A THREE DIMENSIONAL KINEMATICAL MODEL OF THE HUMAN TIBIAL-FEMORAL JOINT

M. Moustafa H. El-Gamal, T. Awad and A. Nabih
Mechanical Power Engineering Department, Faculty of Engineering,
Alexandria University, Alexandria, Egypt

In this paper a geometrical approach for investigating the knee is suggested and applied. The results of which undoubtedly affect the different research directions the knee is subjected to. Experimental measurements are carried out here on the surfaces of the actual human knee joints. Surfaces are then fitted to the experimental data collected using a high accuracy optimization technique. Regression equations are obtained representing the surfaces of the femur and tibia. The directions of the common normal as well as the path of contact for the mating surfaces are obtained from this mathematical method. This procedure was applied for different relative positions of femur and tibia.

قد تمت دراسة مفصل الركبة من عده جوانب فهناك اتجاه لدراسة الأحمال على المفصل أثناء حركاته المختلفة و كذلك دراسة تأثير السائل السينوفى عند نقاط التلامس و تأثير ألا ربطه و العضلات المحيطة بالمفصل على الحركة و ذلك بتطبيق الطرق التقليدية أو العناصر المحددة .

يقدم هذا البحث نموذج كينماتيكى ثلاثي الأبعاد لحركه المفصل بين عظمتي الساق و الفخذ للانسان و قد تم قياس أسطح مفصل ركبه أدمي و الحصول على معادلات لسطحي المفصل باستخدام طرق ألا مثليه و التي تحقق اقل خطا مطلق في الأبعاد بينها وبين الأسطح الحقيقية. من ذلك تم تحديد العمود المشترك بين السطحين عند نقط التلامس بين السطحين و كذلك مسارات تلك النقط أثناء دوران عظمه الساق حول عظمه الفخذ . ويعتبر هذا النموذج أساس لحساب توزيع القوى على سطحي التلامس أثناء الدوران .

Keywords **:** Knee model, Kinematic, Tibial, Femoral .

INTRODUCTION

The knee is a condylar synovial joint but as regards the range of movements allowed in the joint, it may be classified as a modified hinge synovial joint. It is a large and complicated joint as seen in Figure 1.
The joint consists of:

1. The femur, which is formed of an upper end, a shaft and a lower end.
2. The patella is the largest sesamoid bone in the body, developing in the insertion of the quadriceps femoris. It is a flatted triangular bone with its apex directed downwards and its base directed upwards.
3. The tibia is the medial and stronger bone of the leg. It is a long bone, which has an upper end, a shaft and a lower end.

The knee joint has been studied from many different viewpoints, but most of the interest has been focused on the importance of the role of the quadriceps in knee function. For more clarification of this role, same investigators related the angular position of the knee joint and force generated by the quadriceps mechanism. Others compared forces generated by the quadriceps with respect to different types of muscular contraction and normal versus abnormal [1]. The forces engendered by the flexors of the knee were also studied together with the mechanics of the knee joint [2].

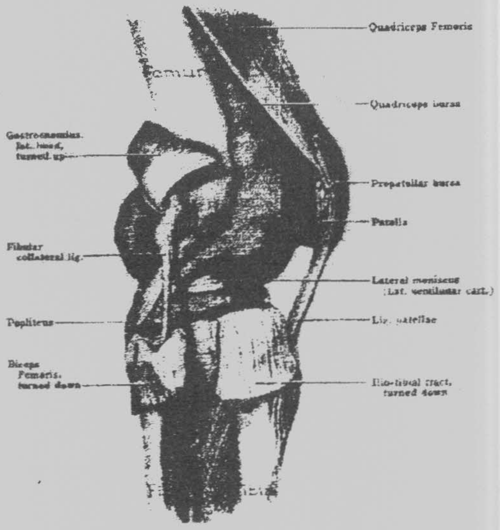

Figure 1 The knee

Mathematical models have proved to be an effective mean to study structural elements of the musculoskeletal system. These tools are most effective in simulating joint activities using the computer rather than costly in vivo experimental procedures. Also, one may list out ideas on the function of the different components of the joint using modeling techniques before performing such tests on vitro or in vivo. However, models must be validated experimentally [3]. Two types of models were identified: phenomenological and anatomical.

