Finding the optimum vehicle replacement policy: a parametric dynamic programming model

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After a certain period of the vehicle age, its elements deteriorate with time, which make the cost of still operating the vehicle monotonically increasing. The logic decision is to determine properly when to replace the current vehicle with another new one. In this paper, the Dynamic Programming (DP) technique is used to solve the practical problem of vehicle replacement. As a real application, the DP technique is used to find the optimum replacement policy for TATA and NASR busses at El-Minia University in the time horizon (1982/1983 to 2002/2003) with the objective to minimize the total cost over the whole planning period. The results of a parametric study is applied on the practical problem to investigate the effect of changing the data elements corresponding to the income, maintenance, and replacement costs on the obtained optimal replacement policy. The parametric study reveals that a subset of those elements can change freely without affecting the obtained optimal policy, while another subset will affect that optimal policy.

إحلال المركبة بأخرى جديدة يحتاج إلى استثمار كبير، ومن هنا فانه من المنطقي التفكير في تحديد الوقت المناسب لإحلال المركبة بأخرى جديد'.

فى هذا البحث تم استخدام أسلوب البرمجة الديناميكية لا يجاد الحل الأمثل لإحلال المركبات فى أحد التطبيقات الواقعية وهو الأتوبيسات المستخدمة فى جامعة المنيا من النوعين تاتا و نصر خلال الفترة الزمنية من العام الجامعي ١٩٨٢/١٩٨٣ إلى العام الجامعي ٢٠٠٢/٢٠٠٣. والهدف من هذه الدراسة هو تقليل التكاليف الكلية لاستخدام الأتوبيسات خلال الفترة الزمنية المخططة.

وقد تم تطبيق نتائج الدراسة الوسيطة (البارامترية) على المشكلة الواقعية وذلك لبحث تأثير التغير في عناصر البيانات المناظرة للعائد المادي من تشغيل الأتوبيسات، وتكاليف الصيانة، وتكاليف الإحلال على سياسة الإحلال المثلى. وقد أظهرت نتائج هذه الدراسة أنه يمكن تقسيم عناصر البيانات إلي مجموعتين الأولى ذات تأثير على سياسة الإحلال المثلى والثانية لا يؤثر تغيرها على هذه السياسة مما يشكل مرونة كبيرة بالنسبة لمتخذ القرار في مثل هذه المشكلة.

Keywords: Dynamic programming, Parametric study, Replacement problem, Vehicle replacement, and Bus operating cost.

1. Introduction

During a vehicle operating-lifetime, some elements are consumed or damaged. There fore, a periodical maintenance, repair, half-overhaul, or full-overhaul is needed. The planned maintenance procedure is carried at known intervals through the vehicle-operating lifetime. The actual effective vehicle lifetime depends on the maintenance procedure, which keep quality for using the vehicle. Due to the vehicle aging, the operating efficiency of its equipment decreases, which will be reflected in increasing the operation and maintenance costs. Moreover, the income charges of the vehicle decreases continually due to the increase of out of working periods. Therefore, in a proper time, replacement process must be carried out for the old vehicle with a new one [1].

To find the optimal time to replace the current vehicle by a new one, the replacement problem needs to be solved. The survey of literatures indicates that the most famous methods are the enumeration, the shortest path, the regeneration point approach, Markov chain, zero-one integer programming, and the dynamic programming technique [2,3, 4].

Kobbacy et al. [5] presented rent models which are a useful class of replacement models that can be used to study the replacement of capital equipment over a long period of time. A method based on Markov chain to simulate the interest rate and discount factor is presented to quantify the sensitivity of rent applied to commercial vehicles. The effect of changing parameters such as the capital cost, maintenance cost, resale value, and tax parameters on the optimal replacement age is indicated.

Karrabakal et al. [6] described a parallel replacement problem in which the economic interdependence among assets is caused by limited funds. They formulate the problem as a 0-1 integer program and develop a branch and bound based on the Lagrangian relaxation methodology. This paper concentrates a special type of replacement problems in which the cash flows of a future asset do not depend on the service conditions nor on the previous replacement decisions.

