ORIENTATION OF POINT ANTENNA ON THE TRIAXIAL ELLIPSOIDAL EARTH TO A COMMUNICATION SATELLITE

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ABSTRACT

Through the Global Positioning System (GPS), precise orientation and positioning are the major device for visualizing spatial motion of the distributed visual satellites in the sky. The optimum geodetic position of an antenna, suitable for observing a movable or geostationary satellite from a point located on the triaxial ellipsoidal earth, is determined herein. The mathematical treatment presented here takes into consideration the equatorial ellipticity (i.e. the earth's triaxiality). The solution depends exclusively on the combination of the advanced relations of the differential geometry with the basics of geodesy. In this manner two approaches are developed herein to extract the orientation elements. Firstly, the necessary formulae are accomplished by applying the basics of the differential geometry to the triaxial ellipsoidal earth. Secondary, the geometric characteristics of the deduced formulae are checked those based on the transformation matrix between the geodetic and local geodetic coordinate systems. A comparison between the present results and those previously based on the spherical earth is introduced. The validity of the present techniques is illustrated with numerical applications. When very precise pointing to communication satellites and other space objects is dictated, the correction of the slight deviations should be taken into consideration. These deviations will cause very significant errors if high gain antennas or electro-optical devices such as lasers are used.

Keywords: Differential geometry, Triaxial ellipsoid, Satellite orbit, Transformation matrix.

INTRODUCTION

Orientation elements of an antenna to a geostationary or movable communication satellite are the main requirement for an ideal observation. Precise values of the azimuth and altitude of the satellite are considered extensive tasks of the geodesy. The solutions of such orientation problems were previously introduced but they were based on the spherical datum of the earth [1]. Obviously, the spherical datum was chosen to avoid the complexity of the mathematical analysis due to the oblatity or triaxiality of the precise datum of the actual earth. To avoid the uncertainty of results due to excluding the oblatity of the earth, some corrections have to be added to the final spherical result [1]. The current work seems to be more consistent since it abandons the ambiguity due to the earth's figure and the satellite's fixed position. The triaxiality of the datum is considered herein for the reference to match the earth, (i.e. triaxial ellipsoidal), [2-4]. Conventionally, the observed satellite is located at a general position, not fixed at the equatorial plane. The satellite is considered to move along normal orbit [5]. The actual path of the satellite along a normal orbit is an ellipse located on the orbital plane, which is fixed in space. The focal point of the orbital ellipse is at the center of the earth. The geometric characteristics of the triaxial datum and satellite orbit are the main devices to develop the generalized exact orientation elements. The present paper offers a precise model valid to the satellite communications with the high gain antennas or electro-optical devices such as lasers.

POSITION VECTOR OF THE EARTH STATION AND MOVABLE SATELLITE

Geometric Representation of the Earth Station

The position vector $\bar{r}(\lambda, \varphi)$ of a current point $P(X, Y, Z)$ on the surface of the ellipsoidal reference system shown in Figure 1 is [2]:

$$\bar{r}(\lambda, \varphi) = \bar{r}(X, Y, Z)$$

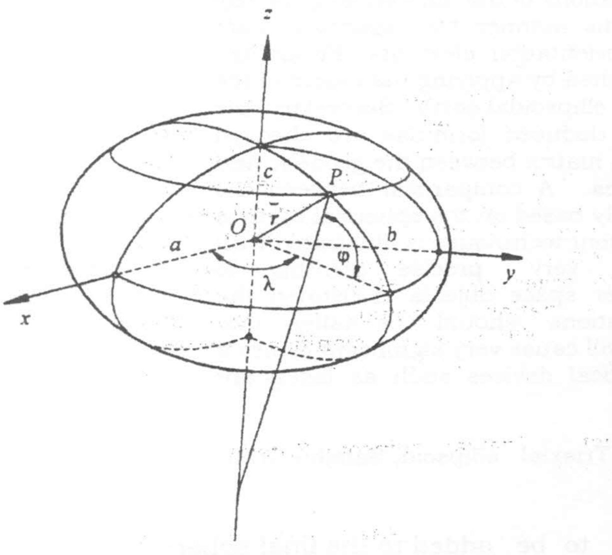

Figure 1 Position vector of an ellipsoidal point p

where:

$$X = \frac{b^2 \cos\lambda \cot\varphi}{[\sin^2\lambda + \frac{b^2}{a^2}\cos^2\lambda]^{\frac{1}{2}}[b^2\cot^2\varphi + c^2(\sin^2\lambda - \frac{b^2}{a^2}\cos^2\lambda)]^{\frac{1}{2}}},$$

$$Y = \frac{b^2 \sin\lambda \cot\varphi}{[\sin^2\lambda + \frac{b^2}{a^2}\cos^2\lambda]^{\frac{1}{2}}[b^2\cot^2\varphi - c^2(\sin^2\lambda - \frac{b^2}{a^2}\cos^2\lambda)]^{\frac{1}{2}}},$$

$$Z = \frac{c^2[\sin^2\lambda + \frac{b^2}{a^2}\cos^2\lambda]^{\frac{1}{2}}}{[b^2\cot^2\varphi - c^2(\sin^2\lambda + \frac{b^2}{a^2}\cos^2\lambda)]^{\frac{1}{2}}} \qquad (1)$$

here a, b and c are the semi axes lengths of the triaxial ellipsoid; $a > b > c$, while λ, φ are the geographic coordinates of the current point on the surface, (Figure 1).

