A PROGRAM PACKAGE FOR LAMBERT CONFORMAL CONICAL PROJECTION, TRANSVERSE MERCATOR PROJECTION AND DIFFERENCE BETWEEN THEM

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ABSTRACT

Since a long time Transverse Mercator Projection (TMP) combined with rectangular grid system are applied in Egypt. Recently Lambert Conformal Conical Projection (LCCP) combined with rectangular grid system were also applied. The development of computers nowadays raises the need for suitable programs to transform from the geodetic system to any of the two above-mentioned systems and also for the reverse case. Sometimes there are two or more maps of the same area (with the same two meridians and two parallels), each prepared at a different time, using different projection, and containing different information. Therefore, it is often useful to know the accuracy with which positions may be directly plotted from one map onto another without using local grid squares and without making the relatively complicated transformation for the different projection. In this paper, a concrete program package is introduced to carry out the transformation for all cases used in both systems. These programs run on any IBM compatible computer with media of Quick Basic language. Additional programs are also included. Firstly, a program for the calculation of differences between the above two projections i.e. LCCP and TMP. Secondly a program is introduced which is concerned with the calculation of similarity fit to maps for the two projections.

Keywords: Lambert conical projection, Transverse Mercator projection, Transformation, Geodetic and plane coordinate systems, Computer program.

INTRODUCTION

This paper is devoted to introduce a set of computer programs for coordinate transformation between geodetic and plane systems, i.e. between Lambert Conformal Conical Projection (LCCP), Transverse Mercator Projection (TMP) and plane systems.

In each case, the transformation formulae (direct and reverse) are introduced, together with other important parameters; via convergence of meridians and point scale factor. Flow charts for the computer programs are also given. The program itself is available through the authors.

Lambert Conformal Conical Projection and Transverse Mercator Projection are conformal projections, so sometimes it is required to transfer data between two different maps covering the same area (surrounded by the same two lines of latitudes and the same two lines of longitudes) but each is in different projection. It is found here necessary to study the accuracy of data transformation and the elements of that transformation.

Finally, if two maps are based on the same ellipsoid and covers the same geographical area, but one is Lambert conformal conical projection and the other is a Transverse Mercator projection, the maximum error of fit is required to be known. A program for the computation of the differences between the two projections and similarity to fit of maps is included, which is sometimes necessary for application during the design and execution of large engineering projects where transformation of data between maps made by different kinds of projections is required.

LAMBERT CONFORMAL CONICAL PROJECTION

Johann Heinrich Lambert developed this projection in 1772 , the same year in which he invented the transverse Mercator projection. As it is a conformal conical projection, meridians are straight lines meeting in a point outside the map limit; parallels are arcs of concentric circles, and both sets of lines meet at a right angles. Scale is true only along the standard parallel; it is compressed between and expanded beyond them.

In the Lambert conformal conical projection formulae, the radius vector distance from the pole to the point appears. This is a large number, and in descending from the pole crosses 10,000,000 m. value at about 30° latitude. Usually it is inconvenient to have grid quantities changing from positive to negative as the grid zero lines are crossed. To avoid this, fixed numerical quantities are added to all grid values, and these quantities are so large that non-of the resultant false coordinates are negative. This introduces a false origin, to the north and west of the true origin.

For the Southern Hemisphere, Figure 1 should be turned upside down. Everything remains the same except the direction of north and west, which are inverted.