Sheu et al. [7] proposed a generalized policy where a system has two types of failures and is replaced at the n-th type I failure (minor failure) or first type II failure (catastrophic failure) or at age T, whichever occurs first. Type I failure is removed by minimal repairs, the cost rate of that repairs can be estimated.

It is shown that the dynamic programming technique can solve the replacement problem for large number of stages (planning periods) with small effort than the other methods [8]. That research involved a comparison between the various methods that can solve the replacement problem, it indicates that the dynamic programming technique can be considered as the most suitable technique for large scale problems from point of view of the solution time.

In Ref. [9], a dynamic programming approach is used to determine the optimal vehicle replacement policy that minimizes the total costs of replacement, operation, and maintenance. A parametric study is introduced to reveal the effect of changing some problem parameters on the optimal solution without the need to resolve new problems from the beginning. The parametric study shows that the set of problem data elements can be divided into two subsets. The first may change freely without affecting the obtained optimal policy, while the change in the other subset will affect the optimal replacement policy.

The real application in this paper is to find the optimal replacement policy for TATA and NASR busses at El-Minia University garage in the period from (1982/1983 to 2002/2003). The parametric study is applied to investigate the effect of changing individual problem data element and also the effect of changing the income, maintenance, and replacement cost rows on the obtained optimal replacement policy.

2. The practical vehicle replacement problem

The El-Minia University garage has 99 vehicles of different types and models, which includes busses, passenger cars, trucks, and tractors. Of these vehicles, there are 19 busses of two types: 16 TATA, and 3 NASR. TATA busses were introduced in 5-groups at different years, and NASR busses were added in 1982. Data collected are the vehicle purchase prices at different vehicle ages, the income charges, the fuel and oil consumption, and the price of maintenance operations.

Due to the government decision of minimizing the imports, TATA was not imported after the year 1986, and old busses had to be replaced by NASR ones. The problem is studied over 20 years, which is estimated as the life-period for the vehicles. The last group of TATA busses had been bought at 1982, so that the replacement study is started at 1983 with 1-year old bus.

The collected data about the income is available through 10-years starting from (1985/1986) to (1995/1996). When plotting these values, it is found that they represent an exponential function of the form: $y = A. (B)^x$, where the values of the constants A and B differ from one group of data to another. The income of the new and used busses through the planning period at different ages of the vehicles TATA&NASR are shown in Table 1. In this table, (B.A.) denotes the Bus Age in years, and (P.Y.) denotes the Policy Year, the upper numbers in the cells of the table are the income of TATA busses, and the lower numbers are the income of NASR busses in (L.E. x 10^3).

Operating and maintenance costs include the fuel and oil consumption, maintenance cost, repair cost, and chassis and engine overhauls. These costs are called the variable costs. The prices for the variable costs are gathered for each bus [10], and busses of the same type and age are collected, and the mean value is obtained to represent that class of busses.

It is found that the cost of overhaul, tires, and batteries is very large compared with the maintenance cost. Overhaul is done every 3 years for TATA busses, so their costs are shared for that period of time. Overhaul is done every year for NASR busses and its price is added to the total cost. There are other costs which are not affected by the bus operation, such as the insurance tax, the benefit prices, perishing values, workers and drivers salaries, management expenses,...etc., these are the fixed costs. The total operating costs represent the sum of all the fixed and variable costs. These total operating costs for TATA and NASR busses are shown in Table 2. In this table, the upper figures are for the TATA busses, while the lower figures are for the NASR busses and all figures are in (L.E. x 10^3).

