EFFECT OF BEAM STIFFNESS ON BEHAVIOR OF REINFORCED CONCRETE SLABS

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ABSTRACT

An experimental program was conducted on simply supported, square, two-way reinforced concrete slabs under the action of uniformly distributed load. The slabs were supported on edge beams with span 1800 x 1800 mm. The influence of the stiffness of an interior (secondary) beam, dividing the slab into two one-way slabs of equal dimensions, on the structural behavior, cracking, and ultimate capacity of such slabs was investigated. The experimental results of five slabs tested in this program with regard to deflections, strains and mode of failure are discussed. The results reveal that interior (secondary) beams with depth equals to at least four times the slab thickness are to be considered acting efficiently in carrying and transferring the slab load.

يشمل البحث دراسة معملية علي خمسة بلاطات خرسانية مسلحة مربعة بأبعاد ١٨٠٠ مم x ١٨٠٠ مم و ترتكز في حوافها علي كمرات خرسانية مسلحة و معرضة إلي أحمال منتظمة متزايدة حتي حمل الكسر . و المتغير الرئيسى في هذه التجارب كان وجود كمرة ثانوية في البلاطات الثلاث الأخيرة فى منتصف أحد الأتجهات فقط بحيث تنصف البلاطة المربعة إلي بلاطتين مستطيلتين بالتساوي كل منهما بأبعاد ٩٠٠ مم x ١٨٠٠ مم ، مع ثبات عرض الكمرة الثانوية بنفس عرض الكمرات الرئيسية و تغير الأرتفاع من ١,٣٣ سمك البلاطة إلي ٣,٣٣ سمك البلاطة . و للمقارنة أيضا تم زيادة الحديد السفلي فى البلاطة الثانية (الغير مزودة بكمرة ثانوية – كدراسة لما يعرف بالكمرة المدفونة) لدراسة تأثيرها في تغير توزيع الأحمال . و تمت مناقشة النتائج و سلوك البلاطات ، و أوضحت النتائج ما يلي :-

البلاطات المزودة بحديد سفلي إضافي في المنتصف لم يحدث بها تغير في سلوكها و شكل الشروخ عن البلاطات الغير مزودة بهذا التسليح و أيضا البلاطات المزودة بكمرات ثانوية ذات عمق يساوي ١,٦٦ سمك البلاطة لم تختلف في سلوكها عن البلاطات بدون كمرات . بزيادة عمق الكمرة بدأ يحدث إختلاف في سلوك البلاطة بحيث بدأت الكمرات الثانوية في نقل جزء من الحمل أقل من الحمل التصميمي للكمرة . و قد أظهرت النتائج أنه في البلاطات المربعة يجب إستخدام كمرات ثانوية بعمق يساوي علي الأقل أربعة أمثال سمك البلاطة حتي يمكن أن يجعل سلوك الكمرات الثانوية مثل الكمرات الرئيسية ، و يجعل الكمرات الثانوية ذات فاعلية في نقل أحمال البلاطات .

Keywords: One-way, Reinforced concrete, Secondary beams, Slabs, Two-way.

INTRODUCTION

A two-way slab is a structure in which each panel is supported along all four edges by beams or walls. For slabs supported along their edges by deep, stiff and monolithic reinforced concrete beams, bending moments in the slabs may be obtained by elastic analyses or code tables may be used. Stiff supports mean stiff enough to be considered unyielding. However, if the concrete edge beams are shallow and flexible, deformations of the floor system along the column lines significantly alters the distribution of moments in the slab panel itself.

The differentiation between stiff or flexible beams is not clear in most reinforced concrete building codes and, practically, beams with depth equals to the slab thickness are used frequently, as in the case of ribbed slabs.

The Egyptian Code ECP 1995 [1] recommends that the depth of the marginal beams for flat slab structures should be at least three times the slab thickness. However, nothing is mentioned in the code about beams in two-way floor slab systems, except that in a different clause, the code states that the breadth of beams supporting floor slabs should not be less than slab thickness and not less than 100 mm, in order to resist the torsional strains.

The ACI building code 318-95 [2] recommends the use of the direct design method for two-way systems. In this method, convenient parameters which relate the stiffness of the beam section to the slab stiffness, had been used. The relative flexural stiffness "α" in either direction of the floor slab is defined as the ratio of flexural stiffness of beam section ($E_c I_b$) to flexural stiffness of slab ($E_c I_s$); i.e. $\alpha = E_c I_b / E_c I_s$, where E_c is the modulus of elasticity of concrete, I_b is the moment of inertia of the T-beam with a portion of the slab on each side of the beam extending a distance equals to the projection of the beam below the slab but not greater than four times the slab thickness, and I_s is the slab moment of inertia with width equals to the distance between the centerlines of the panels on each side of the beam. The flexural stiffness of beam and slab may be based on gross concrete section neglecting reinforcement. The range of parameter α is from zero (no beam) to infinity (rigid support).

