Control of cracking in reinforced concrete flexural members

Mohie Eldin S. Shukry
Structural Engineering Department, Faculty of Engineering, Alexandria University

This paper presents a study of the crack control formulations as recommended by the Egyptian Code of practice for design and construction of reinforced concrete structures. The study reveals that a large difference exists between the two alternate approaches recommended by the code for crack control. The tables given in the code, for the allowable bar diameter without direct calculations, are conservative in most cases while the equation given in the code for the calculation of the maximum allowed bar diameter may be unsafe in some cases. According to the code, and within the practical range of longitudinal steel ratio in ordinary buildings, crack control requirements are generally satisfied for flexural members reinforced with plain bars. For members reinforced with deformed bars and subjected to severe environment a check for crack control requirements is required. A simple formula for the calculation of the allowable bar diameter is proposed in this study and design tables are given. In this formula, most of the variables affecting crack width are considered.

يتناول البحث دراسة اشتراطات حالة التشرخ للعناصر الخرسانية المسلحة كما أوصى بها الكود المصري لتصميم و تنفيذ المنشآت الخرسانية المسلحة لسنة ١٩٩٥ . وقد تمت مقارنة على مثال عددي بين اشتراطات الكود المصري والاشتراطات المذكورة في بعض الكودات الأخرى.

وأوضحت الدراسة وجود اختلاف كبير بين الشروط الواردة بالكود المصري لحالة حد التشرخ - فالجداول المذكورة بالكود (جداول ٤-١٤ و ٤-١٥) للقطر الأقصى لحديد التسليح -فى حالة عدم إجراء حسابات- تعطى قيم صغيرة ومتحفظة لهذه الأقطار - بينما في حالة إجراء الحسابات (المعادلة ٤-٦٦ بالكود) تكون قيم هذه الأقطار كبيرة جدا و غير آمنة في بعض الحالات. وأوصت الدراسة انه بالنسبة للعناصر الخرسانية المسلحة في المباني العادية والمعرضة لظروف بيئية معتدلة لا تحتاج لدراسة حالة التشرخ إذا ما تم الالتزام بقواعد ترتيب أسياخ التسليح.

وقد تم عرض معادلات مبسطة لحساب القطر الأقصى لأسياخ التسليح وتمت مقارنتها مع معادلة الكود المصري مع عمل جداول تصميميه لاختيار القطر الأقصى لحديد التسليح ذات النتوءات في الظروف البيئية المختلفة.

Keywords: building codes, crack width, deformed bars, plain bars, reinforced concrete

1. Introduction

Cracking of concrete members is unavoidable in all reinforced concrete structures. Cracks occur in flexure, even at loads below service loads, because of the low tensile strength of concrete. Cracking of concrete is influenced by many factors. Among these factors are the steel stress, the bond characteristics of the steel bars, the concrete cover, and spacing and distribution of bars. In well-designed reinforced concrete structures, cracks are fine and can not be seen by casual observer, this level of cracking does not affect the appearance of the structure or lead to corrosion of reinforcement. The uses of high strength reinforcing steel and the strength design approaches (or limit state design) have made crack control a very important item [1]. Codes based on ultimate strength design permit indirectly high stresses in steel under service conditions. With the increase of steel stress, which is the most important variable in crack control, crack width is expected to be large and visible. Crack Control means the assurance that cracking of concrete does not affect its appearance or durability. The purpose of crack control calculations is not really to limit cracks to certain rigid maximum values but rather to use reasonable bar details that will keep cracks within a reasonable range.

Codes of practice attempt to control cracking of concrete through several measures such as: good quality impermeable concrete, adequate concrete cover, and imposed limitations on the maximum allowable service load stresses in reinforcing steel. A review of various codes of practice indicates that major differences exist between design for crack control of these codes. Some codes such as the Egyptian Code ECP-95 [2] and the German Code DIN 1045-88 [3] require that limit state of cracking must be imposed. Such imposition requires the limiting of the maximum allowable steel stress and bar diameter as a standard procedure for the design of all types of structures subjected to different exposure conditions. Other codes such as ACI 318-95 [4] and BS 8110-85 [5] state that such measures are only needed for certain types of structures. These are subjected to severe environmental conditions, such as liquid retaining structures. However, for the majority of structures, cracking will automatically be controlled through proper detailing practice specified by the code.

The main objective of the present work is to present a comparative study that shows the main differences between the design for crack control according to various codes of practice. Also, an examination for the approaches recommended by the Egyptian Code for crack control is presented and discussed.

In the following, the crack control requirements and expressions for both the steel stress at service load and the crack width according to four codes of practice; namely, the Egyptian Code ECP-95, DIN 1045-88, ACI 318-95 and BS 8110-85, are presented.