The state of the art of anatomical knee models is summarized as follows:

1- Most models assumed either static or quasi-static conditions. Only one dynamic model is reported. It is a two dimensional model, considering motions to take place in the sagittal plane [4].
2- Few models of the patello-femoral joint are identified. The most comprehensive model reported is two dimensional [5], and accounted for both movement and forces in the sagittal plane.
3- An integrated model of the human knee joint describing the interactions between the tibia, femur, patella, and fibula do not exist in either two-dimensional or three-dimensional formats [3].

Jonsson, Karrholm and Elmqvist [6] found that during the active extension of the knee, from passive flexion at 30°, the tibial initially rotates internally followed by an external rotation. At the same time abduction occurs. The tibial intercondylar eminence displaces laterally, distally, and anteriorly. Absence of the Anterior Cruciate Ligament (ACL) probably does not significantly change the tibial rotations, but may cause a more pronounced distal and anterior position of the center of the knee, up to 10°, at which the instability disappears. Hart, Mote and Skinner [7] studied the flexion of knee joint from 0 to 90°.

METHODS AND PROCEDEURES

In designing a true anatomic resurfacing prosthesis for a joint, it is of prime importance to study the articulating joint shapes, internal structure and geometric properties of the bone. This is a key to the better understanding of joint motion. The object of this study is to develop a three-dimensional kinematical model of the tibial-femoral knee joint, which modeled as two rigid bodies representing a moving tibia and fixed femur.

CASE STUDY

Subjects

A normal dead knee joint -(femur and tibia bones) - for a man (50 year of age), with no history of knee pain during his life is studied.

Experiment

Each bone was individually immersed in a transparent polyester resin to fill up any holes or cavitations found in it. The polyester resin used here is known as Siropol8231 and was given suitable time to cure. The bone was then mounted by means of a cylindrical frame in a casing containing black polyester resin with glass fiber in order to facilitate its mounting on the lathe.

The specimen was mounted on the lathe and clamped properly to provide clean cutting. Each cut was adjusted to remove a layer of 1.0 mm thick from the femur, and 0.5 mm from the tibia. The exposed surface profile, external reference lines and the frame counter were photographed using a 35 mm camera. The accuracy of measurement is ±0.01 mm which equal to the accuracy of the lathe.

The photographs were traced using a two dimensional digitizer (Auto master Summagraphics). Each digitized surface profile was reproduced. They are stored in DWG files and converted to DXF files using the AutoCAD software. The X, Y data for each layer were collected from the DXF files. The data files were displayed using the WinWord software and all unnecessary statements appearing in the data are omitted.

A Fortran program SURFMIN was developed and designed to find the optimum equation fitting the two surfaces of the tibia and femur. The optimization technique based on the gradient method of Zettel [8] and modified by Awad [9] is applied in order to minimize the following selected objective function:.

$$\phi = \sum_{N=1}^{N_p} \left| Z_{measured} - Z_{calculated} \right| \qquad (1)$$

Where:

N_p is the number of points to be fitted on the surface.

$Z_{measured}$ is the distance measured in Z direction for a specific measured X, Y values.

$Z_{calculated}$ is the value of the distance obtained from the equation proposed to represent the surface at the same specified X, Y values.

For the sake of good accuracy, two conditions must be satisfied. The number of measured points must be large and the proposed form of equation must have enough number of coefficients. The well-known software such as SURFER or EXCEL can execute only small numbers of points and represents yield rather simple equations. So the available surface fitting software are not suitable for the present study and the need for designing a special program SURFMIN is required to incorporate a large number of fitted points and complex to somewhat equations representing the surfaces.

