MODELING SINGLE SLIDER CRANK MECHANISM AS NONLINEAR MULTIBODY DYNAMIC SYSTEM

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ABSTRACT

The single slider-crank mechanism is mathematically modeled. The multibody dynamic approach is applied to derive the equation of motion of the single slider-crank mechanism, as a nonlinear dynamic system, in the minimal symbolic form. The flexibility in the slider-crank linkage is just considered in the crank axis. Finally, the nonlinear mechanism system is linearized to a linear vibrating system.

Keywords: Machine Dynamics, Multibody Dynamics, Linear Vibration.

INTRODUCTION

The method of multibody dynamic system [1,2] is applied to the single slider-crank mechanism. The mathematical model is constructed, such that the crank and coupler (connecting rod) are represented by their center lines and centers of masses [3]. The slider is considered as a lumped mass, performing translatory motion. Since the crank is subjected to a resisting torque, it is necessary from the practical point of view to mainly consider the flexibility of the crank axis. For simplicity, it is modeled with regard of a viscous bearing friction as a linear torsional spring-damper unit coupled to the crank.

The kinematic study is based on the translation of centers of masses of crank and coupler, and the rotation of each with respect to the mass center [4,5]. The applied forces are represented in symbolic forms, and are specified as the dynamic force acting on the slider, the weights of crank, coupler and slider. The crank is subjected to a resisting torque arised from the torsional spring-damper coupling. The reaction forces and moments are analyzed. The friction is neglected in the coupler turning joints and the slider contact surface.

Newton-Euler equations are applied to the slider-crank linkage to derive the global equations of motion of the dynamic system [3,6]. *D'Alembert's* principle is applied to eliminate the reaction forces and moments, concluding the equation of motion of the mechanism system in the minimal symbolic form [4,7]. The linkage system is finally linearized [2] in order to be treated as linear vibrating system.

MATHEMATICAL MODEL OF THE SLIDER-CRANK MECHANISM

The slider-crank mechanism is undertaken as a one degree of freedom holonomic scleronomic rigid system. The generalized coordinate is the crank rotation angle ϕ, as shown in Figure 1. This angle represents the torsional angle resulting from the flexibility (elasticity) in the crank axis. The flexibility together with an assumed viscous bearing friction is displayed via the torosional action of a linear spring-damper coupling model shown in Figure 1.

An inertial coordinate system represented by the fixed *Cartesian* coordinates x_I, y_I, z_I is assigned as a reference frame [1] to the rigid dynamic system in Figure 1. The centers of masses of crank and coupler C_c and C_p, respectively, are chosen as origins of local coordinate systems [9,10], as indicated in Figure 1. The motion of crank and coupler can be thus specified by the translation of centers of masses and the rotation of each link with respect to the mass center [3,11]. The slider is considered as a lumped mass, since it performs translatory motion.

