ANALYSIS OF TWO-WAY REINFORCED CONCRETE SLABS

Mohie E. Shukry* and *Ahmed F. Hassanien

Structural Engineering Department, Faculty of Engineering,
Alexandria University, Alexandria, Egypt

ABSTRACT

This paper presents a study on the design bending moment values in two-way reinforced concrete slabs as suggested by three different building codes, namely; the 1995 Egyptian Code (ECP-95), the 1985 British Standards (BS 8110-85) and the early 1963 ACI code, and the maximum moment values as obtained from two analytical analyses; a linear finite element program (SAP 90) and a nonlinear finite element program developed by the first author. The results reveal that a large variation occurs between different methods of analyses of two-way slabs. However, designers do not need to use a sophisticated method for the analysis of such slabs since the cracking loads in most cases are lower than the working loads that are usually encountered in typical buildings. Generally, the simplified method suggested by the Egyptian Code agreed well with the finite element analyses.

Keywords : Reinforced concrete, Two-way slabs, Building codes, Finite element analysis

INTRODUCTION

The values of bending moments in two-way slabs with various conditions of continuity of edges are determined using the elastic theory of plates and are tabulated in many textbooks [1]. These values of moments were determined assuming non-deformable edges, and the slabs were analyzed as separate rectangles with specified edge conditions either simply supported or fixed. Various simplified methods have been suggested by different building codes for determing moments, shears and reactions for reinforced concrete slabs. Most codes limit the use of its equations to different conditions such as the span of the adjacent panels is approximately equal. However, for slabs not meeting these conditions, designers used to follow the code equations for analysis of such slabs.

The aim of this study is to compare the values of maximum bending moments as predicted by different building codes and analytical methods with the design values suggested by the Egyptian Code simplified method.

The following is a summary of the equations suggested by three building codes; the Egyptian Code (ECP-95), the British Standards (BS 8110-85) and the ACI code.

The Egyptian Code (ECP-95) [2]

The Egyptian Code presents a simplified method for the calculation of moments in two-way slabs in typical buildings. The moments per unit width may be obtained as follows:

$$M_x = (\alpha\ w\ l_x^2)/k \tag{1-a}$$

$$M_y = (\beta\ w\ l_y^2)/k \tag{1-b}$$

where M_x and M_y are design bending moments per unit width in x and y direction respectively, l_y is the length of longer side of slab and l_x is the length of shorter side of slab, w total design dead and live load per unit area, α and β are coefficients given by the code according to slab boundary conditions, and k is bending moment factor; k = 10 for slabs continuous from one end and k = 12 for slabs continuous from both ends.

BS 8110-85 [3]

When simply supported slabs do not have adequate provisions to resist torsion at the corners and to prevent corners from lifting, the design moments per unit width are obtained by using Grashoff coefficients, which yield high values for bending moments.

In slabs where the corners are prevented from lifting and provision for resisting torsion is made, the design moments per unit width are given by the following equations:

$$M_x = \alpha' \, w \, l_x^2 \qquad (2\text{-a})$$
$$M_y = \beta' \, w \, l_x^2 \qquad (2\text{-b})$$

where α' and β' are coefficients given by the code (Table 3-15, Reference 3) according to boundary conditions and the aspect ratio l_y / l_x.

The BS states that where the analysis is carried out for the single load case of all spans loaded, the code allows the negative moments to be reduced by 20 % with a consequential increase in the span moments. Also, the BS recommends, for restrained slabs with unequal conditions at adjacent panels, the use of the negative moments at supports, using Equations 2, as fixed end moments and the distribution of these moments across the supports according to the relative stiffness of adjacent spans, giving new support moments with the adjustment of mid-span moments.

ACI 1995 [4]

According to this code, all two-way reinforced concrete slab systems are to be analyzed and designed according to one unified method (i.e. direct design method). However, the complexity of the generalized approach particularly for systems that do not meet the requirements permitting analysis by this method has led many engineers to use the ACI 1963 code (moment coefficients by Marcus) for the special case of two-way slabs supported on four sides of each slab panel by relatively deep, stiff edge beams.