Due to the difficulty of the proposed equations and the inclusion to a 10 and more coefficients to be optimized, the objective function may have many local minima. The program is designed to keep searching for the global minimum to attain good accuracy between the equation representing the surface and the real measured surface it self. Many forms of equations are proposed and optimized. The surfaces obtained from that equation failed to realize accurate fitting with the measured surfaces.

The optimized Equation 2 represents the femur surface optimized, while equation 3 is the optimized relation for the tibia surface. The accuracy for Equation 2 is 91.4% and for Equation 3 is 91.65%. These equations may be written as,

$$\begin{aligned} Z_F = & -3.2x^2e^{-y} + 0.0659y^2e^{-x} - 0.00085xy \\ & + 2.88x - 2.32y + 0.0135x^3 - 0.0058y^3 \\ & - .344x^2 + 0.2y^2 + 0.1x^2ye^{-xy} + .02xy^2e^{-xy} \\ & - 0.0496\sin(x)\cos(y) + 0.059\sin e^{-x} + 1.412 \end{aligned} \qquad (2)$$

$$\begin{aligned} Z_T = & -21.06x^2e^{-y} + 18.8y^2e^{-x} + 0.0165xy \\ & + 4.82x - 4.36y + 0.0109x^3 - 0.0069y^3 \\ & - .405x^2 + 0.3y^2 + .01x^2ye^{-xy} + .02xy^2e - xy \\ & - 0.104\sin(x)\cos(y) - 0.00484\sin e^{-x} + 0.16 \end{aligned} \qquad (3)$$

In order to draw the two surfaces obtained a MATLAB program MATSURF is developed. This program can plot each surface separately or both together and can allow also a rotation of one surface relative to the other. Plots of the contours for the two surfaces are options. The femur surface is given in Figure 2. The tibia surface is given in Figure 3.

PATH OF CONTACT OF THE TIBIAL-FEMORAL KNEE JOINT

It is required to get the path of contact between the tibia and femur during 90° rotation from the in line position of the two bones. This is great of importance in any force analysis of the joint.

Procedure

To obtain the path of contact of tibial-femoral joint, it is required to rotate the tibia relative to the femur and with the help of a suitably designed optimization program named PATHFOR the points that realize the same coordinates and the same common normal (direction cosines) are the contact points.

The procedures are repeated every 2.5° of the rotation of the tibia relative to the femur to obtain the path of contact between zero and a maximum angle of rotation of 90°.

The derivatives of the femur and coordinates Z_F and Z_T are required for calculating the common normal to the surface.

