mean and variance σ^2 [14]. Furthermore, if $X_1 \ldots X_n$ are independent Gaussian random variables for which X_i has mean $\overline{X_i}$ and variance σ_i^2, then $X_1 \ldots X_n$ is a Gaussian random variable with mean $\overline{X_1} + \cdots + \overline{X_n}$ and variance $\sigma_1^2 + \cdots + \sigma_n^2$. Hence, it can be concluded that each of $B_{ic}(0), B_{is}(0), B_{ic}(j)$, and $B_{is}(j)$ is a zero-mean Gaussian random variable or zero, and B_{ic}, and B_{is} are both zero-mean Gaussian random variables.

The remainder of this paper is concerned with performance evaluation of slow frequency hopped spread spectrum signals due to wide band interference, which has not been previously analyzed in the open literature. Particular emphasis is given below to different possible scenarios of co-channel interference.

Case I: The transmitted information bit (B_t) which is currently being received and processed by the receiver is assumed not to be hit by its previous bit or signals from other users.

$$B_{ic} = B_{ic}(0) + \sum_{j=1}^{N-1} B_{ic}(j) + B_{ic}(N) \qquad (8)$$

$$i = 0, 1$$

$$B_{is} = B_{is}(0) + \sum_{j=1}^{N-1} B_{is}(j) + B_{is}(N) \qquad i = 0, 1 \tag{9}$$

Notice that $B_{ic}(0)$ and $B_{is}(0)$ both have variance of $(T_b/2)\sigma^2$, and $B_{ic}(j)$ and $B_{is}(j)$ both have variance of $(T_b - t_j)^2/2T_b\,\sigma^2$. Thus B_{ic} and B_{is} both have variance of $(T_b/2)\sigma^2 + \sigma_a^2 + N_o/2$ where

$$\sigma_a^2 = \sum_{j=1}^{N-1}\left[(T_b - t_j)^2/2T_b\right]\sigma^2 .$$

Notice further that since B_{ic} and B_{is} are independent zero-mean Gaussian random variable with the same variance, R_i is a Rayleigh random variable with probability density function;

$$f_{Ri}(r_i) = \begin{cases} \dfrac{r_i}{\frac{T_b}{2}\sigma^2 + \sigma_a^2 + N_o/2} \exp\left[-\dfrac{r_i^2}{T_b\sigma^2 + 2\sigma_a^2 + N_o}\right], & r_i \geq 0 \\ 0, & r_i < 0. \end{cases} \tag{10}$$

Consequently errors occur when $r_{\bar{i}} > r_i$ i.e.,

$$P_e(t_1, t_2, \ldots t_{N-1}) = \int_0^\infty f_{Ri}(r_i) \int_{r_i}^\infty f_{R\bar{i}}(r_{\bar{i}})\, dr_{\bar{i}}\, dr_i = \frac{N_o}{T_b\sigma^2 + 2\sigma_a^2 + 2N_o} \tag{11}$$

Case II. The transmitted information bit (B_t) which is currently being received and processed by the receiver is assumed to be hit by it previous bit (B_{t-1}).

The condition for this to occur is both B_t and B_{t-1} are transmitted on the same frequency channel, i.e, memory channel as opposed to memoryless channel. When both $B_t = B_{t-1} = i$ have the same value, then

$$B_{ic} = B_{ic}(0) + \sum_{j=1}^{N-1} B_{ic}(j) - B_{ic}(i,j) - B_{ic}(N) \tag{12}$$

and

$$B_{is} = B_{is}(0) + \sum_{j=1}^{N-1} B_{is}(j) + B_{is}(i,j) + B_{is}(N), \tag{13}$$

for i =0, 1, respectively. Then B_{ic} and B_{is} both have variance of $(T_b/2)\sigma^2 + \sigma_a^2 + \sigma_b^2 + N_o/2$

where ,

$$\sigma_a^2 = \sum_{j=1}^{N-1}\left[(T_b - t_j)^2/2\,T_b\right]\sigma^2 \text{ , and}$$

$$\sigma_b^2 = \sum_{j=1}^{N-1}\left(t_j^2/2\,T_b\right)\sigma^2.$$

It follows that R_i and $R_{\bar{i}}$ have the following probability density function;