Practically, due to the irregularity of the surface of the earth, a geodetic height h_I may be existing at the surface point P. The actual coordinates of the earth station related to position P are the geodetic coordinates (X_I, Y_I, Z_I) of point $P(X_P, Y_P, Z_P)$, such that:

$$\begin{Bmatrix} X_I \\ Y_I \\ Z_I \end{Bmatrix} = \begin{Bmatrix} X_P[1 + h_I/(a^2 G_P)] \\ Y_P[1 + h_I/(b^2 G_P)] \\ Z_P[1 + h_I/(c^2 G_P)] \end{Bmatrix} \qquad (2)$$

where:

$$G_P = [\frac{X_P^2}{a^4} + \frac{Y_P^2}{b^4} + \frac{Z_P^2}{c^4}]^{\frac{1}{2}} \qquad (3)$$

Geometric Characteristics of the Satellite Orbit.

The orbital motion of satellite is a result of the earth's gravitational attraction and some other forces acting on the satellite such as the attraction of the sun and the moon in addition to the pressure on the satellites caused by impacting solar radiation particles [5]. Mathematically, the equations of motions for satellite are differential equations that are solved by numerical integration over time. The integration begins with initial conditions, such as the position and velocity of the satellite at some initial epoch [5, 6]. To simplify attempts to study satellite motion, we study the so-called normal orbits. For a normal orbit, a satellite moves along the actual path in an orbital plane fixed in space. In the mathematical strict sense, the actual path [5] of the satellite in the orbital plane is an ellipse as shown in Figure 2. The focal point of the orbital ellipse is fixed at the center of the earth, (Figure 3).

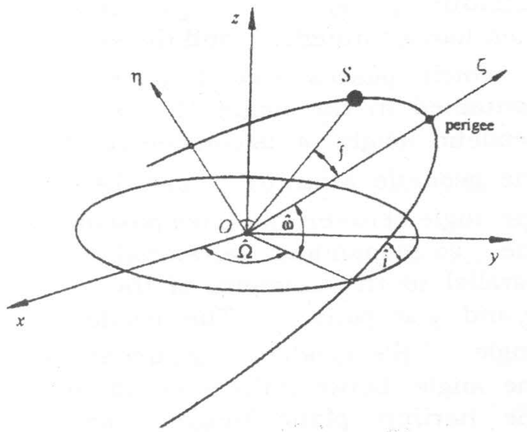

Figure 2 Orientation elements of satellite orbit

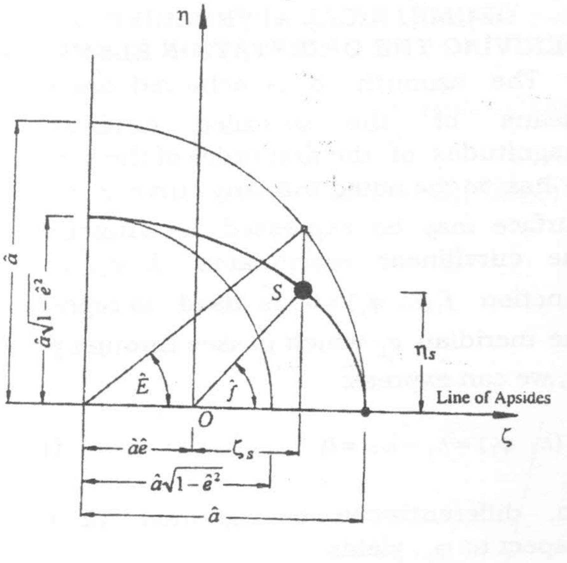

Figure 3 Geometric characteristics of the satellite orbit

Six Keplerian elements are often used to describe the position of satellite in space. These elements [5-7] are, as shown in Figures 2, 3, the semi major axis $\hat{a}$, the eccentricity $\hat{e}$ of the elliptic orbit, the inclination $\hat{i}$ of the orbital plane with respect to the earth's equatorial plane, the right ascension of the ascending node $\hat{\Omega}$ of the orbit, the argument of the perigee $\hat{\omega}$ and the eccentric anomaly $\hat{E}$. The two elements $\hat{a}$ and $\hat{e}$ define the size of the orbital plane while the three elements $\hat{i}$, $\hat{\Omega}$ and $\hat{\omega}$ define the position and slope of the orbital plane in space with respect to the earth's axes. The eccentric anomaly $\hat{E}$ defines the position of the satellite in its orbit at certain epoch. The eccentric anomaly $\hat{E}$ is also defined geometrically as a function of the true anomaly $\hat{f}$ as shown in Figure 3.

The positional coordinates X_S, Y_S and Z_S of a satellite S, referred to the geocentric reference axes, are:

$$\begin{Bmatrix} X_S \\ Y_S \\ Z_S \end{Bmatrix} = [\Re] \begin{Bmatrix} \zeta_S \\ \eta_S \\ 0 \end{Bmatrix} \qquad (4)$$

where

$$[\Re] = \left[\Re_3(-\hat{\Omega})\Re_1(-\hat{i})\Re_3(-\hat{\omega})\right] \qquad (5)$$

and

$\Re_1(-\hat{i})$ and $\Re_3(-\hat{\Omega})$ are the well - known rotation matrices defined by:

$$\Re_1(-\hat{i}) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-\hat{i}) & \sin(-\hat{i}) \\ 0 & -\sin(-\hat{i}) & \cos(-\hat{i}) \end{bmatrix} \qquad (6)$$

$$\Re_3(-\hat{\Omega}) = \begin{bmatrix} \cos(-\hat{\Omega}) & \sin(-\hat{\Omega}) & 0 \\ -\sin(-\hat{\Omega}) & \cos(-\hat{\Omega}) & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (7)$$

The matrix $\Re_3(-\hat{\omega})$ can be accomplished by analogy from Equation 7 when $\hat{\Omega}$ is replaced with $\hat{\omega}$.

