ANALYTICAL COMPUTATION OF A CAPILLARY TUBE DIMENSIONS AS AN EXPANSION DEVICE

N.I.I. Hewedy, Mofreh H. Hamed and Fawzy Sh. Abo-Taleb

Mechanical Power Engineering Department Faculty of Engineering, Menoufia University-EGYPT

ABSTRACT

The primary objective of the current study is to analyze a theoretical model of flow, which simulates homogenous steady two-phase flow of a refrigerant through a capillary tube to predict it's adiabatic performance. The dimensions of the capillary tube for given inlet refrigerant conditions and mass flow rate are calculated to give definite evaporator temperature. Also the choked flow conditions of R-12, R-22 and R-134a through the capillary tube are presented as a function of inlet conditions. The results predict, for a given evaporator temperature, the suitable capillary tube diameter and the mass flow rate of the refrigerant. The model can predict the capillary tube length for various entry parameters. The results of R-134a are compared with those of R-12 and R-22 to provide a base line for judging the performance of R-134a.

Keywords: Two-phase flow, Capillary tube and choked flow.

INTRODUCTION

The capillary tube is the most commonly expansion device used with small refrigerating systems and its application extends up to refrigerating capacities of 10 kW. The capillary tube is a long tube with small diameter, [1. to 6 m long, and 0.5 to 2 mm inner diameter]. The size of the capillary tube, for given inlet conditions of a certain refrigerant, depends mainly on the tube diameter and its length. Thus to provide the required pressure drop between the condenser and evaporator, we have to size the capillary tube properly. The same job may be performed by a short capillary tube of small diameter or a longer tube with a larger diameter.

In one of the first theoretical investigations, Marcy [1] studied capillary tubes of diameter 0.58 mm and lengths 2.028 m and 4.572 m with R-12 and SO₂ as refrigerants. In more recent years, Rizza [2] developed a theoretical flow model for R-22 through capillary tubes of diameters 0.914,

Alexandria Engineering Journal, Vol38, No. 2, A99 – A109, March 1999 ©Faculty of Engineering Alexandria University-Egypt AEJ 1999 1.78 and 2.29 mm and lengths 0.305 and 1.52 m. In 1990, Kuehl and Goldschmidt [3] presented a theoretical model developed in conjunction with an experimental evaluation of adiabatic capillary tube performance with R-22.

Most of the theoretical and experimental studies on flow through capillary tubes are performed on R-12 and R-22 [4-6]. It is necessary to know the behavior of the flow of R-134a as an alternative refrigerant to R-12 in the small refrigerating units. Therefore, the study in this work includes comparisons between the characteristics of flow through capillary tubes using refrigerants R-12, R-22 and R-134a.

SIMULATING MODEL

The conventional approach is to simulate the flow through a capillary tube in the region of two-phase flow, (see Figure 1). The model of calculation given in Reference [5] is based on the governing equations and correlations of the refrigerant properties.

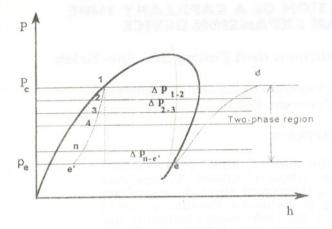


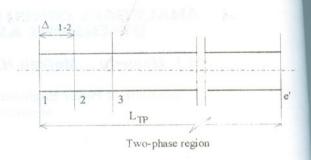
Figure 1 Decremental pressure drops in the capillary tube.

Governing Equations

The equations governing the flow in the capillary tube are the continuity, conservation of momentum and conservation of energy. In modeling the flow, these governing equations have to be satisfied through both the single phase region (saturated liquid) and the two-phase region. The flow model was developed under the following assumptions:

- The flow is one dimensional and steady,
- Homogenous two-phase flow and no
- foreign admixtures at inlet,
- Negligible heat transfer with irreversible adiabatic flow.

In this case, the fundamental equations applicable on a control volume bounded by sections 1 and 2 as in Figure 2, take the form given in the Appendix .