According to ACI code [2], beams with $\alpha_1 l_2 / l_1$ (where l_1 is length of slab panel for which α_1 is being calculated and l_2 is the perpendicular length) equals to or greater than 1.0 shall be proportioned to resist shear caused by factored loads on arbitrary areas bounded by 45° lines from the corners of the panels and the centerlines of the adjacent panels parallel to the long sides. If the stiffness of the interior beam $\alpha_1 l_2 / l_1$ is less than 1.0, the shear on the beam may be obtained by linear interpolation. The remaining fracture of the load is assumed to be transmitted directly by the slab to the four corner supports or columns if any.

It was found [3, 4] that the parameter "α" influenced the straining internal moment of the two-way slab as well as that of the supporting beam. As α increased the slab's internal moment decreased and beam's internal moment increased correspondingly. Also, in case of slabs supported on relatively stiff beams (with beam depth equals to 3.33 times the slab thickness) measured moments (calculated from experimental results of strains) compared well with moment values predicted from building codes [3]. However, in case of slabs supported on relatively shallow beams (with beam depth equals to twice the slab thickness), measured moments were different from those assumed in design.

Another parameter which affects the behavior of two-way slab systems is the relative torsional stiffness, J which relates the torsional stiffness of the beam to the flexural stiffness of the slab spanning across the beam, i.e. $J = GC / E_c I_s$, where G is the modulus of rigidity and C is the torsional inertia of the beam. J= 0 means no beam and J = infinity means clamped edges.

The tests reported in this paper were made to obtain data on the effect of varying the depth of an interior (secondary) beam on the behavior of two-way square floor slabs. It should be noted that the variation in beam depth indicates a variation of its stiffness. In this study, the value of 'α' for the interior beams varied from zero (i.e. no beams) to 7.2 (interior beam with depth equals to 3.33 times the slab thickness) and the value of "$\alpha_1 l_2 / l_1$", which according to the ACI code may be considered as a measure for the beam stiffness, varied from zero to 3.6. The value of $\alpha_1 l_2 / l_1 = 1$ corresponds to an interior beam with depth equals to about 2.34 times the slab thickness.

EXPERIMENTAL PROGRAM

A total of five reinforced concrete slabs were tested to failure under the action of uniformly distributed load. As shown in Figure 1, the slab dimensions were 1900 x 1900 mm, with a thickness, t_s, equals to 60 mm. The slabs were supported at their edges on four reinforced concrete beams, such that the effective span of the slabs was 1800 mm in the two directions. The span/ depth ratio of the slabs was 30 while the limit recommended in ECP [1] for two-way simply supported slabs is 35. The four edge beams were 100 mm wide, 200 mm thick and were vertically supported at corners with an effective span of 1800 mm.

The reinforcement of the slabs consisted of 6 mm diameter plain mild steel bars with an average value of yield stress of 300 N/mm^2 and an ultimate stress of 429 N/mm^2. Bars in all slabs were uniformly spaced in the two orthogonal directions and the reinforcement ratio, ρ was considered constant for all the slabs (in each direction, $\rho = A_s / b\,d = 0.5\ \%$ where $A_s = 28\ mm^2$, $b = 125$ mm, $d = 45$ mm). The bottom reinforcement of each edge beam consisted of 2-ϕ 13mm of high strength steel with yield stress of 394 N/mm^2 and ultimate stress of 648 N/mm^2. The reinforcement ratio ρ for the edge beams was 1.47% while, according to ECP [1], the upper bound for ρ for under-reinforced section (i.e., $\rho_{balanced}$) is 2.07% and the minimum value of ρ is 0.28%. The top reinforcement for the edge beams consisted of 2- ϕ 8mm plain bars.

