STABILITY OF TWO CONSECUTIVE FLOORS WITH INTERMEDIATE FILTERS

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ABSTRACT

A subsidiary weir is proposed to be constructed downstream of each of the existing barrages on the Nile river in Egypt to secure its stability, which might have been affected after the construction of the Aswan High Dam. The present paper is intended to investigate the characteristics of seepage flow beneath two structures with an intermediate filter. The downstream structure (the proposed weir) has a slopping middle apron and two flat aprons in the upstream and downstream sides and is also provided with upstream and downstream cutoffs. The upstream structure (the existing barrage) has upstream and downstream cutoffs. A conformal mapping technique is used to solve the problem. Equations to calculate the uplift pressure distribution acting on both the existing barrage and the suggested weir are obtained; also equations to calculate the values of exit gradient along the intermediate filter and the downstream bed are derived. The seepage flows which seep into and/or drained from the intermediate filter are estimated. The analytical results are verified using experimental measurements performed on electrical analogue model and a very good agreement is noticed. A computer program is designed to compute the seepage flow, the uplift pressure and the exit gradients.

Keywords: Drop structures, Seepage, Conformal mapping, Weirs, Barrages

INTRODUCTION

Due to the degradation along the Nile river, consequent to the construction of the Aswan High Dam, the existing barrages, which had been constructed on the Nile river has become vulnerable to instability. To reduce the difference of water level acting on such barrages, a subsidiary weir is proposed to be constructed downstream of each barrage at some distance from it.

Studies of the seepage characteristics of two hydraulic structures with an intermediate filter have been carried out using conformal mapping technique by several investigators. These studies besides their application to limited cases such as the downstream structure has either a flat floor with or without cutoff [1,2,3], or has an inclined middle apron without cutoff [4], are difficult to be applied by the practicing engineers. An approximate solution which is based on successive conformal mapping method, for floor having inclined middle apron with a single cutoff is available [5]. All these studies considered the existing barrages as a simple floor without cutoffs or having single cutoff. No attention has been made to consider the effect of the floor thickness.

Here, an analytical solution, using conformal mapping transformation, for the problem of seepage beneath two structures with an intermediate filter has been obtained. The downstream structure has a slopping middle apron, with upstream and downstream cutoffs. The upstream structure has upstream and downstream cutoffs. The embedded floor thickness has been taken into consideration. The upstream cut-off is provided to prevent the soil under the structures from slipping into the scour holes, while the downstream one is very important to prevent undermining. Equations to calculate the uplift pressure distribution acting on both the existing barrage and the suggested weir are obtained. Equations to calculate the values of exit gradient along the intermediate filter and the downstream bed are derived. The seepage flows which seep into and/or drained from the intermediate filter are estimated. The analytical results are verified using experimental measurements performed on electrical analogue model and good agreement is noticed.

MATHEMATICAL MODEL

The geometry of a typical two floors with an intermediate filter is shown in Figure 1-a. It also shows the flow domain in the Z-plane. The following symbols are used for the dimensions shown in this figure: L_1 is the length of the upstream floor, CD; S_{10} is the depth of the front face of the upstream cutoff, AU; S_{11} is the depth of the back face of the upstream cutoff, UC; S_{20} is the depth of the front face of the downstream cutoff, DE; S_{21} is the depth of the back face of the downstream cutoff, EF; L_f is the length of the intermediate filter, FG. The downstream floor GJLMNOQB has two horizontal aprons, a middle sloping apron and two end cutoffs; the length of the upstream apron, LM, equals L_2; the length of the downstream apron, NO, equals L_4. The middle apron, MN, makes an angle with the horizontal, and has a projection length equal to L_3. S_{30} is the depth of the front face of the upstream cutoff, GJ; S_{31} is the depth of the back face of the upstream cutoff, JL; S_{40} is the depth of the front face of the downstream cutoff, OQ; S_{41} is the depth of the back face of the downstream cutoff, QB. The drop between upstream and down stream beds equal d. The effective heads on the upstream and downstream floors equal to (H_1-H_2) and (H_2-H_3), respectively. The two floors are founded on a homogeneous isotropic pervious bed extending to infinity in the upstream, downstream, and vertically downward directions.

Denoting $w=\phi+i\psi$, where ψ=stream function, ϕ= velocity potential function and equal to -kH, in which H= the total head, and k= coefficient of permeability.

A complete solution of the problem, w=f(z), is derived. This is done as follows:

1- Mapping the physical plane (z) onto the lower half of an auxiliary semi-infinite t-plane, where t=r+is.