Correlation of the available data

It is clear from the relations given in the Appendix. that. accurate property correlations should be used to arrive to a precise tube length. The friction factor is a function of Reynolds number, which in turn depends largely on the viscosity of the refrigerant. Using mathematical analysis and computer programming, the following equation was developed to calculate the of the vapour and liquid properties refrigerants from the available tabulated saturated values of the properties of each refrigerant in References 7, 8 and 10.

$$\phi_{f,g} = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 \tag{1}$$

where $\phi_{1,g}$, is the property of liquid and vapour, c_0 , c_1 , c_2 , c_3 and c_4 are constants, listed in Table 1, for refrigerants R-12, R-22 and R-134a.

Physical property	Co	C 1	C2	C ₃	C4
P (kPa)	308.663	10.125	0.125671	0.0007197	ben a - ea
$\mathbf{V}_{\mathbf{f}}$ (m ³ /kg)	0.715E-03	1.647E-06	7.306E-09	7.156E-011	-
V_g (m ³ /kg)	0.0552923	-0.0017448	3.473E-05	-4.988E-07	3.488E-09
Hf (kJ/kg)	199.9380	0.924365	0.0010376	0.0	de second
Hg (kJ/kg)	351.4740	0.428860	-0.0006492	-7.051E-06	
μf (Pa.s)	0.2676E-03	-2,471E-06	2.0618E-08	-1.07E-010	-
μ _g (Pa.s)	1.162E-05	4.746E-08	1.075E-010	6.481E-013	-

Table 1 Constants in the correlated relations according to Equation 1.

Analytical Computation of a Capillary Tube Dimensions as an Expansion Device

Physical property	Co	C1	C2	C3	C4
P (kPa)	497.684	16.1897	0.198484	0.0011077	-
v f (m³/kg)	0.778E-03	2.0354E-06	9.8584E-09	1.3318-010	-
$v_g (m^3/kg)$	0.0470568	-0.001495	2.9568E-05	-4.249E-07	3.0099E-09
hr (kJ/kg)	199.9360	1.171390	0.00183959	0.0	-
hg (kJ/kg)	405.3520	0.366790	-0.0015929	-1.463E-05	-
µf (Pa.s)	0.2359E-03	-1.688E-06	1.2706E-08	-7.47E-011	-
μ _g (Pa.s)	1.1952E-05	5.1626E-08	1.941E-010	3.619E-013	-

(b) R-22

Physical property	Co	C1	C ₂	C3	C4
P (kPa)	292.5850	10.6206	0.150691	0.0009019	-
v f (m ³ /kg)	0.7736E-03	1.9439E-06	9.785E-09	7.079E-011	00.00
v _g (m ³ /kg)	0.0693130	-0.0022996	5.0203E-05	-1.188E-06	1.5326E-08
hf (kJ/kg)	49.094400	1.321620	0.0023328	0.0	102013
hg (kJ/kg)	247.79600	0.5852090	-0.0010614	-9.907E-06	
µf (Pa.s)	0.3117E-03	-3.144E-06	2.9939E-08	-2.34E-010	in a sector
μg (Pa.s)	1.2098E-05	4.3660E-08	1.290E-010	6.536E-013	-

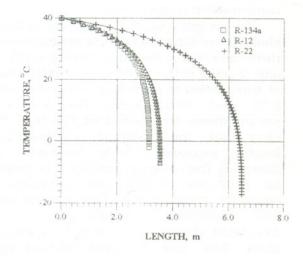
(c) **R-134a**

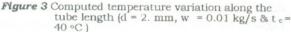
RESULTS AND DISCUSSION

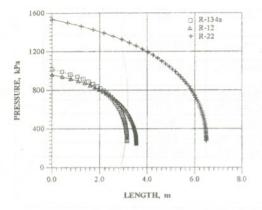
In this study, the computed results are presented for the flow of R-12, R-22 and R-134a, through different capillary tubes of different dimensions at different mass flow rates.

