ULTIMATE RESISTANCE OF PANELED BEAM SYSTEM BY LINEAR PROGRAMMING

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ABSTRACT

An automated procedure by linear programming is used for studying the ultimate resistance of a pure metallic paneled beam structural system. The classical method of plastic analysis is useful only for simple problems, beside the tedious search needed to get the potential mode of collapse. The present study gives an optimum reinforcement for the floor beams and indicates the collapse mode as well as the ultimate load parameter.

Keywords: Linear programming, Paneled beam, Plastic analysis mechanism, Failure mode, Ultimate resistance.

INTRODUCTION

In the classical plastic limit analysis method for a given structural system [1-5], potential collapse mechanisms are postulated and the corresponding load parameters are evaluated by the well-known ways. It is established [6] that the magnitude of the load parameter calculated for any trial mechanism cannot be less than the value causing plastic collapse of the system. If the actual mode of collapse is selected as a trial mechanism, the classical kinematic method correctly evaluates the collapse load. For all other trial mechanisms, the classical method evaluates an unsafe bound to the load. However, the task of generating suitable trial mechanisms for relatively complicated problems is far from simple. Therefore, it is desirable to use an automatic procedure for deriving the collapse mechanism without first guessing such trial mechanisms.

An application of this automatic procedure by linear programming [7] is presented here for studying the ultimate resistance of a pure metallic paneled beam system.

The metallic paneled beam system is a commonly used structure in practice. An example is the arrangement shown in Figure 1 to support a floor system composed of corrugated metallic sheets and concrete cevering in industrial construction. The method of Guyon-Massonnet [8-10] perfectly simulates the elastic behavior of this system.

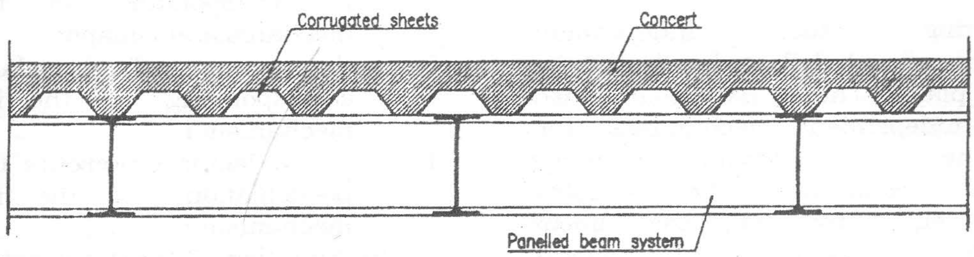

Figure 1 Typical industrial flooring system

The problem of a pure paneled beam system could be treated at two different levels according to either the torsional rigidity is taken into consideration or not. In this study, it is assumed that all the metallic beams have an "I" cross section. The torsional rigidity of such sections is relatively small. It can be assumed, without violating the accuracy, that the resistance offered by the torsionol rigidity is negligible.

Heyman [11] indicated that for a paneled I-beam system supported at its four sides, the error due to this assumption is less than 0.1%. Therefore, the collapse of a paneled I-beam system could be determined according to the theory of plastic analysis by the formation of failure mechanism [12].

LINEAR PROGRAMMING FORMULATION

Solving a plastic analysis problem for a paneled beam system, having a certain configuration, is a process of two stages. First, the collapse load factor λ is to be obtained. In the second stage, the corresponding mode of collapse is determined. The paneled beam system is composed of perfectly plastic prismatic bars with known plastic moment capacity, "Mp_j" at section j, the system resists a combination of service loads having certain magnitudes, directions, and points of application.

In the plastic analysis by linear programming, the analyzed structure is subdivided, by potential plastic hinges, into a number of beam elements as in finite element method. The location of the potential plastic hinges and the number of subdivisions depend on the boundary conditions, loading conditions, and desired accuracy.

Considering the independent displacement of each deflected node and the associated plastic hinge pattern obtained from the independent mechanisms, the corresponding equilibrium energy expressions constitute the equality constraint equations of the linear programming problem. The energy expressions are obtained by equating the internal and external work done.

The linear programming method combines the independent mechanisms in order to achieve the failure mechanism that minimizes the failure load. This process concludes the objective function.

The kinematic approach has been used in this study because of its simplicity, though static method could be also used in the formulation.

