ELASTO-PLASTIC BUCKLING BEHAVIOUR OF HYBRID STEEL COLUMNS

Abd-Elaziz M. Ibrahim, Mohamed A. Diwan and Tarek M. Ibrahim
Structural Engineering Department Alexandria University.

This paper presents a numerical model for the simulation of elasto-plastic behavior of hybrid box columns composed of a mix of steel-52 and steel-37 plates. The results of the numerical approach are compared with those obtained by using the ECCS curves for the cases of homogeneous column material. The spreading of the plastic zones in the column web and flange plates is obtained in both the longitudinal and the transversal directions. A limiting value of the column slenderness ratio is suggested to govern the use of ECCS curves for the estimation of the corresponding hybrid box column resistance.

يعرض البحث نموذج نظرى لتمثيل التصرف المرن – اللدن للأعمدة المعدنية المختلطة من قطاعات صندوقية. والأعمدة المختلطة (Hybrid Coumns) هى التى تكون الأعصاب فيها مثلا من صلب ٣٧ والشفاه من صلب ٥٢ أو العكس.
والنموذج المظرى المقترح يتتبع إنتشار التلدن فى أجزاء العمود سواء فى الأتجاه العرضى أو الأتجاه الطولى طوال فترات التحميل التدريجى للعمود حتى الانهيار . وقد اختبرت كفاءة النموذج النظرى بمقارنة نتائجه مع المنحنيات الاوربية **ECCS** للانبعاج التى تم الحصول عليها من دراسات احصائية لما يزيد عن ١٠٠ تجربة معملية وقد اعطت نتائج جيده فى حالة الاعمدة ذات القطاع المتجانس.
وقد تم فى هذا البحث التوصيه بامكانية استخدام المنحنيات الاوربية ايضا فى حالة الاعمدة ذات القطاعات المختلطة طبقا للشروط التالية:-
أ – فى حالة نسبة نحافة الاعمدة تزيد عن ٨٠
* يمكن استخدام المحنى الاوربى (b) باعتبار أن العمود من صلب ٥٢ متجانس اذا كان القطاع المختلط مكون من شفاه من صلب ٥٢ واعصاب من صلب ٣٧. ويلاحظ هنا ان استخدام العمود المختلط يعتبر استخداما أمثل.
* يمكن استخدام المنحنى الاوربى (b) باعتبار أن العمود من صلب ٣٧ متجانس اذا كان القطاع المختلط مكون من شفاه من صلب ٣٧ واعصاب من صلب ٥٢. ويلاحظ هنا ان استخدام القطاع المختلط لم يعطى اى تحسن فى قدرة تحمل العمود.
ب – فى حالة نسبة نحافة الأعمدة تقل عن ٨٠
* فى هذه الحالة نوصى تجاوزا فى الوقت الراهن باستخدام الاوربى (b) باعتبار العمود من صلب متجانس وبذلك يكون التصميم فى جانب الأمان لاى من نوع من الأعمدة المختلطة. والحقيقة ان تحديد القدرة الفعلية لهذه الأعمدة يحتاج إلى دراسة خاصة لتعديل ثوابت هذه المنحنيات لتتماشى مع الاعمدة المختلطة المتوسطة والقصيرة.

Keywords: Hybrid, Elasto-plastic, Buckling numerical model, Sienderress ratio.

INTRODUCTION

The basic problem of the elasto-plastic stability of axially loaded columns in the plane of symmetry has been well understood since Von Karman's efforts [I] in the early 20'" century. The considerable amount of research work that has been focused on such topic since that time represents remarkable efforts in the development of advanced methods of analysis.

The overall change of geometry of the axially compressed column is usually large enough to be of primary importance.

