STEADY STATE HEAT CONDUCTION IN SOLIDS WITH EXPONENTIAL THERMAL CONDUCTIVITY

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ABSTRACT

The solutions for the non-linear partial differential equation of heat conduction, are obtained in which the thermal conductiviy is temperature dependent and is of exponential form, using Kirchhoff transformation. Two worked problems (homogeneous and non-homogeneous) are considered for illustration.

Keywords: Kirchhoff transformation, finite Fourier transform, exponential thermal conductivity.

INTRODUCTION

The non-linear partial differential equation of heat conduction has been the subject of intensive interest dating from Kirchhoff's work in 1894, where he introduced the transformation which has come to be known as the Kirchhoff transformation. The derivation of the non-linear equation of heat conduction is developed in conduction of heat in solids by Carslaw and Jaeger [1]. They gave solutions to some non-linear problems. In 1930, Van Dusen used the Kirchhoff transformation to simplify the non-linear equation of conduction. Plunkett [2], in 1950, transformed the non-linear time dependent equation of conduction in rectangular system using the Kirchhoff transformation and solved the resulting simplified non-linear equation by a numerical methods. Friedmann [3], in 1957, dealing with the same types of problems, gave additional analytical solutions and further numerical results. Cobble and Ames [4], in 1963, developed a method for solving the linear Poisson's equation in any orthogonal coordinate system. Cobble [5], in 1967, used the Kirchhoff transformation to simplify the non-linear equation of heat conduction where the temperature dependence thermal conductivity is in ploynomial form.

In this paper, the non-linear heat conduction equation in any orthogonal coordinate system is handled by solving a linear partial differential equation, and transforming the solution in such a way, where the temperature dependence thermal conductivity is in exponential form. This technique can be utilized directly using the methods presented in this paper to solve a large class of non-linear conduction problems. The range of non-linear conduction solutions is extended to handle a special class of unsteady state problems in which the specific heat at constant pressure is a specific function of temperature.

MATHEMATICAL FORMULATION OF THE PROBLEM

The steady-state conduction equation for an incompressible solid having a coordinate dependent distributed source [6,7] is :

$$\vec{\nabla}.\left[K(u)\,\vec{\nabla}u\right]+g=0 \tag{1}$$

It can be linearized using Kirchhoff's transformation, namely,

$$V=V(u)=\int_0^u K(s)\,ds \tag{2}$$

Equation 1 takes the form:

$$\nabla^2 V+g=0 \tag{3}$$

TRANSFORMATION OF BOUNDARY CONDITIONS IN AN ORTHOGONAL COORDINATE SYSTEM

The three types of boundary conditions are:

i Dirichlet boundary condition: prescribes temperature on the boundary.
If $K=K(u)=K_o\ F(u)$, where F(u) is in exponential form.

i.e. $$F(u) = \exp(cu) \tag{4}$$

Then, using Equation 2

$$V = V(u) = \int_0^u K_o \exp(cs)ds = \frac{K_o}{c}\left[\exp(cu) - 1\right] \tag{5}$$

Equation 5 can be expressed as:

$$u = \frac{1}{c} \ln\left(1+\frac{cV}{K_o}\right) \tag{6}$$

It can be seen from Equation 6, that if

$V = 0$, then $u = 0$

ii Neumann boundary condition: specifies the rate of change of temperature at points on the boundary in a direction perpendicular to the boundary (directional derivative).

Since Equation 2 holds, then

$$\frac{\partial V}{\partial u_i} = \frac{\partial V}{\partial u}.\frac{\partial u}{\partial u_i} = \frac{d V}{d u}.\frac{\partial u}{\partial u_i} \tag{7}$$

where u_i is a general orthogonal coordinate. Now from Equations 4 and 5 we get

$$\frac{dV}{du_i} = K_o \exp(cu) = K_o\ F(u) = K(u) \tag{8}$$

Substituting from Equation 8 in Equation 7 we obtain

$$\frac{\partial V}{\partial u_i} = K(u).\frac{\partial u}{\partial u_i} \tag{9}$$

then $\frac{\partial u}{\partial u_i} = 0$ follows from $\frac{\partial V}{\partial u_i} = 0$

iii Robin (convection) boundary condition : This is a linear combination of Dirichlet and Neumann boundary conditions. The vector equation for the heat flux (Fourier's law) is

$$\vec{q} = -K\vec{\nabla} u \tag{10}$$

So that the component in a given direction u_i, for orthogonal curvilinear coordinates is.

$$q_{ui}\ \vec{e}_i = -K\frac{1}{h_i}\frac{\partial u}{\partial u_i}\vec{e}_i \tag{11}$$