2- Mapping complex potential plane (w) onto the lower half of the auxiliary semi-infinite t-plane.

3- The w-z relationship is then obtained by combining the derived equations.

Boundary Condition

As shown in Figure 1-a and 1-c, the first stream line, $\psi=0$, coincides with the subsurface contour of the upstream floor AUCDEF. The upstream bed A`A, the intermediate filter FG, and the downstream bed BB` are equipotential lines. If the upstream bed is chosen as a datum, $\Phi_{A'A}=-kH_1$, $\Phi_{FG}=-kH_2$, and $\Phi_{BB'}=-kH_3$ respectively.

Depending on the dimensions of the two structures, the length of the intermediate filter, and the relative value of $(H_2-H_3)/(H_1-H_2)$, three cases can occur:

1- Part of the seepage flow coming from the upstream side is drained through the upstream part of the intermediate filter, FP. The remaining part of the intermediate filter, PG, works as an inlet face. In this case, along the length, FP, the stream function varies between 0 and $-q_1$ whereas along length, PG, it varies between $-q_1$ and $-q_2$, see Figures 1-a, and 1-c.

2- A streamline, $\psi=-q_1$ starting from somewhere at the upstream bed would meet the floor GJLMNOQB, at some point P, where it would divide into two streamlines, one along PG, emerging at G, and the other along PB emerging at B. In this case the entire intermediate filter works as an exit face, along which the value of the stream function varies between 0 and $-q_1$, see Figures 1-d.