The local satellite's coordinates ζ_S, η_S referred to the orbital plane's axes ζ and η as shown in Figure3 are expressed as [7]:

$$\left.\begin{aligned} \zeta_S &= \hat{a}\{\cos(\hat{E}) - \hat{e}\} \\ \eta_S &= \hat{a}\sin(\hat{E})\sqrt{1-(\hat{e})^2} \end{aligned}\right\} \qquad (8)$$

It has to be noted that the eccentric anomaly $\hat{E}$ is evaluated by iteration from as [7]:

$$\hat{E} = \hat{M} + \hat{e}\sin(\hat{E}) \quad (9)$$

where $\hat{M}$ is the mean anomaly expressed by:

$$\hat{M} = \sqrt{\frac{\hat{W}}{(\hat{a})^3}}\,(t-T) \quad (10)$$

in which $\hat{W}$ is the earth's gravitational constant and $(t-T)$ is the difference (in seconds) between the time of perigee passage t and the time of satellite passage T at a certain epoch.

Substituting from Equations 6, 7 into 4, 5 one can find:

$$\begin{Bmatrix} X_S \\ Y_S \\ Z_S \end{Bmatrix} = \begin{Bmatrix} \hat{a}\{\cos(\hat{\Omega})\cos(\hat{\omega}) - \sin(\hat{\omega})\sin(\hat{\Omega})\cos(\hat{i})\}\{\cos(\hat{E}) - \hat{e}\} \\ -\hat{a}\{\cos(\hat{\Omega})\sin(\hat{\omega}) + \cos(\hat{\omega})\sin(\hat{\Omega})\cos(\hat{i})\}\sin(\hat{E})\sqrt{1-(\hat{e})^2} \\ \hat{a}\{\sin(\hat{\Omega})\cos(\hat{\omega}) + \sin(\hat{\omega})\cos(\hat{\Omega})\cos(\hat{i})\}\{\cos(\hat{E}) - \hat{e}\} \\ +\hat{a}\{-\sin(\hat{\Omega})\sin(\hat{\omega}) + \cos(\hat{\omega})\cos(\hat{\Omega})\cos(\hat{i})\}\sin(\hat{E})\sqrt{1-(\hat{e})^2} \\ \hat{a}\{\sin(\hat{i})\sin(\hat{\omega})\}\{\cos(\hat{E}) - \hat{e}\} \\ +\hat{a}\{\sin(\hat{i})\cos(\hat{\omega})\}\sin(\hat{E})\sqrt{1-(\hat{e})^2} \end{Bmatrix} \quad (11)$$

these formulas show the geocentric coordinates of the satellite expressed as functions of the six Keplerian elements.

FORMULATION OF THE PROBLEM

Let the dish antenna be located on the earth's surfaces at a position $P(X_P, Y_P, Z_P)$ and the instantaneous position of the satellite is $S(X_S, Y_S, Z_S)$ as in Figure 4. Furthermore, assume that $\Pi[w, w^*]$ denotes the plane which is specified by two parallel lines w, w^* where w is the normal to the earth at P and w^* passes through the satellite S. Plane Π intersects the elliptic equator at the sub-satellite E. The distance $\overline{PS}$ is the range of the object S (distance from satellite to the antenna P). The geodetic azimuth β is the angle between the meridian g_1 through P and the surface curve g which passes though p and E and is contained in the plane Π. Sometimes, if a geodetic height h_I is considered at point P, the geodetic azimuth β can be defined as the angle between two lines passing through the corresponding terrestrial point and parallel to the tangents of the two curves g_1 and g at point P. The geodetic vertical angle ω (i.e. geodetic altitude) of satellite is the angle between the observation line and the horizon plane (tangent plane) to the surface through point P. The main objective is to determine the azimuth β and the vertical angle ω.

GEOMETRICAL APPROACH FOR DERIVING THE ORIENTATION ELEMENTS

The azimuth β is achieved firstly by means of the so-called fundamental magnitudes of the first order of the surface. It has to be noted that any curve g_i on the surface may be expressed as a function of the curvilinear coordinates λ_i, φ_i. If the function $f_1(\lambda_1, \varphi_1) = 0$ is used to represent the meridian g_1 which passes through point P, we can express:

$$f_1(\lambda_1, \varphi_1) = \lambda_1 - \lambda_P = 0 \quad (12)$$

so, differentiation of Equation 12 with respect to φ_1, yields

$$\frac{\partial \lambda_1}{\partial \varphi_1} = 0 \quad (13)$$

On the other hand, the surface curve g, shown in Figure 4, is the curve of intersection of the plane and the surface. The equation of plane is:

$$\ell X + mY + nZ - R = 0 \quad (14)$$

where:

$$R = \ell X_P - mY_P + nZ_P$$

The magnitudes ℓ, m and n are the direction ratios of the normal to the plane Π so that:

$$\left.\begin{aligned} \ell &= \rho_{PS}\frac{Z_P}{c^2} - \sigma_{PS}\frac{Y_P}{b^2}; \\ m &= -\sigma_{PS}\frac{X_P}{a^2} + \varepsilon_{PS}\frac{Z_P}{c^2}; \\ n &= \varepsilon_{PS}\frac{Y_P}{b^2} - \rho_{PS}\frac{X_P}{a^2}; \end{aligned}\right\} \quad (15)$$

