TRANSVERSE VIBRATION OF PLATES HAVING SPANWISE QUADRATIC THICKNESS VARIATION

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ABSTRACT

Free lateral vibration of rectangular and trapezoidal plates that have parabolic thickness variation along the span of the plate has been studied. An eighteen degrees of freedom triangular plate bending finite element that has quadratic thickness variation in one direction is formulated and used in the analysis. The convergence of the results is ensured by applying several approximations till the results converge to the desired accuracy. Comparisons indicate that the results are in good agreement with those available in the literature. Plates that have clamped, simply supported and cantilevered boundary conditions are investigated for a range of variation of the tip to root thickness ratio. For each ratio, the effects of the parameters, which govern the thickness variation, on the natural frequency coefficients are analyzed.

Keywords Free Transverse Vibration, Rectangular Plates, Trapezoidal Plates, Non-uniform Thickness.

INTRODUCTION

The analysis of the free transverse vibration of plates plays an important role in the design of a variety of structures. When the plates are homogeneous and for some special arrangements of boundary conditions, there are closed form solutions for their natural frequencies. However, when the plate sides are supported in any complicated fashion or when the plate is non uniform, no closed form solutions exist and one has to apply approximate mathematical methods along with numerical analysis. The study of the free lateral oscillations of variable thickness plates has attracted the attention of many investigators. It is known that, an isotropic plate that has linear thickness variation will have a cubical varying flexural rigidity. Therefore, the determination of natural frequencies of such a plate will be more complicated than a plate that has uniform thickness.

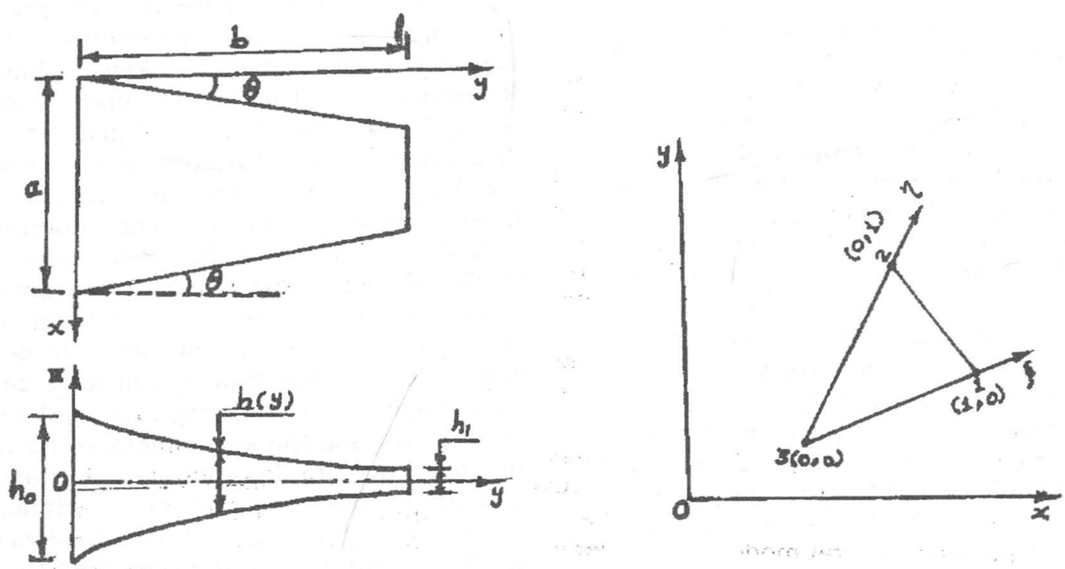
Ashton [1, 2] determined both the natural frequencies and the natural modes for clamped tapered rectangular plates by applying the Rayleigh- Ritz method. Cheung *et al.* [3] analyzed the free vibration of rectangular and other irregular polygonal plates that have linear thickness variation by the finite strip method. Chopra and Durvasula [4] applied the energy method of Lagrange for tapered skew plates. In Reference 5, the variational Galerkine method was used by Filipich *et al.* to determine the fundamental frequency coefficient for tapered rectangular plates that have some different combinations of boundary conditions. In References 6 and 7, the Rayleigh-Ritz method was applied by Laura *et al.* to solve the problem of free vibration of tapered cantilever trapezoidal plates. The differential quadrature method was applied by Kukreti *et al.* [8] for the linearly varying thickness rectangular plates. In Reference 9, the rectangular plate that has an exponential thickness variation was considered by Cortinez et al in the two cases of mixed boundary conditions, the CCCF and the CSSF boundary conditions. The finite element method was used and bilinear approximation of the thickness variation was assumed along the span of the plate. In

