

# SURVEYING PROCEDURE OF MEASURING DIFFERENTIAL DISPLACEMENT

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## ABSTRACT

Differential displacement is a problem which needs to be defined accurately. Surveying observations are required to be taken in order to know the movement of each point of the object. Special surveying techniques are to be adopted using surveying instruments. A mathematical model is to be derived for each case of objects. In this paper definitions, necessity, kind of movement and mathematical model are given. The system of coordinates suitable for determining differential displacement is explained. In previous researches some cases of measuring deformation were explained and this paper gives a new case and object.

**Keywords:** Differential Displacement; Movements, Deformations, Settlement, Monitoring.

## INTRODUCTION

Differential displacement of parts of an object is used to be termed deformation. Expression of deformation is normally used to indicate of changing for the worse disfigurements or perverted form; in other words deformation is a change of state of an object, it not necessary this change to the worse. Therefore, it is preferable to change the term "deformation" to "differential displacement".

If the displacement of all parts of an object is the same in magnitude and direction then the object is moving but differential displacement is the subject of this paper. If parts of a building have a differential displacement then this case is a defect and ought to be measured

## NECESSITY OF MONITORING DIFFERENTIAL DISPLACEMENT

The necessity of monitoring differential displacement is to verify the reception of an observed object to different influences as [1]:

- a. Differential displacement of dams due to the change of water level.
- b. Characteristics of soil under construction.
- c. Sliding of crust like Mokattam hills in Cairo-Egypt.

- d. Increasing or decreasing of water table.
- e. Age of construction such as archaeological buildings.
- f. Sudden accident such as earthquake.
- g. Human action in or around the construction such as sewers, mining, large dams, pollution.... Etc.
- h. Reaction of factory walls and foundation to vibration of machines.
- i. Extension of cities.
- j. Checking product against the standard, i.e. the faults of machine.
- k. Settlement of parts of a construction.

## CLASSIFICATION OF DIFFERENTIAL DISPLACEMENTS

### Kinematic Differential Displacement

Kinematic differential displacement is the change of points of an object referred to an assumed or natural position. The assumed position is the theoretical or mathematical form. The natural position is referred to some natural direction such as the direction of gravity and horizontal line or plane.

The assumed position could be :

- a. Designed part of machine.
- b. The ellipsoidal form of the earth.
- c. Designed civil engineering work.

The natural position could be :

- a. Direction of gravity at any instant.
- b. Geometrical conditions of triangulation networks.

*One set of observation at an instant is enough to determine the kinematic differential displacement.*

### **Dynamic Differential Displacement**

If the kinematic differential displacement is determined through one set of observation at an instant, the dynamic differential displacement is to be determined through periodical measurements. Therefore, dynamic differential displacement is to determine the continuous change of points of an object.

The periodical observations can be taken at regular or irregular periods of times. The time interval between two successive sets of observations is normally determined according to the magnitude and direction of the movement of points of an object, resulting from changing of the shape of an object.

### **Slow and Fast Differential Displacement**

Slow and fast differential displacement is another classification of differential displacement. Classification of slow or fast differential displacement is depending on the object monitored for the dynamic movements of its points.

The slow dynamic differential displacement is normally measured for the geological purpose such as crustal movements, continental drift and tectonic plates.

Fast dynamic differential displacement is measured for man-made construction such as dams, bridges, towers, railways.....etc.

### **SURVEYING PROCEDURE FOR MEASURING DIFFERENTIAL DISPLACEMENT**

The accuracy of normal survey practice is not likely to be sufficient to measure differential displacement. The accuracy required is higher than the normal survey procedure. Therefore, other survey procedures are to be applied.

### **System of Coordinates**

A system of coordinates should be adopted to calculate the coordinates of measurements. The origin of this system has to be defined according to the case. The origin could be taken at one of the object points or a point outside the object. It should be a point, which is believed to be fixed and is not subject of any movements. The origin is not necessarily a physical point existing in the field but could be an imaginary point. For monitoring slow differential displacement the origin is normally taken as a local point, on/in/out the object [2].

One, two or three dimension coordinates are then used depending on the case and requirements of monitoring. One dimension can be used as in case of settlement and levelling. Two dimension of system is used for planimetry monitoring of points on an object.

Normally cartesian three dimensions system of coordinates is used in many cases of monitoring differential displacement. The three axes are taken perpendicular to each other. the vertical axis is always the vertical direction of gravity. The other two axes are in a horizontal plane perpendicular to the vertical axis. One of the horizontal axes has to be taken as a natural direction such as the north direction (geographical or magnetic). The other horizontal direction is always 90° east of north [3,4].