Moments in middle strips in the two directions are given as follows [5]:

$$M_x = C_a \; w \; l_x^2 \qquad (3\text{-a})$$
$$M_y = C_b \; w \; l_y^2 \qquad (3\text{-b})$$

Where C_a and C_b are coefficients depending on boundary conditions and l_y / l_x ratio. For positive moments, two values for the coefficients were given; one for dead load and the other for live load. These coefficients are based on elastic analysis but also account for inelastic redistribution.

When the slabs are supported by relatively shallow, flexible beams, the ACI code recommends two alternative approaches: a semi-empirical direct design method or an approximate elastic analysis known as the equivalent frame method.

CASES OF STUDY

To satisfy the aim of this study, seven roof slabs with different shapes and boundary conditions (as shown in Figure 1) were analyzed using the three code methods, i.e. the ECP-95, the BS 8110-85 and the early code, in addition to Grashoff method for load distribution.

Slabs were also analyzed using a linear elastic analysis [6], which is a commercial program for the static and dynamic analysis of structures. Two cases were considered for each slab; i) slabs supported along their edges on undeformable supports (i.e. line-supported), and ii) slabs supported on edge beams having depth equals to five times the slab thickness and supported on edge columns. Shell elements were used for modeling the slabs and frame elements were used for modeling the edge beams.

Also, a non-linear numerical analysis using the finite element method was used to analyze the slabs. The elements used in this analysis were divided across its thickness into a number of layers with the steel reinforcement smeared into the concrete layer. The analysis takes into consideration cracking of concrete in tension and the non-linear stress-strain relationship of concrete in compression. In this analysis, the slabs supported along their edges on undeformable supports (i.e. line supports). Details of the analysis may be found in Reference 7