Let i be one of the n possible independent mechanisms obtained by virtual displacement of the node i, without violating the kinematic boundary conditions, θ_{ij} be the rotation of the plastic hinge j of the S possible plastic hinges, and e_i be the work done by the unfactored external loads.

The plastic analysis by linear programming using the kinematic method can be formulated as follows.

Find λ_c, θ_j, t_i for j = 1,2,.......... S and i = 1,2,...........n such that:

$$\lambda c = \sum_{j=1}^{S} M\rho_j \; \theta_j = \text{mimimum} \tag{1}$$

subjected to

$$\theta_j - \sum_{i=1}^{n} t_i \; \theta_{ij} = 0 \quad j = 1,2.....S \tag{2}$$

and

$$\sum_{i=1}^{n} t_i e_i = 1 \tag{3}$$

Where,

- λc : the load multiplier corresponding to the actual mechanism at collapse
- Mp_j : the plastic moment resistance of the hinge i
- θ_j : the actual rotation of the plastic hinge j corresponding to the actual mechanism at collapse
- θ_{ij} : the rotation of the plastic hinge j corresponding to the independent mechanism i
- t_i : a factor represents the weight of participation of the independent mechanism i

In Equation 3 the total external virtual work has been arbitrarily set equal to unity. The chosen plastic hinge pattern in the combined mechanism may cause either positive or negative rotations, similarly the participation factors t_i may be either positive or negative. These two variables are written as the difference between two positive variables to meet most of the simplex code requirements.

Therefore, Equations 1, 2 and 3 may be represented in the following form:

$$\text{minimize} \sum_{J=1}^{S} Mpj(\theta JT - \bar{\theta}_j) \quad (4)$$

subjected to

$$(\theta_j^+ - \theta_j^-) - \sum_{i=1}^{n} (t_i^+ - t_i^-)\,\theta_{ij} = o \quad (5)$$

$j=1,2,\ldots S$

and

$$\sum_{i=1}^{n} (t_i^+ - t_i^-)\,e_i = 1 \quad (6)$$

to satisfy the requirement of the simplex code .

PROBLEM DISCRITISATION

The paneled beam system shown in Figure 2 is composed of 10 beams intersecting at panel distances equal to L and hinged at both ends.

The floor is supposed to be loaded at each intersection node by the gravity load P_k . For the case of P_k = constant and Mp_i = constant, the proposed floor could be simplified to study only one quarter of the floor Figure 3.

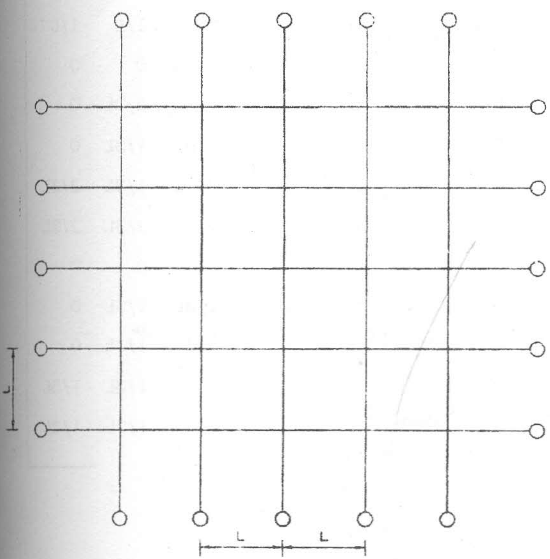

Figure 2 The studied Floor system

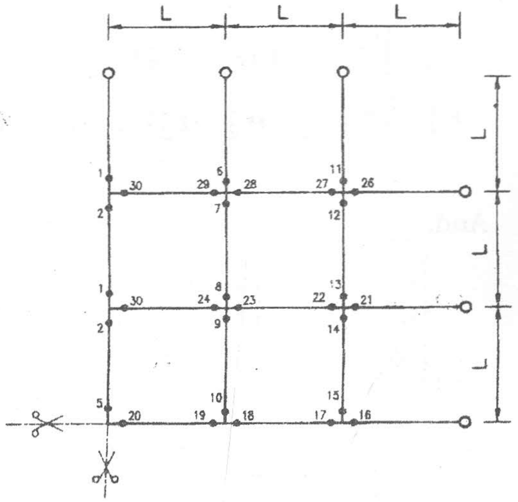

Figure 3 One quarter of the symmetric floor with all possible plastic hinges

All the possible plastic hinges are indicated in Figure 3 (j =1,2,......S), where S = 30.