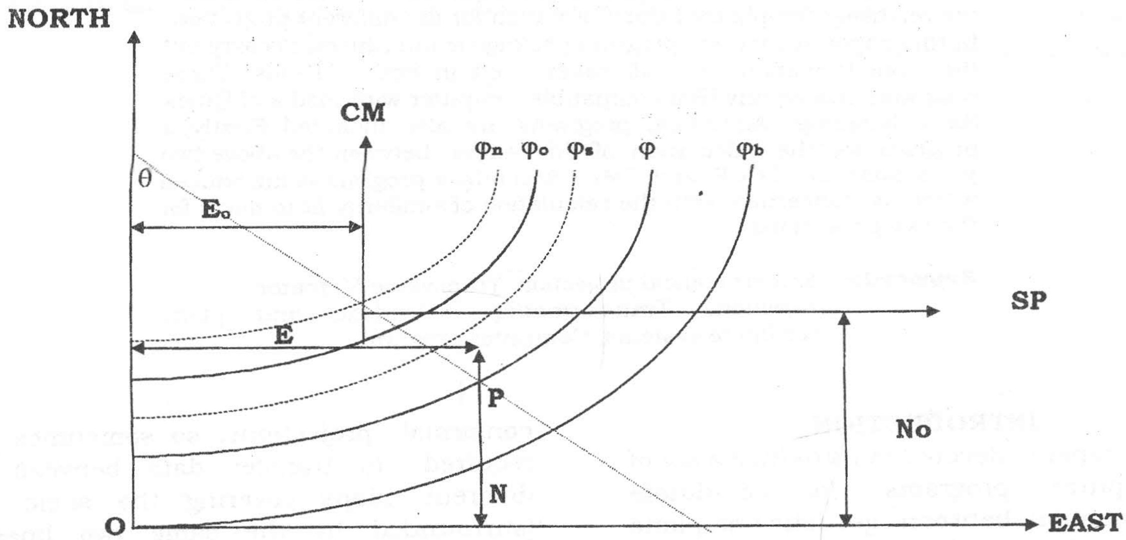

Figure 1 Lambert conical conformal projection

Transformation from LCCP (One Standard Parallel) to Geodetic Coordinates.

$$Q_o = \frac{1}{2}\left[\frac{1+e\sin\varphi_o}{1-e\sin\varphi_o} - e\ln\frac{1+e\sin\varphi_o}{1-e\sin\varphi_o}\right] \quad (1)$$

$$W_o = (1-e^2\sin^2\phi_o)^{1/2} \quad (2)$$

$$K = \frac{a\cos\varphi_o\exp(Q_o\sin\varphi_o)}{W_o\sin\varphi_o} \quad (3)$$

$$N_o = R_b + N_b - R_o \quad (4)$$

$$R_b = k/\exp(Q_o\sin\varphi_o) \quad (5)$$

$$R' = R_b - N + N_b \quad (6)$$

$$E' = E - E_o \quad (7)$$

$$R = (R'^2 + E'^2)^{1/2} \quad (8)$$

$$Q = \ln(K/R)/\sin\varphi_o \quad (9)$$

$$\gamma = \tan^{-1}(E'/R') \quad (10)$$

$$\lambda = \lambda_o - \gamma/\sin\varphi_o \quad (11)$$

$$\sin\varphi = \frac{\exp(2Q) - 1}{\exp(2Q) + 1} \qquad (12)$$

Iterate for $\sin\varphi$ three times, as follows

$$f1 = \frac{1}{2}\left[\ln\frac{1+\sin\varphi}{1-\sin\varphi} - e\ln\frac{1+e\sin\varphi}{1-e\sin\varphi}\right] - Q \qquad (13)$$

$$f2 = \frac{1}{1-\sin^2\varphi} - \frac{e^2}{1-e^2\sin^2\varphi} \qquad (14)$$

Correction to $\sin\varphi = -f_1/f_2$

$$k = \left(1 - e^2\sin^2\varphi\right)^{1/2} R\sin\varphi_o/(a\cos\varphi) \qquad (15)$$

Figure 2 illustrates the flow chart of the program used in transformation between geodetic and LCCP projection. For more details about the derivation of the equations see Reference1.

