PROPOSED TECHNIQUE IN GEODETIC POSITIONING

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ABSTRACT

This paper introduces a proposed method for the computation and the estimation of accuracy of surveying control points. The proposed method is a combination of intersection and resection. Steps of adjustment are given for the cases where an unknown position has been observed from any known positions by means of any two observations. This paper introduces a proposed method for observation and calculation of third order or town triangulation network. Such a method aims at overcoming the problem of time and cost for such operations.

Keywords: The fixation of stations, The position of points, Intersection, Resection, Adjustment.

INTRODUCTION

The fixation of stations is an essential operation in the field of surveying. To determine the position of points linear or angular observations are needed and these observations or measurements should be adjusted to obtain the Most probable values (M.P.V) of the coordinates of stations. Traditionally due to practical region and time limitations beside the invention of EDM and progressing in their techniques most of new stations in processes of condensations and extension should be performed according to the proposed technique. In this paper the development length data with angular observations, in determining position of control points. This paper offers also some useful formulae for evaluation of accuracy of fix which are considered to be essential operations in surveying.

CONCEPT OF THE PROPOSED METHOD

If two observations are made of an angle or line or bearing (azimuth) all with equal precision or unequal precision, besides the fixed stations, can be obtained. Consider, as an example, the following cases observations were taken in Figure 1.

In Figure 1, F.S.... is the fixed station, P is the new station (unknown) α is the observed bearing (azimuth), β_i is the observed angles and S is the observed length.

From the two observed quantities, as shown in Figure 1, the coordinate of the new station can be obtained Namely $\Sigma\ V^TPV = \min$ the fundamental principle of the method of least squares, can be applied.

MATHEMATICAL TREATMENT

Observation equations for the adjustment

Geodetic network adjustment using the observation equation method usually starts with the observation equation [1].

$$A\,x + L = V \qquad (1)$$

where A is a given mxn matrix of coefficients or design matrix, X the nx1 vector of unknown parameters, L the mx1 vector of observations, V the mx1 vector of residuals and m>n.

The residuals are defined by:

$$V = AN^{-1}A^T PL + L \qquad (2)$$

where,

$$N^{-1} = (A^T PA)^{-1}$$

where P is the weight matrix, N is the coefficient of the normal equation matrix.