$$f_{R_i}(r_i) = \begin{cases} \dfrac{r_i}{\frac{T_b}{2}\sigma^2 - \sigma_a^2 - N_o/2} \exp\left[-\dfrac{r_i^2}{T_b\sigma^2 + 2\sigma_a^2 - 2\sigma_b^2 + N_o}\right], & r_i \geq 0, \\ = 0, & r_i < 0 \end{cases} \tag{14}$$

and

$$fR_{\bar{i}}(r_{\bar{i}}) = \begin{cases} \dfrac{r_{\bar{i}}}{N_o/2} \exp\left[-\dfrac{r_{\bar{i}}^2}{N_o}\right], & r_{\bar{i}} \geq 0, \\ 0, & r_{\bar{i}} < 0. \end{cases} \tag{15}$$

Consequently,

$$P_e(t_1, t_2, \ldots t_{N-1}) = \frac{N_o}{T_b\sigma^2 + 2\sigma_a^2 + 2\sigma_b^2 + 2N_o}.$$

When B_t and B_{t-1} have different values, $B_{t-1} = \bar{B}_t = i$, then;

$$B_{ic} = B_{ic}(0) + \sum_{j=1}^{N-1} B_{ic}(j) + B_{ic}(N), \tag{16}$$

and

$$B_{is} = B_{is}(0) + \sum_{j=1}^{N-1} B_{is}(j) + B_{is}(N). \tag{17}$$

Following similar argument, pdf's are given as;

$$f_{Ri}(r_i) = \begin{cases} \dfrac{r_i}{\frac{T_b}{2}\sigma^2 + \sigma_a^2 - N_\circ/2} \\ \exp\left[-\dfrac{r_i^2}{T_b\,\sigma^2 + 2\,\sigma_a^2 - N_\circ}\right], & r_i \geq 0, \\ 0, & r_i < 0, \end{cases} \tag{18}$$

and

$$f_{R_i}(r_i) = \begin{cases} \dfrac{r_i}{\sigma_b^2 + N_\circ/2} \exp\left[-\dfrac{r_i^2}{2\,\sigma_b^2 - N_\circ}\right], & r_i \geq 0, \\ = 0, & r_i < 0. \end{cases} \tag{19}$$

Consequently,

$$P_e = \frac{2\ \sigma_b^2 + N_\circ}{T_b\ \ \sigma^2 + 2\ \ \sigma_a^2 + 2\,\sigma\ _b^2 + 2\,N_\circ} \tag{20}$$

The average error probability P_e, can be derived only for $s \leq 1$ (mainly, due to practical consideration and mathematical simplicity), Where s has been used to denote the number of bits transmitted in a dwell interval, i.e., during each hop at most one bit is transmitted.

If $s < 1$, that is, if the transmission of each bit takes $1/s$ hops, then $T_s = s \times T_b$. The detection of a bit can be done through majority decision rule using the $1/s$ symbols which have been received [15]. Further, let P'_e be the probability that any of these $1/s$ symbols is erroneously detected. Then the actual bit error probability is

$$P_e = \sum_{i=0}^{1/2s} P_e'^{\,1/s-i}\,(1 - P'_e)^i \tag{21}$$

3. Results and discussions

Slow frequency hopping spread spectrum modulation has been claimed to be an efficient technique of coping with co-channel interference. To demonstrate this, it is best to examine the performance of frequency hopped spread spectrum scheme when no co-channel interference exists.

Firstly, when $s \geq 1$, one or more bits are transmitted in a hop, then, except for the very first bit, the detection for each of the following s-1 bits fits within the second scenario discussed in section II. As for the detection of the very first bit is concerned, it is the second scenario if the very last bit of the previous hop and the very first of the current hop have the same value. Otherwise, it is within the first scenario. Hence, the probability of erroneous detection is

$$P_e = \frac{1}{s}\left(\underbrace{\frac{N_b - 1}{N_b} P_e}_{\text{1st case}} + \underbrace{\frac{1}{N_b} P_e}_{\text{2nd case}} + \underbrace{(s-1) P_e}_{\text{2nd case}}\right) \tag{22}$$

Fig. 2, shows P_e Versus $s \geq 1$ under various SNR, when there is no interference and N=5. It is observed that when too many bits are transmitted in one hop, the performance deteriorates very rapidly. Secondly, when s<1 , the result is the same as that obtained in (21).