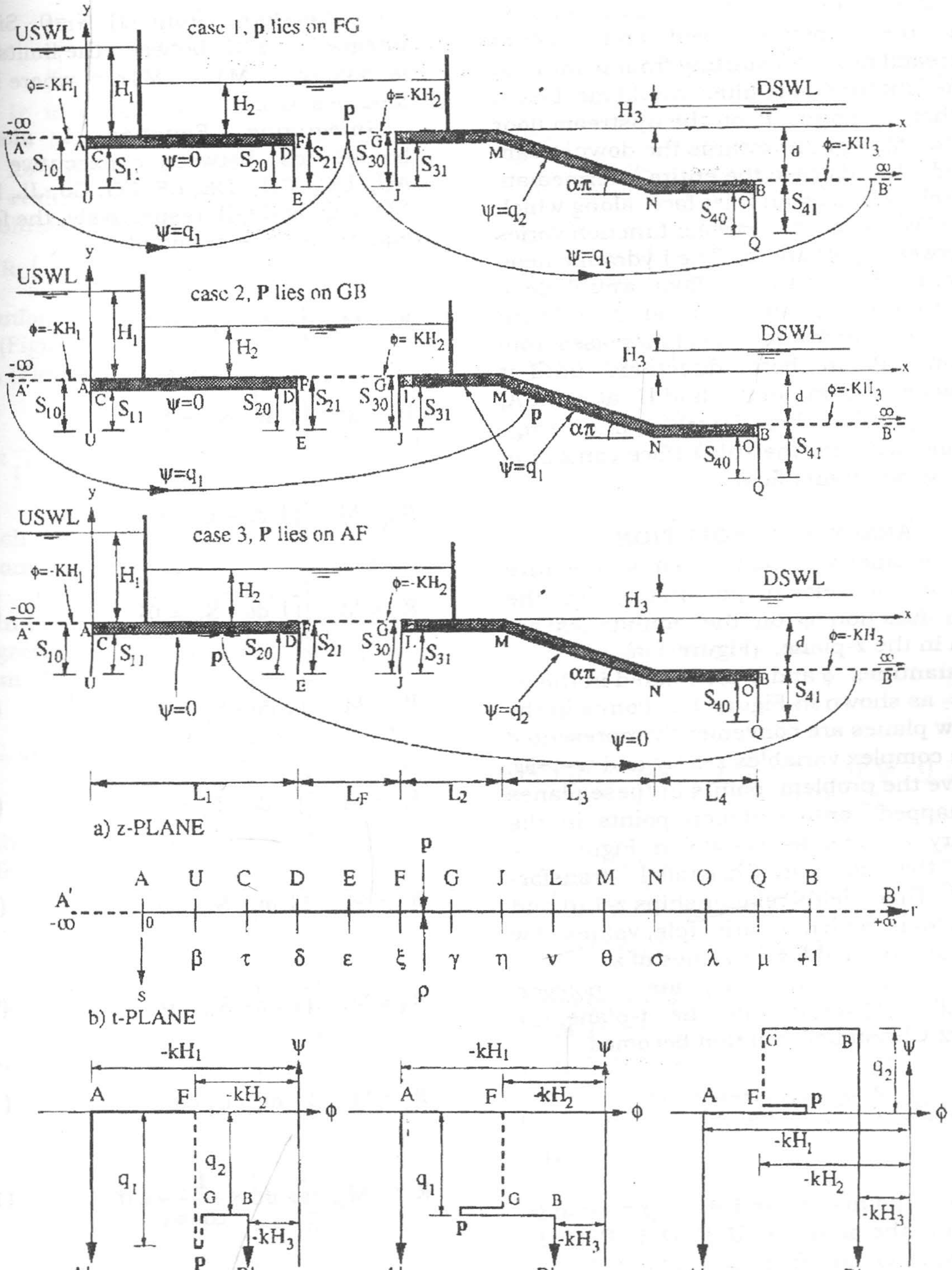

Figure 1 Illustration of the problem

3- A streamline, 0 starting from point A, at the upstream bed and another streamline, $\psi=0$ starting from point F, at the intermediate filter, would meet each other at point, P, on the upstream floor and will leave it towards the downstream bed. In this case the entire intermediate filter works as an inlet face, along which the value of the stream function varies between $q_1=0$ and q_2. The hydraulic head along the upstream floor would be a minimum at point, P, as shown in Figure 1-e. The hydraulic head increases from point, P, in both directions until it reaches H_2 at point F and H_1 at point A, respectively. The stream line $\Psi=q_2$, coincides with the subsurface contour of the downstream floor.

ANALYTICAL SOLUTION

The geometry of the problem and relative values of the velocity potential ψ and the stream function ψ on the boundaries are shown in the z-plane, (Figure 1-a).

The quantities ϕ and ψ are related in the w-plane, as shown in Figure 1-c. Points in the z and w planes are conveniently represented by the complex variables $z=x+iy$ and $w=\phi+i\psi$. To solve the problem, points in these planes are mapped onto common points in the auxiliary t-plane, as shown in Figure 1-b, using the Schwarz-Christoffel transformation. This yields relationships $z=f_1(t)$ and $w=f_2(t)$ from which, in principle, values of w *can be determined for all values of z.*

To map the z-plane polygon AUCDEFGJLMNOQB onto the t-plane, the Schwarz-Christoffel equation becomes

$$z=M_1\int_0^t \frac{(\beta-t)(\varepsilon-t)(\eta-t)(\sigma-t)^{\alpha}(t-\mu)\,dt}{t^{5}(\tau-t)^{5}(\delta-t)^{5}(\xi-t)^{5}(\gamma-t)^{5}(t-\nu)^{5}(t-\theta)^{\alpha}(\lambda-t)^{5}(1-t)^{5}}+N_1 \quad (1)$$

where the points A and B are placed at 0 and 1, and the points A', U, C, D, E, F, G, J, L, M, N, O, Q, and B' lie at $-\infty$ β, τ, δ, ε, ξ, γ, η, ν, θ, σ, λ and ∞, respectively, Figure 1-a,b. The values of these parameters are to be determined. α,π = angle of inclination of the slopping floor MN; and M_1 and N_1 =complex constants. At point A, z=0, and t=0, therefore, from (1) $N_1=0$. Since the integration of (1) between the limits θ and σ is positive, $M1 = M1r^{e-i\pi}$ where M_{1r}= the modules of M_1.

Integrating Equation 1 along the boundaries between consecutive vertices AU, UC, CD, DE, EF, FG, GJ, JL, LM, MN, NO, OQ, and QB, respectively, the following equations were obtained

$$R_1 = M_{1r}\left|\int_0^{\beta} I\,dt\right| - S_{10} = 0; \quad (2)$$

$$R_2 = M_{1r}\left|\int_{\beta}^{\tau} I\,dt\right| - S_{11} = 0 \quad (3)$$

$$R_3 = M_{1r}\left|\int_{\tau}^{\delta} I\,dt\right| - L_1 = 0 \quad (4)$$

$$R_4 = M_{1r}\left|\int_{\delta}^{\varepsilon} I\,dt\right| - S_{20} = 0; \quad (5)$$

$$R_5 = M_{1r}\left|\int_{\varepsilon}^{\xi} I\,dt\right| - S_{21} = 0 \quad (6)$$