The independent mechanisms are presented in Figure 4, where (i = 1,2,...... n) and n = 15

When all beam elements of the paneled floor are prismatic with constant Mp, the objective function which have to be minimized is represented by the form :

$$M_p\Big\{0.5\big[\theta_1^+ + \theta_1^- + \ldots\ldots + \theta_5^+ + \theta_5^-\big] + \big[\theta_6^+ + \theta_6^- + \ldots\ldots + \theta_{15}^+ + \theta_{15}^- + 05\big[\theta_{16}^+ + \theta_{16}^- + \ldots\ldots + \theta_{20}^+ + \theta_{20}^- + \big[\theta_{21}^+ + \theta_{21}^- + \ldots\ldots - \theta_{30}^+ + \theta_{30}^-\big] \quad (7)$$

Where $\theta_1, \theta_2, \ldots, \theta_5$ and $\theta_{16}, \theta_{17}, \ldots, \theta_{20}$ are on the line of symmetry.

The function given by Equation 7 is subjected to the constrains (5) and (6) which are represented by the form (8) and (9):

$$\{\theta\} = [A]\{t\} \quad (8)$$

The matrix A(Sxn) represents the independent rotations of the plastic hinges corresponding to the studied independent mechanisms. The vectors {θ} and {t} represent the unknown rotations and participation parameters, respectively.

$$\{\theta\} = \left[(\theta_1^+ - \theta_1^-)\ (\theta_2^+ - \theta_2^-) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (\theta_{30}^+ - \theta_{30}^-)\right]^T$$

$$\{t\} = \left[(t_1^+ - t_1^-)\ (t_2^+ - t_2^-) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (t_{30}^+ - t_{30}^-)\right]^T$$

And;

$$[A] = \begin{bmatrix}
0 & 0 & 1/L & 0 & 0 & 0 & 0 & 0 & 0 & 1/L & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1/L & 1/L & 0 & 0 & 0 & 0 & 0 & 1/L & 1/L & 0 & 0 & 0 & 0 \\
0 & 0 & 1/L & 1/L & 0 & 0 & 0 & 0 & 0 & 1/L & 1/L & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1/L & 0 & 0 & 0 & 0 & 1/L & 1/L & 1/L & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1/L & 0 & 0 & 0 & 0 & 1/L & 0 & 1/L & 1/L & 0 & 0 & 1/3L \\
0 & 1/L & 0 & 0 & 1/L & 0 & 0 & 0 & 0 & 2/3L & 0 & 0 & 0 & 0 & 0 \\
0 & 1/L & 0 & 0 & 1/L & 0 & 0 & 0 & 0 & 2/3L & 1/L & 0 & 0 & 0 & 0 \\
0 & 1/L & 0 & 0 & 1/L & 0 & 0 & 0 & 1/L & 2/3L & 2/3L & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1/L & 0 & 0 & 1/L & 0 & 0 & 2/3L & 2/3L & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1/L & 0 & 0 & 1/L & 0 & 0 & 2/3L & 1/3L & 0 & 1/3L & 0 \\
1/L & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1/3L & 0 & 0 & 0 & 0 & 0 \\
1/L & 0 & 0 & 0 & 0 & 1/L & 0 & 0 & 0 & 1/3L & 1/3L & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 1/L & 1/L & 0 & 0 & 0 & 0 & 0 & 0 & 1/3L & 2/3L & 0 \\
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0 & 0 & 0 & 1/L & 1/L & 0 & 0 & 0 & 0 & 0 & 2/3L & 0 & 0 & 2/3L & 2/3L \\
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\end{bmatrix}$$