Reference 10, the natural frequency coefficients for five regular polygonal plates, starting from the triangle up to the heptagonal, were determined by thé author. Both linearly and exponentially varying thickness plates were considered. Two triangular plate bending elements were formulated and used in the analysis. In Reference 11, the problem of free vibration of clamped square plate that has parabolic thickness variation in two orthogonal directions was analyzed by Olson and Hazil using both experimental and theoretical methods. In Reference 12, the finite element technique was used by Mukherjee and Mukhopandhyay for both linearly and parabolically varying thickness plates. The isoparametric quadratic plate bending element that has 24-degrees of freedom was employed. For plates that have parabolic thickness variation, only the case of clamped square plate was considered.

In the present work, the problem of free transverse vibration of rectangular and trapezoidal plates that have parabolic thickness variation in the span-wise direction has been studied. The quadratic thickness variation of the plate along one of its mid-plane axes result in varying the flexural rigidity of the plate as a six-degree polynomial of the position coordinate along that axis. The solution of such a problem will be much more complicated than that concerned with both linearly and exponentially varying thickness plates. A new eighteen-degrees of freedom triangular plate bending element that has parabolic thickness variation in one direction is formulated and used in the analysis. The convergence of the results is checked using several different mesh divisions. The solutions for tapered plates, which could be obtained as special cases of those having parabolic thickness variation, are compared and found to be in good agreement with those available in the literature. Tabulated results for several cases of the study has been presented.

THEORETICAL FORMULATION

The considered plate is assumed to have a symmetric trapezoidal platform which, for $\theta=0$, is reduced to a rectangular one. The quadratic thickness variation of the plate is assumed to be in the span-wise direction. i.e., the thickness is a function of the y-coordinate as shown in Figure 1.



a: The plate geometry.   b. The area coordinates

Figure 1 The global and the local system of axes

In the formulation, the x,y coordinates and all the deformations of the plate are nondimensionalized by a characteristic length, which is the chord of the plate (a)at its root. The thickness of the plate is expressed as:

$$h= h_o (1+\beta_1 y+\beta_2 y^2) \quad (1)$$

where h is the plate thickness at any position y, h_o is the thickness at the plate root (y=0) and β_1, β_2 are the parameters which govern the thickness variation. In the special case, when $\beta_1=\beta_2=0$, the plate is reduced to the case of uniform thickness and when $\beta_2=0$, it is reduced to the one that has a linear thickness variation in the span wise direction. Using Equation 1, the plate bending rigidity, D, will be given by

$$D=D_o(1+\beta_1 y+\beta_2 y^2)^3 \quad (2)$$

where $D_o=Eh_o^3/12(1-\nu^2)$ is the flexural rigidity at the root of the plate E is the Young's modulus of elasticity and ν is the Poisson's ratio.

An eighteen degrees of freedom triangular plate bending element that has quadratic thickness variation in one direction is formulated and used in the analysis. The six nodal variables at each of the element three vertices are the deflection w_i ,the slopes w_{xi},w_{yi} and the curvatures w_{xxi}, w_{xyi}, w_{yyi}. (i=1,2,3). The transverse displacement field w within the element is expressed as follows:

$$w(x,y)=\{A\}^T\{\alpha\} \quad (3)$$

where {A} is a column vector, the elements of which, are those of a complete quintic polynomial expressed in terms of the area coordinates ξ ,η and {α} is a column vector consisting of 21α_is interpolation functions to be determined. Detailed formulation of the stiffness and mass matrices of the element is analogous to that presented by the author [10]. The flexural rigidity of the plate element in the present formulation, as could be shown from Equation 2, will be a sixth-degree polynomial in the y-coordinate. Therefore, the formulation of the element matrices here will be much more complicated than that given in Reference 10 for both plates of linear and exponential thickness variation.