1.1 The Egyptian code of practice, ECP-95

Four classes of exposure conditions are considered in the code; namely, **class 1**, structures with tension sides fully protected against corrosive conditions; **class 2**, structures with tension sides unprotected; **class 3**, structures with tension sides unprotected and subjected to high humidity and corrosive conditions; and **class 4**, structures with tension sides subjected to very severe conditions.

The code does not specify any value for the expected crack width for each class of exposure. The Serviceability Limit State of cracking is checked by limiting the crack width through suitable choice of steel stress under service load, bar diameter, and reinforcement cover.

The code gives the following equation (Equation 4-66 of the code) for the limiting bar diameter and steel stress:

$$\phi \leq r\,(\mu_z / f_{sd}^{\,2})\,10^4 \qquad (1)$$

Where ϕ = largest bar diameter of longitudinal reinforcement, mm, f_{sd} = the tensile stress in the bar under the permanently acting service load (normally the dead load), N/mm^2. The effect of any significant restraint against loading in statically indeterminate structures is also to be considered in f_{sd}. Also, in Eq. (1), r is a coefficient which takes account of the bond characteristics of the steel (Table 4-12 of the code), and μ_z is the percentage of tension reinforcement A_s to the cross sectional area of the section below neutral axis A_{ct}; i.e.

$\mu_z = 100\, A_s / A_{ct}$

The code does not require that limit state of cracking to be satisfied by an analysis with Eq. (1) if the tension steel stress f_s under service loads (or the yield stress f_y when ultimate strength design method is used) is reduced below the limiting values given in Table 1 (Tables 4-14 and 4-15 of the code).

1.2. DIN 1045 -88

The early DIN 1045-78 adopted Eq. (1), as recommended by ECP. The recommendations regarding limitations of crack width were revised in DIN 1045-88. Crack control measures will be sufficient provided the design is consistent with the following rules:

a- minimum reinforcement shall generally be used and given as: $\mu_z = 0.4\, f_t / f_s$, where $\mu_z = A_s / A_{bz}$, A_s is the area of reinforcement, A_{bz} is the area of concrete in tension in state I (i.e. full contribution of concrete in tension) and f_t is the concrete tensile strength.

The value of f_s shall be taken from Table 2 as a function of bar size but not larger than 0.8 f_y. According to DIN 1045-88, minimum reinforcement is not required for members in class I in ordinary buildings.

b- The limits of the bar size and the maximum spacing of bars shall be taken from Table 2, as a function of the steel tensile stress, f_s.

Table 1 Limiting diameters in mm for crack control, ECP-95 [2].

Service steel stress f_s, N/mm^2	Steel yield stress for ultimate strength method f_y, N/mm^2		Class 1	Class 2	Class 3 & 4
Plain reinforcing bars:					
140	240		25	22	12
120	201		28	28	18
100	165		32	32	28
Deformed reinforcing bars:					
	f_y : 360 N/mm^2	400 N/mm^2			
220	360	368	12	10	6
200	335	332	16	12	8
180	306	300	25	18	10
160	270	268	32	22	16
140	234	232	--	28	22
120	202	200	--	--	32

Table 2 Limits of the bar diameter, ϕ, in mm and maximum bar spacing in mm, DIN 1045-88 [3] deformed bars, f_y = 420 N/mm^2 or f_y = 500 N/mm^2.

Steel stress f_s, N/mm^2	Bar size[a]		Bar spacing	
	Class I	Class 2, 3 & 4	Class 1	class 2, 3 & 4
400	10	5	--	--
350	16	8	150	70
280	25	12	200	100
240	28	16	250	150
200	36	20	250	200
160	36	28	250	250

[a] The limits of the bar size may be increased in the ratio h / [10 (h - d)], where h = thickness of the member and d = effective depth.

1.3. ACI 318-95

ACI approach is based on Gergely - Lutz [6] equation. For beams with deformed bars, the crack width 'w' at the bottom may be taken as follows:

$$w\ (mm) = C\ (h_2 / h_1)\ (d_c A)^{1/3}\ f_s, \qquad (2)$$

where (see Fig. 1), h_2 = distance from extreme tension fiber to neutral axis, h_1 is the distance from the centroid of the steel to neutral axis, C is the experimental constant = 11 x 10^{-6} mm^2 / N, f_s is the service load stress ≈ 0.6 f_y N/mm^2, d_c is the cover to main steel measured from extreme tension fiber to the center of bar, mm, and A is the average effective area of concrete around each reinforcing bar = 2 $d_s b$ / (number of bars).