If the monitoring takes long time such as monitoring of crustal movements or if the determined object is large such as continent, care should be given to the deflection of the vertical. Also in this case of large object, curvature of the earth has to be considered. In other words geodetic problem should be looking after for monitoring large parts of the earth. But for fast monitoring in such human construction cartesian local three dimension is normally enough.

### **Reference Control Points**

Movements of points are noticed if it is referred to unmoving object. So, in order to detect the differential displacement of points, relatively fixed points have to be

considered. Differential displacement of an object can be defined as movement of points on that object not uniformly relative to each other.

**APPLICATION OF DIFFERENTIAL DISPLACEMENT ON COLUMNS**

Most of structures are supported by columns which carry the load from top to bottom. These columns are normally in the vertical direction of gravity. The columns either tighten to the walls or separate from them. If the building has a large span, then the supported columns have to be separated from the walls and from each other.

Examples of large spans are meeting halls and mosques. Differential displacement of a building affects the columns carrying loads. Therefore measuring the differential displacement of columns gives the data for structure analysis of the building. This will lead to find out the structural defects of the building.

Deformation components can be taken in three directions, two in horizontal plane and the third in the vertical direction.

The origin of the coordinates of a column can be taken in the center of intersection of horizontal plane with the lowest appearing part of the column as shown in Figure 1.

**The three cartesian coordinates from this origin are:**

- a. Vertical axis (gravity direction), Z.
- b. Horizontal magnetic north, X.
- c. Horizontal magnetic east, Y.

This system of coordinates can be called Natural Local Coordinates System (NLCS). This does not change with time in the range of accuracy required. In addition this system is suitable for determining the differential displacement.

The structural analysis needs high accuracy for determining the inclination of columns rather than the direction of inclination. That is why the magnetic bearing for direction of column's inclination is taken. Any measurements taken from point outside of the column is the relation between the system of coordinates of the surveying instrument and the system of coordinate of the column.

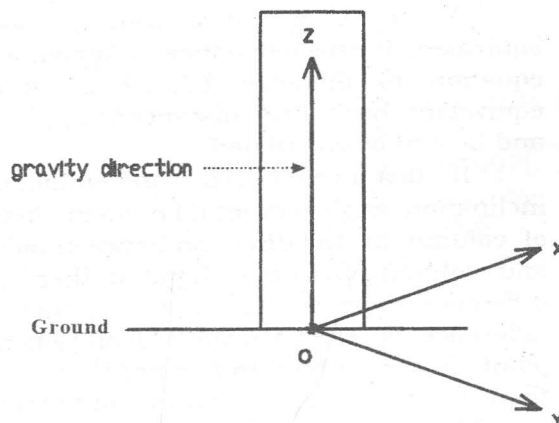


Figure 1 Axes of column

The coordinates system of the surveying instrument is as follow:

- a. Origin is taken as the intersection point of vertical axis (direction of gravity) and the horizontal plane of the instrument.
- b. The vertical axis (gravity direction).
- c. Direction from the origin of instrument to any other known point, horizontally.
- d. The perpendicular horizontal axis is 90° from the axis defined in C, clockwise.

The measurements are distances, horizontal and vertical angles. Then the measurements are taken referred to:

- \* Two points outside the column.
- \* Distances and angles.

Hence the measurements are the relation between the two systems, instrument and columns coordinate systems, then number of measurements should be sufficient to find the transformation parameters.

**MATHEMATICAL MODEL**

Figure 2 shows the observations taken from two points outside the column which are horizontal angles A and B, vertical angles  $v_1, v_2, v_3, v_4$  and space distances  $L_1, L_2, L_3, L_4$ .

The equation used for calculations of transforming the system of coordinates of the surveying instrument (theodolite with EDM) is as follow:

$$L' = L \cos v \quad (1)$$

Where  $L'$  is the horizontal distance equivalent to space distance  $L$ . Applying this equation to distance  $L_1, L_2, L_3$ , and  $L_4$  equivalent horizontal distances  $L_1', L_2', L_3'$  and  $L_4'$  can be calculated.

If distances  $L_1', L_2'$  are equal, then inclination angle  $\theta$  is zero, i.e. no inclination of column in the direction between point B and column. On other hand if there is a difference between  $L_1'$  and  $L_2'$ , then the difference is the horizontal displacement of points I and J seen from point B. If  $L_1'$  less than  $L_2'$  then the inclination outwards of line BJ and vice versa.