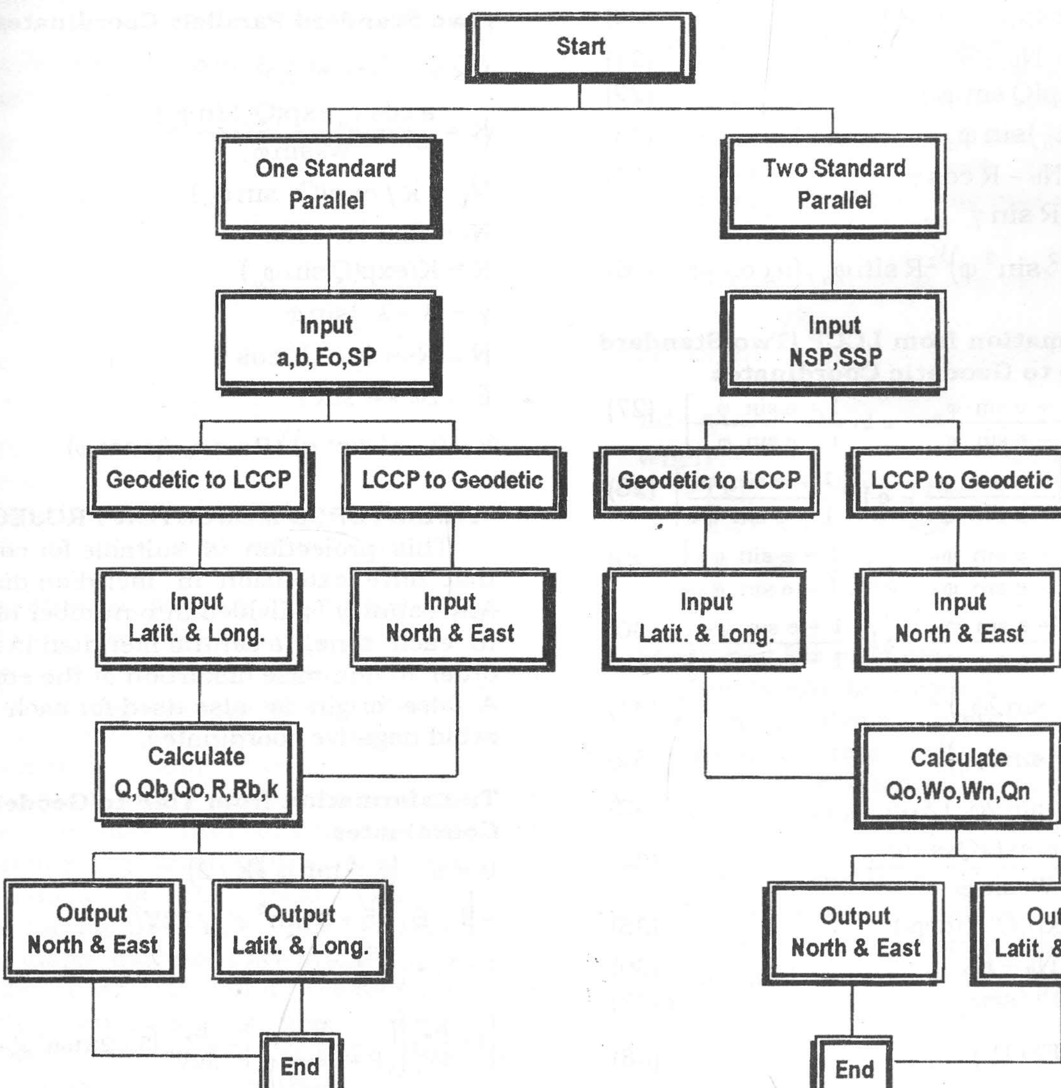

Figure 2 Flow Chart for Transformation Between Geodetic and LCCP

Transformation from Geodetic to LCCP (One Standard Parallel) Coordinates

$$Q_o = \frac{1}{2}\left[\frac{1+e\sin\varphi_o}{1-e\sin\varphi_o} - e\ln\frac{1+e\sin\varphi_o}{1-e\sin\varphi_o}\right] \quad (16)$$

$$Q = \frac{1}{2}\left[\frac{1+e\sin\varphi}{1-e\sin\varphi} - e\ln\frac{1+e\sin\varphi}{1-e\sin\varphi}\right] \quad (17)$$