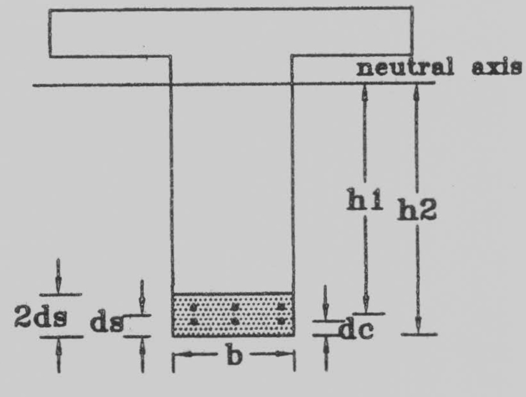

Fig.1. Notations for Eq.(2).

Dividing Eq. (2) by C (h_2/h_1) gives the parameter z; i.e. z = $(d_c A)^{1/3} f_s$ and control of the crack width can thus be obtained by setting an upper limit on the parameter z; i.e. z = 30.6 kN/mm for intermediate environment (w = 0.41 mm) and z = 25.4 kN/mm for severe environment (w = 0.33 mm). When structures are subjected to very aggressive exposure or designed to be watertight, ACI committee 350 [7] limits the value of z to 20.5 for moderate exposure (w = 0.25 mm) and 17.0 for severe exposure (w = 0.20 mm). In both cases, the allowable steel stress f_s is reduced according to both the bar diameter and bar spacing.

1.4. BS 8110-85

The British approach for design of structures for crack control is similar to the American approach but with different formulations for the evaluation of crack width. In the most likely practical situations, bar spacing rules have to ensure that cracking is not serious. However, the British Code quotes a formula similar to that proposed by Beeby [8] for the estimated crack width for concrete in flexural tension:

$$w = 3\, a_{cr}\, \varepsilon_m \,/\, [1 + 2\,(a_{cr} - c_{min}) \,/\, (h - x)], \qquad (3)$$

where a_{cr} = the distance from the point considered to the surface of the nearest longitudinal bar, mm, c_{min} = minimum cover to the tension steel, mm, ε_m = average strain at level where creaking is being considered calculated allowing for the tension stiffening effect of the concrete in the tension zone, x = depth of the neutral axis, h = overall depth of the member, and

$$\varepsilon_m = \varepsilon_1 - b\,(h - x)\,(a' - x) \,/\, [\,3\, E_s\, A_s\, (d - x)\,],$$

where, ε_1 is the strain at level considered; calculated ignoring the stiffening effect of concrete in tension, a' is the distance from the compression face to the point considered, mm, and d is the effective depth, mm.
BS 5337-1976 [9] for liquid-retaining structures defines three classes of exposures; class A exposed to wetting and drying ($w \le 0.1$ mm), class B exposed to continuous contact with water ($w \le 0.2$ mm), and class C not so exposed ($w \le 0.3$ mm) which is the case of ordinary buildings. BS 5337 states that crack width is deemed to be satisfied if the appropriate steel stress and bar spacing requirements are satisfied.

2. Discussion on the approaches recommended by the different codes

2.1. General discussion

(i) Both ECP-95 and DIN 1045-88 states that crack width is a function of the bar diameter. While, in ACI 318-95 and BS 8110-85 the bar diameter is not a major variable but the area of concrete surrounding each reinforcing bar is the important variable. In other words, crack width depends on both bar diameter and bar spacing.

However, the dependence of the crack width on the bar diameter was used in a simplified formula by CEB-FIP 1978 [10] as follows:

$$w_{max} = 1.7\, w_{mean} = 1.7\, [\,0.7\,(f_s \,/\, E_s)\,(\,3\, c_{min} + 0.05\, \phi \,/\, \mu_z\,)] \qquad (4)$$

where, w_{mean} is the mean crack width, mm, μ_z is the ratio of the tension steel area to the effective area of concrete in tension, which depends on the arrangement of bars and it is limited by a line ($c_{min} + 7\,\phi$) from the tension face, and E_s is the modulus of elasticity of steel.
Eq. (4) is similar to that developed by Borges [11] for the maximum crack width in beams reinforced with deformed bars, that is;

$$w_{max} = (2.5 c_{min} + 0.066\phi/\mu)(f_s - 0.75/\mu)/E_s. \qquad (5)$$

Oh et al [12] proposed formulas for the maximum crack width and the crack spacing based on the cracking theory [13], which was developed on basis of the energy criterion of fracture mechanics. These formulas were derived considering all variables affecting crack width. Comprehensive comparisons of the proposed formulas with test data have been made and showed satisfactory agreement. They [12] suggested the following formula for the maximum surface crack width to be used for design purpose:

$$w_{max} = \phi\, a_o\, \varepsilon_s\, (h_2 \,/\, h_1), \qquad (6)$$

where $a_o = 159\,(d_c \,/\, h_2)^{4.5} + 2.83\,(A \,/\, A_{s1})^{1/3}$, ε_s = tensile strain of bars, $A = b\, h_3 \,/ m$, mm^2, m = number of bars in tension, $h_3 = h_2^3$ / (3 h_1^2), mm, A_{s1} = area of each reinforcing bar, mm^2 .