The relation between the global y-coordinate and the local oblique coordinates ξ, η is:

$$y=y_3+y_{13}\xi+y_{23}\eta \quad (4)$$

where $y_{13}=y_1-y_3$, $y_{23}=y_2-y_3$ and 1,2,3 denote the three vertices of the triangular element as shown in Figure1-b.

Substituting for y from Equation 4 into Equations 1 and 2, the following expressions for the plate thickness and the bending rigidity are obtained:

$$h=h_o \sum_{k=1}^{6} S_k \xi^{m_k} \eta^{n_k} \qquad m_k, n_k = 0,1,2 \quad (5)$$

$$D=D_o \sum_{k=1}^{28} R_k \xi^{m_k} \eta^{n_k} \qquad m_k, n_k = 0,1,2, ...6 \quad (6)$$

where S_k and R_k are constants depending on the parameters β_1, β_2 and the global y-coordinates of the element vertices. Using Equations 5 and 6, complete formulation of the element stiffness and mass matrices, which is analogous to that given in Reference 14, could be obtained. The derivation of the equations of motion and the substitution of the boundary conditions will not be presented here since the finite element technique is well known. Some illustrations for Equations 3 to 6 are given in the Appendix.

NUMERICAL SOLUTION AND DISCUSSION

The values of the parameters $\beta_1$and β_2 which govern the parabolic thickness variation of the plate can not be arbitrary chosen. They depend on two other parameters: the aspect ratio (AR=b/a)and the tip to root thickness ratio (h_1/h_o). According to Equation 1:

$$\beta_1 y + \beta_2 y^2 = (h/h_o) -1$$

If $h=h_1$,then $y=y_1=AR$ and the relation between $\beta_1$andβ_2 will be governed by

$$\beta_1 \times AR+\beta_2 \times AR^2 = (h_1/h_o)-1 \quad (7)$$

For assumed values of AR and h_1/h_o, Equation 7 represents a linear relation between β_1 and β_2 from which, for an arbitrary value of β_1, one can determine the corresponding value of β_2 and vise versa.

Study of Convergence

Table 1 indicates the first four natural frequency coefficients, $(\lambda=\omega a^2 \ h_o/D_o$ where ρ is the plate density and ω is the natural frequency of the plate, for a clamped square plate (AR=1) and for two tip to root thickness ratios($h_1/h_o=1/2$ and $h_1/h_o=1/4$). For each pair of values of $\beta_1$andβ_2 , three different mesh divisions are examined, (4×4,5×5and 6×6), which result in successive numbers of elements of 32, 50 and72, respectively. As could be shown, monotonic convergence is achieved through increasing the number of elements and one can expect that using a 6×6 mesh division during the analysis will result in sufficiently accurate solutions.

ACCURACY OF RESULTS

Before undergoing a series of computational work, it is necessary to demonstrate the accuracy of the present solutions. Table 2 indicates the natural frequency coefficients for a square plate that has linear thickness variation along its span. The presented results for such a plate are obtained here as special solutions from those concern the quadratic thickness variation by substituting β_2=0. The results for three different boundary conditions, which are the clamped, the simply supported and the cantilevered edge supports, are compared with those previously published by other investigators. They are found to be in good agreement with them.

Table 1 Convergence of results

h_1/h_o	β_1	β_2	N	λ_1	λ_2	λ_3	λ_4
			4	26.312	53.242	53.799	80.268
0.5	-0.5	0.0	5	26.292	53.061	53.656	79.565
			6	26.292	53.061	53.656	79.565
			4	29.078	58.992	60.423	89.395
0.5	0.0	-0.5	5	29.038	58.777	60.146	88.449
			6	29.038	58.777	60.146	88.449
			4	20.544	40.699	42.158	63.830
0.25	-0.75	0.0	5	20.467	40.244	41.835	62.803
			6	20.467	40.244	41.835	62.803
			4	24.829	50.349	51.538	78.197
0.25	0.0	-0.75	5	24.623	49.609	50.733	76.534
			6	24.623	49.609	50.733	76.534