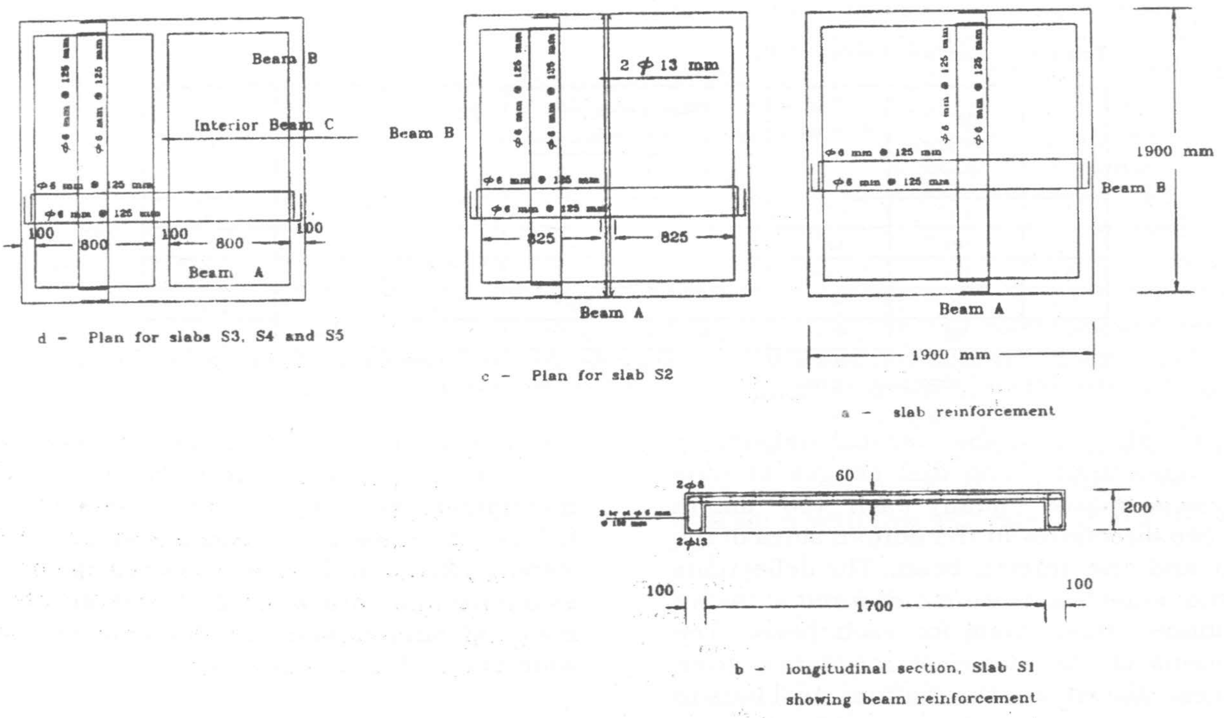


Figure 1 Dimensions and Reinforcement details of slabs

The first slab S1, Figures 1-a and 1-b, was isotropically reinforced in the two directions while for the second slab S2, Figure 1-c, 2-ϕ 13mm steel bars were added to the slab reinforcement at mid-span of one direction. Three of the five slabs; Slabs S3, S4 and S5, had an interior (secondary) beam in one direction dividing the slab into two equal one-way slabs with dimensions 900 x 1800 mm each. The interior beam was supported on the edge beams with a span of 1800 mm. The depth of the interior beam, $h_{sec.}$, was the only variable studied and it varied in the last three specimens as 1.66, 2.5 and 3.33 the slab thickness, respectively. The breadth of the interior beam was kept constant and was equal to 100 mm. The reinforcement for all the interior beams was similar to that for the edge beams (i.e. 2ϕ13mm at bottom and 2-ϕ8 mm at top). Details of the tested slabs are given in Table 1. In the table, the inertia for the interior beams used for the calculation of the flexural parameter "α" was calculated according to the ACI [2].

The concrete used in the specimens consisted of Ordinary Portland Cement, natural sand and broken stones with maximum size of 20 mm, and the mix proportions by weight were 1:1.6 : 2.8, respectively. The respective w/c ratio was 0.45. Three 150 mm cubes were cast for each specimen. The values of concrete strength at time of testing are presented in Table 1.

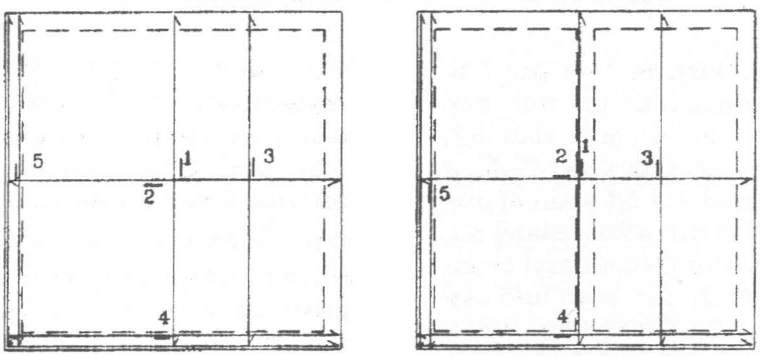
All the slabs were tested in a horizontal position and were loaded at their top surface. The supports at each corner consisted of a 32 mm diameter ball allowing rotation and horizontal straining actions at the two directions. The load was applied to the test specimens at their center by means of a 500 kN capacity hydraulic jack. The jack load was distributed equally by spreading beams in the two directions to sixteen loaded areas on the top surface of the slab, each with dimensions 100x100 mm. Calibrated 500 kN load cell, connected to a strain indicator, was used to enable the determination of the applied load.

Table 1 Properties of tested slabs

Slab	Age of Specimen days	Cube Strength f_{cu} N/mm^2	Dimensions of Secondary Beam, mm		$\frac{h_{sec}}{t_s}$	α	$\alpha_1 L_2 / L_1$
			width	depth			
S1	105	37.0	----	----	----	0	0
S2	40	31.9	----	----	----	0	0
S3	66	44.4	100	100	1.66	0.68	0..34
S4	28	35.5	100	150	2.50	2.72	1.36
S5	38	33.0	100	200	3.33	7.20	3.60

f_{cu}= average value from tests of three 150 mm cubes from the concrete batches used to cast one slab.
h_{sec}= depth of secondary beam. t_s = thickness of slab.