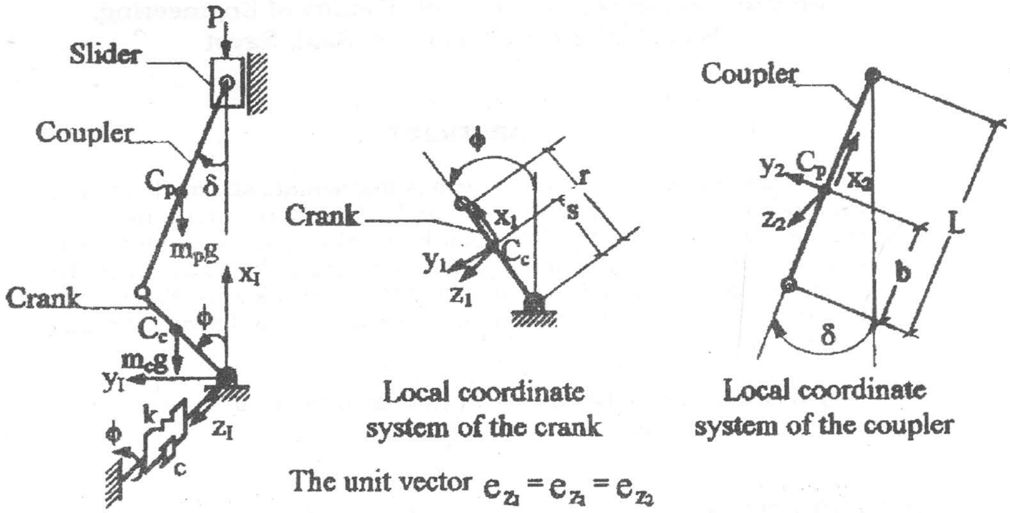

Figure 1 Scheme of the slider-crank mechanism model

KINEMATICS OF THE SLIDER-CRANK MECHANISM

The slider-crank linkage shown in Figure 1 is treated as a closed kinematic chain. A geometric constraint is represented by the mathematical relation between the crank and coupler rotation angles ϕ and δ, respectively. It is derived to be

$$\sin\delta = \frac{r}{L}\sin\phi\,,\ \cos^2\delta = 1 - \frac{r^2}{L^2}\sin^2\phi\,,$$

$$\frac{d\delta}{d\phi} = \delta' = \frac{r}{L}\cos\phi / \cos\delta\,. \qquad (1)$$

$$\frac{d^2\delta}{d\phi^2} = \delta'' = \frac{r}{L}\cdot\frac{\delta'\cos\phi\sin\delta - \sin\phi\cos\delta}{\cos^2\delta}\,.$$

The position of each body relative to the inertial frame is given by 3x1-translation vector and 3x3-rotation tensor [4,11,12]. The crank translation vector and rotation tensor are determined as

$$r_C = \begin{bmatrix} s\cos\phi \\ s\sin\phi \\ 0 \end{bmatrix},\ S_C = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ +\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad (2)$$

respectively, and the position of the coupler further yields

$$r_P = \begin{bmatrix} r\cos\phi + b\cos\delta \\ r\sin\phi - b\sin\delta \\ 0 \end{bmatrix},\ S_P = \begin{bmatrix} \cos\delta & \sin\delta & 0 \\ -\sin\delta & \cos\delta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \qquad (3)$$

where s,r,b and L are clarified in Figures 1 and 2. The slider position is just given by the vector

$$r_S = \begin{bmatrix} r\cos\phi + L\cos\delta \\ 0 \\ 0 \end{bmatrix}. \qquad (4)$$

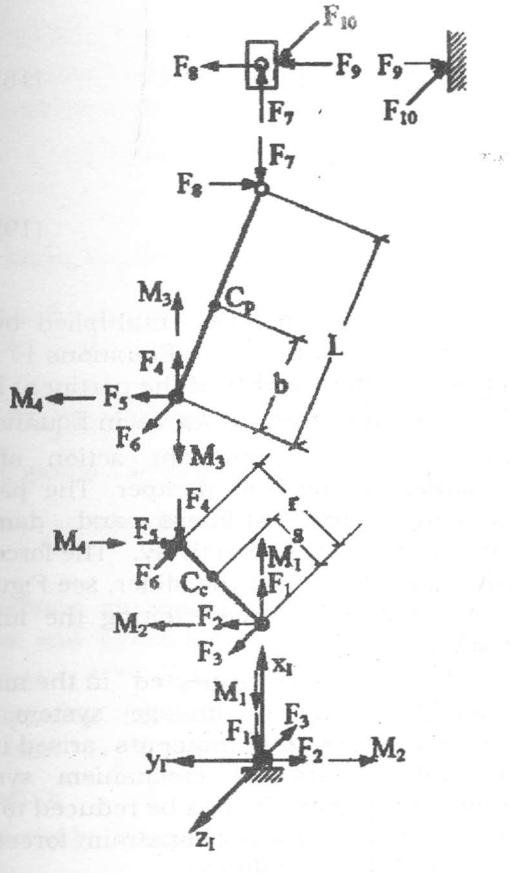

Figure 2 Distribution of reaction forces and moments arised in the slider-crank mechnism

The translational and rotational velocities of each link can be obtained by differentiation with respect to time t as follows :

$$\frac{dr_i}{dt} = v_i = \frac{\partial r_i}{\partial \phi}\dot{\phi} - \frac{\partial r_i}{\partial t} = J_{Ti}\dot{\phi} - \overline{v}_i \ , \ i = c, p, s \ ; \qquad (5)$$

$$\frac{ds_i}{dt} = \omega_i = \frac{\partial s_i}{\partial t}\dot{\phi} + \frac{\partial s_i}{\partial t} = J_{Ri}\dot{\phi} + \varpi_i, \partial\tilde{s}_i = (\partial S_i)S_i^T \qquad (6)$$