(ii) ACI states that for steel stress with $f_y >$ 280 N/mm^2 control of cracking is particularly important, and this indicates that for mild steel plain bars with f_y equals to 240 N/mm^2 crack control may be considered satisfied. Other codes such as DIN limit its formulations to deformed bars.

(iii) In Eq. (1), the value of μ_z is calculated as the ratio of the area of the steel reinforcement to the area of concrete in tension (which is dependent on both the ratio of tension steel μ and the h/d ratio), while in other code formulas, an effective area of concrete in tension surrounding the reinforcing bars is recommended. It should be noted that when ultimate limit state method is used in design, the calculation of μ_z requires an analysis of the section under service load conditions.

(iv) Eq. (1) recommended by ECP shows that crack width is a function of $f_s{}^2$ while, for other codes and formulas, crack width is proportional to f_s or ε_s . Also, the use of the steel service stress under permanent load f_{sd} in Eq. (1) is questionable.

2.2. Numerical comparisons for a typical beam

Appendix 1 presents the results of the crack control calculations for a simply supported reinforced concrete beam according to the four different codes and also according to Eqs. (4, 5, and 6) The results for this simple example indicate the following:

(i) According to ECP-95, the use of the limiting bar diameter ϕ (Eq. 1) satisfies the serviceability limit state of cracking. However, reducing the steel yield stress (or service stress) increased the required area of tension steel by 65 % if same bar diameter is desired. Alternatively, for the same area of steel, bars with maximum diameter of 10 mm should be used (see Table 1). This indicates that some contradiction exists between the two approaches recommended by the code and equation 4-66 in the code is not compatible with the approach of steel stress reduction. Generally, the values for the bar diameter given in Table 1 are too conservative.

(ii) Equation (1) neglects the influence of bar detailing (number and spacing) on crack control. If in the given example, four bars 32 mm diameter (placed on two rows with two bars in each row) are used instead of six bars, 25 mm diameter each, the crack width calculations according to both BS and Eq. (6)). will be larger than the allowable values, (w_{max} = 0.31 mm when using Eq. (3) and 0.35 mm when using Eq. (6). Also, both the bar diameter and bar spacing will not conform with DIN requirements (ϕ_{max} = 28 mm < 32 mm, clear spacing = 166 mm > 150 mm). This indicates that Eq. (1) for this case will be unsafe.

(iii) The crack control in the form of crack width calculations is satisfied according to DIN 1045-88, ACI 318-95, BS 8110-85, and Eqs. (4, 5, and 6).

3. Study of the Egyptian code recommendations

The Egyptian Code ECP-95 recommends two approaches for the design for crack control; either to calculate the maximum allowed bar diameter using Eq. (1) (equation. 4-66 of the code) or to reduce the steel service stress (or the steel yield stress) according to the used bar diameter (Table 1). Fig. 2 shows the relationship between bar diameter ϕ and percentage of longitudinal steel μ. The charts in Fig. 2 were drawn according to Eq. (1) for different values of steel service stress under dead load f_{sd} ranging from 0.45 to 0.9 the allowable service steel stress f_s and for d = 0.9 h. The percentage of steel was chosen in the range of μ_{min} and μ_{max} as recommended in the code and as given in Table 3. For reinforced concrete beams in ordinary buildings, the practical ratios of longitudinal steel $\mu_{practical}$ are also given in Table 3. The values of the limiting bar diameter for f_{sd} = 0.7 f_s are given in Table 4.

Table 3 Values of μ_{min} and μ_{max}, ECP-95 [2].

	f_y =240 N/mm²	f_y =360 N/mm²	f_y = 400N/mm²
$\mu_{m.in}$	0.0025	0.0015	0.0015
μ_{max}[a]	0.0171 –0.0250	0.0100-0.0150	0.0086-0.0129
$\mu_{practical}$[b]	0.0130-0.0200	0.0075-0.0116	0.0064-0.0100

a) Concrete cube strength f_{cu} = 20 N/mm² to 30 N/mm².

b) $\mu = \alpha f_c$, where α is the ratio of the neutral axis depth to the effective depth, f_c is the concrete allowable stress in compression.