$$W_o = (1-e^2\sin^2\varphi_o)^{\frac{1}{2}} \quad (18)$$

$$K = \frac{a\cos\varphi_o\exp(Q_o\sin\varphi_o)}{W_o\sin\varphi_o} \quad (19)$$

$$R_b = k/\exp(Q_o\sin\varphi_o) \quad (20)$$

$$N_o = R_b + N_b - R_o \quad (21)$$

$$R = K/\exp(Q\sin\varphi_o) \quad (22)$$

$$\gamma = (\lambda-\lambda_o)\sin\varphi_o \quad (23)$$

$$N = R_b + N_b - R\cos\gamma \quad (24)$$

$$E = E_o + R\sin\gamma \quad (25)$$

$$k = (1-e^2\sin^2\varphi)^{1/2}R\sin\varphi_o/(a\cos\varphi) \quad (26)$$

Transformation from LCCP (Two Standard Parallels) to Geodetic Coordinates

$$Q_n = \frac{1}{2}\left[\frac{1+e\sin\varphi_n}{1-e\sin\varphi_n} - e\ln\frac{1+e\sin\varphi_n}{1-e\sin\varphi_n}\right] \quad (27)$$

$$Q_s = \frac{1}{2}\left[\frac{1+e\sin\varphi_s}{1-e\sin\varphi_s} - e\ln\frac{1+e\sin\varphi_s}{1-e\sin\varphi_s}\right] \quad (28)$$

$$Q_o = \frac{1}{2}\left[\frac{1+e\sin\varphi_o}{1-e\sin\varphi_o} - e\ln\frac{1+e\sin\varphi_o}{1-e\sin\varphi_o}\right] \quad (29)$$

$$Q_b = \frac{1}{2}\left[\frac{1+e\sin\varphi_b}{1-e\sin\varphi_b} - e\ln\frac{1+e\sin\varphi_b}{1-e\sin\varphi_b}\right] \quad (30)$$

$$W_s = (1-e^2\sin 2\varphi_s)^{1/2} \quad (31)$$

$$W_n = (1-e^2\sin 2\varphi_n)^{1/2} \quad (32)$$

$$W_b = (1-e^2\sin 2\varphi_b)^{1/2} \quad (33)$$

$$K = \frac{a\cos\varphi_o\exp(Q_o\sin\varphi_o)}{W_o\sin\varphi_o} \quad (34)$$

$$R_b = k/\exp(Q_o\sin\varphi_o) \quad (35)$$

$$N_o = R_b + N_b - R_o \quad (36)$$

$$Q = \ln(K/R)/\sin\phi_o \quad (37)$$

$$\gamma = \tan^{-1}(E'/R') \quad (38)$$

$$\lambda = \lambda_o - \gamma/\sin\varphi_o \quad (39)$$

$$\sin\varphi = \frac{\exp(2Q)-1}{\exp(2Q)+1} \quad (40)$$

Iterate for $\sin\varphi$ three times, as follows

$$f1 = \frac{1}{2}\left[\ln\frac{1+\sin\varphi}{1-\sin\varphi} - e\ln\frac{1+e\sin\varphi}{1-e\sin\varphi}\right] - Q \quad (31)$$

$$f2 = \frac{1}{1-\sin^2\varphi} - \frac{e^2}{1-e^2\sin^2\varphi} \quad (42)$$

Correction to $\sin\varphi = -f_1/f_2$

$$k = (1-e^2\sin^2\varphi)^{1/2}R\sin\varphi_o/(a\cos\varphi) \quad (43)$$

Transformation from Geodetic to LCCP (Two Standard Parallels Coordinates)