The same procedure can be followed up in the derivation of the translational and rotational accelerations to be

$$a_i = \frac{dv_i}{dt} = J_{Ti}\ddot{\phi} + \frac{\partial v_i}{\partial \phi}\dot{\phi} + \frac{\partial v_i}{\partial t} = J_{Ti}\ddot{\phi} + \overline{a}_i \ , \qquad (7)$$

$$\alpha_i = \frac{d\omega_i}{dt} = J_{Ri}\ddot{\phi} + \frac{\partial \omega_i}{\partial \phi}\dot{\phi} + \frac{\partial \omega_i}{\partial t} = J_{Ri}\ddot{\phi} + \overline{\alpha} \ , \qquad (8)$$

where J_{Ti} and J_{Ri} are 3xf-*Jacobian* matrices, and f=1 is the number of degrees of freedom for the mechanism system, undertaken in the study. The infinitesimal 3x1-rotation vector ∂s_i in Equation 6 follows from the infinitesimal skew-symmetric 3x3-rotation matrix $\partial\tilde{s}_i$ [3,11]. The partial time-derivatives in Equations 5 to 8 vanish, since the linkage system in Figure 1 is modeled as scleronomic (time invariant) constrained system.

The velocity of the mechanism system can be accordingly read as

$$v_c = \begin{bmatrix} -s\sin\phi \\ s\cos\phi \\ 0 \end{bmatrix}\dot{\phi} = J_{Tc}\dot{\phi} \ . \ \omega_c = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\dot{\phi} = J_{Rc}\dot{\phi} \ . \qquad (9)$$

$$v_p = \begin{bmatrix} -r\sin\phi - b\delta'\sin\delta \\ r\cos\phi - b\delta'\cos\delta \\ 0 \end{bmatrix}\dot{\phi} = J_{Tp}\dot{\phi} \ , \omega p = \begin{bmatrix} 0 \\ 0 \\ -\dot{\delta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -\delta' \end{bmatrix}\dot{\phi} = J_{Rp}\dot{\phi} \ , \qquad (10)$$

$$v_s = \begin{bmatrix} -r\sin\phi - L\delta'\sin\delta \\ 0 \\ 0 \end{bmatrix}\dot{\phi} = J_{Ts}\dot{\phi} \qquad (11)$$

Referring to Equations 1, 9 and 10, the angular velocity vector ω_p of the coupler is parallel to that of the crank, ω_c and apposite in direction, i.e.,

$$\omega_p = -\left(\dot{\delta}\right)e_{z_I} = -\left\{\left(\frac{r}{L}\cos\phi / \cos\delta\right)\dot{\phi}\right\}e_{z_I}$$
$$= -\left(\delta'\dot{\phi}\right)e_{z_I} = -\delta'\omega_c \ , \qquad (12)$$

where e_{z_I} is the unit vector of the fixed coordinate z_I in the inertial frame shown in Figure 1. Further, the system's acceleration yields

$$a_c = J_{Tc}\ddot{\phi} + \begin{bmatrix} -s\cos\phi \\ -s\sin\phi \\ 0 \end{bmatrix}\dot{\phi}^2 = J_{Tc}\ddot{\phi} + \bar{a}_c ,$$

$$\alpha_c = J_{Rc}\ddot{\phi} , \tag{13}$$

$$a_p = J_{Tp}\ddot{\phi} + \begin{bmatrix} -r\cos\phi - b\delta''\sin\delta - b\delta'^2\cos\delta \\ -r\sin\phi - b\delta''\cos\delta - b\delta'^2\sin\delta \\ 0 \end{bmatrix}\dot{\phi}^2$$

$$= J_{Tp}\ddot{\phi} + \bar{a}_p ,$$

$$\alpha_p = J_{Tp}\ddot{\phi} + \begin{bmatrix} 0 \\ 0 \\ -\delta'' \end{bmatrix}\dot{\phi}^2 = J_{Rp}\ddot{\phi} + \bar{\alpha}_p, \tag{14}$$

$$a_s = J_{Ts}\ddot{\phi} + \begin{bmatrix} -r\cos\phi - L\delta''\sin\delta - L\delta'^2\cos\delta \\ 0 \\ 0 \end{bmatrix}\dot{\phi}^2$$

$$= J_{Ts}\ddot{\phi} + \bar{a}_s . \tag{15}$$

NEWTON-EULER EQUATIONS OF MOTION

Newton-Euler Equations [3 and 4] are applied to the mechanism system. Each link is treated separately for the kinetic study, i.e.,

$$m_i a_i = f_i^e + f_i^r , I_i\alpha_i + \tilde{\omega}_i I_i \omega_i$$
$$= l_i^e + l_i^r , i = c,p,s \tag{16}$$