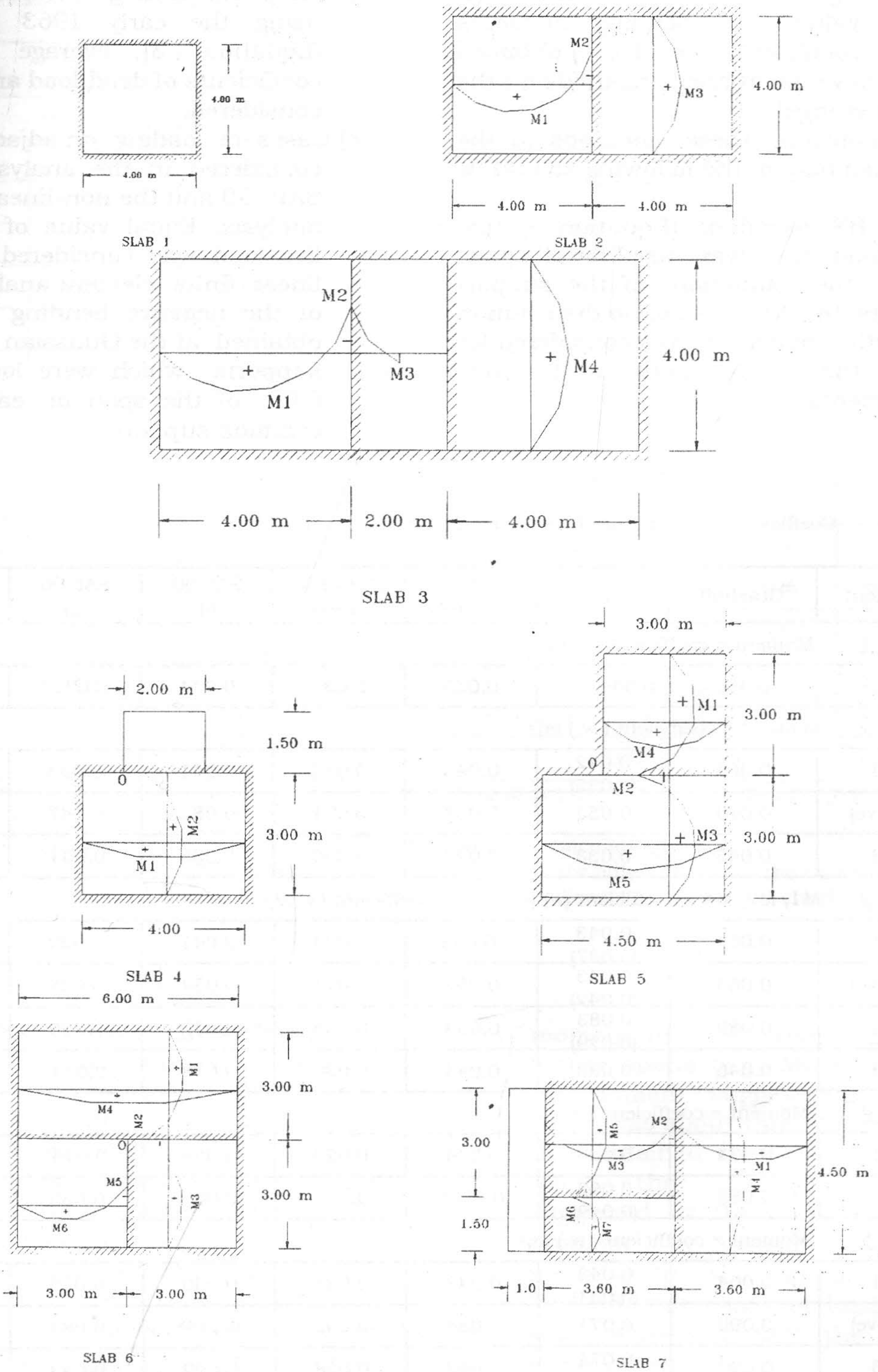

Figure 1 Slabs analyzed in the present study

Table 1 gives the coefficients of the maximum values of bending moments (moment = coefficient * w l_x^2) obtained from the above mentioned methods for the seven slabs studied.

When applying these methods to the slabs studied herein, the following should be noted:

a) For the BS equations (Equation 2), the single load case was used for all spans without the reduction of the support moments by 20 %. Also, no distribution of negative moments was considered for slabs that do not meet code requirements.

b) When calculating the positive moments using the early 1963 ACI equations (Equation 3), average value for the coefficients of dead load and live load was considered.

c) Cases of loading on adjacent slabs were considered in the analysis obtained by SAP 90 and the non-linear finite element analysis. Equal value of dead load and live load was considered. For the non-linear finite element analysis, the values of the negative bending moments were obtained at the Guassian points near the supports which were located at about 0.05 of the span on each side of the common support.

Table 1 Coefficients of maximum bending moments

Moment	Grashoff	ECP-95	BS 8110-85	ACI 1963	SAP 90 (i)	SAP 90 (ii)	non-linear FE
SLAB 1	Moment = coefficient (w) $(4)^2$						
center	0.063	0.044	0.055	0.036	0.044	0.041*	0.042
SLAB 2	Moment = coefficient (w) $(4)^2$						
M1	0.064	0.042 (0.035)	0.043	0.034	0.041	0.034	0.038
M2(-ve)	0.079	0.053	0.057	0.071	0.082	0.047	0.052
M3	0.045	0.033	0.044	0.030	0.035	0.034*	0.033
SLAB 3	M1, M2, M4 = coefficient (w) $(4)^2$, M3 = coefficient (w) $(2)^2$						
M1	0.064	0.043 (0.037)	0.043	0.034	0.041	0.029	0.039
M2(-ve)	0.064	0.043 (0.042)	0.057	0.071	0.054	0.038	0.038
M3	0.083	0.083 (0.029)	0.053	0.053	0.027	0.029	0.026
M4	0.046	0.033	0.044	0.030	0.038	0.031*	0.035
SLAB 4	Moment = coefficient (w) $(3)^2$						
M1	0.034	0.033	0.044	0.025	0.034	0.024*	0.033
M2	0.085	0.062 (0.049)	0.058#	0.052	0.060	0.035	0.058
SLAB 5	Moment = coefficient (w) $(3)^2$						
M1	0.064	0.043 (0.031)	0.043	0.034	0.040	0.025	0.037
M2(-ve)	0.090	0.071	0.084	0.092	0.114$	0.086	0.068
M3	0.090	0.071 (0.062)	0.063	0.058	0.062	0.043	0.059
M4	0.046	0.033	0.044	0.030	0.034	0.026	0.032
M5	0.029	0.033	0.044	0.021	0.031	0.024	0.029