Figure 3 shows the computed variation of the refrigerant temperature along the tube length for the three refrigerants flowing in a tube of diameter, d= 2 mm. Mass flow rate was kept constant at, w = 0.01 kg/s., and condenser temperature at, t = 40 °C. Figure 4 shows the corresponding pressure change along the tube length under the same conditions. The results show that as the refrigerant flows in the tube its temperature and pressure decrease moderately up to near the choking flow condition where the decrement will be more rapidly. Moreover, the choking tube length for the given mass flow rate is longer for R-22 than that for R-12 which has longer length than R-134a. These results have similar trends to that obtained in References 5 and 8.

From these results it can be clearly seen that the variation of temperature and pressure of refrigerants R-12 and R-134a are close to each other, while for R-22 the temperature and pressure values differ largely. Moreover, larger diameter produces lower evaporator temperature and pressure and longer tubes for constant mass flow rate before choking at the same inlet conditions.









The variation of vapour dryness fraction three velocity for the studied and refrigerants for constant mass flow rate (w = same condensation 0.01 kg/s.) and temperature ($t_c = 40$ °C) for two different tube diameters (d = 1.5 and 2 mm.) are shown in Figures 5 to 8. The results show small incremental rate of dryness fraction and velocity near the tube inlet, increase by increasing the tube length and reach the maximum values at the choking condition. Comparisons of Figures 5 and 6 as well as Figures 7 and 8 show that increase of tube diameter increases the dryness fraction, the and the tube length for exit velocity mass flow rate and constant same condensation temperature.

The length of the capillary tube, at choking condition, was computed for flow in three different tube diameters (d = 1.0, 1.5and 2 mm) of the three studied refrigerants at three different mass flow rates (w = 0.005, and 0.015 kg/s). The results of 0.01 computations are plotted in Figure 9. It is clear that, the maximum capillary tube length increases with increasing its diameter. The choking capillary tube lengths for R-12 and R-134a are approximately equal for constant mass flow rate and same inlet conditions. Another plot of the calculated choking tube length against the flow rate for two different tube mass diameters, (d=1.5 and 2 mm) are shown in Figure 10 for the three studied refrigerants. It is clear that, the capillary tube length increases with decrease of mass flow rate.

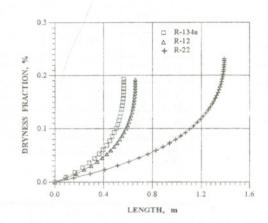
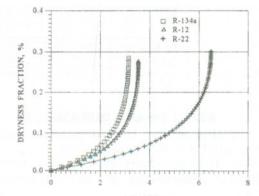
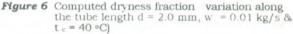
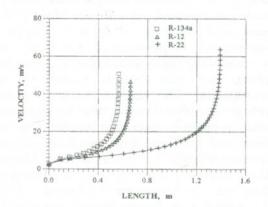


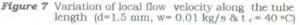
Figure 5 Computed dryness fraction variation along the tube length (d = 1.5 mm, w = 0.01 kg/s & t_c = 40 °C)

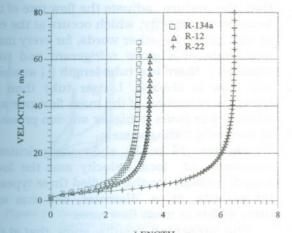




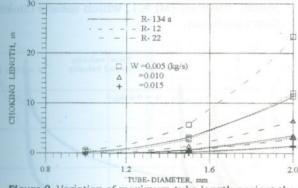


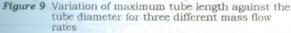


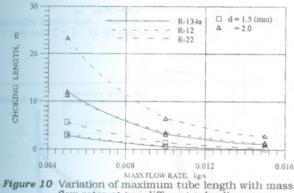












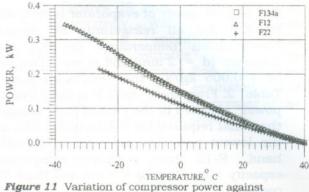
flow rates at two different tube diameters.