The replacement cost at any year means the difference between prices of new and used bus at this year. The prices of new TATA busses are available only for the period of 1978 to 1985, while the prices for other years are calculated from the trend of the corresponding curve. Prices of new NASR busses are available for the period 1982 to 1992, while the values for other years are also predicted from the extension of the corresponding curve. The estimation of a used bus price depends on the opinion, experience, and records of the garage engineers. The estimated price values for the used TATA and NASR busses are presented in Table 3. In this table, the upper values are for TATA busses, and the lower values are for NASR busses. Since the last new TATA busses were imported at 1986, then at 1987, we shall have only 1-year old busses or older. In 1988 we shall have only 2-years old busses or older, and so on for other years. Prices of new busses are shown in Table 4, figures at this table are in (L.E x 10^3).

3. The optimal replacement policy

According to the backward dynamic programming procedure, the total cost at stage j and the preceding stages for keep decision is calculated as follows:

$$F_j(X_j) = M_j(X_j) - I_j(X_j) + F_{j+1}(X_{j+1}).$$

And that for the replacement decision is:

$$F_j(X_j) = M_j(0) - R_j(X_j) - I_j(0) + F_{j+1}(1),$$

where:

F is the total cost,
M is the maintenance cost,
R is the replacement cost,
I is the income,
j is the stage number (specific year in the plan), and
X_j is the age of the vehicle at year j.

Thus, the problem recursive equation can be written as to minimize $F_j(X_j)$, in:

$$F_j(X_j) = M_j(X_j) - I_j(X_j) + F_{j+1}(X_{j+1}),$$ for keep decision, and

$$F_j(X_j) = M_j(0) - R_j(X_j) - I_j(0) + F_{j+1}(1),$$ for replace decision.

At any stage j, j=1, 2,..., 20, the available state variables can be obtained by substituting: $X_j = 1,2,...j-1+y$. Where y denotes the age of the bus at the beginning of the first stage.

A FORTRAN program module based on the backward procedure is written to solve the problem on personal computer. The algorithm of the standard backward procedure for the dynamic programming is explained in references numbers [2,11]. The obtained optimal policy for this problem indicates that the replacement decision should be performed only once after twelve years of operation, the total cost in this case is L.E. 311412. Table 5

Table 1 The income of busses type TATA and NASR (in L.E.x10³).

B.A P.Y.	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1983- 1984	2630 2700	2600 2500																			
1984- 1985	2750 2750	2680 2690	2600 2500																		
1985- 1986	3000 3030	2850 2880	2750 2730	2680 2580																	
1986- 1987	3200 3150	3100 3100	3000 3000	2750 2750	2900 2600																
1987- 1988	3300 3280	3220 3210	3170 3170	3115 3015	3078 2878	3123 2623															
1988- 1989	3450 3500	3380 3420	3311 3331	3275 3175	3236 3036	3178 2878	3082 2682														
1989- 1990	3800 3600	3750 3450	3660 3360	3588 3288	3453 3153	3391 2991	3288 2788	3186 2586													
1990- 1991	4150 3950	4000 3800	3960 3660	3854 3554	3743 3443	3665 3265	3554 3054	3443 2843	3332 2632												
1991- 1992	4595 4195	4471 4071	4350 3950	4222 3832	4118 3718	3998 3598	3822 3322	3718 3218	3598 3098	3359 2859											
1992- 1993	4520 4220	4400 4100	4300 4000	4200 3900	4060 3760	3962 3662	3850 3550	3781 3481	3650 3250	3400 3000	3140 2840										
1993- 1994	6750 6250	6525 6125	6300 5000	6086 4886	5771 4771	5506 4606	5344 4444	5161 3961	4606 3506	4261 3261	3922 2922	3698 2698									
1994- 1995	7200 7000	6900 6700	6621 5521	6316 5316	5960 5160	5618 5018	5338 4838	5011 4211	4761 4061	4511 3811	4398 3698	4246 3446	4005 3005								
1995- 1996	7500 7500	7050 7050	6700 5900	6488 5788	6030 5630	5773 5473	5473 5273	5277 4777	5017 4517	4825 4325	4668 4168	4430 3930	4365 3665	4198 3498							
1996- 1997	8000 8000	7800 7500	7500 5400	7305 6305	7002 6102	6771 5971	6449 5749	6159 5459	5890 5290	5515 5015	5339 4949	5051 4651	4760 4460	4582 4282	4200 4000						
1997- 1998	9000 8600	8840 8040	8570 6970	8270 6770	7960 6460	7618 6318	7271 6171	7000 5800	6822 5522	6538 5238	6197 5197	5752 5000	5664 4864	5001 4701	4600 4400	4200 4100					
1998- 1999	9150 8800	8900 8130	8600 7300	8330 6820	8500 6600	7500 6410	7400 6160	7070 5940	6900 5700	6590 5320	6270 5300	5830 5100	5500 4830	5100 4750	4720 4520	4440 4220	4200 3900				
1999- 2000	9300 9000	9000 8200	8700 7500	8470 6870	8105 6705	7718 6518	7471 6271	7159 6059	6979 5879	6638 5438	6297 5397	5892 5192	5401 5001	5201 4801	4800 4600	4650 4350	4500 4100	4350 3950			
2000- 2001	9500 9200	9200 8500	8900 7600	8686 7486	8343 7343	7815 7115	7541 6841	7360 6560	7100 6200	6796 5896	6386 5886	6064 5664	5661 5461	5328 5228	5000 5000	4860 4760	4700 4500	4560 4360	4430 4100		
2001- 2002	9700 9700	9400 9000	9000 8000	8858 8158	8500 7900	8067 7767	7662 7462	7450 7150	7200 6800	6891 6491	6419 6419	6179 6179	5962 5962	5708 5708	5300 5500	5000 5300	4850 5150	4700 4900	4560 4760	4410 4500	
2002- 2003	11000 11000	10000 10000	9300 9000	8987 8887	8650 8650	8477 8477	8137 8137	7720 7720	7400 7400	7025 7025	6998 6998	6739 6739	6507 6507	6232 6232	6000 6000	5700 5800	5300 5600	5000 5400	4800 5200	4650 4950	4500 4700