Table 1 Coefficients of maximum bending moments (*Cont'd*)

Moment	Grashoff	ECP-95	BS 8110-85	ACI 1963	SAP 90 (i)	SAP 90 (ii)	non-linear FE
SLAB 6	Moment = coefficient (w) $(3)^2$						
M1	0.100	0.100 (0.091)	0.074	0.066	0.070	0.052	0.060
M2(-ve)	0.100	0.100 (0.084)	0.098	0.097	0.086#	0.081	0.088
M3	0.050	0.035 (0.021)	0.034	0.030	0.033	0.026	0.030
M4	0.000	0.000	0.044	0.016	0.020	0.020	0.028
M5(-ve)	0.063	0.044	0.047	0.050	0.055	0.049	0.045
M6	0.050	0.035 (0.029)	0.036	0.030	0.027	0.025	0.031
SLAB 7	Moment = coefficient (w) L^2, where L = 3.6 m for M1, M2, M4, L = 3.0 m for M3, M5, M6 and L = 1.5 m for M7						
M1	0.081	0.057 (0.054)	0.055	0.048	0.051	0.038	0.050
M2(-ve)	0.081	0.057 (0.051)	0.074	0.086	0.071	0.060	0.060
M3	0.055	0.037 (0.020)	0.027	0.022	0.036	0.026	0.029
M4	0.038	0.033	0.044	0.027	0.031	0.026	0.031
M5	0.055	0.038 (0.030)	0.042	0.035	0.035	0.037	0.031
M6(-ve)	0.056	0.042	0.056	0.074	0.047	0.036	0.032
M7	0.100	0.100 (0.077)	0.067	0.066	0.085	0.049	0.063

* coefficient of negative moment at slab edge = 0.024 for Slab 1, 0.032 for M2 in Slab 2, 0.036 for M4 in Slab 3, 0.053 for M1 in Slab 4

( ) cases of loading with the use of three moments equation for calculating moments

\# According to BS, for cantilevers of a length exceeding 1/3 of the adjacent panels, the condition of minimum load on the cantilever and maximum load on the adjacent panel must be checked

$ maximum negative moment occurs at point O.

RESULTS OF THE COMPARISON

The values of maximum bending moments for the slabs studied herein obtained from the finite element analyses and the design moments obtained from the previously mentioned code equations are compared to those obtained using the simplified method recommended by the Egyptian Code in Table 2 and Figure 2.

The comparison indicates the following :

a) Generally, the values of moments as obtained by the Egyptian Code Equations 1-a and 1-b compared well with those obtained by the non-linear FE analysis in most cases with a maximum difference of 24 %, except for the following :

i- moment in short spans surrounded by longer spans (e.g. M3 in Slab 3), where the moment suggested by the Egyptian Code was about three times higher than that obtained by the analysis. However, using the distribution coefficients suggested by the code and solving the problem by considering the middle strips of the slab as continuous beam for different cases of loading resulted good agreement with the analysis, as given in Table 1.

ii- moment in short span of One-Way slabs (e.g. M1 in Slab 6 with $l_y / l_x = 2.0$ and M7 in Slab 7 with $l_y / l_x = 2.4$) where the value obtained by the non-linear FE analysis was about 0.6 that obtained by the Egyptian Code.