Table 2 Comparisons of results

Boundary Condition	Reference	β_1	λ_1	λ_2	λ_3	λ_4
	Present		39.511	80.533	80.597	118.933
CCCC	7	0.2	39.51	80.52	80.59	118.87
	8		39.55	-	-	-
	Present		42.911	87.300	87.539	129.295
CCCC	7	0.4	42.91	87.28	87.53	129.22
	8		42.94	-	-	-
	Present		21.69	54.159	54.199	86.728
SSSS	7	0.2	21.69	54.16	54.20	86.75
	8		21.70	-	-	-
CFFF	Present	0.4	3.354	9.279	23.917	33.216
	10		3.354	-	-	-

In Table 3, the results for clamped square plate for four tip to root thickness ratios are presented. For each value of h_1/h_o, the frequency coefficients are determined for some different pairs of related values of β_1 and β_2. The set of results correspond to the value of $h_1/h_o=1$ and $\beta_1=\beta_2=0$ represents the solutions of the uniform thickness clamped square plate. One concludes that, decreasing the thickness of the plate along the span ($h_1/h_o<1$) results in corresponding decrease of the natural frequency coefficients. The effects of the parameters β_1 and β_2 could be explained as follows: The parameter $\beta_1$represents the linear thickness variation of the plate while the parameter β_2 governs its quadratic thickness variation. For a certain value of $h_1/h_o<1$ and AR=1, the two parameters are linearly dependent according to Equation 7. When $\beta_2=0$, the variation of the thickness is purely linear and the thickness of the plate at any position y is determined from the relation $h=h_o\,[1-(1-h_1/h_o)y]$. When $\beta_1=0$, the thickness of the plate is given from the relation $h=h_o[1-(1-h_1/h_o)y^2]$. The term $(1-h_1/h_o)$ represents the reduction of the thickness in one side of the mid-plane along the span. Since the aspect ratio of the plate is unity, then the non-dimensionalized coordinate y is governed by the inequality $y \le 1$and hence $y^2 \le y$. In accordance, the reduction of the thickness in the case of its linear variation is larger than that corresponding to its purely quadratic variation. Since the rigidity of the plate is a function of its thickness, $D=Eh^3/12(1-\nu^2)$, then the case when $\beta_2=0$ will result in a value of the rigidity which is less than its corresponding value for $\beta_1=0$. Therefore, the value of λ for any ratio $h_1/h_o<1$ and $\beta_2=0$ must be less than that corresponding to the same ratio h_1/h_o and $\beta_1=0$. As an example, for $h_1/h_o=3/4,\beta_2=0$and$\beta_1=-1/4$, the value of $\lambda_1$is 31.342 and for $h_1/h_o=3/4,\beta_1=0,\beta_2=-1/4$, then $\lambda_1=32.713$. It is also noticed that the difference between the corresponding values of λ_1 increases as the ratio h_1/h_o decreases. For example, the value of $\lambda_1$for $h_1/h_o=1/4,\beta_2=0,\beta_1=-3/4$ is 20.467 while, for $h_1/h_o=1/4$, $\beta_1=0,\beta_2=-3/4$, it is 24.623.

Table 3. Natural frequency coefficients for clamped square plates

H_1/h_o	β_1	β_2	λ_1	λ_2	λ_3	λ_4
1.0	0.0	0.0	35.987	73.403	73.409	108.28
	0.0	-0.25	32.713	66.509	67.381	98.825
0.75	-0.125	-0.125	32.027	65.227	65.597	96.619
	-0.25	0.0	31.342	63.809	63.937	94.402
	0.0	-0.5	29.038	58.777	60.146	88.444
0.5	-0.25	-0.25	27.663	56.233	56.609	84.029
	-0.5	0.0	26.292	53.061	53.656	79.565
	0.0	-0.75	24.623	49.609	50.733	76.534
0.25	-0.5	-0.25	21.846	43.742	44.454	67.426
	-0.75	0.0	20.467	40.244	41.835	62.803