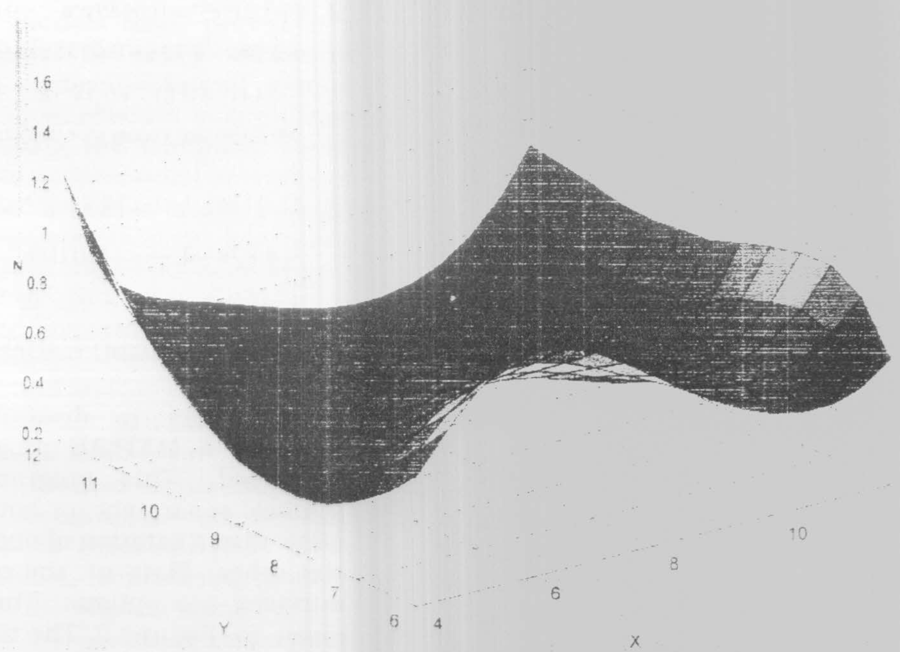

Figure 2 Surface of the femur

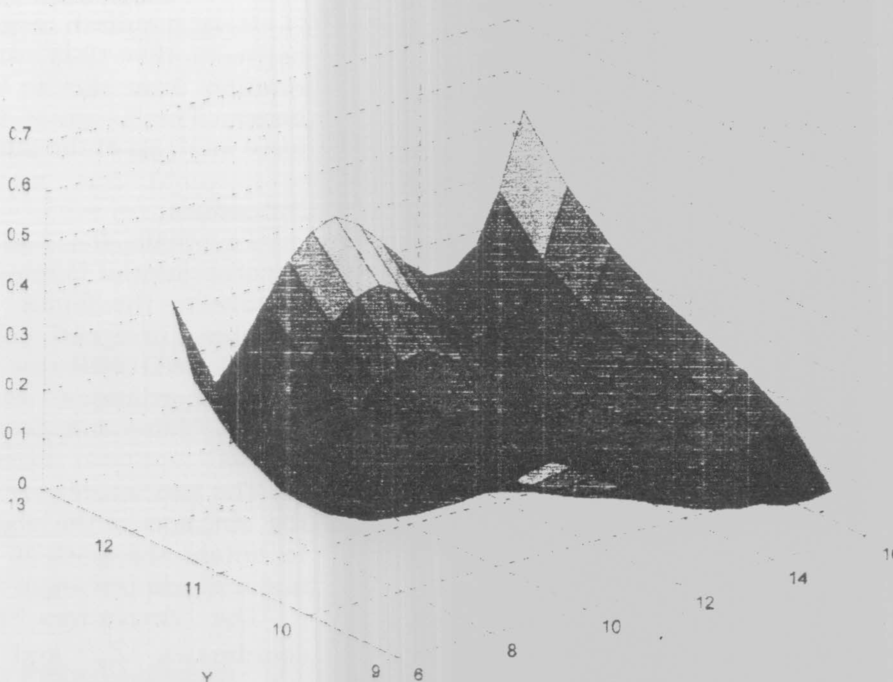

Figure 3 Surface of the tibia

The direction cosines of the common normal for tibia and femur at the points of the contact must be equal and are calculated as follows:

The vector in the direction of the common normal of femur may be written as.

$$\vec{N}_F = A_F i + B_F j + C_F k \qquad (4)$$

and the vector in the direction of the common normal of tibia may be written as.

$$\vec{N}_T = A_T i + B_T j + C_T k \qquad (5)$$

Where:

$$A_F = \frac{\partial Z_F}{\partial X} \quad \text{and} \quad A_T = \frac{\partial Z_T}{\partial X}$$

$$B_F = \frac{\partial Z_F}{\partial Y} \qquad B_T = \frac{\partial Z_T}{\partial Y}$$

$$C_F = \frac{\partial Z_F}{\partial Z} \qquad C_T = \frac{\partial Z_T}{\partial Z}$$

The direction cosines for femur are.

$$\cos\alpha_F = \frac{A_F}{\sqrt{A_F^2 + B_F^2 + C_F^2}}$$

$$\cos\beta_F = \frac{B_F}{\sqrt{A_F^2 + B_F^2 + C_F^2}} \qquad (6)$$

$$\cos\gamma_F = \frac{C_F}{\sqrt{A_F^2 + B_F^2 + C_F^2}}$$

The direction cosines for tibia are.

