POTENTIAL OF USING SPLINE CURVE FEATURES IN THE PHOTOGRAMMETRIC PROCESSES

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ABSTRACT

Linear features (straight line, circle, curve and others) are considered as a new approach to extract information instead of points; which are abundant and accessible in the image space; in order to treat and resolve photogrammetric problems. In this paper, emphasis on spline curve feature is investigated. This investigation includes representation of the spline curve feature in the 3D object space, the mathematical model, and the relation between the object space and the projected straight line feature on the image space. Extensive experiments using several sets of synthetic data were carried out. Results obtained from these experiments emphasize the potential of using spline curve features as control features in photogrammetric processes.

Keywords: Photogrammetry, Linear Features, Spline, Curve Feature, and Control Curve

INTRODUCTION

During the last decade, the use of linear features plays an important role in some of the photogrammetric processes. Mulawa and Mikhail [4-6] are considered as pioneers in this field. According to Boor [1] and Greville [7], there are several mathematical representations to the spline curve.

By definition, the word spline, is a mechanical device consisting of a strip or rod of some flexible materials and used as draftsman's tool to draw jointless smooth curves through fixed points. The interpolating spline is a mathematical tool suited for the same purpose. Continuous piecewise polynomials are used for interpolation of data to produce smooth curves that exactly interpolate between the given data set. The piecewise nature of the spline permits the flexibility to model a broad class of functions. A cubic spline is a spline constructed of piecewise third-order polynomials, which passes through a set of control points (nodes).

Given a data set of *(m)* nodes (control points) $[(X_1, Y_1), (X_2, Y_2), (X_3, Y_3) \ldots X_m, Y_m)]$ within the range between the starting point *(a)* and the end point *(b)*, there are *(m-1)* intervals between them, these intervals are known as spans. In the cubic polynomial, each span has four coefficients to be determined.

$$S_Y(X_i)=a_{0j}+a_{1j}(X_i)+a_{2j}(X_i)^2+a_{3j}(X_i)^3 \qquad (1)$$

Where,

$1 \le j \le m-1$ and $1 < i \le m.$

These give *(4m-4)* parameters that are needed to describe the spline. From the continuity inherent in the nature of the spline and also from the first and second derivatives at the *(m-2)* interior nodes, there are *3 (m-2)* equations for the spline *(S)*. The requirement that the spline *(S)* passes through the nodes $[(X_1, Y_1), (X_2, Y_2), (X_3, Y_3) \ldots\ldots X_m, Y_m)]$ gives another *(m)* equations. Hence, the total number of equations is *(4m-6)*. Thus, two conditions are needed for the solution of the spline *(S)*. In this case, the natural spline condition $S_Y^{//}(X_1) = 0$ and the $S_Y^{//}(X_m) = 0$ are used (curvature at the stating and end points equal zero).

The above analysis is considered only as interpolation spline. If the control points (nodes) contain random errors, then the computed spline *(S)* could result in a poor estimate. Therefore, it is necessary to reformulate the condition equations to allow a redundant data set to be used with the least squares adjustment in order to reduce the effect of random errors.

MATHEMATICAL MODELS

Curve *(C)* in space may be represented with respect to a 3D Cartesian coordinate system as follows:

$$y = f(x) \qquad z = g(x) \tag{2}$$

Where, $y = f(x)$ is the projection of *(C)* into the xy-plane, and $Z = g(x)$ is the projection of *(C)* into the *xz*-plane. In this research, a new development of the description of a spline curve feature*(S)* is presented. A spline S: $\{S_1, S_2, S_3, \ldots\ldots\ S_{m-1}\}$ consists of two-dimensional cubic polynomials $S_Y(X_i)$ and $S_Z(X_i)$ from the three dimensional data (X_i, Y_i, Z_i) and may be represented as follows:

$$S_j = \begin{bmatrix} (X_i) \\ S_Y(X_i) \\ S_Z(X_i) \end{bmatrix} = \begin{bmatrix} (X_i) \\ a_{0j}+a_{1j}(X_i)+a_{2j}(X_i)^2+a_{3j}(X_i)^3 \\ b_{0j}+b_{1j}(X_i)+b_{2J}(X_i)^2+b_{3J}(X_i)^3 \end{bmatrix} \tag{3}$$