b) The results obtained by the linear program SAP 90 compared well with those obtained by the Egyptian Code except for the three values of moments mentioned above in (a) but the following should be noted :

i- for case (i); i.e. slabs supported on undeformable supports and for continuous slabs, very high values of negative moments occurred at the supports but were limited to a small distance (e.g. Slab 2), after which these values dropped considerably. For this reason, the negative moments obtained by the analysis were much higher than those predicted by the ECP-95 by a difference ranged from 12 to 60 %. The maximum difference was recorded for Slab 5 where the maximum negative moment occurred at junction O, as can be seen in Figure 1. The increase of negative moments at these junctions, for Slab 4, Slab 5 and Slab 6, over those obtained at middle strips was 14, 16 and 43 % respectively. The values of the positive moments ranged from 0.77 to 1.15 of those obtained by the ECP-95.

ii- for case (ii); i.e. slabs supported on beams and at simply supported edges, negative moments occurred and these moments affected the values of span moments (e.g. Slab 2, Slab 3, and Slab 4), therefore the values of span moments were much less than those obtained by the ECP-95 by a difference reached 56 %. However, the values of negative moments at intermediate supports agreed well with the code values with a maximum difference of 20%.

c) The use of Grashoff coefficients for load distribution resulted higher moments than those predicted by ECP-95 by about 15 to 50 % except for Slab 5 with the ratio l_y / l_x was 1.5 (or according to the ECP-95, r = 1.724), where the value of moment M5 obtained by Grashoff method was less than that obtained by the code by 12% and in Slab 4 with the ratio l_y/l_x = 1.33 (or r = 1.53), where the value of moment M1 in the long span obtained by Grashoff method was nearly equal to that obtained by the ECP-95.

d) The use of BS equations (Equations 2-a and 2-b) overestimated the values of maximum positive moments in the direction of simply supported edges by 33 % over the design values obtained by ECP-95. Other values of positive moments agreed well with the Egyptian Code predictions. For negative moments, the BS equations gave higher values by a difference ranged from 2 to 33 %, since no reduction was carried out according to BS code

e) The use of early 1963 ACI code resulted in all slabs positive moments lower than those predicted by the ECP-95 by a difference ranged from 8 to 40 %. However, the values of negative moments were higher than those predicted by the ECP-95 by a difference ranged from 14 to 76 %.

f) For the long span of One-Way slabs, both the ECP-95 and Grashoff method gave zero value for the moment while other methods gave values for moment in that direction ranged from 0.25 to 0.59 the value of moment in the short span. It should be noted that the ECP-95 recommends the use of 20 % of the steel area required for short span in the longer one.

g) The cracking loads for some of the slabs studied herein were obtained using the non-linear FE analysis and are given in Table 3. These loads were obtained assuming slab thickness = 100 mm with reinforcement 1 ϕ 8 mm @ 150 mm in the two direction of each panel. The material properties were: f_{cu} = 25 N/mm^2, f_t = 3 N/mm^2 and mild steel with f_y = 240 N/mm ^{2} .The table indicates that the cracking load for such slabs was always higher than the working load for slabs in ordinary buildings which ranges between 5 to 8 kN/mm^2.