The magnitudes $\varepsilon_{PS}, \rho_{PS}$ and σ_{PS} (direction ratios of line PS) are:

$$\varepsilon_{PS} = \left\{ \hat{a}\{\cos(\hat{\Omega})\cos(\hat{\omega}) - \sin(\hat{\omega})\sin(\hat{\Omega})\cos(\hat{i})\}\{\cos(\hat{E}) - \hat{e}\} - \hat{a}\{\cos(\hat{\Omega})\sin(\hat{\omega}) + \cos(\hat{\omega})\sin(\hat{\Omega})\cos(\hat{i})\}\sin(\hat{E})\sqrt{1-(\hat{e})^2} \right\} - X_P \,.$$

$$\rho_{PS} = \left\{ \hat{a}\{\sin(\hat{\Omega})\cos(\hat{\omega}) + \sin(\hat{\omega})\cos(\hat{\Omega})\cos(\hat{i})\}\{\cos(\hat{E}) - \hat{e}\} + \hat{a}\{-\sin(\hat{\Omega})\sin(\hat{\omega}) + \cos(\hat{\omega})\cos(\hat{\Omega})\cos(\hat{i})\}\sin(\hat{E})\sqrt{1-(\hat{e})^2} \right\} - Y_P \,.$$

$$\sigma_{PS} = \left\{ \hat{a}\{\sin(\hat{i})\sin(\hat{\omega})\}\{\cos(\hat{E}) - \hat{e}\} + \hat{a}\{\sin(\hat{i})\cos(\hat{\omega})\}\sin(\hat{E})\sqrt{1-(\hat{e})^2} \right\} - Z_P$$

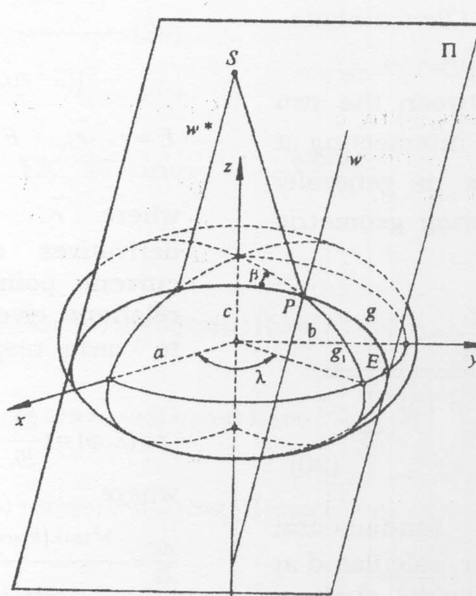

Figure 4 Location of the dish anttena

The function $f(\lambda,\varphi)=0$ of the surface curve g can be obtained by the substitution of Equation 1 of the surface into Equation 14 of the plane Π. Thus

$$f(\lambda,\varphi)=b^2(\ell\cos\lambda-m\sin\lambda)+n\,c^2[\sin^2\lambda-\frac{b^2}{a^2}\cos^2\lambda]\tan\varphi$$
$$-R[b^2-c^2(\sin^2\lambda-\frac{b^2}{a^2}\cos^2\lambda)\tan^2\varphi]^{\frac{1}{2}}[\sin^2\lambda+\frac{b^2}{a^2}\cos^2\lambda]^{\frac{1}{2}}=0 \qquad (16)$$

The differentiation of the function $f(\lambda,\varphi)=0$ is:

$$df=\frac{\partial f}{\partial\lambda}d\lambda+\frac{\partial f}{\partial\varphi}d\varphi=0$$

hence:

$$\frac{d\lambda}{d\varphi}=-(\frac{\partial f}{\partial\varphi})\Big/(\frac{\partial f}{\partial\lambda}) \qquad (17)$$

where:

$$\frac{\partial f}{\partial\varphi}=n\,c^2[\sin^2\lambda+\frac{b^2}{a^2}\cos^2\lambda]\sec^2\varphi-R\,c^2\frac{[\sin^2\lambda+\frac{b^2}{a^2}\cos^2\lambda]^{\frac{3}{2}}\tan\varphi\sec^2\varphi}{[b^2+c^2(\sin^2\lambda+\frac{b^2}{a^2}\cos^2\lambda)\tan^2\varphi]^{\frac{1}{2}}} \qquad (18)$$

and

$$\frac{\partial f}{\partial\lambda}=-(\ell\sin\lambda+m\cos\lambda)b^2+\frac{n\,c^2}{2}\frac{(1-\frac{b^2}{a^2})\tan\varphi\sin2\lambda}{[\sin^2\lambda+\frac{b^2}{a^2}\cos^2\lambda]^{\frac{1}{2}}}-\frac{R}{2}\frac{[b^2+c^2(\sin^2\lambda+\frac{b^2}{a^2}\cos^2\lambda)\tan^2\varphi]^{\frac{1}{2}}[(1-\frac{b^2}{a^2})\sin2\lambda]}{[\sin^2\lambda+\frac{b^2}{a^2}\cos^2\lambda]^{\frac{1}{2}}}$$
$$-\frac{R\,c^2}{2}\frac{[(1-\frac{b^2}{a^2})\sin2\lambda][\sin^2\lambda+\frac{b^2}{a^2}\cos^2\lambda]^{\frac{1}{2}}\tan^2\varphi}{[b^2+c^2(\sin^2\lambda+\frac{b^2}{a^2}\cos^2\lambda)\tan^2\varphi]^{\frac{1}{2}}} \qquad (19)$$