It becomes necessary, in deriving collapse loads,

to follow the detailed behavior of the column under a gradually increasing axial force combined with transversal bending moment. Theoretically, a column composed of perfect eisto-plastic material with the stress-strain relationship shown in Figure 1 should remain straight until either the Euler buckling load PE or the yield stress cy, is reached. However, this idealized problem has a little significance for real columns, in which unavoidable imperfections (eccentric load, initial deformation, residual stresses, etc.) exist.

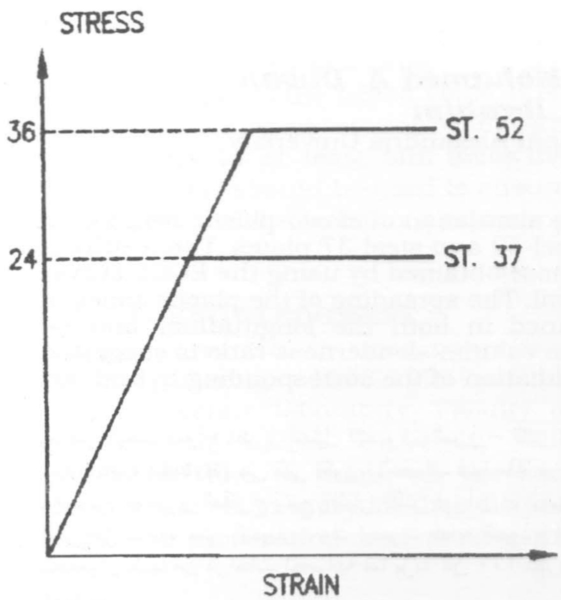


Figure 1 I deal stress-strain relationship

Three decades ago, the European Convention of Constructional Steelwork (ECCS) has carried out an extensive experimental program. The main objèctive of the program was to establish a concrete reference for column buckling load. The buckling of initially imperfect , concentrically loaded hinged-ends columns was investigated. The test series have been statistically treated in such a way that a group of column buckling curves was developed [2]. According to the ECCS recommendations, the strength of the most commonly used structural cross-section shapes are expressed by three column bucking curves a, b, and c as shown in Figure 2. Curve "a" gives the maximum column strength for tubular cross-sections. Curve "b" addresses welded box sections. The third curve "c" is used to calculate the minor-axis buckling load for columns having wide flange 1-cross-sections. Other column cross-sections are assigned to each curve Figure 2 as described by Beer and others [2, 3 and 4].

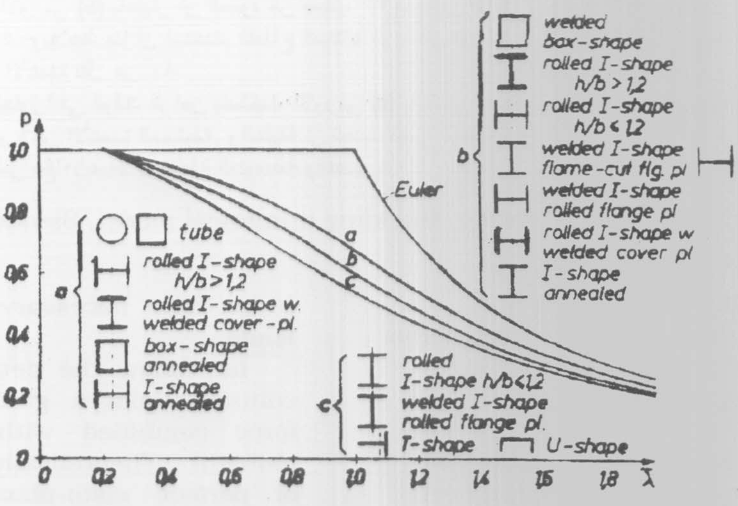


Figure 2 Column bucking strength according to ECCS

The ultimate load is given by the relation [2]:

$$\frac{P_u}{P_y} = \frac{1}{k_o} \qquad (1)$$

$$P_y = A.\sigma_y \qquad (2)$$

Where A is the cross-sectional area of the column and σ_y is the yield stress of the column material.