$$Ws = (1-e^2\sin^2\varphi_s)^{\frac{1}{2}} \quad (44)$$

$$K = \frac{a\cos\varphi_o\exp(Q_o\sin\varphi_o)}{W_o\sin\varphi_o} \quad (45)$$

$$R_b = k/\exp(Q_o\sin\varphi_o) \quad (46)$$

$$N_o = R_b + N_b - R_o \quad (47)$$

$$R = K/\exp(Q\sin\varphi_o) \quad (48)$$

$$\gamma = (\lambda-\lambda_o)\sin\varphi_o \quad (49)$$

$$N = R_b + N_b - R\cos\gamma \quad (50)$$

$$E = E_o + R\sin\gamma \quad (51)$$

$$k = (1-e^2\sin^2\varphi)^{1/2}R\sin\varphi_o/(a\cos\varphi) \quad (52)$$

TRANSVERSE MERCATOR PROJECTION

This projection is suitable for countries that have extension in meridian direction. Any country is divided into number of zones. In each zone, a central meridian is used in order to minimize distortion at the edge of it. A false origin is also used for each zone to avoid negative coordinates.

Transformation from TMP to Geodetic Coordinates

$$\varphi = \varphi' - (E_a{}^2\tan\varphi')(K/2) - [1 - E_a{}^2(5+3\tan^2\varphi')]/12V^2 \quad (53)$$

$$\lambda = \lambda_o + \frac{E_a}{V\cos\varphi'} \left\{1 - \frac{E_a{}^2}{6V^2}\left[\left(\frac{V}{P+2\tan^2\varphi'}\right) - \frac{E_a{}^2}{20V^2}(5+28\tan^2\varphi' + 24\tan^4\varphi')\right]\right\} \quad (54)$$

$$K = K_o(1+0.5E_a{}^2Q) \quad (55)$$

$$\gamma = \frac{E_a \tan\varphi'}{V}\left[1 - \frac{E_a^{\,2}}{3V^2}\left(1 + \tan 2\varphi' - \frac{V}{P-1} - 2\left(\frac{V}{P-1}\right)^2\right)\right] \quad (56)$$

Where:

$E_a = E - F_E$

$V = a\,K_o / (1 - e \sin\varphi')^{0.5}$

$P = [(a - b)/a]^{0.5}$

Figure 3 illustrates the flow chart of the program used in transformation between geodetic and TM projection. For more details about the derivation of the equations see Reference 2.

Transformation from Geodetic to TMP Coordinates

$$N = S_\varphi + VJ^2 \tan\varphi + \frac{1}{24} VJ^4 \tan\varphi(5 - \tan^2\varphi) \quad (57)$$

$$E = VJ + \left(\frac{VJ^3}{6}\right)\left[\left(\frac{V}{R_m}\right) - \tan^2\varphi\right] \quad (58)$$

$$+\left(\frac{VJ^5}{120}\right)(5 - 18\tan^2\varphi + \tan^4\varphi)$$

$$\gamma = a_1 J\left[1 + J^2\left(b_1 + c_1 J^2\right)\right] \quad (59)$$

$$K = K_o\left[1 + f_1 J^2\left(1 + g_1 J^2\right)\right] \quad (60)$$

where:

$V = a\,k_o / (1 - e^2 \sin^2\varphi)^{0.5}$

$J = \delta\lambda \cos\varphi \sin 1''$

a_1, b_1, c_1 and g_1 are constants [3].

DIFFERENCES DUE TO PROJECTION

There are often two or more maps of the same area (bounded by the same two meridians and two parallels), each prepared at a different time, using different projections, and containing different information. If the maps have rectangular grids, one may transfer data from one map to another by locating the data relative to the sides of the local Grid Square surrounding the data. This is tedious, and both maps may not have grids. Therefore, it is often useful to know the accuracy with which positions may be directly plotted from one map onto another without using local grid squares and without making the relatively complicated transformation for the different projection.

The maximum discrepancy normally occurs at one or more corners of the quadrangles and varies not only with the map size and the projection but also with the distance between the quadrangle and the standard parallels or central meridians of the projection used.

The allowable discrepancy depends on the purpose of the combined map, e.g. whether the available maps are on paper or on stable-base material, and on the accuracy with which the original map data were obtained or plotted.