For all the slabs, vertical deflections were measured using dial gauges at nine locations spaced equally each 450 mm in the two directions at the bottom sides of the slab and the interior beam. The deflections of the edge beams were measured at twelve locations; three dials for each beam. The locations of the electrical resistance strain gauges, placed on the bottom steel bars to measure the steel strains, are shown in Fig. 2. The load was applied incrementally (in increments of 10 kN) from zero up to failure. After each increment, all readings were taken and the specimens were examined for cracks. The ultimate load, mode of failure, and the cracking pattern were noted after final failure.



a. Slabs S1 and S2 b. Slabs S3 , S4 and S5

Figure 2 Location of strain gauges on bottom steel bars

EXPERIMENTAL RESULTS

General Behavior of Test Slabs, Mode of Failure, and Ultimate Loads

Observed values of load at first cracking; P_{cr} , first yield of steel reinforcement; P_y , and ultimate loads; P_u, for the five slabs are given in Table 2.

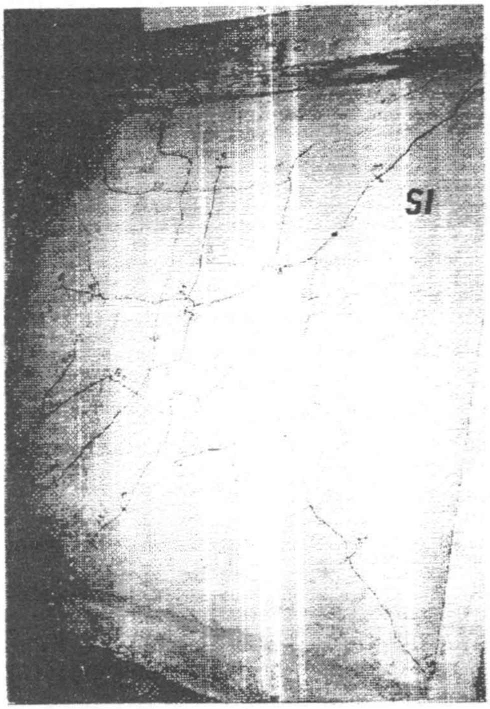
Generally, flexural cracks appeared in the edge beams at loads lower than those at which slab cracking commenced. Visible flexural cracks in edge beams (except beams of S1) appeared at 20 to 26 % of P_u. Flexural cracks on the tension side of the slabs appeared at loads ranged between 33% and 72% of their ultimate loads. The lowest ratio of slab cracking load was recorded for slab S2, with concrete strength relatively low, and the highest ratio was recorded for slab S5, with an interior beam having depth equals to 3.33 times the slab thickness. Cracking of the interior beams occurred at 19 to 31 % of P_u.; the lowest ratio was for the interior beam in slab S5 Cracks formed at the top surface of slab S2 at the connection of the two edge beams, no other top cracks were recorded.

Crack patterns at ultimate loads are shown for some of the slabs in Figure 3. As shown in the figures, the positive moment cracks, for slabs S1 and S2 (without interior beams) tended to form rectangular patterns near the center of each panel and to follow the diagonals of the panel towards the corners. At ultimate loads, these slabs had clear signs of the usual diagonal yield patterns. While in slabs S3, S4 and S5, longitudinal cracks parallel to the interior beams formed at a distance varied from 300 to 400 mm each side from the interior beams. Near the ultimate load of slabs S3 and S4, , diagonal cracks formed at corners and extended to the limits of the longitudinal primary cracks. At ultimate load, hair cracks were observed at the middle part of the slab (perpendicular to the interior beam), and these cracks spread from one side to the other side passing the interior beam. However, no diagonal cracks appeared in slab S5. The distance of the longitudinal cracks may represent the portion of load transferred to the interior beam, i.e. for slabs S3, S4 and S5, about 33% to 45% of the total load was transferred to the interior beam and the remainder of the load was transferred to the edge beams. In the last three slabs which had secondary beam, two vertical cracks appeared near the ultimate load in beams A at the connection of beam A with beam C (see Figures 3-c and 3-d).

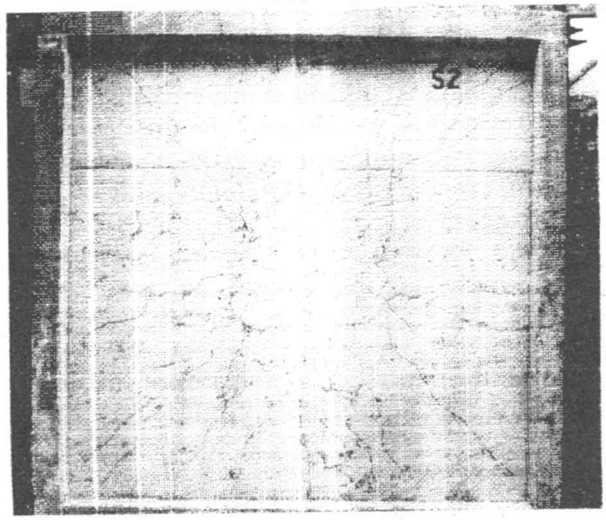
Table 2 Results of the Test Slabs

Slab	Cracking Load, P_{cr} (kN)			Load at First Steel Yield, P_y (kN)				Ultimate Load P_u (kN)	Mode of Failure
	slab	edge beam A	Interior beam C	slab	Edge beam A	Edge beam B	interior beam C		
S1	80.0	80.0	----	NR	NR	NR	---	190.0 #	Corner Failure
S2	100.0	60.0	-----	**No yield**	225.0	NW	267.5+	300.0	Flexure; slab
S3	130.0	60.0	180	176.0*	180.0	222.5	227.0	235.0	Flexure;beam A
S4	130.0	50.0	80.0	180.0*	160.0	240.0	192.5	255.0	Flexure; beam A
S5	190.0	70.0	50.0	**No yield**	NW	260.0	170.0	265.0	Beam A and Slab

NR = Strain not recorded NW = Gauge did not work well
+ gauge no. 1, Figure 2 * gauge no. 2, Figure 2
# Low value due to premature failure of end anchorage of edge beams.