According to the previous analysis in the 2D interpolation spline, there are no redundancies available in the mathematical model to reduce the effect of random errors. Hence, some of the control points may be considered as nodes and the others as support points for the interpolation spline. This method is called the form fitting spline. Let a data set of *(n)* control points be available, a subset of *(m)* control points are selected from this data set as nodes. A spline *(S)* that consists of *(m-1)* spans with *(m)* nodes, in which two of them are considered as external nodes and *(m-2)* as interior nodes is considered to fit this data set. The exterior nodes will be called as end points because of their unique positions on the spline *(S)*. From its characteristics, the spline *(S)* must be joined continuously and also have a continuous first and second derivatives at the interior nodes, this requires:

$$S:\begin{Bmatrix} S_J(X_{J+1}) = S_{J+1}(X_{J+1}) \\ S'_J(X_{J+1}) = S'_{J+1}(X_{J+1}) \\ S''_J(X_{J+1}) = S''_{J+1}(X_{J+1}) \end{Bmatrix} \tag{4}$$

For all $1 \le j \le m\text{-}2$.

Also, the natural spline conditions are applied at the end points

$$S:\begin{cases} S''_1 = 0 \\ S''_m = 0 \end{cases} \tag{5}$$

The constraints given by (4) and (5) will be called the boundary constraints of the spline *(S)*. A spline S: $\{S_1, S_2, S_3, \ldots\ldots\ S_{m-1}\}$ subject to the boundary constraints is called *a Standard Form Spline.*

Form Fitting for the Spline Feature (S)

Control points may be divided into two types: support points *(n)* and nodes *(m)*. The support points are used in the form fitting between the nodes, while the nodes are used to satisfy the spline interpolation. Figure 1 depicts the form fitting of the ground spline curve It will be assumed that the nodes selection done by a person performing the adjustment or by an automated process. Given *(n)* support points, only the starting and end support points must be considered as nodes. It is important to determine the independent parameters in standard form spline *(S)* that consists of *(m-1)* spans. Each span or cubic polynomial (S_j) contributes *(8)* parameters $[a_{0J}, a_{1J}, a_{2J}, a_{3J}, b_{0J}, b_{1J}, b_{2J}, b_{3J}]$. A standard form spline *(S)* with *(m-1)* spans may contribute *8(m-1)* parameters and since there are *(m-1)* spans, this implies that there are *8(m-1)* variables in the standard form spline *(S)*. From Equation 4, there are *(6)* constraints per interior nodes, yielding *6(m-2)* constraints. And from Equation 5, there are *(4)* constraints on the end points. Therefore, a set of *(6m-8)* constraints may be written in case of a standard form spline *(S)*. Considering the total number of unknown parameters and the total number of constraints implies that there are *(2m)* independent parameters in the standard form spline (S_y).

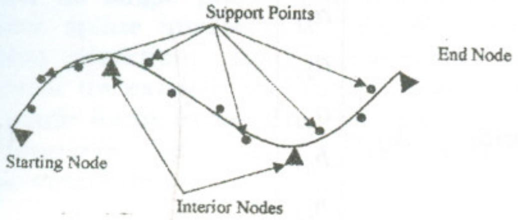

Figure 1 From fitting for ground spline curve

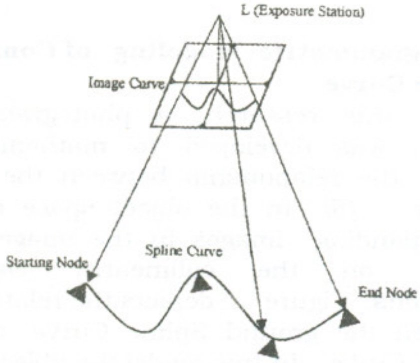

Figure 2 Relationship between ground spline curve and image curve

Least Squares Adjustment for Form Fitting Spline Curve

In this section, the treatment of the mathematical model for the form fitting spline curve is based on the general least squares approach with constraints. The vector of unknown parameters *(X)* that must be solved for an estimation of a spline *(S)* is the vector of parameters of the cubic polynomials $(\hat{S}_1, \hat{S}_2, \hat{S}_3, \ldots\ldots \hat{S}_j)$. The value of the parameters $(\hat{X}_{n1}, \hat{X}_{n(m)})$ which correspond to the end support points are assigned as $(\hat{X}_{n1} = X_1)$ and $(\hat{X}_{n(m)} = X_n)$. The condition equations are written by attaching an index *(j)* that corresponds to the span number in a point-wise fashion as:

$$F_{1i} = Y_i - (a_{0J} + a_{1J} * X_i + a_{2J} * X_i^2 + a_{3J} * X_i^3) = 0 \qquad (6)$$

$$F_{2i} = Z_i - (b_{0J} + b_{1J} * X_i + b_{2J} * X_i^2 + b_{3J} * X_i^3) = 0$$

For all control points, where $i \ni j$ and $1 < i \leq n-1$ and $1 \leq j \leq m-1$.

The general least square with constraints is used to determine the estimated parameters. The constraints that must be imposed on the spline *(S)* are written to utilize the boundary conditions. Two sets of constraints are written for both the start and end points as follows:

$$F_{C1} = \begin{bmatrix} 2a_{21} + 6a_{31}(X_1) \\ 2b_{21} + 6b_{31}(X_1) \end{bmatrix} = 0.0 \qquad (7)$$

$$F_{Cm} = \begin{bmatrix} 2a_{2m} + 6a_{3m}(X_m) \\ 2b_{2m} + 6b_{3m}(X_m) \end{bmatrix} = 0.0 \qquad (8)$$

Moreover, another set of constraints are written for the interior nodes as follows:

$$F_{CJ} = \begin{bmatrix} [(a_{0j} + a_{1j}(X_{j+1}) + a_{2j}(X_{j+1})^2 + a_{3j}(X_{j+1})^3) - (a_{0j+1} + a_{1j+1}(X_{j+1}) + a_{2j+1}(X_{j+1})^2 + a_{3j+1}(X_{j+1})^3)] \\ [(b_{0j} + b_{1j}(X_{j+1}) + b_{2j}(X_{j+1})^2 + b_{3j}(X_{j+1})^3) - (b_{0j+1} + b_{1j+1}(X_{j+1}) + b_{2j+1}(X_{j+1})^2 + b_{3j+1}(X_{j+1})^3)] \\ [(a_{1j} + 2a_{2j}(X_{j+1}) + 3a_{3j}(X_{j+1})^2) - (a_{1j+1} + 2a_{2j+1}(X_{j+1})^2 + 3a_{3j+1}(X_{j+1})^2)] \\ [(b_{1j} + 2b_{2j}(X_{j+1}) + 3b_{3j}(X_{j+1})^2) - (b_{1j+1} + 2b_{2j+1}(X_{j+1})^2 + 3b_{3j+1}(X_{j+1})^2)] \\ [(2a_{2j} + 6a_{3j}(X_{i+1})) - (2a_{2j+1} + 6a_{3j+1}(X_{i-1}))] \\ [(2b_{2j} + 6b_{3j}(X_{i+1})) - (2b_{2j+1} + 6b_{3j+1}(X_{i+1}))] \end{bmatrix} = 0.0 \qquad (9)$$

Where, $1 \leq j \leq m-2$

Let the vector X:

$$X=\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_J \end{bmatrix} \quad \text{For all spans } 1 \leq j \leq m-1. \quad \text{and,} \quad X_J=\begin{bmatrix} a_{0J} \\ a_{1J} \\ a_{2J} \\ a_{3J} \\ b_{0J} \\ b_{1J} \\ b_{2J} \\ b_{3J} \end{bmatrix} \tag{10}$$

Where (X) refer to the spline parameters. General least squares with constraints technique is used to solve for this vector of unknown parameters as follows:

$$A^*V + B^* \Delta = F$$
$$C^*\Delta = Fc \tag{11}$$

Where Δ is the vector of corrections to the approximate values of the unknown parameters.