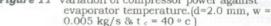
Moreover, the variation of length required to change the diameter from 1.5 mm to 2 mm is larger in smaller mass flow rates than for larger ones. Finally the effect of the type of refrigerant on the refrigerating capacity (R.C), and the power required for compressor, (P) are shown in Figures 11 to

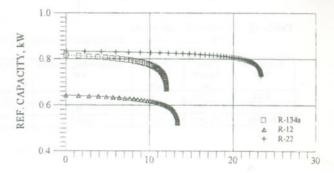
13. The refrigerating capacity and the power required for the compressor are calculated from the following relations:

$$R.C = w(h_e - h_{e'})$$
⁽²⁾

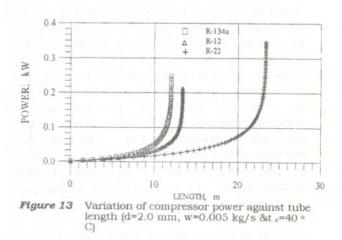
$$P = w(h_d - h_e) \tag{3}$$







LENGTH, m Variation of refrigerating capacity against Figure 12 tube length. (d=2.0 mm, w = 0.005 kg/s & t $c = 40 \circ C$



Here the compression process is assumed to be isentropic.

The coefficient of performance of the refrigeration unit (C.O.P) is given by the relation:

$$C.O.P=R.C/P \tag{4}$$

For a case study of evaporator temperature (-10°C) for all refrigerants at constant condensation temperature (40 °C), tube diameter (d = 2 mm) and at mass flow rate, (w = 0.005 kg/s) the results are given in Table 2. From these data, it is clear that the power required in the case of R-22 is higher than that required for R-12 and R-134a, and the lowest power is for R-12. On the other hand, R-12 has a lower refrigerating capacity than that for R-134a and R-22. The coefficient of performance for the three refrigerants do not differ too much.

Table 2 Results of case study $(t_c = 40^\circ c, t_e = -10^\circ c, d = 2.0 \text{ mm and } w = 0.005 \text{ kg/s.})$

Refrigerant type	Max. Length m	Power	Refrigerating capacity kW	C.O.P
R-12	13.268	0.148	0.546	3.69
R-22	23.061	0.202	0.761	3.77
R-134a	11.961	0.195	0.688	3.53

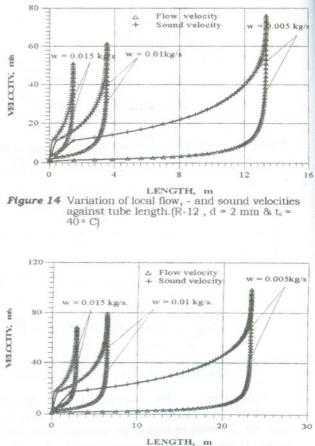
FLOW-CHOKING IN THE CAPILLARY TUBE

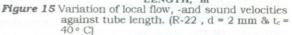
Choked flow in the capillary tube occurs, because of the friction pressure drop, which leads to increase the velocity until it reaches the sonic velocity [5]. The thermodynamic conditions of this flow may be presented by the known Fanno -flow [11]. At the sonic velocity, the Fanno line demands a decrease in entropy for a given mass flow rate, which is violated by the Second law of thermodynamics this for irreversible adiabatic process. The corresponding suction pressure of the refrigeration system, at this condition is the minimum pressure attainable by the capillary tube under such conditions and at which the mass flow rate Further decrease in remains constant. suction pressure for the same tube diameter do not affect the mass flow rate. This means

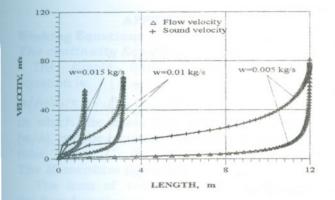
that the nature adjusts the flow rate of the local sonic velocity, which occurs at the exit of the tube. In other words, for every mass flow rate of a given refrigerant and tube diameter, there is a tube length (L) at which the flow is choked. Longer tube than (L) leads to reduction of the mass flow rate and shorter one leads to higher suction pressure in the refrigerating system.