$$\cos\alpha_T = \frac{A_T}{\sqrt{A_T^2 + B_T^2 + C_T^2}}$$

$$\cos\beta_T = \frac{B_T}{\sqrt{A_T^2 + B_T^2 + C_T^2}} \qquad (7)$$

$$\cos\gamma_T = \frac{C_T}{\sqrt{A_T^2 + B_T^2 + C_T^2}}$$

Naturally the equation of the tibia surface will be changed due to its rotation relative to the femur. The vector joining the two contact points before rotation is not necessarily parallel to the X direction. The tibia was rotated about an axis passing through mid point (X_r, Y_r, Z_r) of the vector joining the two previous point of contact and parallel to X axis with rotation angle equal θ. It is assumed that the motion between the two surfaces is a pure rolling but about that axis. This assumption allows a rotational movement in the (Z-Y) plane and sliding at the points of contact which simulate the real relative movement of the joint.

The equation of tibia after rotation relative to the original coordinates may be written as given by Equation 8.

$$\begin{aligned} f = & -21.06X^2 e^{-((Y-Y_r)\cos\theta-(Z-Z_r)\sin\theta)+Y_r)} + 18.8((Y-Y_r)\cos\theta+(Z-Z_r)\sin\theta)+Y_r)^2 e^{-X-Y_r} \\ & +0.0165X((Y-Y_r)\cos\theta+(Z-Z_r)\sin\theta)+Y_r)+4.8X-4.36((Y-Y_r)\cos\theta+(Z-Z_r) \\ & \sin\theta)-4.36Y_r+.01X^3-.0069((Y-Y_r)\cos\theta+(Z-Z_r)\sin\theta)+Y_r)^3-0.405X^2 \\ & +0.303((Y-Y_r)\cos\theta+(Z-Z_r)\sin\theta)+Y_r)^2+0.1((Y-Y_r)\cos\theta+(Z-Z_r)\sin\theta)+Y_r) \\ & X^2 e^{-((Y-Y_r)\cos\theta-(Z-Z_r)\sin\theta)-Y_r).X} + 0.2((Y-Y_r)\cos\theta+(Z--Z_r)\sin\theta)+Y_r)^2 \\ & e^{-((Y-Y_r)\cos\theta-(Z-Z_r)\sin\theta)-Y_r).X} X - 0.104\cos((Y-Y_r)\cos\theta+(Z-Z_r)\sin\theta)+Y_r)\sin X \\ & -4.84E-03\sin(e^X)+0.16+(Y-Y_r)\sin\theta-(Z-Z_r)\cos\theta)-Z_r \\ & =0.0 \end{aligned} \qquad (8)$$

Where:

θ is a total angle of rotation.

Y_r and Z_r are the coordinates of the middle point of the vector $\vec{N}$.

The software PATHFOR is designed in FORTRAN language to search for the points of contact between tibia and femur. Those points provide the same coordinate position and same direction cosines. This can be realized by minimizing the following objective function at each angle of rotation of tibia relative to the femur.

$$\phi_{min} = |\cos\alpha_F - \cos\alpha_T| + |\cos\beta_F - \cos\beta_T| + |\cos\gamma_F - \cos\gamma_T| + |f(X,Y,Z)_F - f(X,Y,Z)_T| \quad (9)$$

TIBIAL-FEMORAL CONTACT CHARACTERISTICS

Figure 4 shows the pathway of the medial and lateral femoral contact points between the femur and the tibia. As the angle of fluxion increases the lateral contact point moves distally on the femur without moving significantly medially or laterally until 25° of flexion. Then it moves medially from 25° to 40° of joint flexion, followed by a lateral movement as the joint flexion angle increases. The medial contact moves distally on the femur, but moves medially from full flexion to about 5° of joint flexion. As joint flexion angle increases to 25° the medial contact moves laterally followed by medial movement for further increase of flexion angle.