Table 4 Limiting bar diameter ϕ, mm, for crack control according to Eq. (1).
$f_{sd} = 0.7 f_s$, $d = 0.9 h$

a- plain bars

<table>
<tr><td rowspan="2">f_s, N/mm²</td><td>100 μ</td><td>0.25</td><td>0.30</td><td>0.40</td><td>0.50</td><td>0.60</td><td>0.70</td><td>0.80</td><td>0.90</td></tr>
<tr><td>μz / Class</td><td>0.29</td><td>0.35</td><td>0.49</td><td>0.63</td><td>0.78</td><td>0.94</td><td>1.10</td><td>1.27</td></tr>
<tr><td rowspan="3">140</td><td>I</td><td>18</td><td>22</td><td>28</td><td colspan="5">32</td></tr>
<tr><td>II</td><td>12</td><td>14</td><td>20</td><td>25</td><td colspan="4">32</td></tr>
<tr><td>III & IV</td><td>8</td><td>10</td><td>12</td><td>16</td><td>20</td><td>25</td><td>28</td><td>32</td></tr>
<tr><td rowspan="3">120</td><td>I</td><td>25</td><td>28</td><td colspan="6">32</td></tr>
<tr><td>II</td><td>16</td><td>20</td><td>28</td><td colspan="5">32</td></tr>
<tr><td>III & IV</td><td>10</td><td>12</td><td>18</td><td>22</td><td colspan="3">28</td><td>32</td></tr>
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<tr><td>II</td><td>22</td><td>28</td><td colspan="6">32</td></tr>
<tr><td>III & IV</td><td>14</td><td>18</td><td>25</td><td colspan="5">32</td></tr>
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<tr><td>III & IV</td><td>18</td><td>22</td><td></td><td>28</td><td colspan="4">32</td></tr>
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Table 4 Limiting bar diameter ϕ, mm, for crack control according to Eq. (1).
$f_{sd} = 0.7\ f_s$, $d = 0.9\ h$ (Continue).

b-Deformed bars

f_s, N/mm²	100 μ	0.15	0.275	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1
	μ_z / Class	0.16	0.32	0.35	0.49	0.63	0.78	0.94	1.01	1.18	1.44	1.62
220	I	8	16	18	25				32			
	II	6	10	12	16	20	25			32		
	III & IV	4	6	8	10	12	16	20	22	25	28	32
200	I	10	20	22	28				32			
	II	6	12	14	20	25			32			
	III & IV	4	8	10	12	16	20	22	28		32	
180	I	12	22	25				32				
	II	8	16	18	25				32			
	III & IV	6	10	12	16	20	25	28			32	
160	I	16	28					32				
	II	10	20	22				32				
	III & IV	6	12	14	18	25			32			
140	I	20						32				
	II	12	25	28				32				
	III & IV	8	16	18	25				32			
120	I	28						32				
	II	18						32				
	III & IV	12	22	25				32				
100	I						32					
	II	25						32				
	III & IV	16						32				