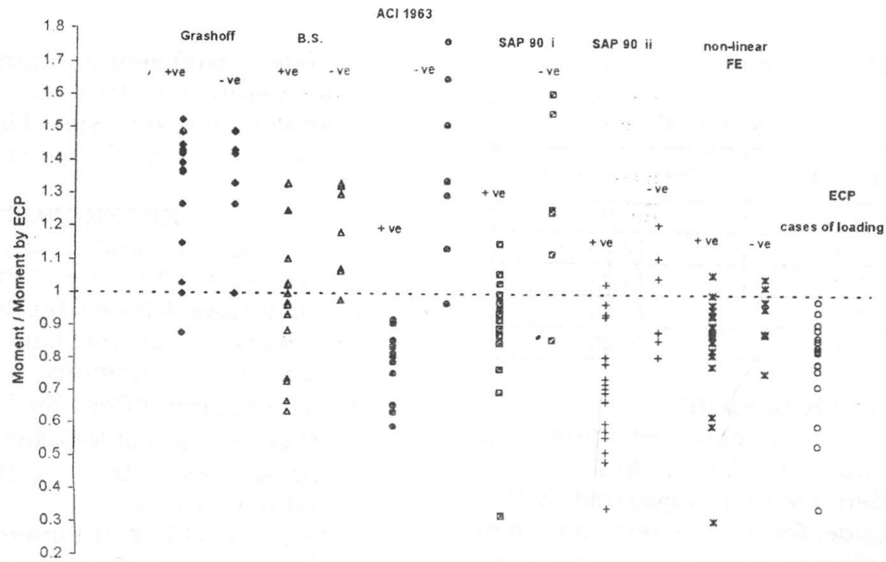

Figure 2 Comparison of different methods with the Egyptian Code

Table 2 Comparison of methods of analysis with the Egyptian code

Slab	Moment	Grashoff / ECP	BS / ECP	ACI / ECP	SAP 90 (i) / ECP	SAP 90 (ii) / ECP	non-lin. FE / ECP
1	M	1.43	1.25	0.82	1.00	0.93	0.96
2	M1	1.52	1.02	0.81	0.98	0.81	0.91
	M2	1.49	1.08	1.34	1.55	0.89	0.98
	M3	1.36	1.33	0.91	1.06	1.03	1.00
3	M1	1.49	1.00	0.79	0.95	0.67	0.91
	M2	1.49	1.33	1.65	1.26	0.88	0.88
	M3	1.00	0.64	0.64	0.33	0.35	0.31
	M4	1.39	1.33	0.91	1.15	0.94	1.06
4	M1	1.03	1.33	0.76	1.03	0.73	1.00
	M2	1.37	0.94	0.84	0.97	0.56	0.94
5	M1	1.49	1.00	0.79	0.93	0.58	0.86
	M2	1.27	1.18	1.30	1.61	1.21	0.96
	M3	1.27	0.89	0.82	0.87	0.61	0.83
	M4	1.39	1.33	0.91	1.03	0.79	0.97
	M5	0.88	1.33	0.64	0.94	0.73	0.88
6	M1	1.00	0.74	0.66	0.70	0.52	0.60
	M2	1.00	0.98	0.97	0.86	0.81	0.88
	M3	1.43	0.97	0.86	0.94	0.74	0.86
	M5	1.43	1.07	1.14	1.25	1.11	1.02
	M6	1.43	1.03	0.86	0.77	0.71	0.89
7	M1	1.42	0.97	0.84	0.90	0.67	0.88
	M2	1.42	1.31	1.51	1.25	1.05	1.05
	M3	1.49	0.73	1.59	0.97	0.70	0.78
	M4	1.15	1.33	0.82	0.94	0.79	0.94
	M5	1.45	1.11	0.92	0.92	0.97	0.82
	M6	1.33	1.33	1.76	1.12	0.86	0.76
	M7	1.00	0.67	0.66	0.85	0.49	0.63

Table 3 Cracking loads for slabs

Slab	Cracking load, kN/mm²	
	top surface	bottom surface
Slab 3	10.00	10.00
Slab 4	6.25	12.50
Slab 5	11.25	11.25
Slab 6	8.75	11.25
Slab 7	8.75	10.00

CONCLUSIONS

From the results obtained from this study, the following may be concluded:

1. The simplified method suggested by the Egyptian Code for the determination of design moments in two-way slabs agreed well with the elastic analysis using the Finite Element method but modeling of the supporting beam in the analysis whether line support or beam element greatly affect the values of negative and positive moments.
2. There is a large variation between the values of the design moments as suggested by the Egyptian Code and those recommended by the British Standards or used in regular designs by the early 1963 ACI code. The difference in the values of moments varies for positive and negative moments.
3. Generally the cracking loads for slabs used in typical buildings are higher than working loads for such slabs, therefore there is no need for designers to use sophisticated methods for the analysis of these slabs and the use of the simplified method recommended by the Egyptian Code is sufficient for obtaining the design moments for two-way slabs. However, large variation in span lengths should be noted in reinforcement arrangement.