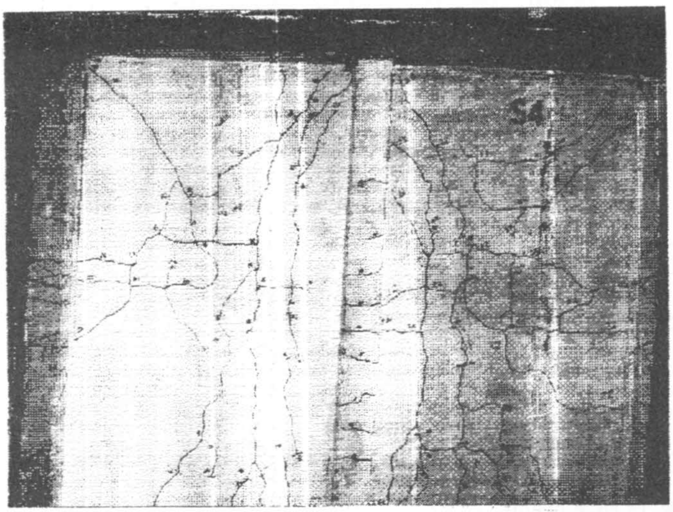


a-S1

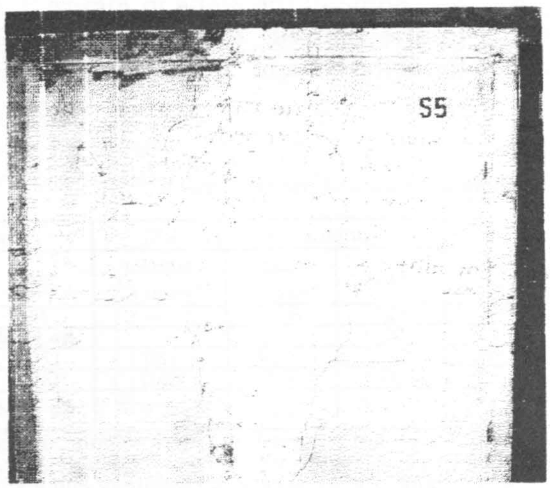


b-S2

Figure 3 Crack patterns of slabs



C-S4



d-S5

Figure 3 (Cont'd) Crack patterns of slabs

Slab S1 failed at one corner due to poor reinforcement anchorage between beams A and B, wherein a bottom diagonal crack developed at the connection and extended vertically along the beams' edge. This may be due to the accumulated torsion moment transferred from the slab which resulted positive moment at the edge of beams. To avoid this phenomenon in subsequent tests, the bottom steel reinforcement of each beam was extended in the adjacent beam with sufficient anchorage length. For other slabs,

flexural failure occurred in edge beam A in addition to the failure at the connection between the interior (secondary) beam C and the edge beam A.
The maximum value of the ultimate load P_u was recorded for slab S2 (without interior beam) for which the behavior was identical to two-way slabs . No difference in the value of P_u was recorded for slabs S3, S4 and S5 since failure occurred mainly in the edge beam (beam A).

The theoretical ultimate loads for the slabs were calculated using the formulas recommended by the ECP [1] with the removal of the material partial factors (γ_c and γ_s) and assuming the following: a uniformly distributed load , slabs are simply supported on edge beams, f_{cu} = 32 N/mm^2 , f_y = 300 N/mm^2 for slabs and f_y = 400 N/mm^2 for beams, d = 50 mm for slabs and d = 180 mm for beams. The calculated ultimate load: P_u for the slabs is 81.2 kN and P_u for beams is 144 kN. All the slabs sustained loads higher than the theoretical ones by an average value of 1.83 (except S1, where P_u experimental/P_u theoretical was only 1.32, because of premature failure of end anchorage of edge beams)

Slab Deflections

The load-deflection curves for all the test slabs are shown in Figure 4. As shown in Figures 4-a and 4-b, for slabs S1 and S2 (without interior beams) the values of deflection recorded for edge beam A were very close to those recorded for edge beam B which indicates that equal load was transferred from the slabs to the edge beams. However, for other slabs with interior beams (Figures 4-c, 4-d, and 4-e) the deflection of beam A was higher than that for beam B which indicates that more load was transferred to beam A through the interior beam C. The deflection of beam B in S5 was very small.
The maximum recorded values of deflection for slabs and beams are given in Table 3. The table indicates that as "α" increased the maximum recorded deflection for the interior beam C increased which indicates that more load was transferred to the beam. From the beginning of loading to prior of significant yield load of slabs S2, S3, S4, and S5 (i.e., up to a load equals to 150 kN), it may observed that the recorded deflections at the mid-quarter of the specimen (dial No. 3, Figure 4) were very close to the recorded deflection at mid-span of interior beam. This means that the presence of secondary beams, with depth varied from 1.66 to 3.33 the slab thickness, did not significantly change the elastic behavior of the slab, and the center deflection of the system is approximately equal to that of mid-quarter even for the slab without interior beam.