The inertia of each body in Equations 16 is represented by a scalar mass m_i and a 3x3-inertia tensor I_i with respect to the center of mass C_i. The terms f_i^e, f_i^r and l_i^e, l_i^r are defined as 3x1-vectors summarizing the applied and constraint forces and torques, respectively.

The applied forces and torques acting on each link of the slider-crank linkage in Figure 1 can be detailed in the following can be detailed in the following vector forms:

$$f_c^e = \begin{bmatrix} -m_c g \\ 0 \\ 0 \end{bmatrix}, l_c^e = \begin{bmatrix} 0 \\ 0 \\ -k\phi - c\dot{\phi} \end{bmatrix}, \tag{17}$$

$$f_p^e = \begin{bmatrix} -m_p g \\ 0 \\ 0 \end{bmatrix}, l_p^e = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \tag{18}$$

$$f_s^e = \begin{bmatrix} -P - m_s g \\ 0 \\ 0 \end{bmatrix} . \tag{19}$$

The scalar masses multiplied by the gravity acceleration g in Equations 17 to 19 represent the weights of the pertinent links. The dynamic torques $k\phi$, $c\dot{\phi}$ in Equation 17 arise from the resistant action of the torsional spring and damper. The passive elements yield stiffness and damping constants k, c, respectively. The force P in Equation 19 acts on the slider, see Figure 1, as a dynamic force exciting the linkage system.

The friction is neglected in the moving kinematic pairs of linkage system. The reaction forces and moments arised in the kinematic pairs of mechanism system, shown in Figure 2, can be reduced to 3x1-vectors of generalized constraint forces and torques [3,4,6] as follows:

$$f_i^r = D_i g^r , i = c,p,s ; l_i^r = L_i g^r , i = c,p; \tag{20}$$

where:

$$g^r = \left[F_1 | F_2 | .... | F_9 | F_{10} | M_1 | M_2 | M_3 | M_4\right]^T \tag{21}$$

is 14x1-vector summarizing the reaction forces and moments illustrated in Figure 2. The matrices D_i, L_i in Equation 20 are defined as 3x14-distribution matrices, clarified in the Appendix.

The inertia tensors I_c, I_p are arranged to be transformed from

$$I'_c = \begin{bmatrix} I_{cx_1} & 0 & 0 \\ 0 & I_{cy_1} & 0 \\ 0 & 0 & I_{cz_1} \end{bmatrix}, I'_p = \begin{bmatrix} I_{px_2} & 0 & 0 \\ 0 & I_{py_2} & 0 \\ 0 & 0 & I_{pz_2} \end{bmatrix} \tag{22}$$

with respect to the local coordinate systems (body fixed frames) of the crank and the coupler, respectively, to the inertial frame of the mechanism system, such that

$$I_c = S_c I'_c S_c^T,\ I_p = S_p I'_p S_p^T \qquad (23)$$

The global equations [3,6,11] of the mechanism system can be derived to be

$$\overline{M} \overline{J} \ddot{\phi} + \overline{q}^c = \overline{q}^e + \overline{Q} g^r, \qquad (24)$$

where

$$\overline{M} = \text{diag}\left\{ m_c E \middle| m_p E \middle| m_s E \middle| I_c \middle| I_p \right\} \qquad (25)$$

is block diagonal matrix, and E is 3x3-identity matrix, and

$$\overline{J} = \left[J_{Tc}^T \middle| J_{Tp}^T \middle| J_{Ts}^T \middle| J_{Rc}^T \middle| J_{Rp}^T \right]^T \qquad (26)$$