The constant K_o is giver by [2]:

$$k_o \frac{2\overline{\lambda}^{2}}{1+\alpha(\overline{\lambda}-0.2)+\overline{\lambda}^{2}-\sqrt{\left[1+\alpha(\overline{\lambda}-0.2)+\overline{\lambda}^{2}\right]^{2}-4\overline{\lambda}^{2}}} \qquad (3)$$

$$\text{and } \overline{\lambda} = \frac{\lambda}{\pi}\sqrt{\frac{\sigma_y}{E}} \qquad (4)$$

Where λ is the slenderness ratio of the column ((ℓ /i) and αrepresents the parameter of imperfection as follows:

α = 0.206 for the curve"a"
α = 0.339 for the curve "b"
α = 0.489 for the curve "c"

The results of approximately 100 tests with concentrically loaded columns have included some tests on hybrid columns [5] in which the flange and the web plates were of different steel grades. The recommended ECCS curves have not clearly classified a corresponding curve for such type of column cross-section.

The present paper is intended to provide a tool for the simulation of the behavior of axially loaded symmetrical steel box column with initial imperfection and a hybrid cross-section (Figure 3).

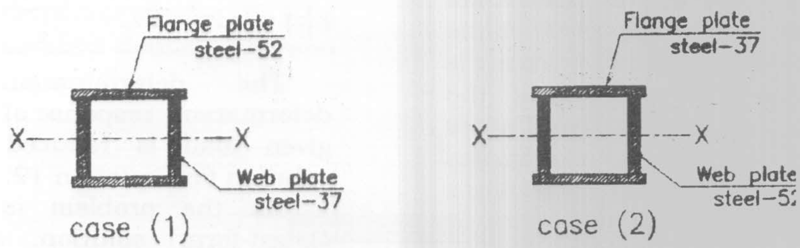


Figure 3 Hybrid column cross-section

NUMERICAL MODEL

The computation of the deflected shape of any column, beyond the elastic limit, is rather complicated. This is because the bending stiffness "El" of the column is not constant to be a function of the bending moment M and the load P. The column may yield over part of the cross-section if the compression load or the deflections are large enough. The bending stiffness El shall then be reduced. The value of the yield stress and the geometry of the cross-section will also affect the relation between M, P and El. As the bending moment is not constant over the length of the column, the bending stiffness El varies along the column length. The relationship between M, P and El can be determined for each particular section if the stress-strain diagram, the distribution of σy over the cross-section, and the residual stress distribution are known.

Assumptions

The following assumptions are adopted in the analysis:

- Plane cross-sections remain plane after bending.
- The stress-strain relations for steel-37 and steel-52 have an elastic-perfectly plastic form Figure 1
- The deformations are considered small.
- Yielding is governed by normal stress only.

The normal strain ε (η)at a point 'n of a certain cross-section situated at a distance z from the origin, and the deformations (v) of the pole of this cross-section are related by the following formula:

$$\varepsilon (\eta) =\varepsilon_o V'' \eta+ \varepsilon_r(\eta) \quad (5)$$

Where ε_r is the residual strain and s. is the centered strain.

If the strain ε (η) is inferior to the yield strain of the material, the fiber is still elastic, and thus,

$$\sigma(\eta) =E(\eta) [\varepsilon_o- v'' \eta] +\sigma_r(\eta) \quad (6)$$

Where $\sigma(\eta)$ is the normal stress at a point (η) of the cross section situated at a distance z from the origin, $E(\eta)$ is the Young's modulus at point (η), and $\sigma_{r(\eta)}$ is the residual stress at point (η)