Fitting LCCP to TMP Maps

If both quadrangle maps are based on the same ellipsoid and cover the same geographical area, but one is a Lambert conformal conical and the other is transverse Mercator, the maximum error of fit is as follows:

$$\Delta X = RSx \cos\varphi\left[D - \left(\frac{K_{o3}}{2}\right)(S_y + \varphi - \varphi_o)^2\right] \quad (61)$$

$$\Delta Y = RS_y\left[D - \left(\frac{K_{o3}}{2}\right)(\varphi - \varphi_o)(S_y + \varphi - \varphi_o) + CS_x\right] \quad (62)$$

$$\Delta d = \sqrt{(\Delta X)^2 + (\Delta Y)^2} \quad (63)$$

Where:

$C = (k_{o2}/2)(\lambda - \lambda_{o2})\cos^2\varphi$

$D = k_{o2} - K_{o3} + (S_x + \lambda - \lambda_{o2})$

$\varphi_o = (\varphi_1 - \varphi_2)/2$

$k_{o3} = 1 - (\varphi_1 - \varphi_2)^2/8$

Similarity Fit of Maps

The foregoing discrepancies result from direct lying of one quadrangle on the other, involving only rotation and translation. For a better fit, one quadrangle may be enlarged or reduced before fitting, to obtain similarity fit.

For a similarity fit of a Lambert conformal conical projection quadrangle to a Transverse Mercator projection, the former is enlarged (or reduced) by factor K as follows:

$$K = \left\{K_{02}\left[1 + (\lambda - \lambda_{o2})^2(\cos 2\varphi)\right]\right\} \Big/ \left\{K_{o3}\left[1 + (\varphi - \varphi_o)^2/2\right]\right\} \quad (64)$$