$$R_6 = M_{1r}\left|\int_{\varsigma}^{\gamma} I\,dt\right| - L_f = 0 \quad (7)$$

$$R_7 = M_{1r}\left|\int_{\gamma}^{\eta} I\,dt\right| - S_{30} = 0; \quad (8)$$

$$R_8 = M_{1r}\left|\int_{\eta}^{\nu} I\,dt\right| - S_{31} = 0 \quad (9)$$

$$R_9 = M_{1r}\left|\int_{\upsilon}^{\theta} I\,dt\right| - L_2 = 0 \quad (10)$$

$$R_{10} = M_{1r}\left|\int_{\theta}^{\sigma} I\,dt\right| - \frac{L_3}{\cos\pi\alpha} = 0 \quad (11)$$

$$R_{11} = M_{1r}\left|\int_{\sigma}^{\lambda} I\,dt\right| - L_4 = 0 \quad (12)$$

$$R_{12} = M_{1r}\left|\int_{\lambda}^{\mu} I\,dt\right| - S_{40} = 0; \quad (13)$$

$$R_{13} = M_{1r} \left| \int_{\mu}^{1} I \, dt \right| - S_{41} = 0 \qquad (14)$$

where I= integrand of Equation 1. The unknowns M_{1r}, β, τ, δ, ε, ξ, γ, η, ν, θ, σ, λ and μ for a particular hydraulic structure, can be evaluated by minimizing the sum of the absolutes of the residuals E

$$E = \sum_{i=1}^{13} |R_i| \qquad (15)$$

The conformal transformation of the w-plane, (Figures 1-c, 1-d and 1-e), onto the t-plane, (Figure 1-b), was given by

$$\frac{dw}{dt} = M_2 \frac{(\rho - t)}{t^{.5}(\xi - t)^{.5}(\gamma - t)^{.5}(1 - t)^{.5}} \qquad (16)$$

in which M_2 = complex constant. The corresponding between the various points on the w-plane and the t-plane are as shown in Figures 1-c, 1-d, 1-e and 1-b.

Integrating Equation 16 along the upstream floor, AUCDEF, from 0 to t, one qets

$$\phi + kH_1 = M_2 \int_0^t \frac{(\rho - t)\, dt}{t^{.5}(\xi - t)^{.5}(\gamma - t)^{.5}(1 - t)^{.5}} \qquad (17)$$

in which ϕ=-kH, at point F, ϕ = $-kH_2$ and t= ξ, therefore

$$kH_1 - kH_2 = M_2 \int_0^{\xi} \frac{(\rho - t)\, dt}{t^{.5}(\xi - t)^{.5}(\gamma - t)^{.5}(1 - t)^{.5}} \qquad (18)$$

Integrating Equation 16 in the portion, GJLMNOQB, from γ to t, the following equation was obtained

$$\phi + kH_2 = M_2 \int_{\gamma}^{t} \frac{(\rho - t)\, dt}{t^{.5}(\xi - t)^{.5}(\gamma - t)^{.5}(1 - t)^{.5}} \qquad (19)$$

At point B, ϕ = $-kH_3$ and t= 1, therefore

$$kH_2 - kH_3 = M_2 \int_{\gamma}^{1} \frac{(\rho - t)\, dt}{t^{.5}(\xi - t)^{.5}(\gamma - t)^{.5}(1 - t)^{.5}} \qquad (20)$$

The two constants M_2 and ρ can be determine from Equations 18 and 20 for known values of ξ, γ.

Uplift Pressures

Knowing the values of M_{1r}, β, τ, δ, ε, ξ, γ, η, ν, θ, σ, λ, μ. M_2 and ρ, the uplift pressure can be determined at any point lying on the upstream floor (AUCDEF) or on the downstream floor (GJLMNOQB) by substituting the corresponding value of t in Equations 17 and 19 respectively. The corresponding location of the desired point on the z-plane can be obtained from Equation 1.

Exit Gradient

In addition to the uplift pressure, it is also important to know the hydraulic gradient along the intermediate filter and/or on the downstream bed. The exit gradient at any point is given by [6]

$$I_{exit} = \frac{i}{k} \cdot \frac{dw}{dt} \cdot \frac{dt}{dz} \qquad (21)$$

Using Equations 1, 16, and 21, the exit gradient was obtained as

$$I_{exit} = \frac{i}{k} \cdot \frac{M_2}{M_1} \cdot \frac{(\tau - t)^{.5}(\delta - t)^{.5}(t - \nu)^{.5}(t - \theta)^{\alpha}(\lambda - t)^{.5}(\rho - t)}{(\beta - t)(\varepsilon - t)(\eta - t)(\sigma - t)^{\alpha}(t - \mu)} \qquad (22)$$

The maximum exit gradients occur at points F, G, and B, (Figure 1-a) can be found by inserting t= ζ, γ, and 1, respectively into Equation 22.