is the global Jacobian matrix. The vector

$$\overline{q}^c = \left[m_c \bar{a}_c^T \middle| m_p \bar{a}_p^T \middle| m_s \bar{a}_s^T \middle| (I_c \bar{\alpha}_c - \tilde{\omega}_c I_c \omega_c)^T \middle| (I_p \bar{\alpha}_p + \tilde{\omega}_p I_p \omega_p)^T \right]^T \qquad (27)$$

is defined as the global vector of *Coriolis* forces and gyroscopic moments. The terms $(I_c \bar{\alpha}_c - \tilde{\omega}_c I_c \omega_c)$ and $\tilde{\omega}_p I_p \omega_p$ vanish, since $\overline{\alpha}_c = 0$ in Equation 13 and the vectors of angular velocities ω_c, ω_p are parallel to those of moments of momentum $I_c\omega_c, I_p\omega_p$, respectively. Further,

$$\overline{q}^e = \left[f_c^{eT} \middle| f_p^{eT} \middle| f_s^{eT} \middle| l_c^{eT} \middle| l_p^{eT} \right]^T \qquad (28)$$

is the global vector of applied forces and torques, and

$$\overline{Q} = \left[D_c^T \middle| D_p^T \middle| D_s^T \middle| L_c^T \middle| L_p^T \right]^T \qquad (29)$$

is the global distribution matrix, indicated in the Appendix.

EQUATION OF MOTION OF THE SLIDER-CRANK MECHANISM

The application of *D'Alembert's* principle by premultiplication of the global Equation 24 from hte left with transposed global Jacobian matrix $\overline{J}^T$ will eliminate the constraint forces and torques [4,9,11], since $J^T \overline{Q} = 0$.

A scalar differential equation of motion in the minimal symbolic form can be thus derived for the nonlinear scleronomic mechanism system to be

$$M\ddot{\phi} + K\dot{\phi}^2 = q, \qquad (30)$$

where :

$$M = \overline{J}^T \overline{MJ} = m_c s^2 + m_p \left[r^2 - 2rb\delta' \cos(\phi + \delta) + b^2 \delta'^2 \right] + m_s(-r\sin\phi - L\delta'\sin\delta)^2 + I_{cz_1} + I_{pz_2}\delta'^2 .$$

$$K\dot{\phi}^2 = \overline{J}^T \overline{q}^c = \left\{ m_p \left(rb\left[\delta'(1+\delta') \sin(\phi+\delta) - \delta'' \cos(\phi+\delta) \right] + b^2 \delta' \delta'' \right) + m_s \left(\left[r\sin\phi + L\delta'\sin\delta \right] \left[r\cos\phi + L\left(\delta''\sin\delta + \delta'^2 \cos\delta \right) \right] \right) + I_{pz_2}\delta'\delta'' \right\} \dot{\phi}^2 .$$

$$q = m_c g s \sin\phi + m_p g(r\sin\phi + b\delta'\sin\delta) + m_s g(r\sin\phi + L\delta'\sin\delta) + P(r\sin\phi + L\delta'\sin\delta) - k\phi - c\dot{\phi} .$$

LINEARIZATION OF THE MECHANISM SYSTEM

The motion of the slider-crank mechanism, regarded in the study, is too small due to the infinitesimal rotary motion of the crank. The flexibility of the crank axis just permits the crank to infinitesimally oscillate. The behavior of the mechanism system, as a consequence, is assumed to be linear. It yields

$$\sin\phi \approx \phi,\ \cos\phi \approx 1,\ \phi^2 \approx 0,\ \dot{\phi}^2 \approx 0,\ \phi\dot{\phi} \approx 0 \qquad (31)$$

for the infinitesimal crank rotation angle ϕ, and

$$\sin\delta \approx \frac{r}{L}\phi;\ \cos\delta \approx 1,\ \delta' \approx \frac{r}{L} \qquad (32)$$

with the reference to Equation 1. The equation of motion of the mechanism system as a linear vibrating system [8] will be:

$$\mu\ddot{\phi} + \chi\dot{\phi} + \xi\phi = M_e, \qquad (33)$$

where:

$$\mu = m_c s^2 + m_p r^2 \left(1 - \frac{b}{L} \right)^2 + I_{cz_1} + I_{pz_2} \cdot \frac{r^2}{L^2};$$

$$\chi = c,$$

$$\xi = k - \left[m_c g s + m_p g r\left(1 + \frac{b}{L} \right) + m_s g r\left(1 + \frac{r}{L} \right) \right].$$

$$M_e = \Pr\left(1 + \frac{r}{L} \right)\phi .$$

It is revealed from Equation 33 that the mechanism system yields a crank rotation angle ϕ = 0 at the stable equilibrium position. Equation 33 facilitates the application of the numerical simulation to analyze and study the dynamic behavior of the mechanism system as a linear vibrating system.