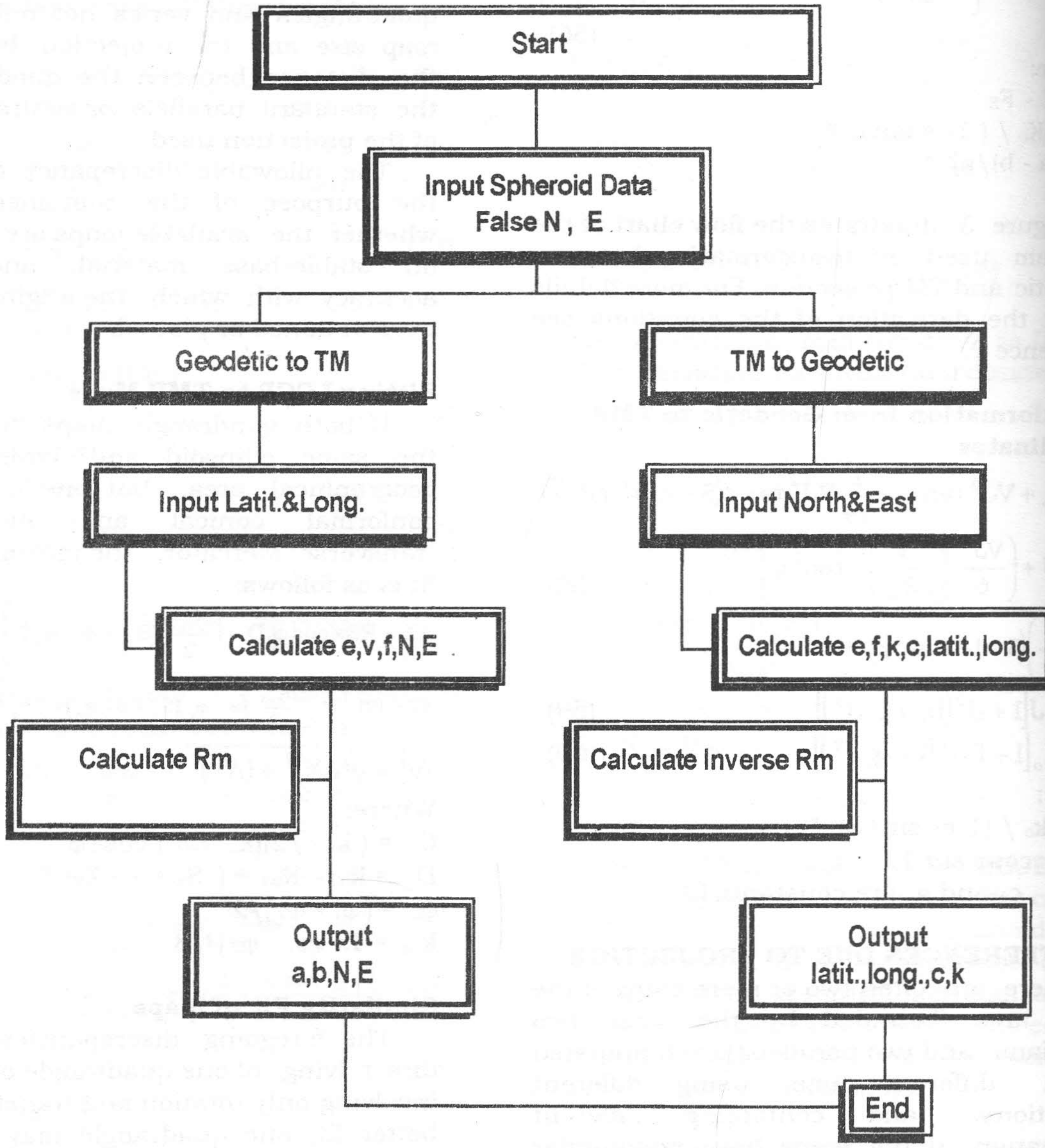

Figure 3 Flow Chart for Transformation Between Geodetic and TMP

APPLICATIONS

A numerical example for each case is given (using the included programs) to verify those programs. Comparing the results of the attached software with those in the mentioned references makes the verification.

Application (1):Transformation from Geodetic to LCCP and vise versa

Geodetic data: [2]

a =6378137.000 m.

b = 6456752.000 m.

False northing of intersection of C.M. with S.P. <0>=0

Northing value of latitude of false origin at C.M.<0>=0

False easting of C.M.<0>=0

Line scale factor<1>=1

Longitude of central meridian = 81 00 00
False origin latitude=24 20 00
λ = 29 00 00.000
φ = 84 30 00.000

LCCP data:
Northing = -7361910.466103235 m.
Easting = 444716.5954193413 m.
Convergence = 00 03 31.204
Scale factor = 0.9981249620546698

Application(2):Transformation from Geodetic to TMP and vise versa
Geodetic data : [2]
Latitude of point = 22 37 15.134
Longitude of point = 32 20 06.689
Longitude of origin = 31 00 00.000
False northing of origin 1810000 m.
False easting of C.M. = 615000 m.
a = 6378200 m. , b = 6356818 m.
C.M. scale factor = 1

TMP data :
Northing of point = 4313194.417 m.
Easting of point = 752276.677 m.
Convergence = -01 29 11.195
Point scale factor = 1.000231402

Application(3): Discrepancies due to projection and Similarity fit
TMP data :
Central meridional longitude = -99 00 00.000 W
Half angular size in longitude = 225 seconds
Half angular size in latitude = 225 secondsCentral meridional scale factor 0.9996

LCCP data :
Northen standard parallel = 26 10 00
Southern standard parallel = 27 50 00
Latitude at center of map = 26 03 45 N
Longitude at center of map =-97 33 45 W
Spheroid data : (4)
a= 6378200 m. ,b= 6356818 m.

Discrepancies due to projection
North _ East = 1.3491740 m.
South _ West = 1.9106817 m.
South _ East = 1.5931605 m.
North _ West = 1.6719077 m.

Similarity fit = 0.9998258

CONCLUSION

In this paper a study of Lambert conformal conical projection, transverse Mercator projection and the difference between them are introduced together with the relevant programs designed to be used by surveyors and engineers who are concerned with planning and executing of large accurate projects, regardless of knowing how to solve the equations included in the programs.