Seepage Flow

Case 1: $\xi < \rho < \gamma$

The seepage flow which seeps into (q_1) and/or drained (q_2) from the intermediate filter may be obtained from the following equations:

$$-iq_1 = M_2 \int_{\xi}^{\rho} \frac{(\rho - t)\, dt}{t^{.5}(\xi - t)^{.5}(\gamma - t)^{.5}(1 - t)^{.5}} \qquad (23)$$

$$-iq_2 = M_2 \int_{\xi}^{\gamma} \frac{(\rho - t)\, dt}{t^{.5}(\xi - t)^{.5}(\gamma - t)^{.5}(1 - t)^{.5}} \qquad (24)$$

Case 2: $\xi < \rho < \gamma$

The seepage flow which seeps into (q_1) the intermediate filter may be obtained from the following equation:

$$-iq_1 = M_2 \int_{\xi}^{\gamma} \frac{(\rho - t)\,dt}{t^{.5}(\xi - t)^{.5}(\gamma - t)^{.5}(1-t)^{.5}} \qquad (25)$$

Case 3: $\xi < \rho < \gamma$

The seepage flow which drained (q_2) from the intermediate filter may be obtained from the following equation:

$$iq_2 = M_2 \int_{\xi}^{\gamma} \frac{(\rho - t)\,dt}{t^{.5}(\xi - t)^{.5}(\gamma - t)^{.5}(1-t)^{.5}} \qquad (26)$$

EXPERIMENTAL VERIFICATION

In order to verify the analytical solutions, some experimental results were acquired using the method of electrical analogy [7,8]. A model was formed from a sheet of electrically conductive paper and incorporated into a Whetstone's bridge circuit. The outer boundary of the conductive sheet has been cut as an elliptic shape to simulate the last stream line to minimize the errors in the experimental results due to the artificial boundaries. The model was cut with an extra 1 cm deep strip along the upper boundaries and this strip was coated with highly conductive silver paint to form the equipotential boundaries. Wires were soldered directly to these boundaries to avoid the problem of variable contact resistance. Three comparative studies were made using values of S_{21}/L_1= 0.10, 0.15, 0.2 and fixed values of L_f/L_1= 2.5, S_{10}/L_1= 0.1 L_2/L_1= 0.15 , L_3/L_1= 0.25, L_4/L_1= 0.3, d/L_1= 0.05, S_{30}/L_1= 0.1 , and S_{41}/L_1= 0.15. The relative thickness of upstream and downstream floors (T/L_1) were considered 0.025. For these ratios, the values of $(H_2-H_3)/(H_1-H_2)$ are 0.5, 1.0, 1.5, and 2.0 respectively. For each S_{20}/L_1 value, the relative potential at the key point were measured. The relative potential at the key points were calculated also using the presented analytical solution. This resulted in the pairs of relationships shown in Figure 2. Upon observing the results in this figure one can immediately see that a good agreement between experimental and theoretical results prevails. The maximum value of the percentage error between the experimental results and the analytical results is less than 5%.

RESULTS AND DISCUSSION

The equations derived herein have been used for computations of uplift pressures acting on both upstream and downstream floors, seepage discharge and exit gradients along the intermediate filter and the downstream bed. These calculations involved the evaluation of many integrals, which were computed on a digital computer by numerical methods. The random search method [9], with the aid of Gauss-Chebyshev formula [10], have been used to find out the state variables involved in Equation 1 for a physical dimensions of the structures. The uplift pressures at key points corresponding to these physical dimensions of the structures are determined from Equations 17 and 19 after determining the values of M_2 and ρ from Equations 18 and 20. The obtained results are used to illustrate the effect of the change in filter length, dimensions of the structure, and relative effective head $(H_2-H_3)/(H_1-H_2)$ on the seepage characteristics.

Effect of Ratio of Effective Heads

The net uplift pressures acting under the upstream floor are significantly decreased as the ratio of effective heads $(H_2-H_3)/(H_1-H_2)$ increased, (Figure 2-a).

Figure 2-b shows that the increase in the value of ratio of effective head results in an appreciable decrease in the value of uplift pressure acting on the down stream floor.