The maximum values of deflection recorded in the present study for the slabs, near ultimate load, represent an average value of span/100. In general, building codes for concrete structures do not specify permissible deflection for two-way slabs but give limitations on span/depth ratios. If permissible deflection for two-way slabs under service loads is considered as L/250 (i.e. 7.2 mm in the present study for L = 1800 mm) which is the value recommended for beams and one-way slabs in ECP, the values of loads corresponding to such permissible deflections are given in Table 3. The average value of theses loads was about 0.57 P_u for both the slabs and beam C, and 0.72 P_u for beam A. The maximum recorded values of deflections for the slabs were 0.25 to 0.42 times the slab thickness.

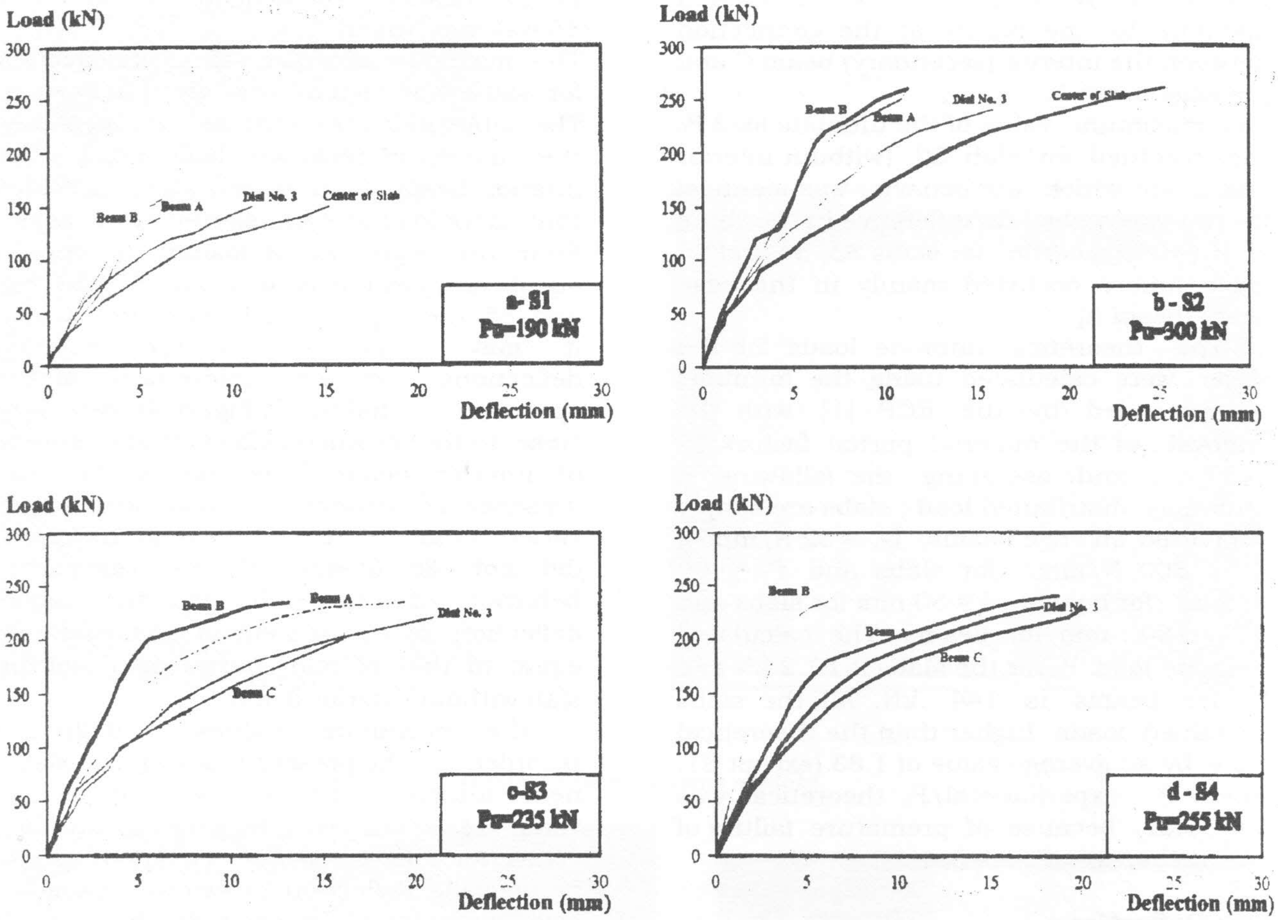


Figure 4 Load – Deflection Relationships

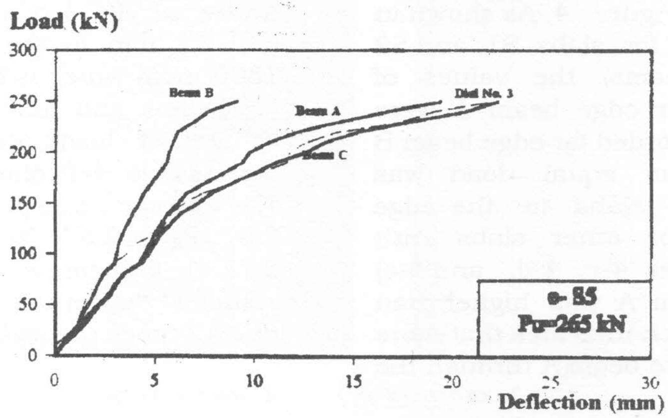


***Figure 4* (Cont'd)** Load – Deflection Relationships

Table 3 Maximum recorded values of deflection

Slab	P_u, kN	Slab		Beam A		Beam B		Interior Beam C	
		Δ, mm	load at deflection of 1/250 span, kN	Δ, mm	load at deflection of 1/250 span, kN	Δ, mm	Load at deflection of 1/250 span, kN	Δ, mm	load at deflection of 1/250 span, kN
S1	190	15.2	107.5	6.5	$>P_u$	6.5	P_u	---	---
S2	300	25.3	135.0	11.3	215.0	11.2	217.5	---	---
S3	235	20.9*	140.0*	19.4	182.5	13.1	210.0	16.5	130.0
S4	255	17.8*	165.0*	18.7	187.5	6.5	P_u	20.1	150.0
S5	265	21.9*	160.0*	19.4	170.0	10.1	230.0	22.3	150.0

* at quarter of slabs