Photogrammetric Modeling of Connected Spline Curve

In this research, a photogrammetric model was developed to mathematically model the relationship between the spline feature (S) in the object space and its corresponding images in the image space based on the collinearity condition equations. Figure 2 depicts the relationship between the ground Spline Curve and the Image Curve. In this model the object space point (p_i) is replaced by a parametric form of the spline feature as follows:

$$x-x_0=-f\frac{m_{11}(X_p-X_c)+m_{12}(a_0+a_1X_p+a_2X_p^2+a_3X_p^3-Y_c)+m_{13}(b_0+b_1X_p+b_2X_p^2+b_3X_p^3-Z_c)}{m_{31}(X_p-X_c)+m_{32}(a_0+a_1X_p+a_2X_p^2+a_3X_p^3-Y_c)+m_{33}(b_0+b_1X_p+b_2X_p^2+b_3X_p^3-Z_c)}$$

$$y-y_0=-f\frac{m_{21}(X_p-X_c)+m_{22}(a_0+a_1X_p+a_2X_p^2+a_3X_p^3-Y_c)+m_{23}(b_0+b_1X_p+b_2X_p^2+b_3X_p^3-Z_c)}{m_{31}(X_p-X_c)+m_{32}(a_0+a_1X_p+a_2X_p^2+a_3X_p^3-Y_c)+m_{33}(b_0+b_1X_p+b_2X_p^2+b_3X_p^3-Z_c)} \tag{12}$$

Where,

$a_0, a_1, a_2, a_3, b_0, b_1, b_2, b_3$ are the coefficients of each span in the spline

X_p are the X- coordinates of the control point.

Least Squares Adjustment for the Spline Curve

General least squares technique is used to solve several photogrammetric processes such as single photo resection and object space spline intersection. In case of single photo resection, the main objective is to recover the exterior orientation parameters of a single frame image. The vector of unknown parameters *(X)* may be written in the following form:

$$X = \begin{bmatrix} \dot{X} & \ddot{X} \end{bmatrix} \quad (13)$$

Where,

$$\dot{X} = \begin{bmatrix} X_L \\ Y_L \\ Z_L \\ \omega \\ \phi \\ \kappa \end{bmatrix}_k \quad \text{For photo } k \quad (14)$$

and

$$\ddot{X} = \begin{bmatrix} \ddot{X}_2 \\ \vdots \\ \ddot{X}_i \\ \vdots \\ \ddot{X}_{n-1} \end{bmatrix} \quad \text{For support points in all spans.} \quad (15)$$

The linearized form of the condition equations may be written as follows:

$$AV + \dot{B}\dot{\Delta} + \ddot{B}\ddot{\Delta} = f \quad (16)$$

Baring in mind that the spline features are considered as control features with known coefficients.

In case of object space intersection of a spline, the inner and exterior orientation parameters of the imaging system are considered as known values. The vector of unknown parameters *(X)* may be written in the following form:

$$X = \begin{bmatrix} \dot{X} & \ddot{X} \end{bmatrix} \quad (17)$$

Where,

$$\dot{X} = \begin{bmatrix} \dot{X}_1 \\ \vdots \\ \dot{X}_j \\ \vdots \\ \dot{X}_{m-1} \end{bmatrix} \quad \text{For all spans} \quad 1 \le j \le m-1$$

$$\text{and,} \quad \dot{X}_J = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ b_0 \\ b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad (18)$$

$\dot{X}$ refer to the spline curve coefficients, and

$$\ddot{X} = \begin{bmatrix} \ddot{X}_2 \\ \vdots \\ \ddot{X}_i \\ \vdots \\ \ddot{X}_{n-1} \end{bmatrix} \quad (19)$$

For support points in all spans.

Where,

$\ddot{X}_i$ is the *X*-coordinate of the support point *Xi.*

The linearized final form of the condition equations may be written as follows:

$$\begin{aligned} AV + \dot{B}\dot{\Delta} + \ddot{B}\ddot{\Delta} &= f \\ \dot{C}\dot{\Delta} &= f_c \end{aligned} \quad (20)$$

The object space intersection of a spline curve is more complex than the resection process, due to lack of information about the nature of the spline in the object space. A suggested procedure is introduced to perform the intersection process of a spline in the object space. This procedure depends on the information available in the overlapping pair of photographs that contain images of the spline. The object space intersection process may be performed through several steps as follows:

[a] Select a segment of the curve from the left photograph and determine the two end points of this segment.