Fig. 3, Shows P_e versus SNR under several values of N, when there is no co-channel interference and s=1/3. It is observed clearly the advantage of using a fast frequency hopping system.

Fig. 4, Shows P_e versus SNR under various values of M and N, while the value of s is taken to be one. The results provided are obtained from computer simulations to support the analytical treatment introduced in this paper. It should be noticed that M+1 represents the total number of users which are simultaneously sharing the communication channel.

Form Fig. 4, it is evident that when SNR is small, large N gives lower P_e. This conclusion can be explained as follows; when SNR is low, the detection is affected mainly by the channel noise. Although the detection is also affected by the hit from a previous bit, the hit occurs rarely when the number of hopping channel is large enough. In other words, most of the time, the detection is dominated only by the channel noise. The co-channel interference signal of the bit in this case tends to reduce the influence from the channel noise. This phenomenon is also observed and reported by Moeneclaey and Wang [16]. On the other hand, when SNR is high, the channel noise does not exist, and errors in detection occurs only when a hit occurs. Hence, more intersymbol interference is introduced when N is higher.

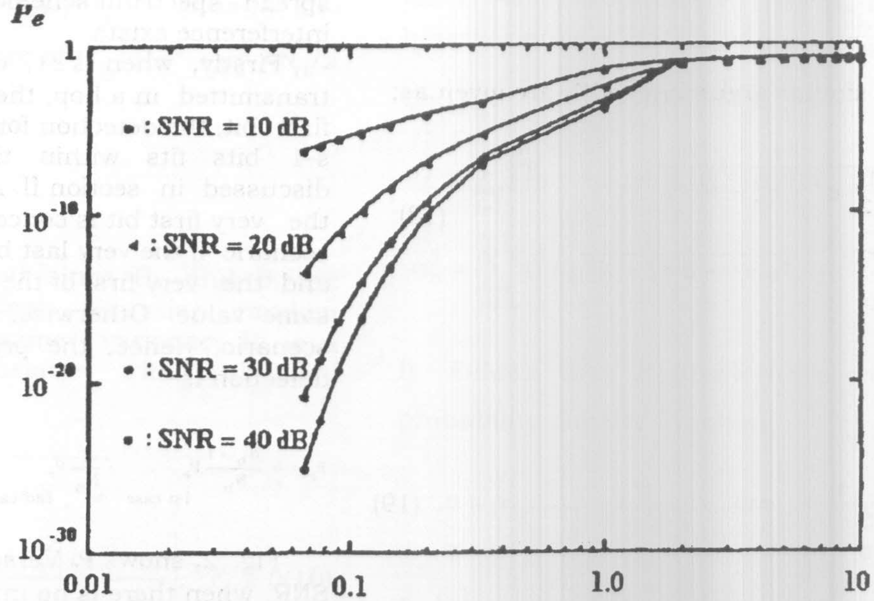

Fig. 2. P_e versus S for N = 5 and M = 0.