As shown in Figure 5-a, the load-deflection relationship at center of slabs was approximately identical, i.e., having the same initial stiffness. The behavior for slab S2, with band reinforcement, was slightly stiffer than that for the uniformly reinforced slab, S1. Similar results were obtained by Taylor *et al.* [5], who also stated that the use of variable spacing for reinforcement did not lead to a higher load-carrying capacity. However, deflection of beam A (Figure 5-b) was higher for slabs with interior beams.

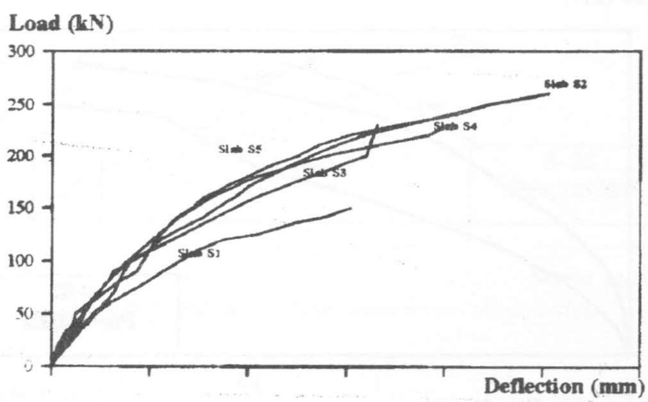


Figure 5-a Deflection at center of slabs

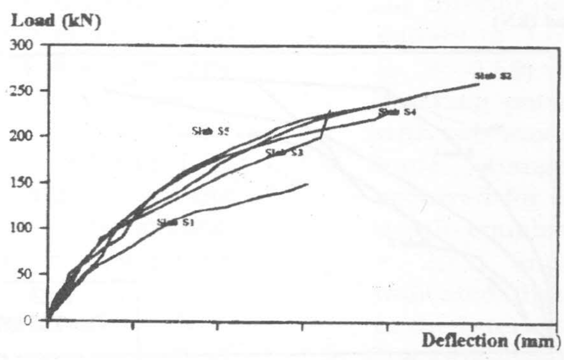


Figure 5-b Deflection of Beam A

Strains in Reinforcement

Figures 6-a to 6-d show the load-steel strain relationship for the tested slabs while Table 2 gives the values of loads at steel yield. At the ultimate load, nearly all of the flexural reinforcement in beams had yielded. Yield of steel reinforcement in beam A occurred at 63 to 77 % of P_u and that for beam B occurred at about 95 % of P_u. Steel yield for the interior beam C occurred at 64 to 97 % of P_u, the lowest ratio was for S5 ,which had the maximum value of "α" and the highest ratio was for S3 with the minimum value of "α". This indicates that for S5, more load was transferred to the interior beam. Figure 6 indicates that for gauge no. 2, placed at slab center perpendicular to the interior beam, with the increase of the interior beam stiffness, the strain decreased. This means a reduction in slab moment in that direction but strains remained positive (tension bottom). Also, for gauge no. 3, placed at quarter of the slab and parallel to the interior beam, with the increase of interior beam stiffness, strain reduced and this means a reduction of moment in that direction.