Azimuth β is the angle between the two curves g_1 and g, which are intersecting at point $P(\lambda_P,\varphi_P)$. This angle is generally defined [8,9] by the following geometric relation:

$$\cos\beta=\frac{E\frac{\partial\lambda_1}{\partial\varphi_1}\frac{\partial\lambda}{\partial\varphi}+F(\frac{\partial\lambda_1}{\partial\varphi_1}+\frac{\partial\lambda}{\partial\varphi})+G}{\sqrt{E(\frac{\partial\lambda_1}{\partial\varphi_1})^2+2F\frac{\partial\lambda_1}{\partial\varphi_1}+G}\sqrt{E(\frac{\partial\lambda}{\partial\varphi})^2+2F\frac{\partial\lambda}{\partial\varphi}+G}} \qquad (20)$$

Where E,F and G are the fundamental magnitudes of the first order, calculated at point $P(\lambda_P,\varphi_P)$ on the ellipsoidal surface. These magnitudes are given in Reference 8 as:

$$E=\bar{r}_\lambda\cdot\bar{r}_\lambda,\quad F=\bar{r}_\lambda\cdot\bar{r}_\varphi,\quad G=\bar{r}_\varphi\cdot\bar{r}_\varphi \qquad (21)$$

where $\bar{r}_\lambda$ and $\bar{r}_\varphi$ are the first partial derivatives of the position vector of the current point (X, Y, Z), defined in the relations given by equation 1, with respect to λ and φ respectively. Thus

$$\bar{r}_\lambda(\lambda,\varphi)=(\frac{\partial X}{\partial\lambda},\frac{\partial Y}{\partial\lambda},\frac{\partial Z}{\partial\lambda}) \qquad (22)$$

where

$$\frac{\partial X}{\partial\lambda}=-\frac{b^2\tan\lambda[b^2\sec^2\lambda-c^2\tan^2\varphi(\sin^2\lambda-\frac{b^2}{a^2}\cos^2\lambda)(1-\frac{b^2}{a^2}+\sec^2\lambda)]}{[\tan^2\lambda-\frac{b^2}{a^2}]^{\frac{3}{2}}[b^2-c^2\tan^2\varphi(\sin^2\lambda-\frac{b^2}{a^2}\cos^2\lambda)]^{\frac{3}{2}}} \qquad (23)$$

$$\frac{\partial Y}{\partial \lambda} = \frac{b^2 \cot\lambda [\frac{b^2}{a^2} \operatorname{cosec}^2\lambda - c^2 \tan^2\varphi (\sin^2\lambda + \frac{b^2}{a^2}\cos^2\lambda)][\frac{b^2}{a^2}\operatorname{cosec}^2\lambda - (1 - \frac{b^2}{a^2})]}{[1 - \frac{b^2}{a^2}\cot^2\lambda]^{\frac{3}{2}} [b^2 - c^2 \tan^2\varphi (\sin^2\lambda + \frac{b^2}{a^2}\cos^2\lambda)]^{\frac{3}{2}}} \tag{24}$$

$$\frac{\partial Z}{\partial \lambda} = \frac{c^2 b^2 \cot^2\varphi \cos\lambda \sin\lambda (1 - \frac{b^2}{a^2})[\sin^2\lambda + \frac{b^2}{a^2}\cos^2\lambda]^{\frac{1}{2}}}{[b^2 \cot^2\varphi + c^2 (\sin^2\lambda + \frac{b^2}{a^2}\cos^2\lambda)]^{\frac{3}{2}}} \tag{25}$$

and

$$\bar{r}_\varphi(\lambda, \varphi) = (\frac{\partial X}{\partial \varphi}, \frac{\partial Y}{\partial \varphi}, \frac{\partial Z}{\partial \varphi}) \tag{26}$$

where

$$\frac{\partial X}{\partial \varphi} = -\frac{b^2 c^2 \tan\varphi \sec^2\varphi \cos^2\lambda\, [\tan^2\lambda + \frac{b^2}{a^2}]^{\frac{1}{2}}}{[b^2 + c^2 \tan^2\varphi (\sin^2\lambda + \frac{b^2}{a^2}\cos^2\lambda)]^{\frac{3}{2}}} \tag{27}$$

$$\frac{\partial Y}{\partial \varphi} = -\frac{b^2 c^2 \tan\varphi \sec^2\varphi \cos^2\lambda\, [1 + \frac{b^2}{a^2}\cot^2\lambda]^{\frac{1}{2}}}{[b^2 + c^2 \tan^2\varphi (\sin^2\lambda + \frac{b^2}{a^2}\cos^2\lambda)]^{\frac{3}{2}}} \tag{28}$$

$$\frac{\partial Z}{\partial \varphi} = \frac{c^2 b^2 \cot\varphi \operatorname{cosec}^2\varphi [\sin^2\lambda + \frac{b^2}{a^2}\cos^2\lambda]^{\frac{1}{2}}}{[b^2 \cot^2\varphi + c^2 (\sin^2\lambda + \frac{b^2}{a^2}\cos^2\lambda)]^{\frac{3}{2}}} \tag{29}$$

Substitution of Equations 12, 17 into Equation 20, gives:

$$\cos\beta = \frac{G(\frac{\partial f}{\partial \lambda}) - F(\frac{\partial f}{\partial \varphi})}{\sqrt{G\left(E(\frac{\partial f}{\partial \varphi})^2 - 2F(\frac{\partial f}{\partial \varphi})(\frac{\partial f}{\partial \lambda}) + G(\frac{\partial f}{\partial \lambda})^2 \right)}} \tag{30}$$