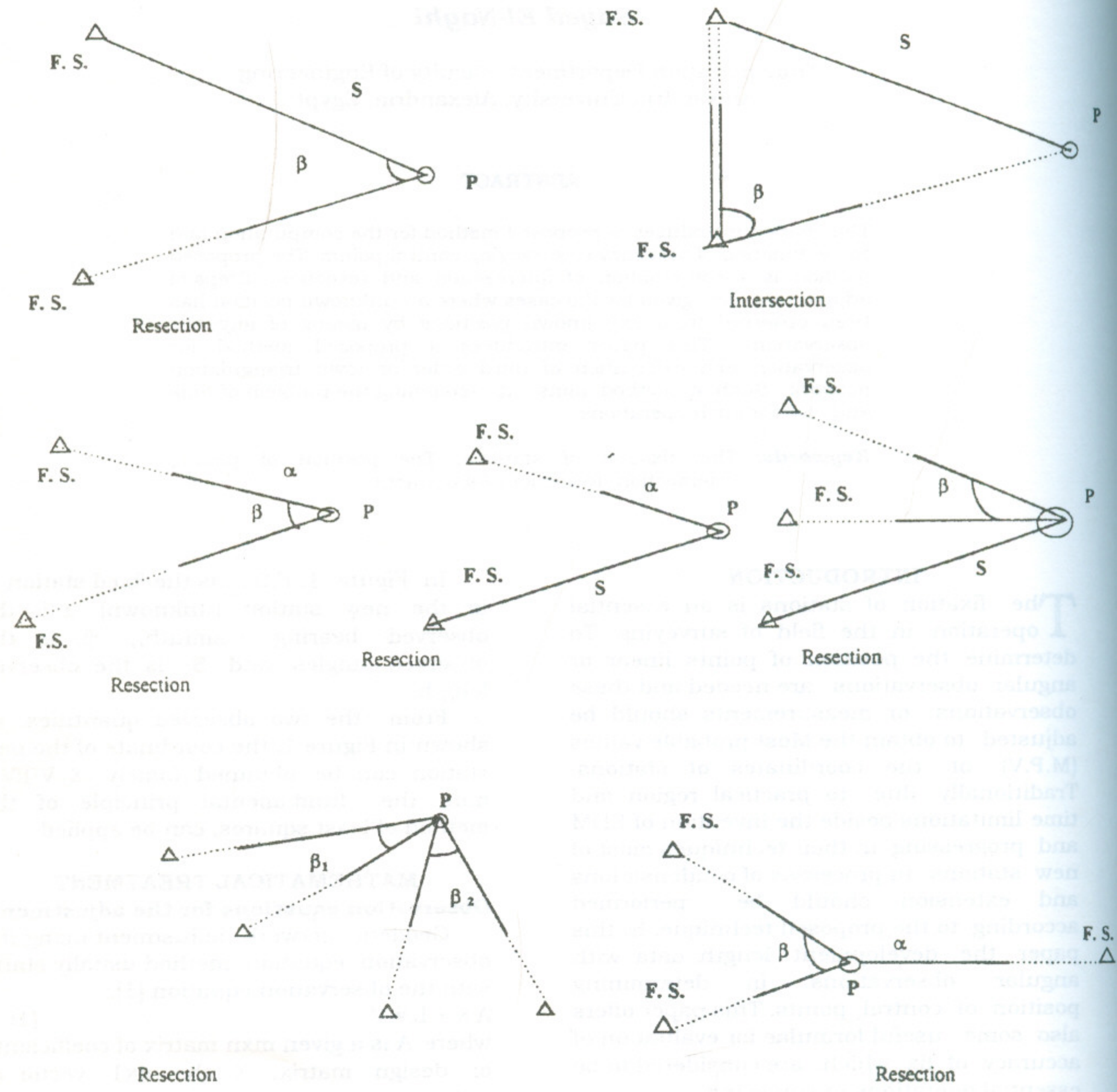

Figure 1 Different configurations for fixing a new station in figure: α....Observed bearing S....Observed length β_i.... Observed angles

If the number of unknowns ($\delta x, \delta y$ for new station) equal the number of observations (two observation), case under study, this situation is expressed formally as [2,3].

$$N^{-1} = A^{-1} P^{-1} (A^{-1})^T \quad (3)$$

By substituting value N^{-1} in Equation 3 into Equation 2, we get:

$$V = - AA^{-1} P^{-1} (A^{-1}. A^T PL + L = 0 \quad (4)$$

Substituting in Equation 1 from Equation 4, we get

$$A x + L = 0 \quad (5)$$

or

$$Ax = -L \quad (6)$$

or in the general form

$$X = - A^{-1} L \quad (7)$$

Establishment of method

Mathematical formulae will be derived to establish the relation connecting unknown parameters values in various cases of observations. Therefore Equation 1 can be set up as:

$$X = \begin{vmatrix} \delta x \\ \delta y \end{vmatrix}; A = \begin{vmatrix} a_1 b_1 \\ a_2 b_2 \end{vmatrix}; L = \begin{vmatrix} \ell_1 \\ \ell_2 \end{vmatrix} \quad (8)$$

and A^{-1} can be computed from;

$$A^{-1} = \frac{1}{D} \begin{vmatrix} b_2 - b_1 \\ -a_2 a_1 \end{vmatrix}, \quad (9)$$

where D is the determinant of A, given by

$$D = |A| = (a_1 b_2 - a_2 b_1) \quad (10)$$

From what followed, unknown parameters values can be obtained. Variation of coordinates for the new station (P) δx_p and δy_p using the following from:

$$\delta x_p = \frac{1}{D} (b_1 \ell_2 - b_2 \ell_1)$$

$$\delta y_p = \frac{1}{D} (a_2 \ell_1 - a_1 \ell_2) \quad (11)$$

where the coefficients a_i, b_i depend on the configuration of the network, whereas ℓ_i are dependent on the values of observations.

Formation of the Coefficients

It should be noticed that, as shown in Figure 1 several cases of observations can be taken:

General case

Figure 1 shows all the possible configurations for fixing a station using two observations where in all cases "P" represents a new station and T represents a fixed station. The distance S and the bearing α are observed.