The cases of simply supported square plates are analyzed and the results are presented in Table 4. The behavior of the natural frequency coefficients along with the variation of the parameters h_1/h_o,$\beta_1$and β_2 is found to be nearly the same as that of the cases of clamped square plates. In Table 5, the solutions for cantilevered square plates are given. The variation of the natural frequency coefficients along with the variation of the ratio h_1/h_o is found to be completely different from that of the two previous cases of clamped and simply supported boundary conditions. It is found that, for any value of $h_1/h_o<1$, the values of λ are greater than those corresponding to the uniform thickness plate. Such a behavior may be explained as follows: The reduction of the thickness along the span of the plate tends to decrease both the stiffness and the mass of the plate. According to the Rayliegh's method, which roughly approximates the system to one that has a single degree of freedom, the fundamental natural frequency coefficient is predicted from the relation $\omega=\sqrt{K/M}$ where K is the stiffness and M is the mass of the system. The variation of both K and M due to the reduction of the thickness may result in values of ω which are greater than that corresponds to the uniform thickness plate.

Table 4 Natural frequency coefficients for simply supported square plates

h_1/h_o	β_1	β_2	λ_1	λ_2	λ_3	λ_4
1.0	0.0	0.0	19.739	49.344	49.347	78.935
	0.0	-0.25	18.041	45.118	45.202	72.144
0.75	-0.125	-0.125	17.635	44.070	44.072	70.509
	-0.25	0.0	17.230	42.938	43.020	68.870
	0.0	-0.5	16.216	40.423	40.432	64.807
0.5	-0.25	-0.25	15.408	38.151	38.360	61.537
	-0.5	0.0	14.604	35.877	36.264	58.242
	0.0	-0.75	14.111	34.549	34.885	56.470
0.25	-0.5	-0.25	12.524	29.966	30.744	49.832
	-0.75	0.0	11.739	27.666	28.679	46.461

Table 5 Natural frequency coefficients for cantilevered square plates

h_1/h_o	β_1	β_2	λ_1	λ_2	λ_3	λ_4
1.0	0.0	0.0	3.473	8.511	21.295	27.200
	0.0	-0.25	3.715	8.385	20.449	24.656
0.75	-0.125	-0.125	3.653	8.188	19.965	24.039
	-0.25	0.0	3.588	7.990	19.481	23.423
	0.0	-0.5	4.049	8.297	19.504	22.301
0.5	-0.25	-0.25	3.917	7.872	18.460	20.991
	-0.5	0.0	3.770	7.435	17.413	19.687
	0.0	-0.75	4.559	8.360	18.406	20.484
0.25	-0.5	-0.25	4.287	7.413	15.944	17.715
	-0.75	0.0	4.115	6.907	14.676	16.344

In Tables 6 and 7, the results for two clamped rectangular plates, of moderately large and moderately small aspect ratios, respectively, are presented. Since the increase of the aspect ratio tends to increase the plate area and hence reduces its stiffness and increases its mass, it is expected that, the larger the aspect ratio, the smaller the natural frequency of the plate and vise versa. The variation of λ along with the variation of the parameters h_1/h_o, β_1 and β_2 is found to be similar to that concerns the clamped square plate.

The problem of the cantilevered symmetric trapezoidal plate is also considered. For a plate of AR=1, the results for three different values of θ are given in Table 8. It is noticed that, the increase of the value of θ tends to increase the corresponding value of λ for all tip to root thickness ratios. The reason of such behavior may be explained as follows: When θ increases, the rectangular plate (θ=0) tends to be a triangular one that has a larger stiffness, hence its natural frequency will increase. It is also found that, the variation of λ along with the three parameters h_1/h_o, β_1 and β_2 is similar to that happened in the case of the cantilevered rectangular plate.

Table 6 Natural frequency coefficients for clamped rectangular plates(AR=2.5)

h_1/h_o	β_1	β_2	λ_1	λ_2	λ_3	λ_4
1.0	0.0	0.0	23.645	27.810	35.427	46.698
	0.0	-0.04	21.652	25.572	32.406	42.624
0.75	-0.05	-0.02	21.032	24.944	31.664	41.670
	-0.1	0.0	20.409	24.313	30.919	40.711
	0.0	-0.08	18.863	23.272	29.160	38.114
0.5	-0.1	-0.04	17.693	21.964	27.680	36.222
	-0.2	0.0	16.511	20.645	26.183	34.309
	0.0	-0.12	14.982	20.456	25.500	32.807
0.25	-0.2	-0.04	12.847	17.679	22.485	29.036
	-0.3	0.0	11.764	16.277	20.937	27.111