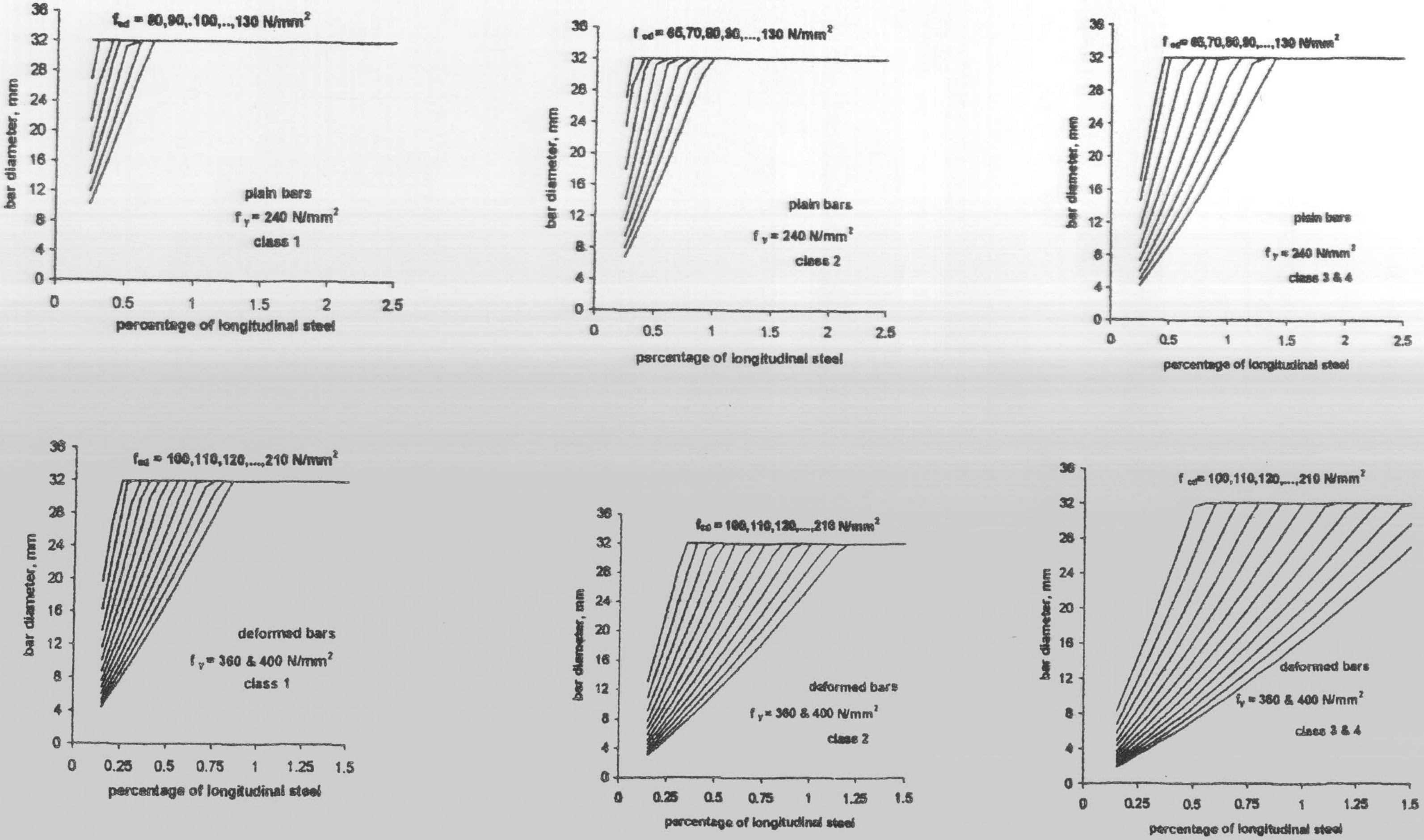

Fig. 2 Relationship between bar diameter and percentage of steel, accenting to eq. (1).

Study of both Fig. 2 and Table 4 indicates the following:

(i) Equation (1) is always satisfied (and hence crack control) for reinforced concrete beams reinforced with plain bars (f_y = 240 N/mm²) for all classes of exposure conditions. However, a check for the maximum allowed bar diameter may be necessary for sections with very low ratio of reinforcement; up to 0.006 (0.24 to 0.35 μ_{max}) for members in class 1 and 2 and up to 0.009 (0.36 to 0.50 μ_{max}) for members in class 3 and 4. These ratios of reinforcement are less than the practical values given in Table 3.

(ii) For reinforced concrete beams reinforced with deformed bars (f_y = 360 and 400 N/mm²), Eq. (1) should be checked for members in class 3 and 4. For other classes, a check for the crack width is required only for steel ratio up to 0.0045 (0.30 to 0.50 μ_{max}) for class 1 and up to 0.0065 (0.45 to 0.75 μ_{max}) for class 2. Again, the above ratios are less than the practical ones.

Generally, Fig. 2 indicates that, within the practical ratio of reinforcement, Eq. (1) gives large bar diameter for most cases of exposure conditions.

4. Suggestions for a simplified formula

Equation (6) [12] was chosen in this study for the crack control requirements in flexural members since it represents most of the variables affecting crack width, such that; concrete cover, longitudinal steel ratio, steel tensile stress, bar diameter, bar spacing, number of bars, and the depth of the effective area of concrete in tension. The values for the limiting bar diameter, as obtained from Eq. (6) for members reinforced with deformed bars, for different values of μ, are plotted in Fig. 3, and tested against Eq. (1) recommended by the Egyptian Code. Fig. 3 indicates that Eq. (6) gives smaller values of ϕ compared to those obtained by Eq. (1), especially at the practical values of the ratio of reinforcement μ.

Simplified formulas, based on Eq. (6), were obtained for the relationship between the maximum allowed bar diameter ϕ_{max} and the steel service stress f_s. These formulas were based on the following assumptions: w_{max} = 0.4 mm for class 1, 0.3 mm for class 2, and 0.2 mm for class 3 and 4, the effective depth d = 0.9 h or d = 0.95 h, E_s = 200 kN/mm², and the first term for a_0 is small and could be neglected. For different values of μ (ranging from μ_{min} and μ_{max}), average values for (h_2 / h_1) and (h_3 / h_1) were used. Thus, the simplified formulas for calculating ϕ_{max} will take the following forms:

for members in class 1;
for d = 0.9 d, $\phi_{max} = 7380\,(\mu)^{1/3} / f_s$ and for d = 0.95 d, $\phi_{max} = 8800\,(\mu)^{1/3} / f_s$ (7-a),

for members in class 2;
for d = 0.9 d, $\phi_{max} = 5535\,(\mu)^{1/3} / f_s$ and for d = 0.95 h, $\phi_{max} = 6600\,(\mu)^{1/3} / f_s$ (7-c),

for members in class 3 & 4;
for d = 0.9 h, $\phi_{max} = 3690\,(\mu)^{1/3} / f_s$ and for d = 0.95 h, $\phi_{max} = 4400\,(\mu)^{1/3} / f_s$ (7-e).