CONCLUSIONS

The single slider-crank mechanism is modeled as a nonlinear scleronomic holonomic system by the method of multibody rigid systems. The flexibility is just considered in the crank axis, and simply represented by the action of linear spring-damper coupling. This results in an oscillating motion of the crank, assumed to be infinitesimal. *Newton-Euler* equations with *D'Alembert's* principle are applied to derive the equation of motion of the linkage system in the minimal symbolic form. The system is finally linearized to a linear vibrating system in order to be easily undertaken in the numerical simulation.

APPENDIX

Arrangement of the Global Distribution Matrix

$$\bar{Q} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -s\sin\phi & 0 & 0 & -(r-s)\sin\phi & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & s\cos\phi & 0 & 0 & (r-s)\cos\phi & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ s\sin\phi & -s\sin\phi & 0 & (r-s)\sin\phi & -(r-s)\cos\phi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & b\sin\delta & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & b\sin\delta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -b\sin\delta & -b\cos\delta & 0 & -(L-b)\sin\delta & -(L-b)\cos\delta & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} D_c \\ \\ \\ D_p \\ \\ \\ D_s \\ \\ \\ L_c \\ \\ \\ L_p \\ \\ \\ \end{matrix}$$

$$\bar{Q} = \left[D_c^T \,\vdots\, D_p^T \,\vdots\, D_s^T \,\vdots\, L_c^T \,\vdots\, L_p^T \right]^T$$

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Received March 10. 1999
Accepted August 21. 1999

عمل نموذج للآلة الترددية البسيطة كنظام ديناميكي غير خطي متعدد الأجسام

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ملخص البحث

تعتبر الآلة الترددية البسيطة بأجزائها، المرفق (الكرنك) وذراع التوصيل والمكبس، من الأنظمة الديناميكية الغير خطية متعددة الأجسام. وقد تم عمل النموذج التخطيطي لهذه الآلة باعتبار الكرنك وذراع التوصيل كأعضاء (أجسام) ممثلة بمحاورها المركزية، والمكبس ككتلة مركزة، الأمر الذى سهل عمل النمذجة الرياضية لهذه الآلة كميكانزم ذى أعضاء جاسئة. الا أن المرونة قد أخذت فى الاعتبار عند تمثيل عامود المرفق (الكرنك) بصورة مبسطة كياى خطى، مرفق معه على التوازى خامد للصدمات خطى ايضا كنموذج رياضى يمثل الاحتكاك اللزج فى كراسى التحميل لعامود المرفق.

وقد تمت الدراسة الكينماتيكية للأعضاء باعتبار انتقال مركزى ثقل كل من الكرنك وذراع التوصيل وانتقال المكبس فقط ككتلة مركزة، وذلك بتحديد متجه الموضع لكل عضو، لدراسة الإزاحة والسرعة والعجلة. ويتم ذلك منسوبا الى نظام احداثيات كارتيزية ثابت الوضع والإتجاه حيث تم تحديد نقطة الأصل له لتكون منطبقة على مركز دوران الكرنك. وقد تم اعتبار دوران كل من الكرنك وذراع التوصيل مأخوذا بالنسبة لمركزى الثقل على الترتيب.

وبتطبيق معادلات نيوتن- أويلر لإيجاد المعادلات العامة للحركة للنظام الديناميكى، ثم تطبيق مبدأ دالمبرت لحذف ردود الأفعال كقوى وعزوم، مع إهمال الإحتكاك فى الأزواج الدورانية، أمكن إستخلاص معادلة الحركة للآلة الترددية البسيطة كنظام خطى لايعتمد على الزمن فى صورة رمزية مختصرة، وبافتراض أن زاوية دوران الكرنك الناشئة من وجود المرونة فى عامود الكرنك متضائلة القيمة، أمكن ايضا تبسيط معادلة الحركة فى صورة خطية.