As the flexion angle increases, the medial and lateral contact points do not change significantly, as shown in Figure 5.

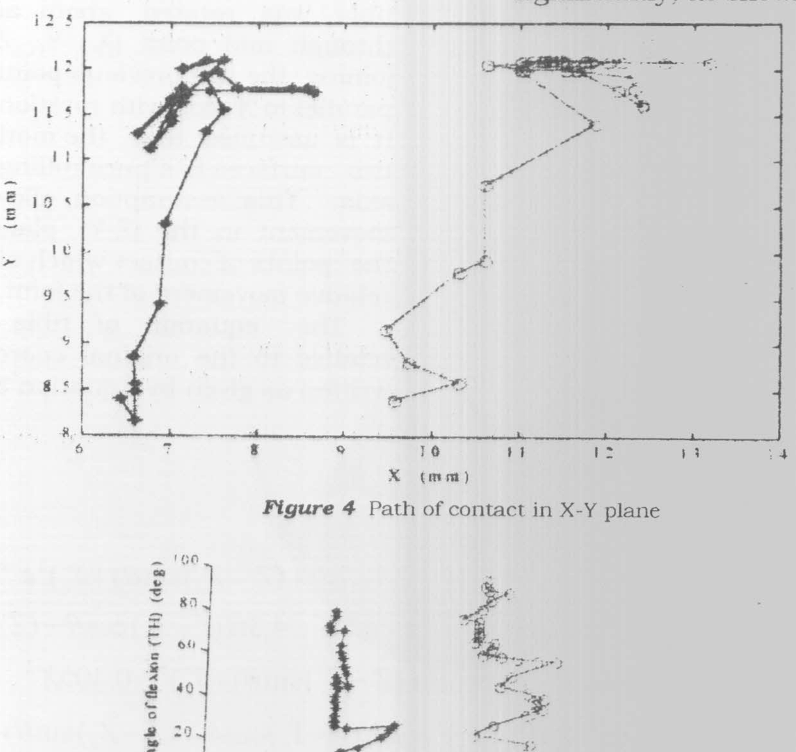

Figure 4 Path of contact in X-Y plane

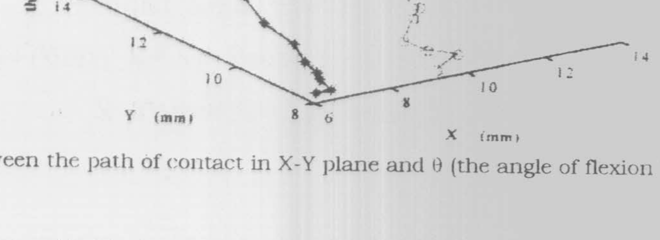

Figure 5 Relation between the path of contact in X-Y plane and θ (the angle of flexion between femur and tibia)

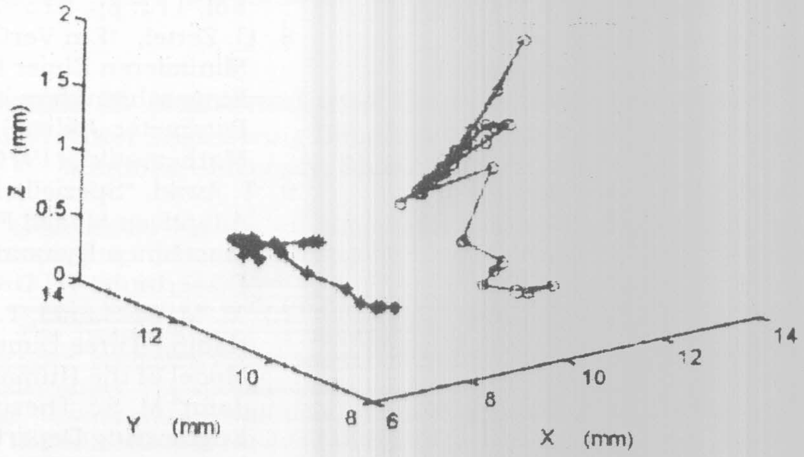

Figure 6 path of contact in three dimensions

For the lateral points of contact small changes in Z direction take place as joint flexion angle increases. But for the medial points the smooth changes in the Z direction are the general trend up to 40° followed by a high jump, then a drop to almost the same previous levels. (see Figure 6).