Effect of Relative Filter Length (L_f/L_1)

Figure 3-a shows that net uplift pressures acting on the upstream floor are slightly increase as the relative filter length is increased. On the other hand, the values of net uplift pressures acting along the downstream floor significantly decrease as the filter length is increased. Lengthening

the intermediate filter to a certain extent, namely $L_f/L_1 = 1.0$ in Figure 3-b, results in a small reduction in the exit gradient at point F. For longer filter length, however this trend reverses and an appreciable increase of the exit gradient at point F is noticed. Figure 3-b also shows that exit gradient at point B decreases significantly as the relative filter length is increased.

Figures 3-a and 3-b show that increasing the full length of the filter (L_f) beyond five times the length of the upstream floor (L_1) will have a negligible effect on the uplift pressures acting on both upstream and downstream floors and on the exit gradients at points F and B. Hence, for $L_f/L_1 > 5.0$, both structures can be treated as a separate structure.

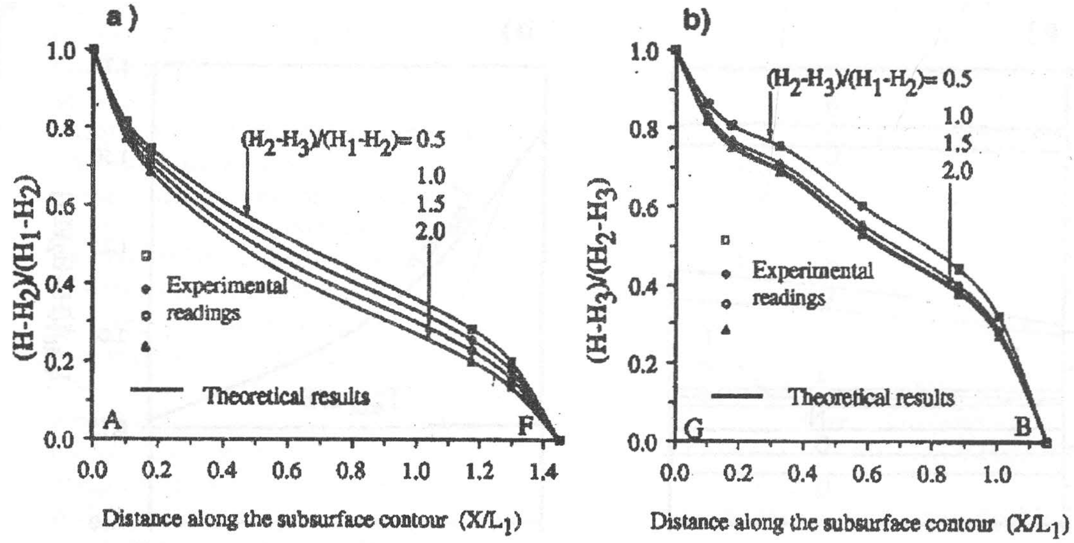

Figure 2 Comparison between experimental and theoretical results for relative uplift pressure acting along the subsurface contour of: a) upstream floor; b) downstream floor, $S_{21} / L_1 = 0.15$

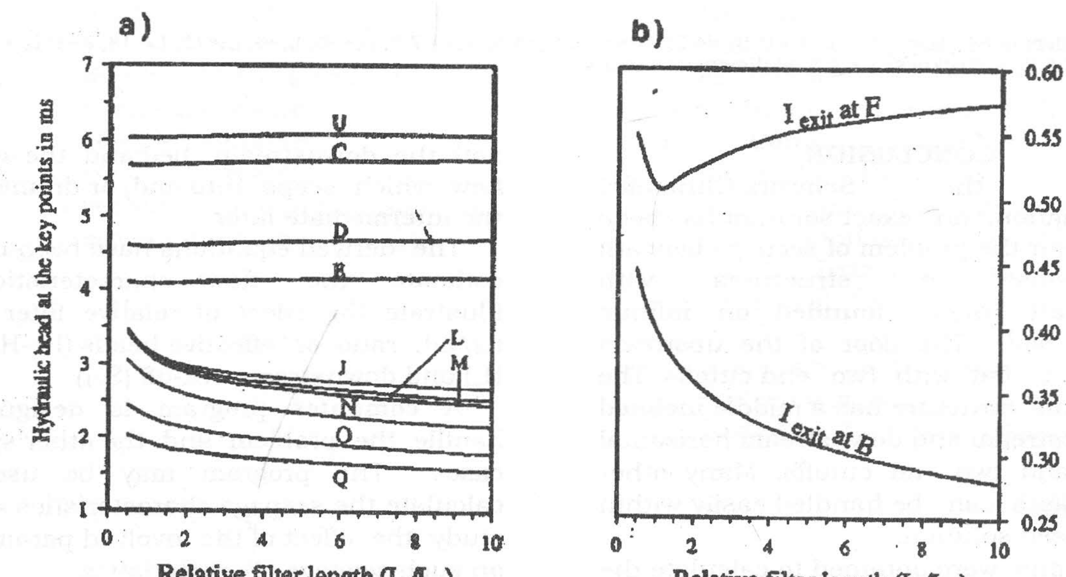

Figure 3 Effect of filter length: H_1=7; H_2=3; H_3=1; S_{11}=3; S_{20}=6; S_{31}=4; S_{40} = 4; L_1=20; L_2=7; L_3=15; L_4=20; d=2; and floor thickness t = 1.0 all dimensions in ms