Table 2 Total operating costs for TATA and NASR busses (in L.E.x10^3).

B.A. P.Y.	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1983- 1984	7109 7009	7659 7249																			
1984- 1985	7349 7359	7909 7569	7979 7779																		
1985- 1986	7579 7529	8279 7749	8479 7979	8629 8219																	
1986- 1987	8049 7779	8629 7999	8779 8259	8899 8519	8979 8849																
1987- 1988	8599 7999	9129 9559	9529 9819	9599 10009	9769 10239	9862 10389															
1988- 1989	9099 8219	9439 9979	9739 10079	9949 10169	10009 10279	10099 10439	10250 10629														
1989- 1990	9379 8429	9659 10029	9889 10179	10049 10229	10129 10389	10169 10519	10289 10739	10462 10999													
1990- 1991	9579 8679	9809 10099	9969 10209	10159 10329	10219 10529	10329 10749	10479 10959	10629 11239	10825 11489												
1991- 1992	9779 8979	9959 10179	10129 10429	10209 10579	10279 10729	10379 10979	10479 11219	11279 11429	11629 11659	12193 11939											
1992- 1993	9989 9229	10179 10489	10239 10799	10359 11030	10569 11410	10899 11679	11189 11880	12179 12099	13120 12309	14204 12529	14957 12819										
1993- 1994	10149 9479	10319 10839	10439 11129	10679 11379	10989 11729	11359 12004	11789 12490	12679 12689	13441 12919	14699 13299	15239 13629	15324 13979									
1994- 1995	10339 9729	10529 11619	10739 11879	11129 12109	11539 12392	11959 12690	12659 13200	13589 13399	14479 13594	15409 13919	15799 14269	16079 14549	16743 14719								
1995- 1996	10649 9959	10969 12320	11129 12429	11629 12609	12139 13179	12759 13489	13479 13900	14299 13999	15109 14279	15919 14599	16179 14949	16529 15329	17179 15579	17463 15829							
1996- 1997	10919 10199	11359 13729	11679 13879	12229 14009	12889 14679	13579 14890	14359 15379	15139 15499	16009 15679	17049 16279	17799 17779	18179 18019	18679 18269	19279 18519	19729 18749						
1997- 1998	11429 10479	11849 14379	12299 14579	12919 14709	13689 15439	14429 15679	15279 16029	16039 16079	16999 16379	17709 16939	18279 18559	18779 18879	19129 21239	19879 23629	20379 26219	20879 28749					
1998- 1999	11929 10779	12359 14879	12989 15379	13629 16379	14469 16879	15239 17179	16069 17479	16879 17579	17789 17879	18429 18379	18789 19879	19729 21379	20139 23066	21628 25259	22079 27649	22579 30159	23179 32759				
1999- 2000	12329 11079	12949 22079	13669 23079	14439 23579	15379 24579	16129 24979	17009 25279	17969 25579	18999 26079	20079 26679	20569 28079	23319 29079	26549 31079	28339 33479	30079 35969	31079 38319	32079 40129	33079 42349			
2000- 2001	13019 11379	13739 26579	14529 27279	15389 27779	16299 28279	17239 28879	18349 29579	19689 30279	21989 31079	26079 31279	27789 32479	30779 33279	32729 35279	33379 38229	34279 40279	35279 42799	36279 45269	37279 47859	38279 50119		
2001- 2002	13809 11779	14689 31879	15579 32479	16449 33079	17519 33479	18749 34279	20229 34879	22179 35479	25569 35979	30129 36679	32169 37779	34129 38479	35679 40479	37179 43829	38279 46079	40129 48479	41479 51079	42479 53469	43979 55919	45479 58229	
2002- 2003	14749 12179	15679 37979	16629 38679	17779 39379	19209 39779	20819 40479	22909 41179	25639 41679	28909 42179	32089 42679	34179 43979	36579 44679	38279 46679	40379 50429	42189 52179	44279 54679	46579 57179	48679 59689	50679 62189	52179 64339	53679 66969