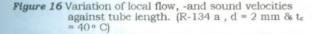
Figures 14 to 16 show the calculated variations of local velocity and the local sound velocity for the studied three types of refrigerants at a diameter, d=2 mm and three different mass flow rates.

The presented results show that at the end of the tube length, the local flow velocity is equal to the local sound velocity, i.e. Mach number (M) = 1, which means choked flow condition.

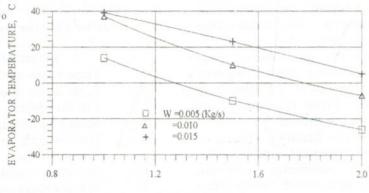






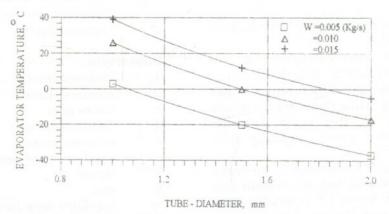


Figures 17 to 19 show the variation of the evaporator temperature against the capillary tube diameter at constant condensation temperature (t $_{\circ}$ = 40 $^{\circ}$ C) for three different mass flow rates (w = 0.005, 0.01 and 0.015 kg/s.) at choking conditions. In Figure 17 the results are plotted for refrigerant R-12. and in Figures 18 and 19 for R-22 and R-With the aid of these 134a respectively. specified for a evaporator curves. temperature, the suitable capillary tube diameter and the mass flow rate of the refrigerant can be evaluated and consequently the suitable compressor of the refrigeration system can be selected.



TUBE - DIAMETER, mm







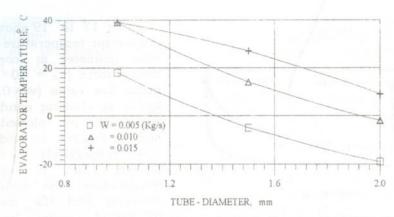


Figure 19 Variation of evaporator temperature against tube diameter for R-134a at $t_c = 40 \circ C$.

CONCLUSION

A theoretical model for predicting capillary tube performance, based on irreversible adiabatic homogeneous two-phase flow assumptions has been studied.

The model predicts the required capillary tube length, diameter and power required for compressor for any inlet conditions to the capillary tube. From this study the following conclusions are drawn:

- 1- For the same inlet conditions to the capillary tube and at constant condensation temperature, the maximum choking capillary tube length and the evaporator temperature for R-22 is greater than that for R-12 and R134a. On the other hand, R-22 has a lower flow velocity through the capillary tube than that for R-12 and R134a.
- 2- The behavior of the flow of R134a is similar to that of R-12.
- 3- For the same inlet conditions and at constant condensation temperature, the maximum choking tube length increases as the capillary tube diameter increases but decreases as the mass flow rate increases at the same capillary tube diameter.
- 4- For the same inlet conditions of the three refrigerants investigated, the compressor power required for R-12 is smaller than that for R-22 and R134a at the same evaporator temperature. On the other hand, the flow of R-22 has a higher refrigerating capacity than that of R-134a and R-12.

5- The evaporator temperature increases as the capillary tube diameter decreases and also increases as the mass flow rate increases for the same inlet conditions.

NOMENCLATURE

- A: Tube cross sectional area, m²
- d : Tube inner diameter, m
- f: Friction factor, -
- h: Specific enthalpy, J/kg
- L: Tube length, m
- p: Pressure, Pa
- Re: Reynolds number, -
- T: Temperature, K
- V: Velocity, m/s
- v: Specific volume, m³/kg
- w: Mass flow rate, kg/s.
- x: Dryness fraction, -
- μ: Dynamic viscosity, Pa.s.
- ρ: Density, kg/m³

Subscripts

- 1: element inlet
- 2: element outlet
- c: condenser
- d: compressor exit
- e: evaporator exit
- e': capillary tube actual exit
- f: liquid
- g: gas
- es capillary tube isentropic exit
- Tp: Two-phase
- m: mean

APPENDIX

Working Equations The continuity Equation

The simple form of the continuity equation is:

 $w/A=V_1/v_1=V_2/V_2$ A-1

in which the mass flow rate per unit area (w/A), will be constant through the capillary tube.