Effect of Downstream Cutoff of the Upstream Structure (S_{21})

A perusal of Figure 4 indicates that the uplift pressure acting on the upstream floor increases with an increase in depth of the downstream cutoff. Exit gradient at the downstream end of the upstream structure, point F decreases with an increase in the depth of the downstream cutoff.

Increasing of downstream cutoff, S_{21} has a negligible effect on the uplift pressure acting on the upstream and downstream structures and on the exit gradient at the downstream end and on the intermediate filter.

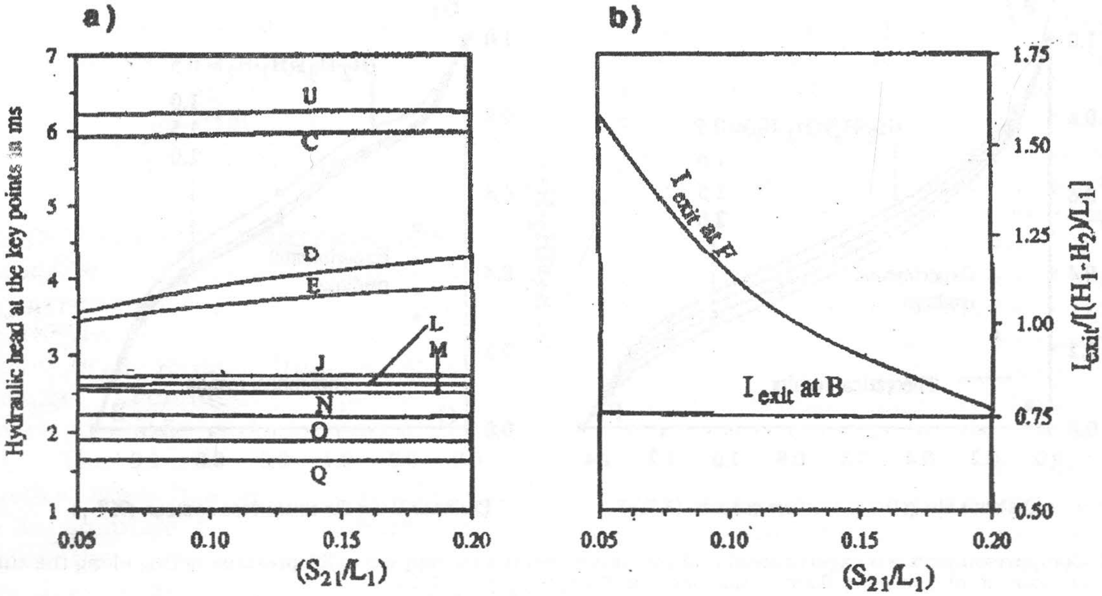

Figure 4 Effect of S_{21}: $H_1=7$; $H_2=3$; $H_3=1$; $S_{11}=4.5$; $S_{31}=4.5$; $S_{31}=4.5$; $S_{40}=7.5$; $L_1=60$; $L_2=9$; $L_3=15$; $L_4=18$; $d=3$; $L_f=150$ and floor thickness $t=1.5$ all dimensions in ms

CONCLUSION

Using the Schwarz-Christoffel transformation, an exact solution has been obtained for the problem of seepage beneath two consecutive structures with intermediate filter, founded on infinite pervious soil. The floor of the upstream structure is flat with two end cutoffs. The downstream structure has a middle inclined apron, upstream and downstream horizontal aprons and two end cutoffs. Many other special cases can be handled easily within *the proposed solution.*

Equations were obtained to calculate the uplift pressures acting on both the existing barrage and the subsidiary weir, the exit gradients along both the intermediate filter and the downstream bed and the seepage flow which seeps into and/or drained from the intermediate filter.

The derived equations have been used to estimate the flow characteristics, to illustrate the effect of relative filter length (L_f/L_1), ratio of effective heads $(H_2-H_3)/(H_1-H_2)$ and downstream cut-off (S_{21}).