Table 3 Prices of the used TATA and NASR busses (in L.E. 10^3).

B.A. P.Y.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1983- 1984	30 25																			
1984- 1985	34 26	29 21																		
1985- 1986	38 30	33 25	27 20																	
1986- 1987	40 34	36 28	30 22	24 18																
1987- 1988	42 38	38 33	33 29	28 24	22 19															
1988- 1989	42	43 37	38 32	34 26	29 20	23 17														
1989- 1990	55	50	42 44	36 38	31 31	27 24	22 20													
1990- 1991	88	80	72	41 64	35 55	30 48	26 40	21 30												
1991- 1992	115	105	90	80	40 71	34 60	30 50	25 40	20 29											
1992- 1993	130	120	108	100	88	41 77	35 68	30 56	25 45	20 32										
1993- 1994	140	131	120	108	96	45 83	40 70	35 60	30 48	25 37	20 30									
1994- 1995	150	140	131	120	109	99	88	78	65	53	41	30								
1995- 1996	155	145	135	124	113	105	96	87	77	69	60	52	40							
1996- 1997	161	154	145	136	127	119	110	102	90	80	71	60	49	38						
1997- 1998	165	158	149	140	132	124	117	110	103	96	88	80	72	63	50					
1998- 1999	178	165	155	146	137	130	121	113	105	99	91	83	75	67	55	45				
1999- 2000	180	170	161	152	143	134	125	117	109	102	94	88	80	72	65	55	46			
2000- 2001	184	175	166	158	150	141	132	125	118	110	101	92	85	76	68	60	51	43		
2001- 2002	186	176	167	159	152	144	135	128	120	113	105	95	88	79	70	62	55	46	35	
2002- 2003	190	180	171	163	155	148	139	130	121	116	108	98	90	82	72	65	58	50	40	32

Table 4 Prices of new busses (in L.E.x10^3).

Year	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	2000	01	02
TATA	34	40	43	45																
NASR	30	34	36	40	45	50	64	95	128	145	152	158	165	171	176	181	186	192	196	200

Table 5 Total cost for different replacement stages (in L.E.x10^3).