When the strain $\varepsilon(\eta)$ is equal to or larger than the yield strain $(\sigma_y/E,\eta)$ then;

$$\sigma(\eta) = \pm\, \sigma_y \tag{7}$$

The equilibrium of the internal forces in the column section is given by the following two equations:

$$P= \int_A \sigma_{(\eta)} dA \tag{8}$$

$$M_{(\eta)\,int}\ \int_A \sigma_{(\eta)}\, \eta\, dA \tag{9}$$

In order to examine the external forces, a column in a deflected configuration is considered. At an arbitrary section ($z = z_c$) as shown in Figure 4, the deflections of the centroid of the local coordinates ξ and η are u and v, respectively, where u=0.0 as the problem of the out-of-plane deformations is not considered due to the geometry of the used cross-section. As the local coordinates (ξ , η) are parallel to the global coordinates (x, y), the bending moment M, at a section z produced by the applied axial force P is given by:

$$M_{z(ext)}\ P(v_o +v) \tag{10}$$

Where v_o is the initial deformation expressed by the function:

$$v_o = \bar{v} \sin(\pi z / L) \tag{11}$$

In which $\bar{v}$ is the initial deformation at mid-span of the column taken equal to L/1000. This value represents a worst condition as recorded in practice [6 and 7].

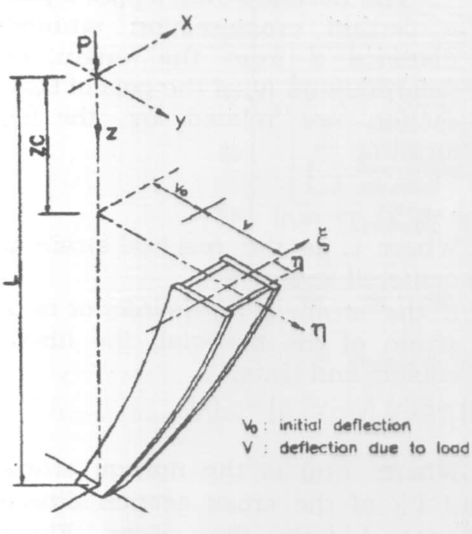


Figure 4 Deformed column

In Equation 10, "v" represents the additional deformation due to external axial loading. The differential equations are obtained by equating the internal moments to the external moments in the deformed configuration.

The flexural stiffness about x-axis is not constant as it changes according to the distribution of the yielded portion of the cross-section. If all the nonlinear terms of displacements are neglected and the effect of residual stresses and initial geometrical imperfections are considered, the differential equation of equilibrium becomes:

$$E(z)\, v'' + Pv = Pv_o \tag{12}$$

The determination of the load-deformation response of the column under a given load is reduced for the sake of a solution for Equation 12.

As the problem is not approaching closed-form solution, a finite difference procedure is therefore employed. The column is divided into "in" segments of equal lengths. The derivatives in Equation 12 are replaced by central differences at the pivotal points.

The deformations corresponding to a given increment of external forces are computed by an iterative procedure. in the first iteration, an increment of the deformations is computed by using the flexural stiffness corresponding to the previous known deformations. In the next iteration, another increment of deformation is computed by using the flexural stiffness of the updated deformations. The unbalanced forces can be computed by comparing the internal forces at the updated deformations to the external applied forces for each iteration. Iterations for eliminating the unbalanced forces should continue until the unbalanced forces are negligible.

The flexural stiffness is calculated numerically by dividing the cross-section into finite elements as shown in Figure 5. The strain and stress in each element are computed as the average values at its centroid. The flexural stiffness can be expressed as:

$$(EI)_i \left(\frac{-\int_A \sigma_{(\eta)}(\eta) dA}{v''} \right)_i \quad , i = 1, m \qquad (13)$$

Where "m" is the number of finite divisions in the longitudinal direction. The ultimate load is determined by the instability condition that is imminent when a small increment of the load results in a large increase in deformations.