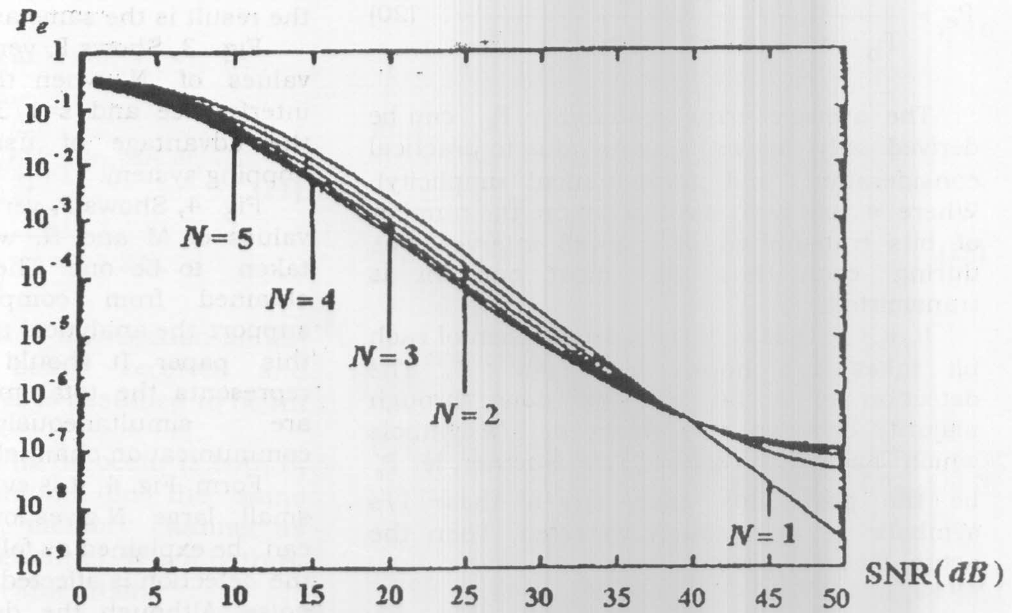

Fig. 3. P_e versus SNR for S = 1/3 and M = 0.

Another conclusion which is observed in Figure 4, is the accuracy of the approximation worsens as M increases. This can be explained in reference to the calculation of P_e, when a bit is hit by two or more users, its detection is always considered as erroneous, while in fact, the detection may have chance to be correct (0.5).

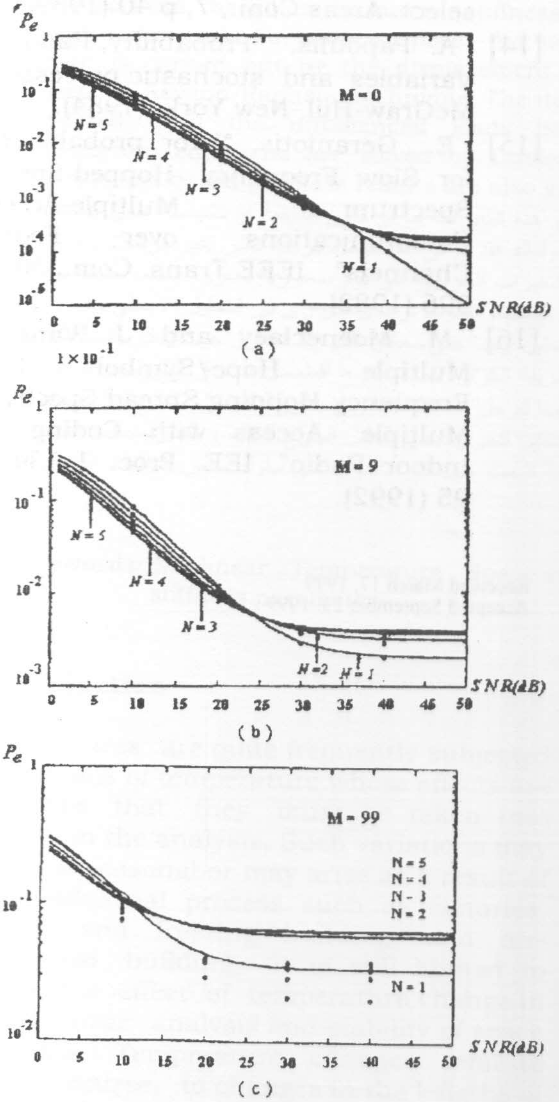

Fig. 4. Pe versus SNR under various M values with S=1.

Figure 5, presents a result of fast frequency hopping scheme under co-channel interference. It is observed much better performance for fast hopping. However, this improvement is obtained at the expense of wider bandwidth and more sophisticated synchronization.

4. Conclusions

In this paper, the performance of a frequency hopped spread spectrum system when co-channel interference exists has been studied. The measure of performance employed is the probability of erroneous detection P_e. The major contribution is the derivation of mathematical expressions for P_e for several different scenarios of system parameters.

Rayleigh fading is assumed to model the channel, though exist other possibilities such as Rician and Nakagmi.

The results obtained in the paper show some degree of inaccuracy when compared with computer simulation, as the number of interfering users becomes too large. Otherwise the results obtained should serve satisfactorily.

Most importantly, a fundamental conclusion is that, frequency hopping spread spectrum, preferably fast hopping, indeed is an efficient scheme in coping with co-channel interference.

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