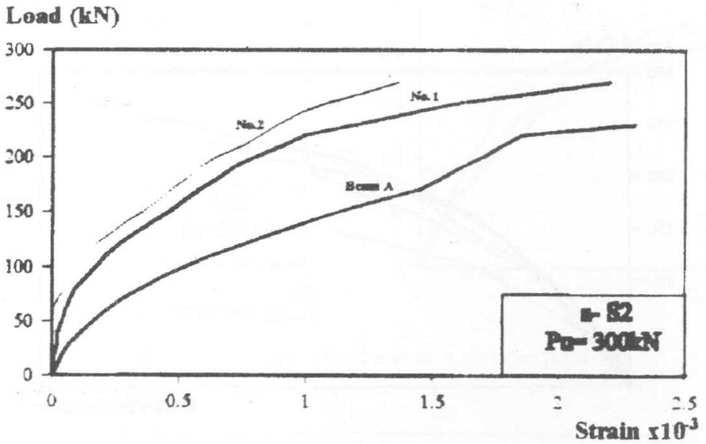
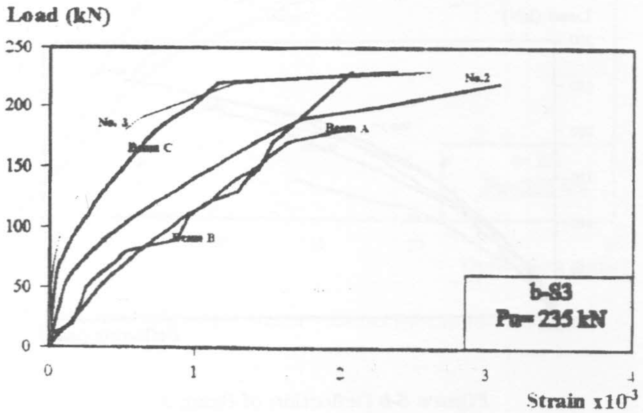


Figure 6 Load-steel strain relationship



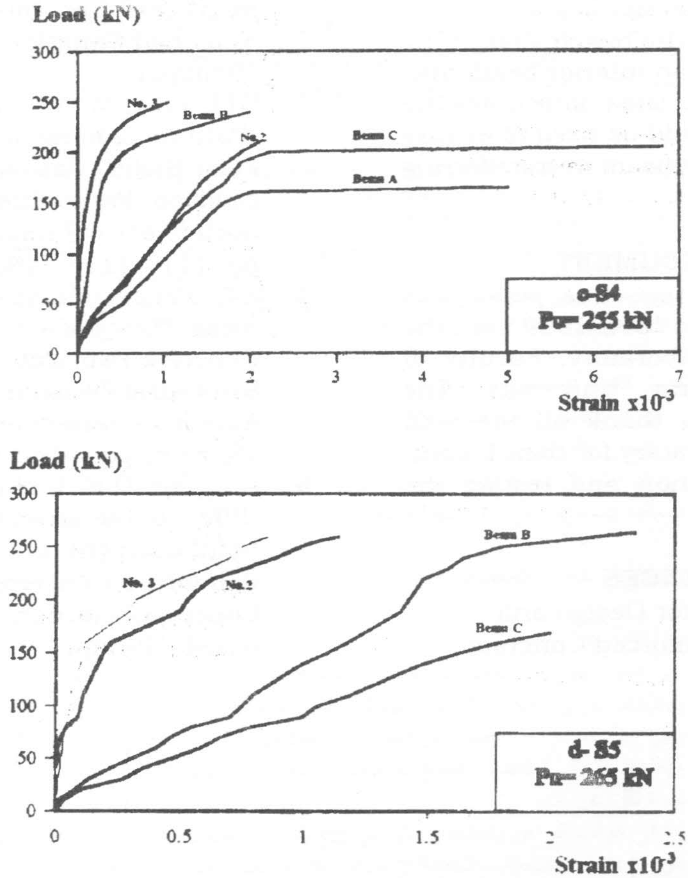


Figure 6 (Cont'd) Load-steel strain relationship

CONCLUSIONS

Five tests were carried out on square slabs supported on four edge beams, the first was an isotropic two-way slab, and the second had an additional band reinforcement in the mid-span in one direction. For the last three tests, a secondary beam replaced the band reinforcement and its depth varied from 1.66 to 3.33 the thickness of the slabs. From the results of the present experimental investigation, the following general points can be concluded :

1. Band reinforcement in slab had no effect on behavior and cracking pattern of two-way square slabs.
2. In the elastic range of loading, the recorded deflections at center of slabs with interior beam were approximately the same; i.e.. secondary beam with depth up to 3.33 the slab thickness did not significantly change the elastic behavior of the slab.
3. For interior (secondary) beam with depth equals to 1.67 times the slab thickness (α = 0.68) no change in behavior or cracking pattern occurred in comparison with two-way slabs without such beams.
4. Some changes in cracking pattern occurred for slab with interior beam with depth equals to 2.5 the slab thickness (α = 2.72) and deflections and strains indicated that some load was transferred to the interior beam.
5. Cracking pattern for the slab with interior beam with depth equals to 3.33 times the slab thickness (α=7.2) indicate that about 33 to 45 % of the load was transferred to the interior beams.

Theoretically, 50 % of the load should be transferred to the interior beam.

6. From the results of the tested slabs, it is recommended that an interior beam with depth equals to at least four times the slab thickness should be used to ensure the efficiency of the beam in transferring the slab load.

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