Table 5 gives the values of the bar diameter calculated according to Eq. (7). Also, given in the table are the values of μ_z, the ratio of the area of steel to the effective area of concrete in tension. The values of μ_z are only 45% of those obtained according to the Egyptian Code. Table 5 is given only for deformed bars since for plain bars with small values of f_s, large bar diameters were obtained.

Table 5 Maximum allowed bar diameter according to Eq. (7).

a. Deformed, d=0.9h

f_s, N/mm ^{2}	100 μ	0.15	0.30	0.45	0.60	0.75	0.90	1.05	1.20	1.35	1.50
	μ_z / class	0.38	0.80	1.25	1.71	2.21	2.71	3.23	3.76	4.29	4.86
220	I	18	22	25	28	30			32		
	II	14	16	18	20	22	24	25	25	28	28
	III & IV	8	10	12	14	14	16	16	18	18	20
200	I	20	25	28	32			32			
	II	14	18	20	22	25	25	28	30	30	32
	III & IV	10	12	14	16	16	18	18	20	20	20
180	I	22	28				32				
	II	16	20	22	25	28	30	30		32	
	III & IV	10	14	16	16	18	20	20	22	22	24
160	I	25	30				32				
	II	18	22	25	28			32			
	III & IV	12	16	18	20	20	22	22	25	25	25
140	I	28					32				
	II	20	25	30				32			
	III & IV	14	18	20	22	24	25	25	28	28	30
120	I					32					
	II	25	30				32				
	III & IV	16	20	22	25	28	30	30		32	

b. Deformed bars, d=0.95 h

f_s, N/mm ^{2}	100 μ	0.15	0.30	0.45	0.60	0.75	0.90	1.05	1.20	1.35	1.50
	μ_z / class	0.46	0.99	1.56	2.18	2.83	3.50	4.21	4.96	5.73	6.51
220	I	20	25	30				32			
	II	16	20	22	25	25	28	30		32	
	III & IV	10	14	14	16	18	18	20	20	22	22
200	I	24	30					32			
	II	18	22	25	28	30			32		
	III & IV	12	14	16	18	20	22	22	24	24	25
180	I	25					32				
	II	20	25	28	30			32			
	III & IV	12	16	18	20	22	24	25	25	28	28
160	I	28					32				
	II	22	28	30				32			
	III & IV	14	18	20	22	25	25	28	30	30	32
140	I						32				
	II	25					32				
	III & IV	16	20	24	25	28	30		32		
120	I						32				
	II	30					32				
	III & IV	20	25	28	30			32			

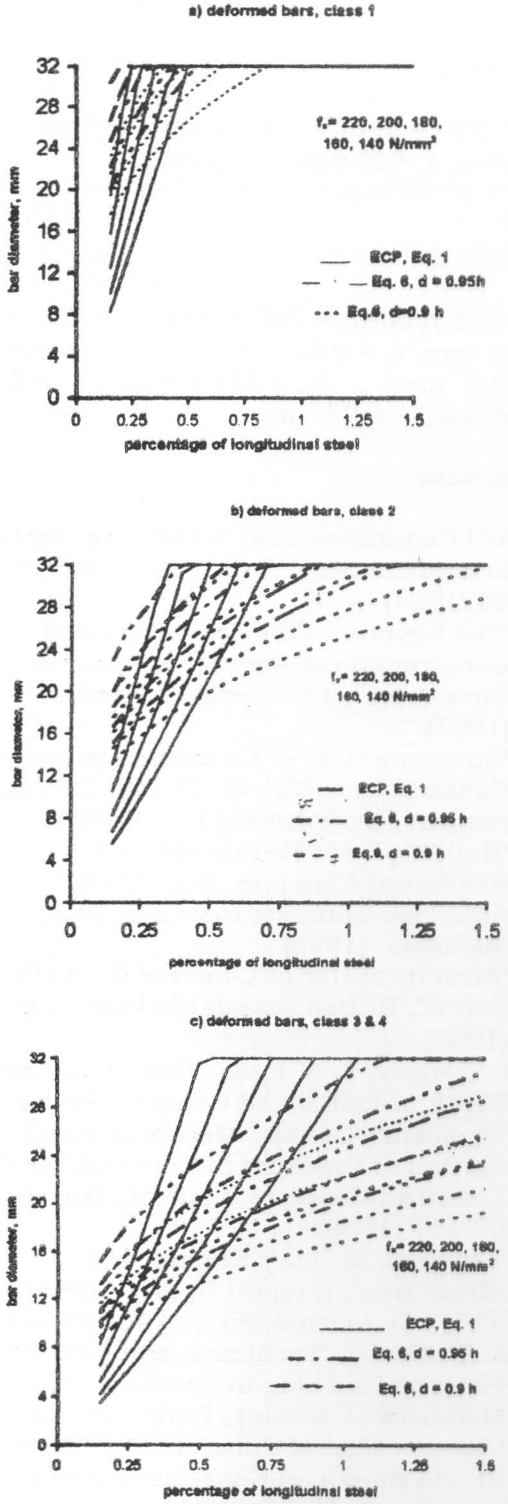

Fig. 3. Relationship between bar diameter and percentage of steel, Eqs. (1 and 6)