From the above data the required coefficients (a,b and ℓ) can be computed from the formula

$$a = - \cos \alpha_{P-T}; \quad b = - \sin \alpha_{P-T}; \quad \ell = S' - S_{P-T}; \quad (12)$$

where

$$Sl = \sqrt{(X_T - X_{p^o})^2 + (Y_T - Y_{p^o})^2}; \quad (13)$$

In the above form:

X_T, Y_T are the coordinates of fixed station T; and

X_{P^o}, Y_{P^o} are the provisional coordinates of the new station P.

And similarly for the observed bearing the required coefficients a,b and ℓ can be obtained as follows

$$a = - \rho \frac{\sin \alpha_{P-T}}{S_{P-T}},$$

$$b = - \rho \frac{\cos \alpha_{P-T}}{S_{P-T}}; \quad (14)$$

$$\ell = \alpha' - \alpha_{P-T};$$

where

$$\alpha' = \text{arc tg} \frac{X_T - X_{p^o}}{y_T - Y_{p^o}}. \quad (15)$$

Sample case

In this case, as shown in Figure 2, the observations are angles (β_1 or β_2 or β_3)

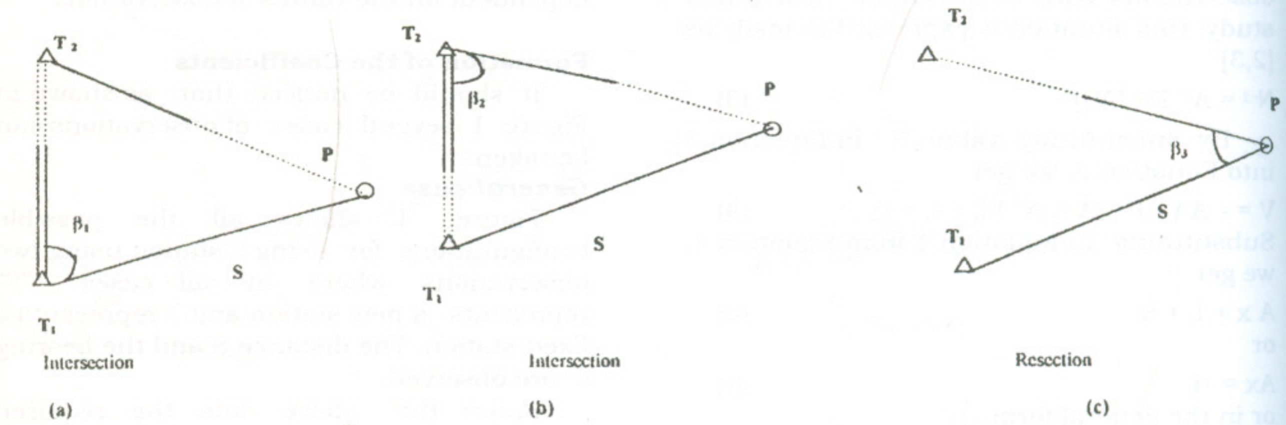

Figure 2 Sample case care of the three tested cases. In Figure: T_1T_2 are fixed statucs; P is the station; s is the observed length; β_1 are the observed angles

a) For observed angle β_1

The coefficients are

$$a = \rho\frac{\sin\alpha_{P-T1}}{S_{P-T1}},$$

$$b = -\rho\frac{\cos\alpha_{P-T1}}{S_{P-T2}}; \quad (16)$$

$\ell = \beta'_1 - \beta_1;$

where

$$\beta'_1 = \alpha^{\copyright}_{T1-P} - \alpha_{T1-T2} \quad (17)$$

b) For observed angle β_2

The required coefficients are:

$$a = -\rho\frac{\sin\alpha_{P-T2}}{S_{P-T2}},$$

$$b = \rho\frac{\cos\alpha_{P-T2}}{S_{P-T2}}; \quad (18)$$

$\ell = \beta_2' - \beta_2$

where

$$\beta'_2 = \alpha_{T2-T1} - \alpha'_{T2-P} \quad (19)$$

c) For observed angle β_3

The required coefficients are:

$$a = \rho\left(\frac{\sin\alpha_{P-T2}}{S^{\copyright}_{P-T2}} - \frac{\sin\alpha_{P-T1}}{S_{P-T1}}\right),$$

$$b = -\rho\left(\frac{\cos\alpha_{P-T2}}{S^{\copyright}_{P-T2}} - \frac{\cos\alpha_{P-T1}}{S_{P-T1}}\right); \quad (20)$$

$\ell = \beta'_3 - \beta_3$

where

$$\beta'_3 = \alpha'_{P-T2} - \varepsilon_{T1-T2} \quad (21)$$

and

$$S'_{p-T2} = \sqrt{(X_{T2} - X^{o}_{p})^2 + (Y_{T2} - Y^{o}_{P})^2} \quad (22)$$

In all the above cases computation of the bearing α' and distance S' are based on coordinates of fixed stations (T1, T2) and the provisional coordinates of the new station (P).