DISCUSSION AND CONCLUSION

As a result of this work two equations representing the surfaces of the femur and tibia of the human knee joint are obtained. This makes it possible to find the normals to these surfaces and hence the path of contact between the two bones is predicted. The form of two paths undoubfully affect the force analysis for the tibial femoral joint.

The two paths of contact are not similar because of the following reasons

1. The angle of femoral torsion which takes place when the longest axes of the upper and lower ends of the femur are not in the line with each other. The neck of femur is directed upwards and medially with a slight forward inclination. Accordingly, the long axis of the neck forms an angle of femoral torsion with the transverse axis of the lower end.
2. The effect of the cartilage between the two bones is neglected because it is somewhat difficult to find it in vivo and the donation system does not exist.
3. The effect of muscles and ligaments are neglected.

For a more accurate path of contact, the future work should include the following:-

1. A more general kinematic analysis taking into consideration the real paths of contact between patella, tibia and femur.
2. Force analysis model, which takes into account the effect of ligaments and the real directions of reactions between the mating surfaces of the human knee *joint.
3. A more realistic model of the knee joint considering the elasticity of the mating surfaces and the presence of the synovial fluid.

REFERENCES

1. G.A. Ateshian, S.D. Kwak, L.J. Soslowsky and V.C.A. Mow, "Stereophotogrammetic Method for Determining in Situ Contact Areas in Diarthrodial Joints, and a Comparison With Other Methods", J. Biomechanics, Vol. 27, No. 1. pp. 111-124. (1994).
2. G.L. Smidt, "Biomechanical Analysis of Knee Flexion and Extension, J. Biomechanics, Vol. 6, pp. 79-92, (1973).
3. J.D. Reuben, J.S. Souick, R.J. Schrager, P.S. Walker and A.L. Boland, "Three Dimensional Dynamic Motion Analysis of the Anterior a Qaruciate Ligament Deficient Knee Joint", The American

Journal of Sports Medicine, Vol. 17, No. 4, pp. 463-471, (1989).
4. M.H. Moeinzadeh, A.E. Engin and A. Akkas, "Two-Dimensional Dynamic Modeling of Human Knee Joint, J. Biomechanics, Vol. 16, No. 4, pp. 253 - 264.(1983).
5. M.S. Hefzy, and E.S. Grood, "Review of Knee Models", Appl. Mech. Rev, Vol. 41, No. I, pp. 1-13, (1988).
6. H. Jonsson, J. Karrholm, and L.G. Elmqvist, "Kinematics of Active Knee Extension After Tear of the Anterior Cruciate Ligament", The American Journal of Sports Medicine, Vol. 17, No. 6, pp.796 - 802.
7. R.A. Hart, C.D. Mote and H.B. Skinner, "A Finite Helical Axis as a Landmark of Kinematics Reference of the Knee, Journal of Biomechanical Engineering, Vol. 113, pp. 215 -221, (1991).
8. G. Zettel, "Ein Verfahrm Zum Minimieren Eimer Funktion bei Eingeschrenktem Variationsbereich der Parameter-Journal, Numererisch Mathematik", (1970).
9. T. Awad, "Spezielle Probleme der Adaptiven Modell Findung in der Maschinen Dynamik", Ph.D. Dissertation TU Dresden, (1978).
10. M. M. El-Gamal, T. Awad and A.A. Nabih, "Three Dimensional Kinematical Model of the Human Tibial-Femoral Joint" M. Sc. Thesis, Mechanical Engineering Department, Alexandria University, (1999).

Received August 10. 1999
Accepted November 16. 1999