Table 7, Natural frequency coefficients for clamped rectangular plates(AR=0.4)

h_1/h_o	β_1	β_2	λ_1	λ_2	λ_3	λ_4
1.0	0.0	0.0	147.78	173.81	221.42	291.87
	0.0	-1.5625	132.02	156.38	200.85	266.43
0.75	-0.25	-0.9375	130.69	154.38	197.65	261.51
	-0.625	0.0	128.70	151.39	192.86	254.14
	0.0	-3.125	114.61	137.09	177.95	237.67
0.5	-0.75	-1.25	110.61	131.07	168.28	222.85
	-1.25	0.0	107.95	127.60	161.85	212.47
	0.0	-4.6875	94.67	115.03	151.50	203.80
0.25	-1.0	-2.1875	88.99	106.55	138.08	183.42
	-1.875	0.0	84.13	99.26	126.45	165.69

Table 8 Natural frequency coefficients for cantilevered trapezoidal plates(AR=1.0)

h_1/h_o	β_1	β_2	θ	λ_1	λ_2	λ_3	λ_4
			5	3.663	10.070	21.768	33.813
1.0	0.0	0.0	10	3.910	12.207	22.217	37.640
			15	4.262	15.300	22.793	43.266
			5	3.910	9.918	20.955	31.001
0.75	0.0	-0.25	10	4.163	12.033	21.396	34.441
			15	4.523	15.111	21.943	39.821
			5	3.774	9.450	19.970	29.695
0.75	-0.25	0.0	10	4.017	11.470	20.381	33.029
			15	4.363	14.426	20.899	38.067
			5	4.250	9.788	20.149	27.982
0.5	0.0	-0.5	10	4.511	11.864	20.568	31.189
			15	4.879	14.920	21.082	36.069
			5	3.951	8.773	18.014	24.740
0.5	-0.5	0.0	10	4.187	10.648	18.388	28.122
			15	4.527	13.446	18.843	32.442
			5	4.769	9.780	19.472	24.707
0.25	0.0	-0.75	10	5.038	11.776	19.898	27.900
			15	5.415	14.765	20.378	32.030
			5	4.291	8.065	15.837	19.377
0.25	-0.75	0.0	10	4.518	9.726	16.197	22.820
			15	4.845	12.297	16.580	26.148

CONCLUSION

An eighteen degrees of freedom triangular plate bending element with parabolic thickness variation in one direction has been formulated. It has been employed in the free vibration analysis of rectangular and trapezoidal plates that have quadratic thickness variation in the span wise direction. For clamped and simply supported plates, it is found that the reduction of the thickness along the span reduces their natural frequencies, while for cantilevered plates an opposite behavior is happened. The effect of the parameters that govern the role of thickness variation is studied. It is concluded that, for a plate that has a certain tip to root thickness ratio and a certain aspect ratio, the natural frequencies in the case of purely linear thickness variation will be lower than those corresponding to the case of purely quadratic thickness variation. For trapezoidal plates, it is found that, the increase of the sweep back angle θ tends to make the plate to be much more stiff.

APPENDIX

Equation 3 represents the shape function of the eighteen degrees of freedom triangular plate bending element. This equation is a standard one for such a conforming element.

$$w=\{A\}^T\{\alpha\}=\alpha_1+\alpha_2\xi+\alpha_3\eta+\alpha_4\xi^2+\alpha_5\xi\eta+\alpha_6\eta^2+\text{---------------}+\alpha_{21}\eta^5. \quad \text{(A-1)}$$

This equation contains the 21-α_i's unknowns that must be determined. After substituting for the expressions of the 18-degrees of freedom of the element and carrying out the required transformations between the global and the local systems of coordinates, eighteen equations between the 21 unknowns are obtained. The other three equations result from suppressing the normal slope at the three mid-sides points of the three edges of the triangle by assuming that the normal slope along an edge is being a cubic function of the edgewise direction coordinate.

Equation 3 is not the corner stone of the present work. This equation could be used either the plate has uniform thickness or variable thickness. The main purpose of the present work is to study the free transverse oscillations of the plate when it has a quadratic thickness variation. In Reference 10, two categories of plates were analyzed, the first one which have linear thickness variation and the second were those having exponential thickness variation.