The efficiency of the theoretical model is verified by comparing its results with those obtained by utilizing the curve "b" of the ECCS recommendations. Figure 6 shows the variation of the dimensionless ratio (P_u / P_y) in terms of the modified slenderness ratio $\bar{\lambda}$. The column has a welded box cross-section of steel-37. Another comparison between the results of the model and the ECCS recommendations is shown in Figure 7 for similar box column made of steel-52. Both comparisons, for homogeneous column sections of steel-37 and steel-52, show that the results of the model are in good agreement with curve "b" of the ECCS recommendations.

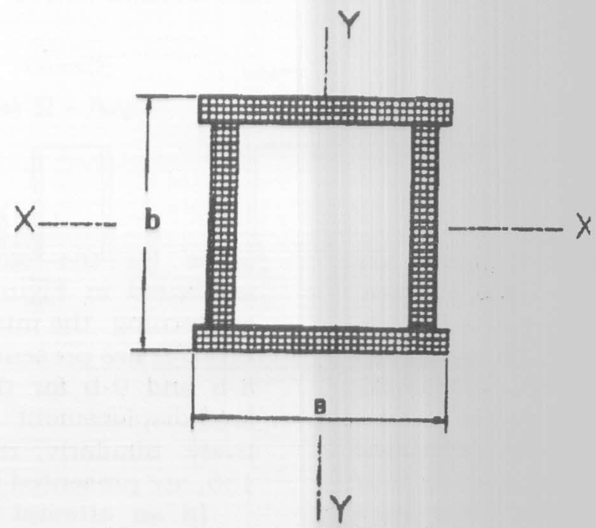


Figure 5 Cross section discritisation

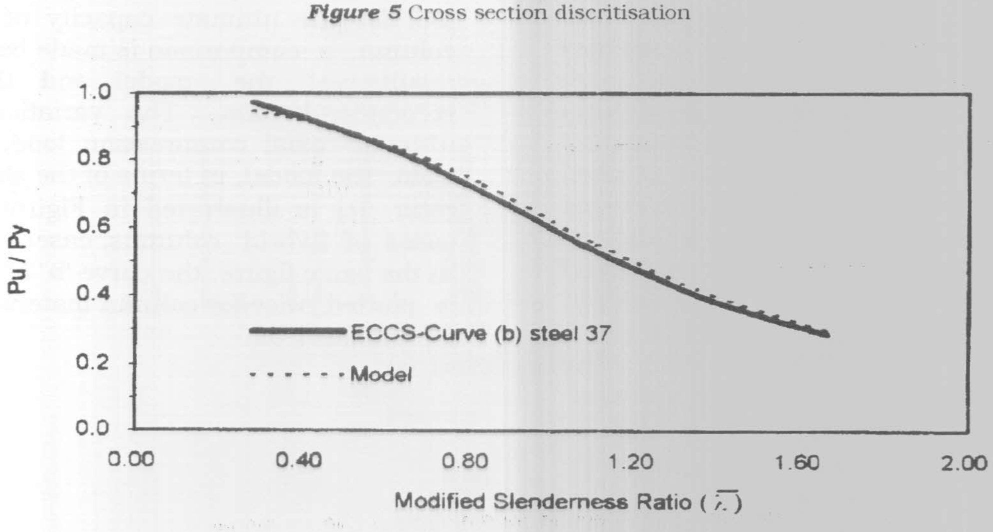


Figure 6 Homogeneous box-column resistance-steel 37

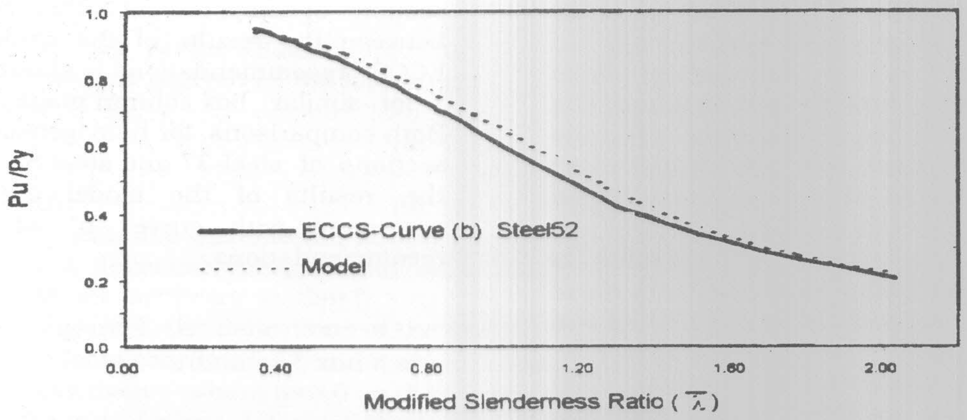


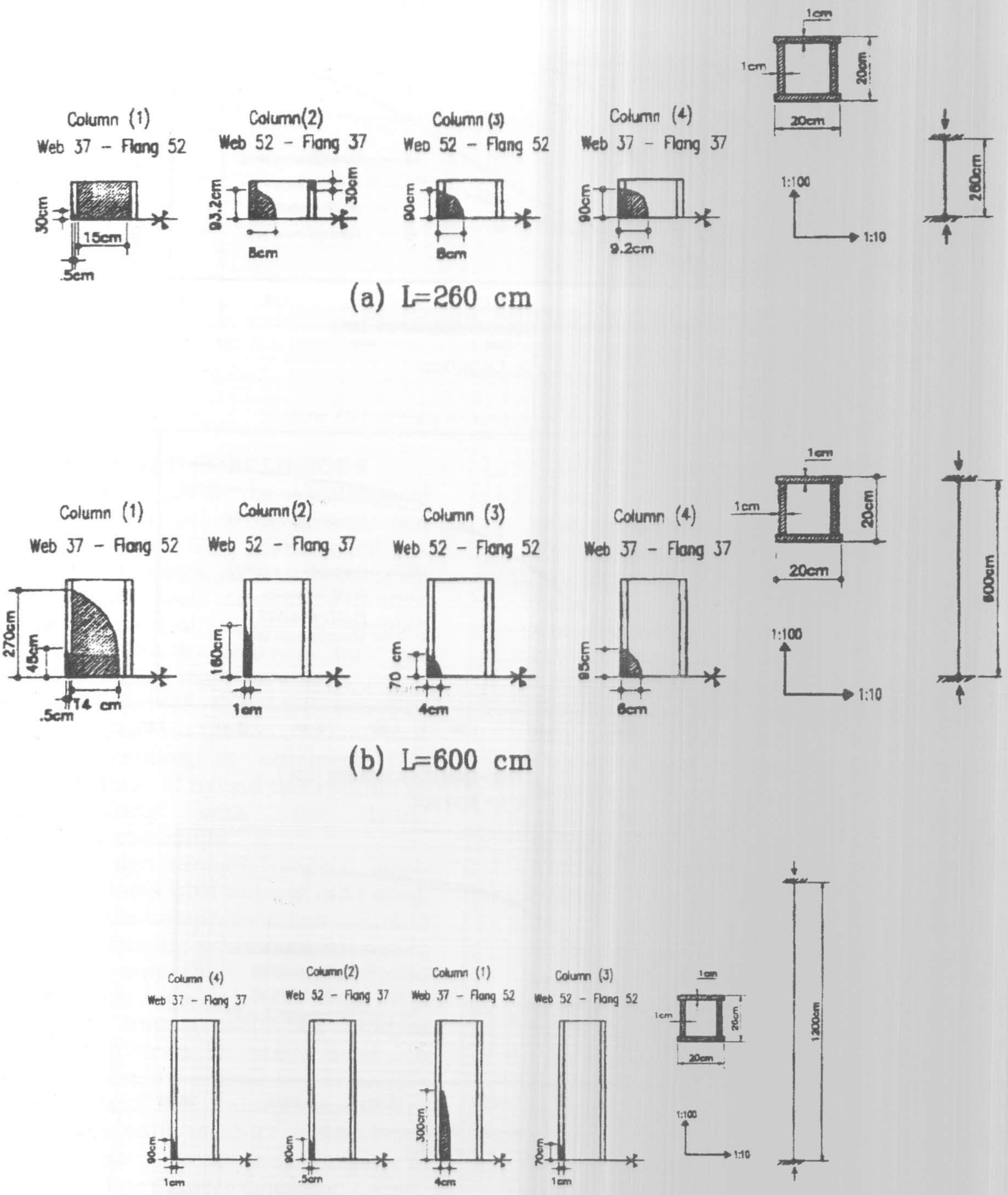
Figure 7 Homogeneous box-column resistance-steel 52