The vertical angle ω is the angle between the tangent plane through P and the observation line IS. The observation line is directed from the terrestrial point $I(X_I, Y_I, Z_I)$ to a satellite $S(X_S, Y_S, Z_S)$, Figure 5. The direction cosines of the normal to the ellipsoid at P, are denoted by the symbols ℓ_w, m_w, n_w and given by [2] as:

$$\ell_w = \frac{\frac{b^2}{a^2}\cos\lambda\cos\varphi}{Q}, \quad m_w = \frac{\sin\lambda\cos\varphi}{Q}; \quad \text{and } n_w = \frac{[\sin^2\lambda + \frac{b^2}{a^2}\cos^2\lambda]\sin\varphi}{Q} \tag{31}$$

Where

$$Q = \sqrt{[\sin^2\lambda + \frac{b^2}{a^2}\cos^2\lambda]^2 + (1 - \frac{b^2}{a^2})^2 \sin^2\lambda \cos^2\lambda \cos^2\varphi} \tag{32}$$

If the direction ratios of the observation line IS are denoted by ε_{IS}, ρ_{IS}, σ_{IS}, the vertical angle ω will be:

$$\omega = \sin^{-1} \frac{\ell_w \varepsilon_{IS} + m_w \rho_{IS} + n_w \sigma_{IS}}{[(\varepsilon_{IS})^2 + (\rho_{IS})^2 + (\sigma_{IS})^2]^{\frac{1}{2}}} \tag{33}$$

where

$$\varepsilon_{IS} = \left\{ \begin{array}{l} \hat{a}\{\cos(\hat{\Omega})\cos(\hat{\omega}) - \sin(\hat{\omega})\sin(\hat{\Omega})\cos(\hat{i})\}\{\cos(\hat{E}) - \hat{e}\} \\ - \hat{a}\{\cos(\hat{\Omega})\sin(\hat{\omega}) + \cos(\hat{\omega})\sin(\hat{\Omega})\cos(\hat{i})\}\sin(\hat{E})\sqrt{1 - (\hat{e})^2} \end{array} \right\} - \left[X_P (1 + \frac{h_I}{a^2 G_P}) \right] \tag{34}$$

$$\rho_{IS} = \left\{ \begin{array}{l} \hat{a}\{\sin(\hat{\Omega})\cos(\hat{\omega}) + \sin(\hat{\omega})\cos(\hat{\Omega})\cos(\hat{i})\}\{\cos(\hat{E}) - \hat{e}\} \\ + \hat{a}\{-\sin(\hat{\Omega})\sin(\hat{\omega}) + \cos(\hat{\omega})\cos(\hat{\Omega})\cos(\hat{i})\}\sin(\hat{E})\sqrt{1 - (\hat{e})^2} \end{array} \right\} - \left[Y_P (1 + \frac{h_I}{a^2 G_P}) \right] \tag{35}$$

$$\sigma_{IS} = \left\{ \hat{a}\{\sin(\hat{i})\sin(\hat{\omega})\}\{\cos(\hat{E}) - \hat{e}\} + \hat{a}\{\sin(\hat{i})\cos(\hat{\omega})\}\sin(\hat{E})\sqrt{1-(\hat{e})^2} \right\} - \left[Z_P(1+\frac{h_I}{a^2 G_P}) \right] \quad (36)$$

The terrestrial coordinate system [10, 11] is utilized herein to verify the validity of the previously accomplished formulae. The mutual transformation between the local geocentric terrestrial coordinate system $x.y.z$ and the local coordinate system $u.v.w$, Figure 5, is our device for satisfying the validity of the last method based on the conception of the differential geometry.

The reference axes $u.v$ and w of the local coordinates system are considered at the origin I. Here w is the normal to the surface through the terrestrial point $I(X_I, Y_I, Z_I)$ while v is parallel to the tangent to meridian at $P(X_P.Y_P.Z_P)$. Therefore u is taken in a direction that completes the left-handed system of coordinates $u.v$ and w. The direction cosines $\ell_v.m_v.n_v$ and $\ell_u.m_u.n_u$ of the two directions v and u are respectively given by [2] as:

$$\ell_v = -\sin\varphi\cos\lambda, \quad m_v = -\sin\varphi\sin\lambda, \quad n_v = \cos\varphi \quad \text{and} \quad (37)$$

$$\ell_u = -\frac{\sin\lambda[1-(1-\frac{b^2}{a^2})\cos^2\lambda\sin^2\varphi]}{Q}, \quad m_u = \frac{\cos\lambda[\frac{b^2}{a^2}+(1-\frac{b^2}{a^2})\sin^2\lambda\sin^2\varphi]}{Q}, \quad n_u = -\frac{(1-\frac{b^2}{a^2})\sin 2\lambda\sin 2\varphi}{4Q}$$