To calculate the provisional coordinates of a new station P two methods can be performed. The first one can be derived by scaling from cadastral maps of large scale (1:2000, 1:5000, 1: 1000). The second method can be obtained using the following formula:

$$X_p{}^{o} = X_T + S\sin\alpha \quad (23)$$
$$Y_p{}^{o} = Y_T + S\cos\alpha$$

PRECISION AND ACCURACY

For evaluating the accuracy of adjusted value for the unknown station P the Mean square Error (M.S.E) "σ" for corrected coordinates must be obtained this values. The required (M.S.E) , δ_x, δy, for coordinates of a new station P will be related to M.S.E , δ_1 δ_2 , for observations values.

To find the δ_x and δ_y the following form can be used:

$$\left.\begin{aligned} \delta_x^2 &= \frac{1}{D^2}(b_1^2\ \delta_2^2 + b_2^2\ \delta_1) \\ \delta_y^2 &= \frac{1}{D^2}(a_1^2\ \delta_2^2 + a_2^2\ \delta_1^2) \end{aligned}\right\} \qquad (24)$$

For better accuracy the position of the new station can be obtained using the variances covariance as follows:

$$tg2\theta = \frac{2Q_{12}}{Q_{11} - Q_{22}},$$

$$w = \sqrt{(Q_{11} - Q_{22})^2 + 4Q_{12}^2}\ ;$$

$$U^2 = \frac{1}{2D^2}\ (Q_{11} + Q_{22} + W); \qquad (25)$$

$$V^2 = \frac{1}{2D^2}\ (Q_{11} + Q_{22} - W);$$

where θ is the azimuth the semi-major axis at the x-direction. U and V are the semi-major and semi-minor axis of the ellipse error respectively; and where Q_{11}, Q_{22}, Q_{12} are weight coefficients given as [4, 5]:

$$\begin{aligned} Q_{11} &= b_1^2\ \delta_2^2 + b_2^2\ \delta_1^2; \\ Q_{22} &= a_1^2\ \delta_2^2 + a_2^2\ \delta_1^2; \qquad (26) \\ Q_{12} &= -\ (a_1\ b_1\delta_2^2 + a_2\ b_2\delta_1^2) \end{aligned}$$

CONCLUSIONS

This paper offers an important mean of dealing with positioning of new stations. In this study it can be easily seen that the computed values have enough accuracy of third order or town triangulation networks. In addition, the proposed method is quite simple as every station could be calculated independently, while in other techniques the network adjustment should be computed as one blocked unit which complicate the solution. The simplicity of this system arises from the fact that each station can be handled separately. More over the required error ellipse parameters can be evaluated from the variance and covariances value.

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Received August 26, 1998
Accepted December 31, 1998

أسلوب مقترح للمواقع الجيوديسية

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ملخص البحث

تحديد النقط والمواقع الجيوديسية هى الهدف الاساسى فى الاعمال المساحية - ولذلك توجد طرق كثيرة عن كيفية تحديد هذه النقط والمواقع . وتختلف الطرق تبعا لسباب كثيرة منها الاجهزة المستخدمة وكذلك طبيعة سطح الارض.

وهذا البحث هو عبارة عن تقديم طريقة وأسلوب جديد وتقييم النقط الجيوديسية وذلك بادماج طريقى الامامى والتقاطع العكسى وهما من الطرق الشائعة الاستعمال فى المجال المساحى .

وفى البحث دراسة نظرية وضبط للطريق المقترحة الجديدة وكذلك تقييم دقة النقط الجديدة باستخدام رصدتين فقط اما من النقطة الجديدة أو من النقطة الثابتة تبعا للظروف المتاحة للارصاد.