Equation 1 represents the assumed thickness variation. This equation is actually the corner stone of the present work. In Reference 10, the thickness variation was given by: $h = h_o(1+\beta y)$ and

$h = h_o e^{\beta y}$ but here it is given by: $h = h_o(1+\beta_1 y+\beta_2 y^2)$.

Equation 2 gives the varying bending rigidity of the plate. It is a function of the span-wise y-coordinate and contains the variable y up to the six degree. $D=D_O(1+\beta_1 y+\beta_2 y^2)^3$.

Equation 4 indicates the relation between the global y-coordinate and the local ξ, η coordinates. $y=y_3+y_{13}\xi+y_{23}\eta$. If Equation 4 is substituted into Equation 1, then the thickness will be given as a quadratic polynomial of ξ, η as expressed in Equation 5. This polynomial could be explicitly expressed as follows:

$$h = h_o(S_1+S_2\xi+S_3\eta+S_4\xi^2+S_5\xi\eta+S_6\eta^2) = h_o\sum_{k=1}^{6} S_k\,\xi^{m_k}\,\eta^{n_k} \quad \text{(A-2)}$$

The total number of terms is six, m_k and n_k are the powers of ξ and η respectively. The substitution of Equation 4 into Equation 2 gives the flexural rigidity D of the plate as a six-degree polynomial of ξ, η that contains 28 terms and each of the powers m_k and n_k takes the values from zero to six. The explicit form of this equation is given as follows:

$$D = D_O(R_1+R_2\xi+R_3\eta+R_4\xi^2+R_5\xi\eta+R_6\eta^2+R_7\xi^3+R_8\xi^2\eta+R_9\xi\eta^2+R_{10}\eta^3+\text{------}+R_{22}\xi^6+\text{----}+R_{28}\eta^6) \quad \text{(A-3)}$$

where R_k are constants depending on the parameters $\beta 1$, $\beta 2$ and the global y-coordinates of the element vertices.

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الاهتزازات الحرة للألواح ذات السمك المتغير تربيعيا

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ملخص البحث

يختص هذا البحث بدراسة الاهتزازات الحرة للألواح ذات السمك المتغير بصورة تربيعية مع أحد محورى المستوى المتوسط للوح وذلك لتعيين تردداتها الطبيعية. وقد استهل البحث بمقدمة موجزة عما تم انجازه فى هذا المجال من دراسات سابقة ثم العرض المختصر للطريقة الرياضية المستخدمة هنا وهى طريقة العناصر المحددة حيث تم استحداث عنصر محدد مثلثى الشكل يتغير سمكه بصورة تربيعية مع أحد محورى مستواه المتوسط وتم استخدام هذا العنصر الجديد فى الدراسة.

اختصت الدراسة فى هذا البحث بتعيين الترددات الطبيعية للألواح ذات الشكل المستطيل والشكل شبة المنحرف وقد تمت دراسة تقارب الحلول ومقارنة النتائج فى بعض الحالات الخاصة بنتائج الباحثين السابقين المتاحة والتى تم الحصول عليها بطرق رياضية أخرى حيث ظهر التطابق مه هذه النتائج بصورة كبيرة.

وقد خلص البحث الى بعض النتائج الهامة والتى يمكن تلخيصها فى الآتى:

١. فى حالتى الألواح المثبتة تثبيتا تاما أو المرتكزة ارتكازا بسيطا على محيط اللوح كلما زادت نسبة تقليل السمك على طوال اللوح كلما قلت قيم تردداته الطبيعية.

٢. للوح ذاو نسبة أبعاد معينة ونسبة سمك معينة (نسبة السمك = السمك عند نهاية اللوح ؟السمك فى بدايته) وجد أن تأثير المعامل الذى يحكم التغير التربيعى للسمك على الترددات الطبيعية للوح أكبر من تأثير المعامل الذى يحكم التغير الخطى للوح

٣. للوح ذو الشكل شبة المنحرف كلما زادت زاويتى انحرافه كلما ذات تردداته الطبيعية وذلك لأى نسبة سمك