Two cases of hybrid box columns are investigated. Case (1) has 200x 10 mm. flange plates of steel-52 and 180 x 10 mm. web plates of steel-37. Case (2) has flange plates of steel-37 and web plates of steel-52 with the same dimensions of case (1).

The proposed model is used to determine the ultimate axial compression load for four cases of column cross-sections. Two cases have hybrid column cross-section as shown in Figure 3. The third and fourth cases have homogeneous column cross- sections, made of steel-52 and steel-37 respectively, with the same geometry of the hybrid columns.

Three column lengths are studied representing short, intermediate, and long columns. Figure 8-a shows the spreading of the plasticized zones in the longitudinal and transversal directions at the ultimate load for the short column, λ= 33, for each of the four cases of column cross-section. The corresponding load-displacement relationships for the same column length are presented in Figure9-a. the study results of concerning the intermediate column length, λ = 77, are presented graphically in Figures 8-b and 9-b for the plasticized zones and load-displacement relationships, respectively. Similarly, results of long column, λ 155, are presented in Figures 8-c and 9-c.

In an attempt to develop an estimation tool for the ultimate capacity of a hybrid column, a comparison is made between the results of the model and the ECCS recommendations. The variation of the ultimate axial compression load, obtained from the model, in terms of the slenderness ratio (λ) is illustrated in Figure 10fortwo cases of hybrid columns, cases (1) and (2). In the same figure, the curve 'b" of the ECCS is plotted twice for column material of steel-37 and steel-52.