From the above study, one can conclude the following:

1. A complete study highlighting the differences between different versions of Lambert conformal conical projection, i.e. one standard parallel and two standard parallels, was performed. Attention is given to the use of transformation equations and evaluation of various parameters related to the projections.
2. Complete and accurate computer programs were written in a QBASIC language, for the direct and inverse transformation. Also these programs are concerned with the calculation of important parameters such as scale factor and convergence of meridians.
3. Lambert Conformal Conical Projection (LCCP) main disadvantages so far as the surveyor is concerned are the difficulties over the scale factor and arc to chord corrections, and for these reasons it is being superseded by the Transverse Mercator Projection.
4. The largest discrepancy in direct fit is less than 4 m. and in similarity fit is less than 0.3 m. at full scale, or less than 0.2 mm. and 0.013 mm., respectively, at a map of scale 1:25,000

NOMENCLATURE

i. LCCP

a = Major semiaxis of spheroid.
b = Minor semiaxis of spheroid.
E_o = False easting of central meridian.
φ_o = Standard parallel.
φ_b = False origin latitude.
φ = Latitude of a point.
λ = Longitude of a point.

λ_o = Longitude of the central meridian.
N_o = Northing of intersection of central meridian with standard parallel.
e = First eccentricity of the spheroid.
Q_b = Isometric latitude of false origin.
Q_o = isometric latitude of standard parallel.
Q = isometric latitude of a point.
KK = Mapping radius at the equator.
R = Mapping radius.
k_d = point scale factor.
R_b = Mapping radius at false origin latitude.
γ = Convergence.
N = Northing of the point.
E = Easting of the point.
φ_n = Northern standard parallel.
φ_s = Southern standard parallel.

ii.TMP

a = Major semiaxis of spheroid.
b = Minor semiaxis of spheroid.
e = First eccentricity of spheroid.
e' = Second eccentricity of spheroid.
f = flattening.
φ = Latitude of a point.
λ = Longitude of a point.
φ_o = Latitude of central meridian.
λ_o = Longitude of central meridian.
φ' = Latitude of foot point.
K = Point scale factor.
K_o = Central meridian scale factor.
r = radius of rectifying sphere.
V = Radius of curvature in prime vertical.
R_m = Meridional radius of curvature.
S = Meridional distance.
F_E = False easting coordinates.
$S_{\varphi 30}$ = Meridional distance for latitude = 30° North.
γ = Convergence.
R_f = Meridional distance for latitude = f°.
I = Degree in one radian.
N = Northing coordinate of a point.
E = Easting coordinate of a point.
N_o = Northing coordinate of origin.
E_o = Easting coordinate of origin.

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A Program Package for Lambert Conformal Conical Projection, Transverse Mercator Projection and Difference Between Them

مجموعة البرامج الخاصة بكل من إسقاط لامبرت المخروطى التشابهى وإسقاط مركيتور المستعرض والفرق بينها

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ملخص البحث

فى هذا البحث تم دراسة إسقاط مركيتور المستعرض وكذلك إسقاط لامبرت المخروطى التشابهى وشبكة الإسقاط الممثلة لكل مسقط وكذلك الفرق بين النظامين فى الإسقاط حيث تم استخدام هذين النظامين فى إسقاط الخرائط فى مصر.

هذا وقد تم فى هذا البحث إعداد مجموعة من البرامج للاستخدام بأجهزة الكمبيوتر الحديثة وذلك حتى يمكن استخدامها بواسطة المهتمين بالمساحة والمهندسين والقائمين على إعداد المشاريع القومية الكبرى دون الدخول فى تفاصيل ومشاكل حل معادلات التحويل المعقدة وعليه يمكن استخدام هذه البرامج على أى جهاز كمبيوتر متوافق مع نظام **IBM** واستخدام لغة **(Quick Basic)**.

وقد أنتهى البحث الى ضرورة الحيطة فى استخدام معادلات التحويل للأنواع المختلفة لإسقاط لامبرت المخروطى التشابهى ومراعاة اختيار البارامترات المختلفة لكل نوع وكذلك طريقة حسابها المباشرة بواسطة البرامج المعدة فى البحث.