The Momentum Equation

The form of the momentum equation in the flow direction is:

$$w(V_2-V_1) = (p_1-p_2) \tau_{wall} \pi d\Delta L_{1-2}$$
 A-2

Where (p_1-p_2) is the pressure drop across the incremental length ΔL_{1-2} , d is the inside tube diameter and τ_{wall} is the average wall shear stress over ΔL_{1-2} , (see figure-2).

Inserting the coefficient of friction fm in this equation according to Darcy-Weisbach definition [9],

$$f_{m} = 8\tau_{wall} / \rho_{m} V_{m}^{2} = 8(\tau_{wall} / V_{m}) (A/w) \quad A-3$$

Then, $w(V_{2} - V_{1}) = (p_{1} - p_{2})A - \pi w d\Delta L_{1-2} \frac{f_{m}V_{m}}{8A}$
A-4

Here, f_m is taken as a mean value along ΔL_{1-2} and V_m is the average velocity between sections 1 and 2 which are given by the equations:

$$f_{m} = (\frac{f_{1} + f_{2}}{2})$$
 A-5
 $V_{m} = (\frac{V_{1} + V_{2}}{2})$ A-6

In the low Reynolds number range of turbulent flow through straight tubes [9], Blasius equation for the frictional coefficient may be used:

$$f = \frac{0.33}{R_e^{0.25}}$$
 A-7

where : R_e is the Reynolds number which equals (Vd/vµ).

The Energy Equation

The simplified form of the energy equation applied to the control volume in Figure 2 is:

$$10^{3} \mathbf{h}_{1} + V_{1}^{2}/2 = 10^{3} \mathbf{h}_{2} + V_{2}^{2}/2$$
 A-8

Calculation procedure

As the refrigerant flows through the capillary tube, its pressure and temperature decrease as in Figure 1. On the other hand, the fraction of vapour x continuously increases, so for homogenous flow at any point, then:

$$\mathbf{h} = \mathbf{h}_{f} + (\mathbf{h}_{g} - \mathbf{h}_{f})\mathbf{x} \qquad \qquad \mathbf{A} - 9$$

$$\mu = \mu_f + (\mu_g - \mu_f) \mathbf{x}$$
 A-11

In order to calculate the fraction of vapour x, the continuity (Equation A-1) and the energy (Equation A-8) are combined as:

$$10^{3}h_{2} + v_{2}^{2}w^{2}/(2A^{2}) = 10^{3}h_{1} + V_{1}^{2}/2$$
 A-12

Here h is in kJ/kg.

Substituting Equations A-9 and A-10 in Equation A-12, one may obtain:-

$$\frac{10^{3} h_{f2} + 10^{3} (h_{g2} - h_{f2}) x + (v_{f2} + (v_{g2} - v_{f2}) x)^{2}}{w^{2} / (2A^{2}) = 10^{3} h_{1} + V_{1}^{2} / 2} A-13$$

The resultant equation can be put in the quadratic form: $ax^2 + bx + c = 0$, where a, b and c are constants.