Stage	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Cost	340	337	333	326	320	318	316	315	314	311	328	334	347	355	369	383	400

represents the total cost (in L.E. x 10^3) when the bus is replaced only once at different stages j, where j = 4, 5, ..., 20 years. From this table, it can be seen that the optimal policy saves 14.3% compared with the decision when replacement is performed after 17 years, and will save 31% if it is performed after 20 years.

4. Parametric study results

The optimal replacement policy depends upon the problem data. If there is a change in one or more of the data elements, the problem should be resolved once more to obtain the new optimal solution. In such cases the importance of using the parametric study is clear, it can reveal the effect of changing some data elements on the optimal solution without the need to resolve new problems from the beginning. Moreover this approach will determine the range of change for those elements without affecting the obtained optimal policy.

The optimal policy consists of a set of alternatives, one at each stage. Some of the data elements exist in the optimal policy; and those elements will be called the In-Line Elements (I.L.E.) and denoted by $\O_{in}$. All other elements which do not exist in the optimal policy will be called the Out-Line Elements (O.L.E.) and denoted by $\O_{out}$. The set of all data elements is denoted as $\O = \O_{in} + \O_{out}$. Considering a state variable S at stage j in the optimal policy, the (I.L.E.) are defined as: $\O_{in} = \{I_j(S), M_j(S), R_j(S), I_j(0), \text{and } M_j(0)\}$.

The parametric dynamic programming study is applied to the practical replacement problem, the results can be summarized as follows:

4.1 Change in data elements

Generally, the set of data elements can be classified into two sub-sets, one can be increased, while the other can be decreased without any change in the optimal policy, to find the new total cost, the required equations are formulated [9]. The summary of the elements that could be changed without affecting the optimal policy, and the new optimal costs Z_{new} are presented in Table 6.

Table 7 summarizes the parametric study results applied to three different stages: 1,13, and 17. Stages 1 and 17 have a keep decision, while stage 13 has a replace decision. In this table, the elements in the Increase position can be increased arbitrarily without changing the optimal policy, while the elements in the decrease position can be decreased freely without changing the optimal policy. For example, in stage number 1, it can be seen that $M_1(0)$, $I_1(1)$, and $R_1(1)$ can be increased arbitrary and $I_1(0)$, and $M_1(1)$ can be decreased freely without affecting the pre-obtained optimal replacement policy.

The parametric study indicates that the optimal policy will be changed only once allover the range of change of any data element. As an example for obtaining the new optimal policy, assume that the old income of the new bus at the year 1992 is increased by λ_1. The parameter λ_1 is assumed to take an arbitrary large value, for example 10,000; so the new income is given generally by:

$$I = I + \lambda_1, \text{ then:}$$

$$I_{10}(0) = I_{10}(0) + 10{,}000.$$

Table 6 Summary of the effect of changing problem data elements.

Changeable Element	In Line Elements		Out Line Elements
Decision	Keep	Replace	Keep or replace
Increasing	1) $M_j(0)$, $R_j(S)$ 2) $I_j(S)$	$M_j(S)$, $I_j(0)$	$M_j(X_j)$, $R_j(X_j)$
Decreasing	1) $I_j(0)$, 2) $M_j(S)$	$I_j(S)$, $M_j(0)$, $R_j(S)$	$I_j(X_j)$
Stages	$j = 1, 2, \ldots, N$		$J = 2, 3, \ldots, N$
States	$S > 0$		$S > 0$
Z_{new}	1) $Z_{new} = Z_{old}$ 2) $Z_{new} = Z_{old} - \lambda$		$Z_{new} = Z_{old}$

Table 7 Examples of the effect of changing some data elements.