A computer program is designed to handle the problem and the other special cases. The program may be used to calculate the seepage characteristics and to *study the effect of the involved parameters* on such seepage characteristics.

Random search method is found to be useful in solving conformal mapping of flow domain with several vertices.

NOMENCLATURE

d = the drop between upstream and downstream beds;
H_1 = head in upstream bed measured above upstream bed level;
H_2 = head in filter measured above upstream bed level;
H_3 = head in downstream bed measured above upstream bed level;
i = imaginary unit:
I_{exit} = exit gradient;
k = coefficient of permeability;
L_1 = floor length of the upstream structure;
L_2 = length of the upstream apron of the downstream structure;
L_3 = sloping projection of the middle apron of the downstream structure;
L_4 = length of the tail apron of the downstream structure;
L_f = intermediate filter length;
M_1, M_2 = complex constants;
M_{1r}, β, τ, δ, ε, ξ, γ, η, ν, θ, σ, λ, μ and ρ state variables;
N_1 = complex constant;
p = stagnation point;
P = uplift pressure;
q = discharge per unit width;
R_1,...., R_{13} = residuals;
S_{10}, S_{20}, S_{30}, S_{40} = depths of front face of cutoffs;
S_{11}, S_{21}, S_{31}, S_{41} = depths of back face of cutoffs;
t = complex variable;
w = complex variable;
z = complex variable;
ϕ = velocity potential function; and
ψ = stream function;

REFERENCES

1. A.S Chawla, "Stability of Structure with Intermediate Filters", J. Hyd. Div., ASCE, 101 (HY2), Pap. 11139, pp. 223-241, (1975).
2. M.A. Elganainy, "Flow Underneath a Pair of Structures with Intermediate Filter on a Drained Stratum," *Journal of* Appl. Math. Modeling, Vol. 10, pp. 394- 400, (1986).
3. Kumar, B. Singh and A.S. Chawla, "Design of Structure with Intermediate Filters", J.Hyd. Div., ASCE 112, (HY3), pp. 206-219, (1986).
4. M.A. Elganainy, "Seepage Underneath Barrages with Downstream Subsidiary Weirs," Journal of Appl. Math. Modeling, Vol. 11, pp. 423- 431, (1987).
5. R.I. Nasr, "Stability of Barrages with Subsidiary Glacis Weirs, Alexandria Engineering Journal, Vol. 36 No. 1, pp. C61-71, (1997).
6. M.E. Harr, "Groundwater and Seepage," Mc Graw-Hill, New York, (1962).
7. S. Muthukumaran and V.C. Kulandaiswamy, "End effects in models for seepage below weirs," Journal of Hydraulic division, ASCE, Vol. 98, No. Hy3, pp. 541-552, (1972).
8. V.I. Vaidyanathan, "The Application of Electrical Analogy to the design of Hydraulic structures," Publication No. 55, Central Board of Irrigation and Power, New Delhi, India, (1955).
9. P.K. Swamee, G.C. Mishra and A.A.S. Salem, "Optimal Design of Sloping Weir", J. Irrig. and Drain. Engrg., ASCE, Vol. 122, No. 4, pp. 248-255, (1996).
10. Francis Scheid, "Theory and Problems of Numerical Analysis," Mc Graw-Hill, New York, (1968).

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أتزان منشأى حجز مياه متتالين بينهما مرشح

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ملخص البحث

يختص البحث بدراسة خصائص التسرب تحت منشأى حجز مياه بينهما مرشح. المنشأ الخلفى يحتوى على جزء مائل فى المنتصف وفى بدايته ونهايته جزء أفقى وستائر لوحية فى الامام والخلف. المنشأ الأمامى عبارةعن فرشة افقيه وستائر لوحية فى الامام والخلف ايضاً.تم أخذ سمك الفرشه للمنشأ الامامى والخلفى فى الاعتبار.

استخدمت نظرية التحويل المتطابق لحل المسأله. تم استنتاج معادلات لحساب الضغوط على طول فرشتى المنشأ الامامية والخلفية ولحساب الميول الهيدروليكيه, وكذلك تم حساب كمية التصرف المتسرب الى و من المرشح. تم التأكد من دقة النتائج معملياً باستخدام التشابه الكهربى.

ايضاً تم تصميم برنامج كمبيوتر للتعامل مع المسأله موضوع البحث. يمكن الاستفاده من نتائج هذا البحث فى دراسة التسرب تحت القناطر المقامه على النيل فى حالات وجود هدار خلفى مساعد.