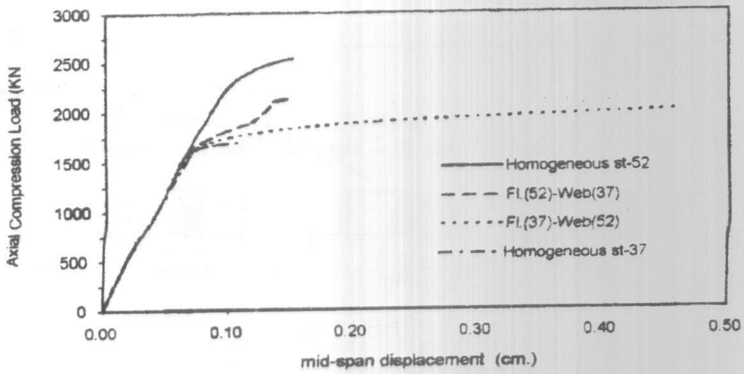


(a) L=260 cm

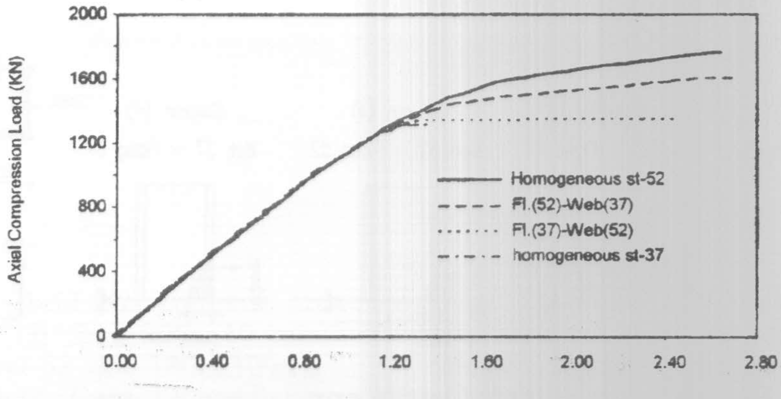
(b) L=600 cm

(c) L=1200 cm

Figure 8 Plastic zones

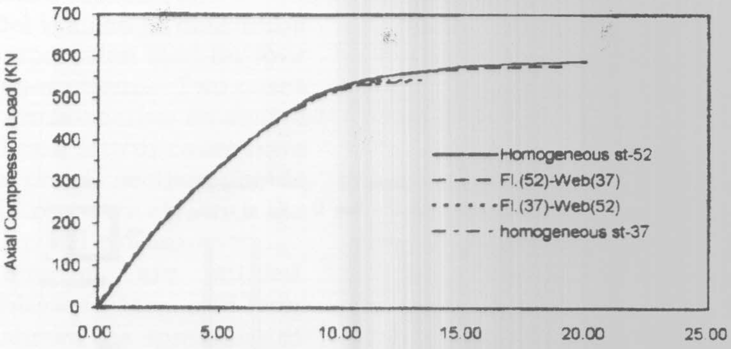


a: L= 260cm



mid-span displacement (cm.)

b: L= 600 cm



c: L= 1200cm

Figure 9 Load-displacement relationships

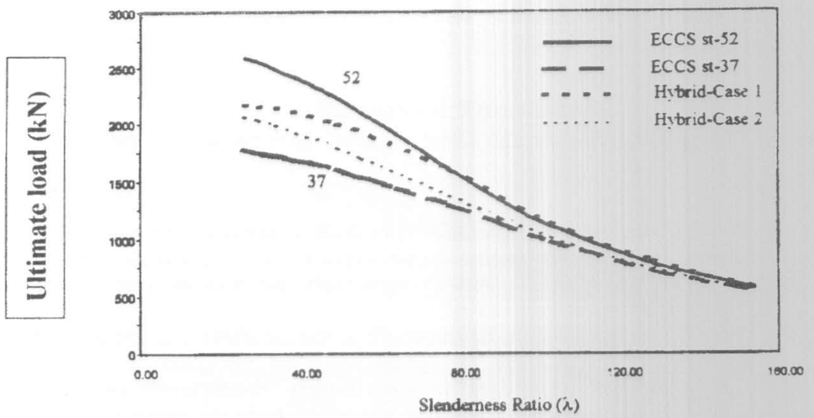


Figure 10 Buckling resistance of hybrid column

SUMMARY AND CONCLUSIONS

A numerical model for the simulation of elasto-plastic behavior of a hybrid box column is presented. The investigated box column has web plates made of steel- 37 and flange plates made of steel-52 or vice-versa. The numerical model is verified through the ECCS curves for the homogeneous column material cases.

The distribution of the plastic zones in the column plates, in the longitudinal and transversal directions, is obtained. The ultimate resistance of hybrid box column is determined along with the load-displacement relationships.

The use of high strength material, steel-52, as flange plates with webs of mild steel, steel-37, for axially compressed box columns increases the ultimate column resistance in a more efficient way than the case of mild steel flanges with high strength webs. The ECCS curve for homogeneous box columns made of steel-52 can be safely used to estimate the ultimate resistance of axially compressed hybrid box columns made of steel-52 flanges with steel-37 webs if the slenderness ratio "λ" of the column is higher than 80. For Shorter columns, $\lambda < 80$, corresponding ECCS curve of steel-37 may be conservatively used. Further investigations still needed to provide a closer approach for the determination of the ultimate axial resistance of such short columns. Cross-section geometries other than box sections are probable headlines for future research work.

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