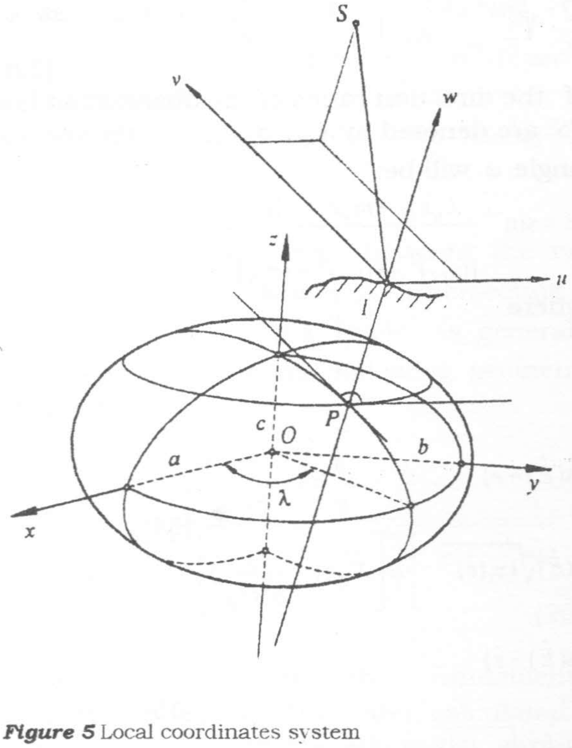

Figure 5 Local coordinates system

The geocentric coordinates $(X.Y.Z)$ can be transformed to the local geodetic coordinates $(U.V.W)$ according to Reference 11 as follows:

$$\begin{Bmatrix} U \\ V \\ W \end{Bmatrix} = [\hbar] \begin{Bmatrix} X - X_I \\ Y - Y_I \\ Z - Z_I \end{Bmatrix} \quad (38)$$

where $[\hbar]$ is a rotation matrix defined by:

$$[\hbar] = \begin{bmatrix} \ell_u & m_u & n_u \\ \ell_v & m_v & n_v \\ \ell_w & m_w & n_w \end{bmatrix} \quad (39)$$

Applying the satellite position $S(X_S.Y_S.Z_S)$ on the transformation formulae given by Equation 38, one can achieve the local geodetic coordinates of the satellite $S(U_S.V_S.W_S)$ such that:

$$\begin{Bmatrix} U_S \\ V_S \\ W_S \end{Bmatrix} = [\hbar] \begin{Bmatrix} \varepsilon_{IS} \\ \rho_{IS} \\ \sigma_{IS} \end{Bmatrix} \quad (40)$$

Then, the horizontal azimuth and the vertical angle ω are:

$$\beta = \cos^{-1} \frac{V_S}{\sqrt{U_S^2 + V_S^2}} \qquad (41)$$

$$\omega = \sin^{-1} \frac{W_S}{\sqrt{U_S^2 + V_S^2 + W_S^2}} \qquad (42)$$

It has to be noted that, substitution of Equations 29, 30 into 32 proves that the vertical angles calculated from Equation 26 are identical to those calculated from Equation 32. Moreover, a numerical study will be made to verify the reliability of the deuced equations.

PRACTICAL APPLICATIONS

Orientation Elements of an Antenna Directed from Cairo to a Moveable Satellite

Particularly, for an antenna located at a given geodetic position in Cairo $P(31.33333^\circ E, 29.85^\circ N)$; $h_l = 0$, one can find a particular formulation for calculating the orientation elements β_C, ω_C from Cairo as functions of satellite position. The Keplerian elements of the orbit are [7]:

$\hat{a}$ =26561740.4 m. $\hat{e}$ =0.0041338, $\hat{i}$ =63.25 , $\hat{\Omega}$ =148.29 , $\hat{\omega}$ = -23.93 , T=46154.1059 sec.

Introducing the given data of the position of antenna collected with the data of the path of the satellite into Equations 31and 32 and doing the necessary substitutions, one can get:

$$\beta_C = \frac{180}{\pi} \cos^{-1}\left(\frac{-0.501880078(10)^7 \cos(\hat{E}) + 0.26082760(10)^8 \sin(\hat{E}) + 39204.92473}{\sqrt{R_1}} \right)$$

$$\omega_C{}^\circ = \frac{180}{\pi} \sin^{-1}\left(\frac{-0.10585161(10)^8 \cos(\hat{E}) - 0.1893272842(10)^7 - 0.63290666(10)^7 \cos(\hat{E})}{\sqrt{R_2}} \right) \qquad (43)$$

where:

$$R_1 = 0.59348039(10)^{15} \cos^2(\hat{E}) - 0.509894425(10)^{13} \cos(\hat{E}) - 0.70192949(10)^{14} \sin^2(\hat{E}) + 0.11292274(10)^{13} \sin(\hat{E}) - 0.40081195(10)^{14} \cos(\hat{E}) \sin(\hat{E}) + 0.11237994(10)^{11}$$

$$R_2 = 0.70552603(10)^{15} \cos^2(\hat{E}) - 0.12889893(10)^{15} \cos(\hat{E}) - 0.70551397(10)^{15} \sin^2(\hat{E}) + 0.25094527(10)^{14} \sin(\hat{E}) - 100000 \cos(\hat{E}) \sin(\hat{E}) + 0.40068322(10)^{14}$$

The eccentric anomaly $\hat{E}$ defines the position of the satellite in its orbit at certain epoch. The earth's gravitational constant $\hat{W}$ is usually taken 3986005 $(10)^8$ m³/sec², [6,, 10]. Then substitution of the magnitude of $\hat{W}$ on Equations 9 and 10, yields:

$$21682.838\, \hat{E} - 0.0041338 \sin(\hat{E}) = t - 46154.1059 \qquad (44)$$

Where t is the time (in seconds) of perigee passage of satellite.

Numerical Applications

A set of numerical applications is analyzed here for comparing the orientation elements based on the spherical and triaxial ellipsoidal reference surfaces. The spherical datum is of 6378137 m radius [10] while the ellipsoidal dimensions are a=6378137 m, b=(1-1/93800)×a, c=(1-1/297.78)×a [2]. The negative vertical angle means that the observed satellite is invisible.

Application 1

Assuming that a single geostationary satellite located at S(42200000m ,0, 0) to be observed via a group of antennas located along the reference meridian (λ=0) but at different positive geodetic latitudes separated by increments of 10°.