[b] Using the a priori information of the overlapping pair of photographs and applying the coplanarity condition equations, determine the two end points of the selected segment of the curve on the right photograph.

[c] Applying the collinearity condition equations, determine the object space coordinates of the end points of the spline curve.

[d] Consider that the object space spline curve consists of a single span, in which the end boundary constraints are the only constraints that must be applied on the spline curve.

[e] Different image points are selected on the image curve in the streopair.

[f] The eighth parametric coefficients of the spline curve as well as the X-coordinates of all the intersection points will be estimated

[g] An important check must be performed using the span adaptive selection criterion to investigate if the spline curve should be divided into more than one span.

[h] In case where the spline curve needs to be divided, step (e) through (g) are repeated until the spline curve can not be divided.

EXPERIMENTS AND RESULTS

Experiments for Single Photo Resection

Aerial images of a single frame camera at scale *1/5000* were simulated. Three synthetic ground spline curves were generated within the covered area. Figure 3 depicts the geometric configurations for all curves. Experiments with the simulated data were performed using spline curves as control curves where each spline curve is fully determinant. In fact, the minimum number of the image points that are needed to be taken on the image space curve is seven points. This means that each image space point contributes two equations which inherent seven unknown parameters (six parameters for the exterior orientation of the camera and one-parameter for the X-coordinate of the ground control point taken on the spline curve). Three well-distributed spline curves were selected to perform the resection problem. Many tests were performed with different number of points beginning from *(8) to (300)* image points. Different levels of random noise were introduced to the simulated data in all experiments.

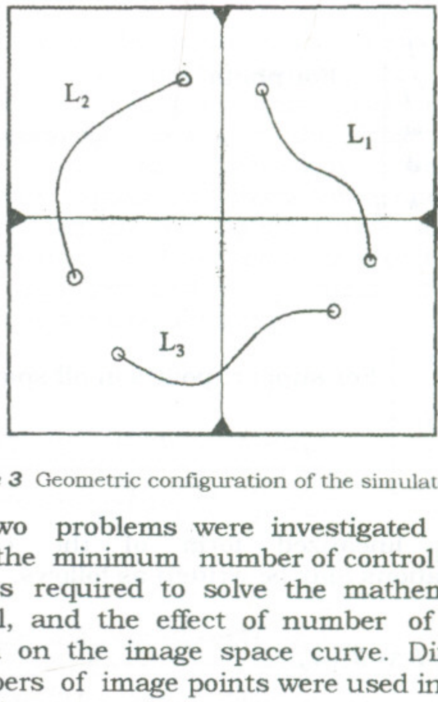

Figure 3 Geometric configuration of the simulated data

Two problems were investigated which are: the minimum number of control spline curves required to solve the mathematical model, and the effect of number of points taken on the image space curve. Different numbers of image points were used in these experiments to recover the exterior orientation parameters of the exposure station. In these experiments different levels of random noise were introduced to the simulated data. The random noise were generated using a Gaussian distribution with mean equals zero and different standard deviations (σ_n). Vector Errors V_E for the exposure station coordinates in all experiments were computed to evaluate their results as follows:

$$V_E = \sqrt{(X_T - X_E)^2 + (Y_T - Y_E)^2 + (Z_T - Z_E)^2} \quad (21)$$

Where,

X_T, Y_T, Z_T are the true coordinates of the exposure station

X_E, Y_E, Z_E are the estimated coordinates of the exposure station

A sample of the results of many tests is presented in Figure 4 which depicts the Vector Error of the exposure station position using three control spline curves with different numbers of points and different levels of random noise. Inspecting this figure, it's obvious that the accuracy of recovering the exposure station parameters increases with the increase of number of points used in the solution. Moreover, the accuracy of the solution depends on the levels of noise introduced to the simulated data.