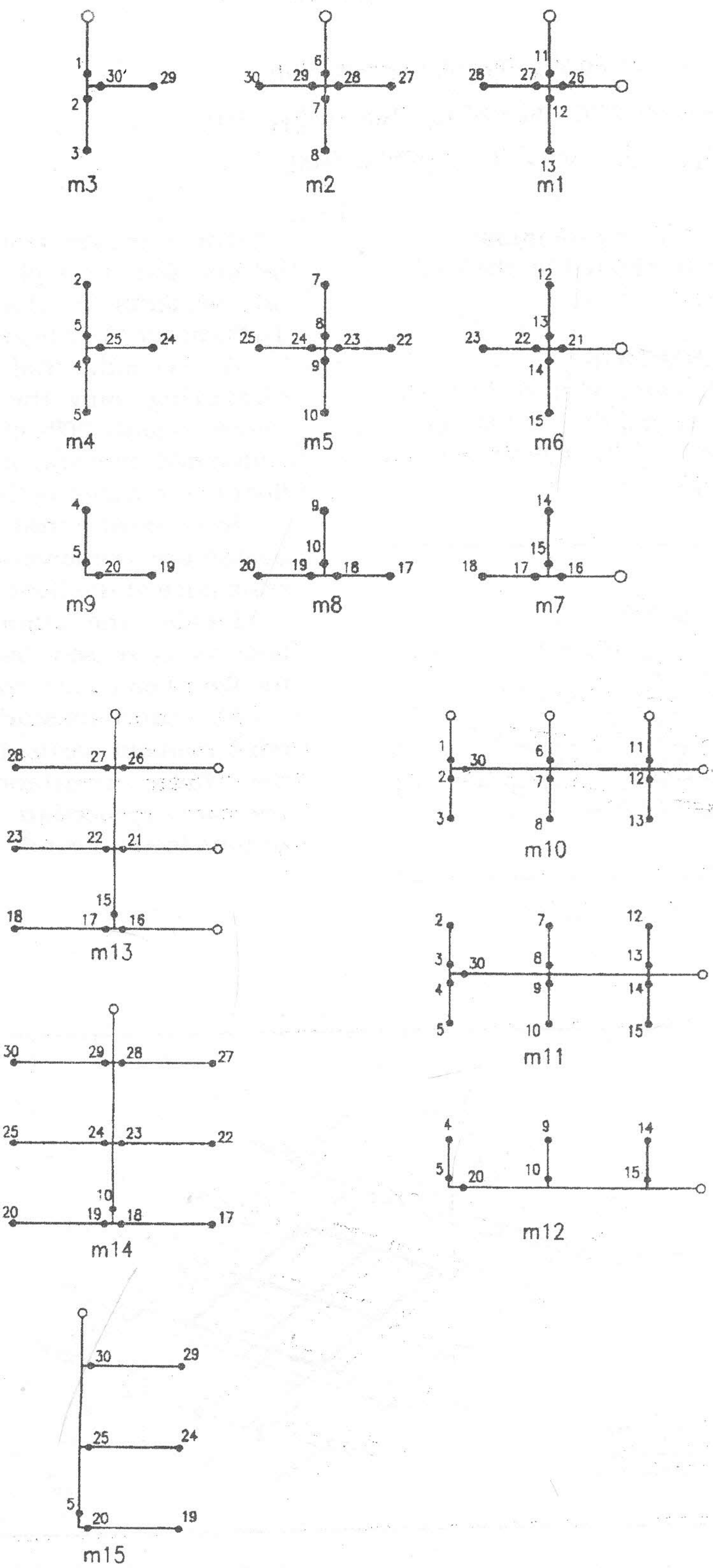

Figure 4 The independent mechanisms

The constraint representing the external energy is given by:

$$\begin{aligned}&(t_1^+ - t_1^-) + (t_2^+ - t_2^-) + 0.5(t_3^+ - t_3^-) + 0.5(t_4^+ - t_4^-) + (t_5^+ - t_5^-) + (t_6^+ - t_6^-)\\ &+ 0.5(t_7^+ - t_7^-) + 0.5(t_8^+ - t_8^-) + 0.25(t_9^+ - t_9^-) + 1.5(t_{10}^+ - t_{10}^-) + 1.5(t_{11}^+ - t_{11}^-)\\ &+ 0.75(t_{12}^+ - t_{12}^-) + 1.5(t_{13}^+ - t_{13}^-) + 1.5(t_{14}^+ - t_{14}^-) + 0.75(t_{15}^+ - t_{15}^-) = 1\end{aligned} \quad (9)$$

This system of linear programming equations is ready to be treated by the well-known simplex code [13].

APPLICATION

For the case of panel distance L=2.0m (floor dimension 12X12m) the load factor λ_c is equal to 0.227Mp. The mechanism of failure is shown in Figure 5.