Stage	Decision	Elements
1	Keep	Increasing: $M_1(0)$, $I_1(1)$, $R_1(1)$. Decreasing: $I_1(0)$, $M_1(1)$.
13	Replace	Increasing: $I_{13}(0)$, $M_{13}(j)$, $j = 1, 2, \ldots, 13$, $R_{13}(j)$, $j = 1, 2, \ldots, 12$. Decreasing: $M_{13}(0)$, $I_{13}(j)$, $j = 1, 2, \ldots, 13$, $R_{13}(13)$.
17	Keep	Increasing: $I_{17}(4)$, $M_{17}(j)$, $j = 0, 1, 2, 3, \ldots, 17$, $R_{17}(j)$, $j = 1, 2, \ldots, 17$. Decreasing: $M_{17}(4)$, $I_{17}(j)$, $j = 0, 1, 2, 3, \ldots, 17$.

The problem is resolved to find the new optimal policy, which indicates that the bus should be replaced only once at the beginning of the tenth year. The new cost Z_{new} = 305,227. To calculate the new cost for any change λ in the data element, the critical value, which indicates the point at which the optimal policy will be changed, is calculated as follows:

$$\lambda^* = Z_{new} - Z_{old} + \lambda_1$$
$$= 305227 - 311412 + 10000 = 3,815$$

Then, the new optimal cost is calculated from the equation:

$$Z_{new}(\lambda) = Z_{old} + \lambda_1 - \lambda = 295,227 - \lambda$$

The value of λ^* indicates that the old optimal policy will remain unchanged when the income $I_{10}(0)$ is increased by a value 0 $\lambda <$ 3,815. For all values of λ 3,815; the optimal policy will change only once, and the new cost is given by the last equation.

4.2 Change in elements of one row

It is shown that the optimal policy does not change if all elements of one or more rows corresponding to the income and / or the maintenance costs are increased or decreased by the same value [9]. In such cases, the new optimal cost will be:

$$Znew = Zold \pm \sum_{i=1}^{n} \lambda i,$$

Where:

i is the stage at which the change takes place,

λ is the value added to the element.

The sign (+) is used for maintenance rows, while the sign (-) is used for the income rows.

As an example, assume that the total operating cost at year 1993 is increased by λ_1=L.E.3,000, and the income of the bus is increased by λ_2 = L.E.1,000, then:

$$I_{11}(x_{11}) = I_{11}(x_{11}) + 1000$$

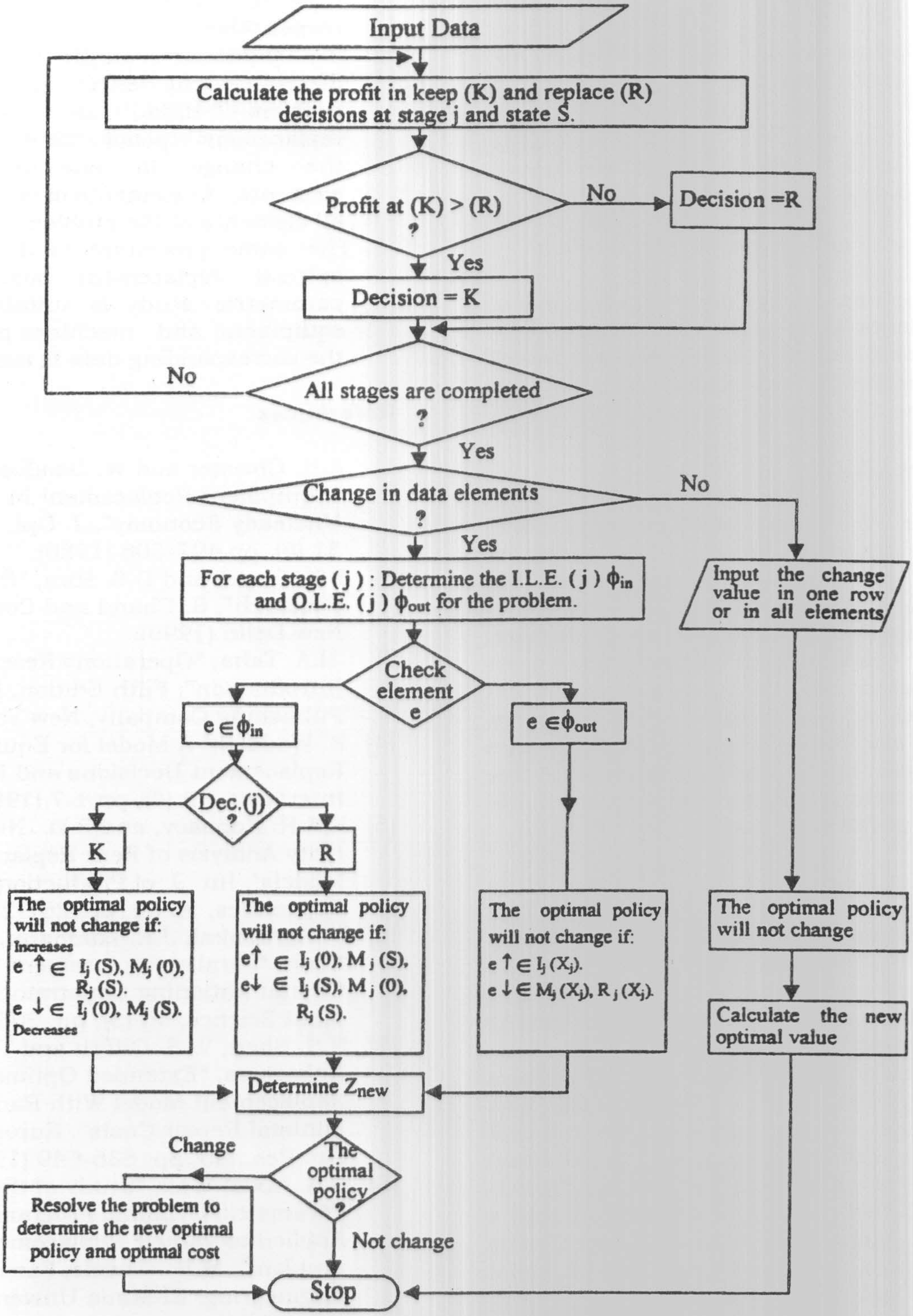

Fig. 1. The flowchart for the solution algorithm.

$M_{11}(x_{11}) = M_{11}(x_{11}) + 3000.$

The optimal policy will not change, and the value of the total cost will be:

$$Z_{new} = Z_{old} + \lambda_2 - \lambda_1$$
$$= 311{,}412 + 3{,}000 - 1{,}000 = \text{L.E. } 313{,}412$$

4.3 Change of all elements of the problem

The optimal policy does not change if all the elements (operating cost, maintenance cost, income charge, and replacement cost) are multiplied by the same value $\lambda > 0$. In such cases, the optimal value will be [9]:

$$Z_{new} = (1+\lambda).Z_{old}.$$

Figure 1 shows the corresponding flowchart for the procedure to solve the dynamic programming problem and the parametric analysis involved.

As an example, suppose that the total operating costs and the replacement costs are increased by 20% due to excess charges in the fuel, oil, and vehicle spare part prices. Suppose also that the ticket price for busses is increased by the same percentage. In this case, the optimal replacement policy will not change, but the total cost will be:

$$Z_{new} = (1+\lambda).Z_{old}$$
$$= 1.2(311{,}412) = \text{L.E. } 373{,}694. \text{ L.E.}$$

5. Conclusions

1. The dynamic programming approach is a suitable technique when applied to the replacement problem of El-Minia University busses in the interval from (1982/1983) to (2002/2003). The objective was to minimize the total cost allover the planning period. The optimal policy is to keep the TATA bus for 12-years, then it should be replaced with a NASR bus which should be kept for the remaining period. The obtained optimal policy can save (14.33%) and (31%) of the total cost compared with the old replacement policy when the bus was replaced after 17 or 20 years respectively.
2. The parametric study is applied to determine the effect of changing the problem data on the optimal replacement policy. This study covers the change in one or more data elements, in elements of one row, and in all elements of the problem.
3. The same procedure to determine the optimal replacement policy and the parametric study is suitable for other equipment and machines provided that the corresponding data is available.

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