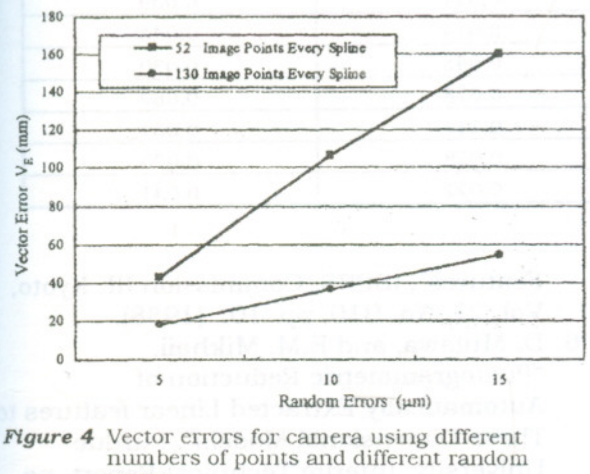

Figure 4 Vector errors for camera using different numbers of points and different random errors

Experiments for Ground Spline Curve Intersection

A synthetic ground spline curve of four spans was generated within an overlapping area of a pair of aerial photographs. In the intersection process, The unknown parameters are the parametric coefficients of the spline curve in addition to the *X*-coordinates of the support points. Different numbers of image points were taken on the projected spline curve ranges from *40 to 176* points associated with random noise values of standardeviation equals *(10)* microns. Two problems were investigated which are: the minimum number of the image points required to solve the mathematical model, and the effect of the number of image points taken on the image space curve. Many tests were performed using *40* and *176* image points to recover the parametric coefficients of the spline curve. As an evaluation of the results, the coordinates of ten support points taken on the spline curve, were computed. Vector errors for these points were calculated and presented in Table 1 and Table 2. Inspecting these tables, it's obvious that the spline was divided into the same number of spans as the simulated data (four spans). Vector errors for all points in all spans are much lower when more points were used in the solution.

CONCLUSIONS

The following remarks may be concluded as results of this research:

- Spline curve features can be used as control to solve most of the photogrammetric processes.
- A robust algorithm for modeling the relationship between the spline curve feature and the exposure station is developed which effectively handles almost all the photogrammetric processes.
- The developed technique is efficient and accurate in determining the exterior orientation parameters for the aerial images using spline curve as control.
- Decreasing the level of random noise plays an important role in improving the accuracy.
- Increasing the number of points used in the solution plays an important role in improving the accuracy.
- The developed technique is efficient and accurate in determining the parametric coefficients for ground spline curve.

Table 1 Vector errors of the support points for the ground spline curve using (40) image points with (10) microns random noise

V_E for span1 (m)	V_E for span2 (m)	V_E for span3 (m)	V_E for span4 (m)
0.119	0.048	0.018	0.050
0.118	0.071	0.024	0.033
0.118	0.080	0.026	0.029
0.112	0.083	0.025	0.026
0.109	0.080	0.029	0.025
0.104	0.075	0.050	0.026
0.096	0.064	0.075	0.024
0.090	0.049	0.113	0.025
0.067	0.032	0.129	0.030
0.046	0.011	0.130	0.038

Table 2 Vector errors of the support points for the ground spline curve using (176) image points with (10) microns random noise

V_E for span1 (m)	V_E for span2 (m)	V_E for span3 (m)	V_E for span4 (m)
0.019	0.030	0.062	0.023
0.020	0.022	0.027	0.055
0.083	0.043	0.059	0.012
0.029	0.058	0.021	0.039
0.055	0.022	0.014	0.043
0.017	0.015	0.045	0.030
0.019	0.064	0.078	0.082
0.081	0.035	0.117	0.061
0.087	0.015	0.058	0.075
0.058	0.008	0.022	0.041

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مقدرة استخدام المعالم المنحنية فى عمليات المساحة التصويرية

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ملخص البحث

تعتبر المعالم الخطية بما تشمله من خطوط مستقيمة ومنحنيات من التقنيات الحديثة المستخدمة فى عمليات المساحة التصويرية بدلا من نقاط الربط الأرضى القياسية. وفى هذا البحث تم التركيز على استخدام المعالم المنحنية فى حل بعض عمليات المساحة التصويرية ذات الأبعاد الثلاثية. وقد تمت عمل دراسة محاكاة لصور صناعية لمعرفة مدى تأثير استخدام الخطوط المنحنية فى عمليات المساحة التصويرية وأثبتت التجارب مقدرة استخدامها كنظام ثوابت أرضية لازمة لعمليات المساحة التصويرية.