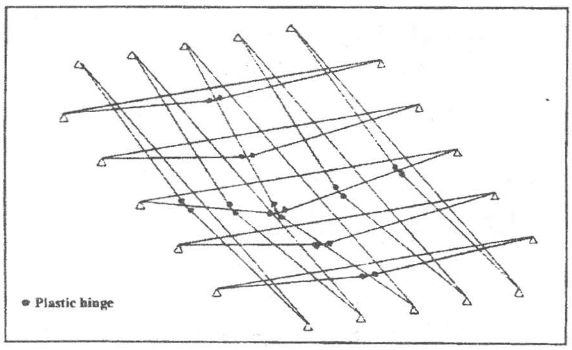

Figure 5 mode of collapse for the original floor

The ultimate resistance of the floor can be doubled if the plastic moment capacity of all elements is doubled, i.e., reinforcing 100% of the floor beams .

A second trial is carried out by reinforcing only the elements indicated in Figure 6 (only 20% of the floor elements) It is concluded that the ultimate resistance of the floor is increased by 56% .

In a third trial, 30% only of the floor beams are reinforced (Figure 7) The ultimate resistance of the floor is increased by 80%

Finally, the ultimate resistance of the floor is increased by 90% when 47% only of the floor beams are reinforced (Figure 8).

The previous study leads to an optimum reinforcement arrangement for increasing the ultimate resistance of the floor system. The same procedure can be carried out for various loading conditions.

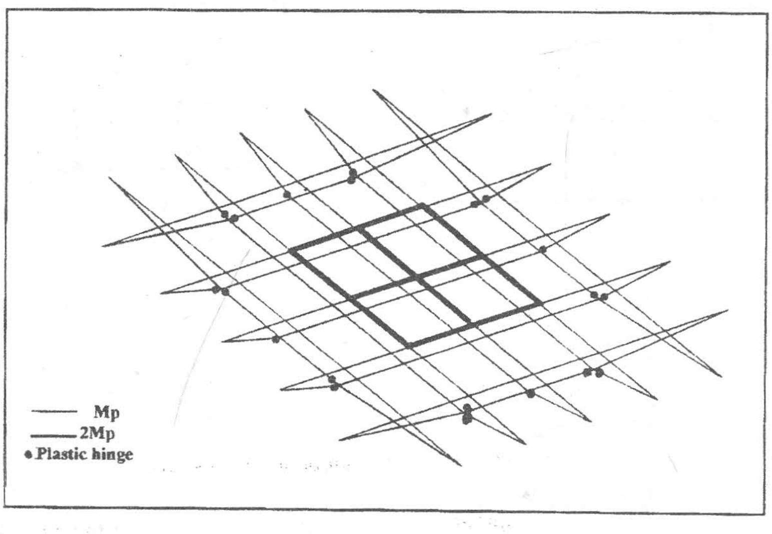

Figure 6 Mode of collapse of the floor (20%of the floor beans are reinforced)

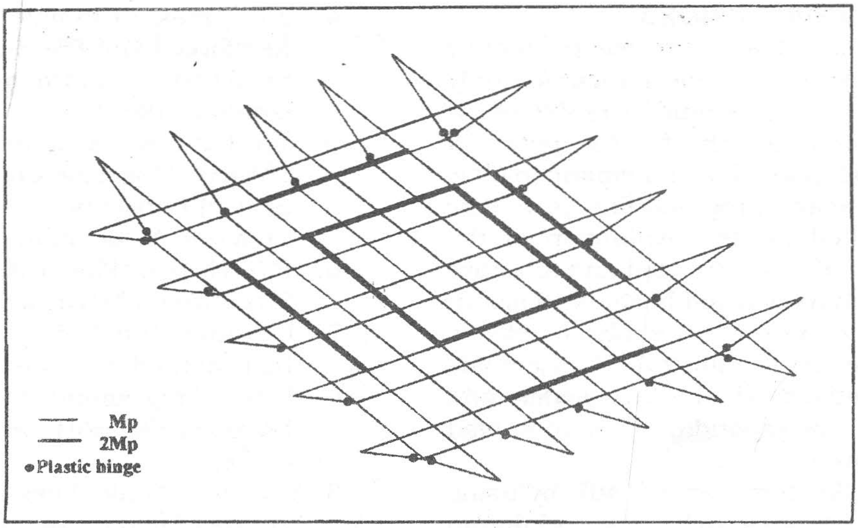

Figure 7 Mode of collapse of the floor (30% of the floor are reinforced)

In the above symmetrical cases, studying one quarter of the floor system was quite enough. For the cases of unsymmetrical loading , or unsymmetrical distribution of the floor beams, the full beam system has to be introduced in the study. The same situation exists when the paneled beam system is discontinued due to arbitrary holes in the floor. In These unsymmetrical cases, the unknowns shall be as follows :

λ_c, θ_j, t_i for j=1,2,.......... S and i= 1,2,...........n Where S = 100 and n = 35.