REFERENCES

1. J. Javarnicky and C. Amerongen, "Tables for Analysis of Plates, Slabs and Diaphragms Based on the Elastic Theory", Bauverlag GmbH., Wiesbaden und Berlin, Germany, (1971).
2. "The Egyptian Code for Design and Construction of Reinforced Concrete Structures, 1995", 1st revision, 3rd Edition (1997) .
3. "Structural Use of Concrete", BS 8110: Part 2, British Standard Institution, (1985).
4. "Building Code Requirements for Reinforced Concrete", ACI 318-95, American Concrete Institute, Detroit, Michigan (1995).
5. A.H. Nilson and G. Winter, "Design of Concrete Structures", 11th edition, McGraw-Hill, (1991).
6. E.L. Wilson and A. Habibulloh, "SAP 90, A Series of Computer Programs for the Static and Dynamic Analysis of Structures", (1989).
7. M.E. Shukry, "Nonlinear Analysis of RC Slabs with Openings", Alexandria Engineering Journal, Vol. 32, No. 1, pp. C41-49, (1993).

Received January 25, 1999
Accepted May 23, 1999

تحليل البلاطات الخرسانية المسلحة ذات الاتجاهين

محيى الدين صلاح شكرى و أحمد فؤاد حسنين
قسم الهندسة الإنشائية - كلية الهندسة - جامعة الإسكندرية

ملخص البحث

يتناول هذا البحث مقارنة لقيم عزوم الانحناء التصميمية فى البلاطات الخرسانية المسلحة ذات الاتجاهين كما تقترحها ثلاثة من المواصفات العالمية للخرسانة المسلحة وهى ؛ الكود المصرى لأعمال الخرسانة المسلحة ١٩٩٥ و المواصفات الإنجليزية ١٩٨٥ و مواصفات معهد الخرسانة الأمريكي ١٩٦٣ . وذلك بالإضافة الى استخدام طريقة العناصر الدقيقة فى برنامجين على الحاسب الالى أحدهما برنامج خطى تجارى وهو SAP 90 والآخر برنامج لا خطى للباحث الأول تم استخدامه فى بحث سابق .

وقد تمت دراسة سبعة أسقف تتكون من بلاطات خرسانية مسلحة ذات أبعاد مختلفة و ذات ظروف ارتكاز مختلفة.

وقد أوضحت نتائج المقارنة ما يلي :

- الطريقة المبسطة المقترحة بالكود المصرى لتحديد قيم العزوم التصميمية فى البلاطات الخرسانية المسلحة تتفق جيدا مع قيم العزوم المستنتجة باستخدام طريقة العناصر الدقيقة ولكن - فى هده الطريقة - تمثيل ركائز البلاطات من الكمرات (سواء ركائز خطية لا تتشكل أو ركائز مرنة) يؤثر على قيم العزوم الموجبة للبلاطات .
- هناك فروق واضحة بين قيم العزوم المستنتجة باستخدام الكود المصرى والقيم المستنتجة باستخدام معادلات المواصفات الإنجليزية أو مواصفات معهد الخرسانة الأمريكي و تتباين هذه الفروق حسب نوع العزوم ؛ موجبة أو سالبة .
- استخدام البرنامج اللاخطى لتحديد قيم أحمال التشريخ للبلاطات التى تم دراستها أوضح أن هذه الأحمال - بوجه عام - أقل من أحمال التشغيل المستخدمة لتصميم البلاطات فى المبانى العادية - وبالتالي لا توجد حاجة لاستخدام طرق دقيقة عن الطريقة المبسطة المقترحة بالكود المصرى لإيجاد العزوم القصوى فى البلاطات الخرسانية المسلحة ذات الاتجاهين .