All parameters in Equation A-13 are known except x which can be solved from the following quadratic equation:

$$x = \left(-b \pm \sqrt{b^2 - 4ac}\right)/2a \qquad A-14$$

where

$$\mathbf{a} = (\mathbf{v}_{g2} - \mathbf{v}_{f2})^2 (\mathbf{w}/\mathbf{A})^2 / 2 \mathbf{b} = 10^3 (\mathbf{h}_{g2} - \mathbf{h}_{f2}) + \mathbf{v}_{f2} (\mathbf{v}_{g2} - \mathbf{v}_{f2}) (\mathbf{w}/\mathbf{A})^2 \mathbf{c} = 10^3 (\mathbf{h}_{f2} - \mathbf{h}_{1}) + (\mathbf{w}/\mathbf{A})^2 \mathbf{v}_{f2}^2 / 2 - \mathbf{V}_{1}^2 / 2$$

For given mass flow rate, w, and the conditions at section 1:

Selecting the temperature t_2 , the parameters p_2 , h_{f2} , h_{g2} , v_{f2} and v_{g2} which are function of t_2 can be known for the refrigerant [7-8]. With the value of x obtained from Equation A-14, the conditions at section 2 can be computed and consequently the value of Reynolds number. Moreover, the mean friction factor along the length Δ L₁₋₂ can be computed from Equation A-5. Finally, substituting Equation A-6 into Equation A-4, the incremental length Δ L₁₋₂ can be computed. The procedure is repeated to compute the next incremental length by

selecting another temperature and starting from the conditions obtained in the last step.

The Second law of thermodynamics in the case of irreversible adiabatic flow provides a constraint on the flow solution. This means that the entropy change in the direction of the flow can never decrease. As the calculated solution proceeds down the capillary tube, the entropy change across each incremental volume is computed and checked to ensure that the change is positive.

The flow solution proceeds downstream until the accumulated capillary tube length equals to the choked length (choked flow conditions). This occurs when the velocity V reaches the sonic velocity (a), which is defined as [11]:

A-15

$$a = \sqrt{(dp/d\rho)}$$

A flow chart of the procedure is given in Figure A-1.

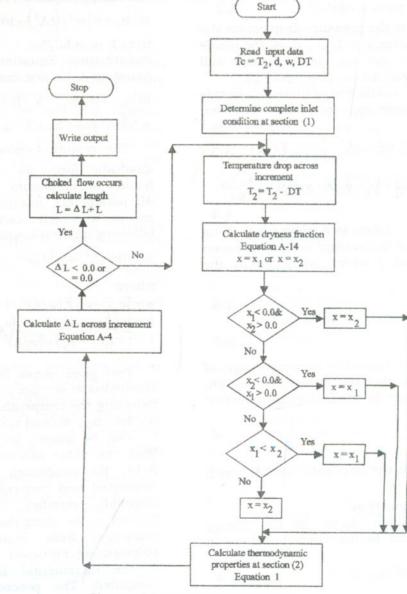


Figure A-1 Computer model flow chart.

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التحليل الرماضي لأمعاد أنبوبة شعربة كوسيلة تمدد

نبيل إبر اهيم هويدى ، مفرح حمادة حامد و فوزى شعبان أبو طالب قسم هـندسة القوى الميكانيكية - جامعة المنوفية

ملخص البحث:

الغرض من هذا البحث هو تحليل موديل رياضى نظرى للسريان الأديباتيكى خلال أنبوبة شعرية. لذا تم حل موديل رياضى لسريان ثنائى الطور متجانس ومستقر لمائع تشغيل خلال أنبوبة شعرية لبيان أداء هذا الأنبوب. تم دراسة أبعداد مختلفة للأنبوبة الشعرية عند ظروف ثابتة لمائع التشغيل عند المدخل وتصرف ثابت لتوفير درجة حرارة محددة للمبخر. أيضا تم دراس ظاهرة خنق السريان لموائع تشغيل مختلفة (R-12 , R-22 , R-12) خلال الأنبوب عند ظروف مختلفة من الدراسة تبين أن للحصول على درجة حرارة محددة للمبخر، يمكن تحديد قطر الأنبوب المناسب، معدل التصرف المناسب، قدرة الضاعط المناسبة لوحدة التبريد وذلك لظروف محتلفة عند المدخل. وقد تم عقد مقارنة لأنواع تقليدية من موائع التشغيل (R-124) لتعطي قاعدة عند اختيار والحكم على أداء مائع التشغيل (R-134)