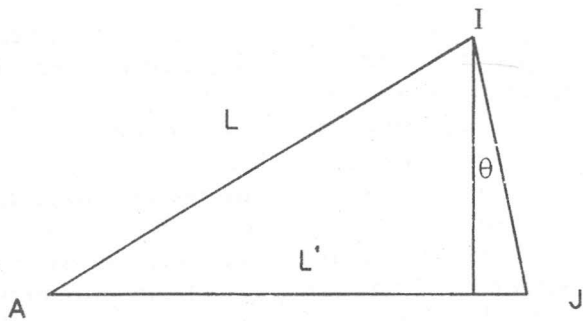
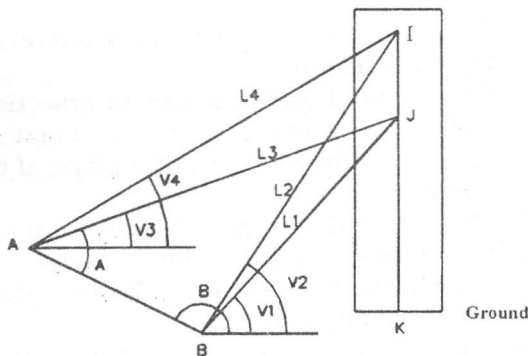


Figure 2 Observation of column's inclination

Similarly the displacement seen from point A is the difference between point I and J from A.

The resultant can be calculated as magnitude and direction from the horizontal projection of triangle A B J in Figure 3 which is drawn as A' B' J'.

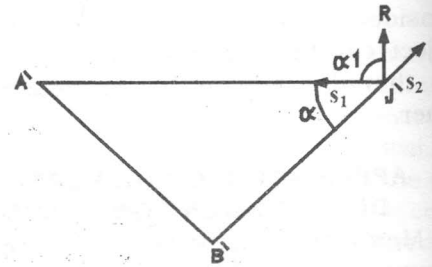


Figure 3 Resultant of inclination

- Calculate the distances of displacement seen from points A and B, denoted by  $S_1$  and  $S_2$  respectively.
- The resultant R is :-  

$$R^2 = S_1^2 + S_2^2 - 2S_1 S_2 \cos \alpha \quad (2)$$
- The angle between the resultant and  $S_1$  is equal to:  

$$\alpha_1 = \tan^{-1} \frac{S_1}{S_2} \quad (3)$$
- If the magnetic bearing of AJ is measured, then the magnetic bearing of R can be calculated according to the direction of  $S_1$  and  $S_2$ .

The angle of inclination of column equivalent to  $S_1$  and  $S_2$  is equal respectively to:

$$\theta_1 = \tan^{-1} S_1/IJ \quad (4)$$

$$\theta_2 = \tan^{-1} S_2/IJ \quad (5)$$

and the resultant  $\theta_3$  is given by :

$$\tan^2 \theta_3 = \tan^2 \theta_1 + \tan^2 \theta_2 - 2 \cos \alpha \tan \theta_1 \tan \theta_2 \quad (6)$$

where  $\theta_1, \theta_2$  and  $\theta_3$  is the angles of inclination.

If it is required to calculate the coordinates, then using angle  $\theta$  and the height of column the cartesian coordinates can be calculated referred to the cartesian coordinates of the column.

Transformation could be done using the classical method where the transformation parameters are calculated.

The transformation parameters are seven: three translation of the origin, three rotation of axes and one is the scale.

### CONCLUSIONS

As shown before measuring differential displacements of points on an object needs, a suitable system of coordinates to be adopted for the case and preferably to use the natural axes. The derived mathematical formulae of coordinates is the relation between system of coordinates of instrument used and the adopted one using surveying measurements. The required accuracy of each coordinate has to be known in advance for the suitability of axes and kind of measurements. Experience shows that each case i.e. each object has special treatment of measurements, instruments and mathematical model.

This paper is the first of a series of papers for this subject in which further cases are given from experience. A graphical solution was found to give results which are accurate enough. So it is recommended to be used in some cases.

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Received April 26, 1999  
Accepted August 21, 1999

## الأسلوب المساحي في قياس التشوهات

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### ملخص البحث

يعتبر قياس التشوهات من الموضوعات الهامة في السنوات الأخيرة ويمكن تقسيم التشوهات في حركة القشرة الأرضية وتشوهات في المباني من ميول في الأعمدة والأبراج والمآذن وكافة الإنشاءات الهندسية. يتناول البحث أهمية وتعريف أنواع التحركات والتشوهات وكذلك المعادلات الرياضية اللازمة لتحديد مقدار هذه التشوهات وأساليب الرصد المساحية التي تم الوصول إليها لتحديد مقدار الميول والتحركات والتشوهات في الإنشاءات الهندسية.