5. Conclusions

From the results obtained in the present study, the following conclusions are made:

(i) There is contradiction between the two approaches recommended by the Egyptian Code for crack control of reinforced concrete members. Generally, equation 4-66 of the code for the limiting bar diameter is satisfied in most cases for structures in ordinary buildings (i.e. classes 1 and 2). However, the other approach of reducing the design steel stress (Tables 4-15 and 4-16 in the code) is conservative and will yield larger percentage of reinforcement or too small bar diameter. For some cases (sections with small number of bars with large spacing), equation 4-66 is unsafe.

(ii) Crack control requirements (by the Egyptian Code and other codes) for members reinforced with plain mild steel bars may be considered as generally satisfied for the practical ratio of reinforcement in beams.

(iii) The use of the rules recommended by other codes (e.g. DIN 1045-88) to control cracking of concrete, through the use of limiting bar diameter together with limiting bar spacing seems to be more convenient and practical in design of reinforced concrete members.

(iv) A simplified formula for the limiting bar diameter for crack control is presented in this study. This formula considers most of the variables affecting the crack width and it is less conservative than the equation given by the Egyptian Code.

Appendix 1

A simply supported reinforced concrete beam of rectangular section and has a span of 12 m is subjected to the following service loads; dead load 20 kN/m and live load 20 kN/m., class 2 of exposure and f_y = 400 N/mm^2, f_{cu} = 30 N/mm^2.

The Egyptian Code ECP-95

The design for the beam section using Ultimate Limit State yields the following:

section dimensions 300 x 1250 mm , d = 1187.5 mm and percentage of reinforcement $\mu = A_s/bd$ = 0.827 % , so use 6 bars 25 mm diameter each with clear cover = 25 mm.
For the Serviceability Limit State:
a) Depth of neutral axis = 462.2 mm, f_{sd} = 118.3 N/mm^2, μ_z = 1.25 %, r = 80 and according to Eq. (1), $\phi \le 71.5$ mm i.e. bars with 25 mm diameter are acceptable
b) Alternatively, the crack control is satisfied by reducing the yield stress of the steel (Table 1) to 232 N/mm^2 if 25 mm diameter bars are used. Redesign of the section gives the required percentage of steel as 1.35 %, which indicates an increase in the required steel by 65 % compared to that calculated in (a).

The German Code DIN 1045-88

a) For n, which is the relationship between the modulii of elasticity of steel and concrete = 10, depth of the neutral axis = 394.6 mm, f_s (under total working loads) = 231.4 N/mm^2 , f_t = 2.41 N/mm^2 , A_{bz} = 175200 mm^2 , μ_{min} = 0.283 % < 0.827%.
b) According to Table 2, ϕ_{max} = 16 [1250 / (1250 - 1187.5)] = 32 mm > 25 mm. Bar clear spacing = 50 mm < 150 mm.

ACI 318-95

h_2 / h_1 = 1.086, A = 6250 mm^2 , d_c = 37.5 mm, f_s = 236.4 N/mm^2 , the maximum crack width w = 0.174 mm and z = 14.57 kN/mm < 30.6 kN/mm for intermediate environment and < 25.4 for severe environment.

BS 8110-85

At the extreme tension fiber at one corner of the section a_{cr} = 40.5 mm , c_{min} = 25 mm, ε_m = 0.00114 and the maximum expected surface crack width w = 0.135 mm < 0.3 mm

At the extreme tension fiber between bars, a_{cr} = 55.0 mm, c_{min} = 25 mm, ε_m = 0.00114 and the maximum expected surface crack width w = 0.183 mm < 0.3 mm.

Equation 4

f_s = 236.4 N/mm^2 , E_s = 200 kN/mm^2 , c_{min} = 25 mm, ϕ = 25 mm, μ_z = 0.0491
w_{max} = 0.141 mm

Equation 5

f_s = 236.4 N/mm^2 , E_s = 200 kN/mm^2 , c_{min} = 25 mm, ϕ = 25 mm, μ = 0.00827
w_{max} = 0.194 mm

Equation 6

d_c = 37.5 mm, h_2 = 787.8 mm, h_1 = 725.3 mm, ϕ = 25 mm, ε_s = 0.001182, h_3 = 309.8 mm, A = 15490.5 mm^2 , A_{s1} = 490.6 mm^2 , a_o = 8.945, hence, w_{max} = 0.287 mm

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