These complicated conditions of the paneled floor could be easily treated by the present technique.

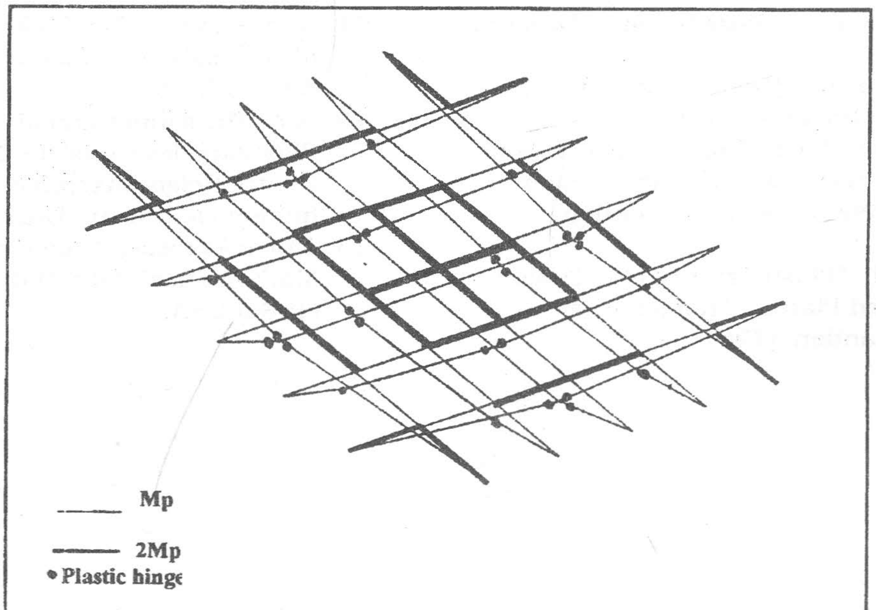

Figure 8 Mode of collapse for the floor (47% of floor beams are reinforced)

CONCLUSIONS

The classical plastic analysis method by traditional independent mechanisms is only convenient for the simplest problems. It requires tedious search for the potential modes of collapse. The kinematic plastic analysis by linear programming is a very powerful method for the evaluation of the collapse load of a complicated and sensitive structures such as paneled beam systems. In this paper, the presented study completely automates the search for the collapse mode of the discretised structural model and provides the corresponding collapse load parameter as well.

In the application carried out by using this automated approach, the minimum reinforcing cost for the paneled beam system is obtained through simple operations.

The ultimate resistance of the floor system can be increased by 56% ,80% and 90% when the cost of floor reinforcing is increased by 20%, 30% and 47%, respectively.

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المقاومة القصوى للكمرات المتقاطعة باستخدام البرمجة الخطية

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ملخص البحث

تم استخدام البرمجة الخطية (Linear Programming) لدراسة المقاومة القصوى لنظام من الكمرات المعدنية المتقاطعة.

ويجدر الأشارة الى التحليل اللدن التقليدى الشائع استخدامه يصلح فقط فى الانظمة البسيطة من كمرتين من متقاطعتين أو أربع كمرات متقاطعة على أكثر تقدير نظرا للمجهود العنيف المطلوب لتحديد آليه الهياره (mechanism) والحمل الأقصى المناظر لها.

والدراسة الحالية تعطى أسلوبا امثل لتقوية نظام من الكمرات المتقاطعة مع تحديد آليه الهياره وحمله الأقصى بأسلوب (آلى) وذلك لأنظمة معقدة من الكمرات المتقاطعة.

وقد تم عمل تطبيق على نظام كمرى يتكون من عشرة كمرات متقاطعة ووجد ان الحمل الأقصى لهذه الكمرات يمكن زيادته بنسبة ٥٦ % ، ٨٠%، ٩٠% عن طريق تقوية جزئية بنسبة من الكمرات الكلية حوالى ٢٠% ، ٣٠% ، ٤٧% على التوالى.