The present results are compared with those based on spherical datum in Reference 1. The vertical angles based on the spherical and ellipsoidal surfaces are calculated and listed in Table 1. Because the differences in the equatorial dimensions of the spherical and ellipsoidal surfaces are very small, the vertical angles based on both datums are slightly deviated. Excluding this small deviation is not accurate if a very precise pointing to a communication satellite is detected via either high gain antennas or electro-optical device such as laser. The results conclude that, the obsrver near latitude 81° shows all true satellites closed to the local horizon. The true satellite of latitude 81° is visible with respect to the ellipsoidal model, but it is not visible with respect to the spherical model. All satellites of latitudes ranging from 82° to 90° are invisible.

Application 2

Many positions of Geostationary satellite located at the equatorial plane with constant range OS= 42200000 m are observed from the earth station $P(\lambda_P = 0, \varphi_P = 45^\circ N)$. The pointing angles based on the spherical and ellipsoidal surfaces are calculated for different positions for the observed satellite. The azimuth and vertical angle are calculated according to the longitude λ_S of the meridional plane on which the satellite is observed. As shown in Table 2, the results are calculated for satellite longitudes λ_S ranging from 0 to -10°. The results show that the observed satellite at a position of longitude λ_S equal to either 80°, 90°, -90° or -80° is invisible.

Table 1 Orientation angles of a point antenna to geostationary satellite for different earth station latitudes with zero longitude

Latitude	Ellipsoidal Observation		Spherical Observation [1]	
	Azimuth	Vertical	Azimuth	Vertical
10	180	78.245476	180	78.2386
20	180	66.573583	180	66.5612
30	180	55.058952	180	55.0434
40	180	43.761930	180	43.7459
50	180	32.725102	180	32.7105
60	180	21.972714	180	21.9605
70	180	11.512381	180	11.5023
80	180	01.338109	180	01.3291
81	180	00.335863	180	negative
82	180	negative	180	negative
90	180	negative	180	negative

Table 2 Orientation angles of point Antenna to Different positions of Geostationary Satellite of Constant Range

Satellite longitude	Ellipsoidal Observation		Spherical Observation [1]	
	Azimuth	Vertical	Azimuth	Vertical
0	180	38.2090577	180	38.1935
10	165.998191	37.2554924	165.9981	37.2411
20	152.745866	34.5138803	152.7637	34.5024
30	140.745203	30.2861361	140.7685	30.2785
40	130.094274	24.9421720	130.1207	24.9386
50	120.653970	18.8281881	120.6821	18.8282
60	112.178865	12.2271530	112.2077	12.2299
70	104.403726	5.35596917	104.4328	5.3605
80	--	negative	--	negative
90	--	negative	--	negative
-90	--	negative	--	negative
-80	--	negative	--	negative
-70	255.59627	5.3559617	255.5627	5.3605
-60	247.821134	12.2271530	247.9723	12.2299
-50	239.346029	18.8281881	239.3179	18.8282
-40	229.905725	24.9421720	229.8792	24.9386
-30	219.254796	30.2861361	219.2315	30.2785
-20	207.254133	34.5138803	207.2363	34.5024
-10	194.011753	37.2554924	194.0019	38.1935

CONCLUSION

The classical approaches for handling the orientation elements for geostationary and movable satellite lack the critical and precise specification of results. This is because the like studies are devoted to the spherical or spheroidal reference systems and fixing the satellite position at the equatorial plane. The present study offers a more precise model for observing a communication satellite. It takes into account the equatorial ellipticity and the spatial mobility of the satellite's position. The model is convenient not only for the ellipsoidal earth but also for many extraterrestrial bodies of our solar system which may be modeled as triaxial ellipsoidal surface.

The effect of the insignificant errors due to equatorial ellipticity should be taken into consideration when very precise pointing to communication satellites and other space objects is dictated, especially if high gain antennas or electro-optical devices such as lasers are used.

The communication through small vertical angles are sensitive to receiver noise due to atmospheric refraction, line of sight obstructions, single reflections with ground or nearby structures and other factors. All possible positive values of vertical angles offer the approximate range of latitude or longitude that an earth station must have in order to communicate successfully with a satellite parked or moved in fixed orbit.

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توجية هوائيات الاستقبال من موضع على سطح الارض الناقصى ثلاثى المحاور إلى قمر صناعى للإتصال

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ملخص البحث

يتناول هذا البحث معالجه عملية التوجيه الدقيق والتسامت لهوائيات الإستقبال بالمحطات الأرضيه - من الأقمار الصناعية.

تم إستنتاج المعادلات الرياضية والعلاقات الجيومترية التى تحكم تسامت وتوجية أطباق البهوائيات الأرضية بإعتبار الشكل الدقيق للأرض وهو السطح الناقصى ثلاثى المحاور.

تم إيضا دراسة وإستنتاج معادلات التوجيه على قمر صناعى ثابت ومتحرك بطرقتين مختلفين ، الأولى إعتمدت على العلاقات الجيومترية التفاضلية لسطح الأرض الناقصى ثلاثى المحاور والثانية إعتمدت على ما هو معروف فى علم الجيوديسيا بمنظومة التحويل بين نظام الإحداثيات الجيوديسية المركزية والجيوديسية المحلية.

فى نهاية البحث تم إجراء دراسة عددية للتحقيق من دقة الطريقة التى لا غنى عنها عند الحاجة إلى نظام رصد تستخدم فيه تكنولوجيا عالية الدقه مثل الرصد بأشعة الليزر.