If this component is convected through a film coefficient h to a medium at zero temperature, then according to Newton's law of cooling (heat transfer proportional to temperature difference), we have

$$q_{ui}\ \vec{e}_i = h(u - 0)\ \vec{e}_i \tag{12}$$

Using Equations 11 and 12, we may write a component identity as

$$-K\frac{1}{h_i}\frac{\partial u}{\partial u_i} = h\,u \tag{13}$$

If twe assume that the heat transfer coefficient takes the form [5]

$$h = h(u) = h_o\ L(u) \tag{14}$$

where L(u) is a function to be determined. Let us consider an equation in V that is linear and similar in form to Equation 13

$$\frac{1}{h_i}\frac{\partial V}{\partial u_i} = -AV \quad , A = \text{constant} \tag{15}$$

Since it was previously required from Equation 2 that V = V(u), then

$$\frac{1}{h_i}\frac{\partial V}{\partial u_i} = \frac{1}{h_i}\frac{\partial V}{\partial u}\cdot\frac{\partial u}{\partial u_i} = \frac{1}{h_i}\frac{dV}{du}\cdot\frac{\partial u}{\partial u_i}$$

$$= \frac{1}{h_i}K_o F(u)\,\frac{\partial u}{\partial u_i}$$

$$= \frac{1}{h_i}K(u)\,\frac{\partial u}{\partial u_i} = \frac{1}{h_i}K\,\frac{\partial u}{\partial u_i} \tag{16}$$

Using Equations 2 and 16 in Equation15, gives

$$\frac{1}{h_i}K\frac{\partial u}{\partial u_i} = -A\int_0^u K(s)ds \tag{17}$$

For Equation 17 to be identical to Equation 13, we must have

$$hu = A\int_0^u K(s)ds$$

$$h_o L(u).u = A K_o \int_0^u F(s)ds$$

It is necessary that

$$AK_o = h_o \rightarrow A = \frac{h_o}{K_o} = \text{constant} \tag{18}$$

$$L(u).\,u = \int_0^u F(s)ds \rightarrow L(u) = \frac{\int_0^u F(s)ds}{u} \tag{19}$$

In addition, from a physical basis , it is necessary that $\lim_{u \to 0} L(u)$ has a finite value.

The objective of a finite value is to avoid singularity, which means contradiction with the boundary conditions.

Thus the convection boundary condition i.e, Equations 17 in any orthogonal coordinate system, will be transformed to:

$$\frac{1}{h_i}K\frac{\partial u}{\partial u_i} = -\frac{h_o}{K_o}.K_o\int_0^u F(s)ds \tag{20}$$

$$= -h_o\, L(u).u = -h(u).u$$

Substituting from Equation 4 in Equations 14 and 19 leads to

$$h(u) = \frac{h_o \int_0^u \exp(cs)ds}{u}$$

$$= \frac{h_o}{cu}\left[\exp(cu) - 1\right] = \frac{h_o}{K_o}\frac{V}{u}$$

From Equation 19, then

$$L(u) = \frac{\int_0^u \exp(cs)ds}{u} = \frac{\exp(cu) - 1}{cu}$$

and so $\lim_{u \to o} L(u) = 1$ is a finite value.

Uniqueness can be established, from Equation 6 and from the physical basis that u is real and $u \geq 0$, and that c is real, then

$$1 + \frac{cV}{K_o} \geq 1 \qquad \therefore\ V \geq 0$$

WORKED PROBLEMS

Problem 1

A rectangular plate $0 \leq x \leq a$, $0 \leq y \leq b$, the boundary at x=0 is kept insulated, the boundary at y=0 is kept at zero temperature, the boundary at x=a dissipates heat by convection into an environment at zero temperature with a heat transfer coefficient h, and finally the boundary at y=b is kept at temperature f(x) as illustrated in Figure 1. It is required to obtain an expression for the steady-state temperature distribution u(x,y) everywhere in the plate.

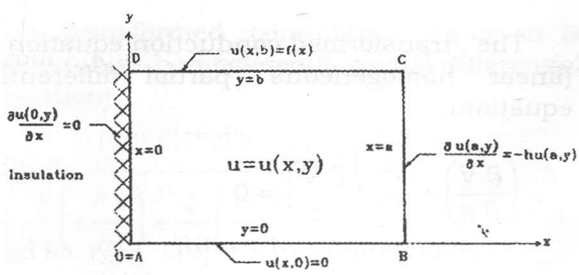

Figure 1 Boundary conditions for a rectangular region considered in problem 1

Solution : To mathematically formulate the problem we have:

The conduction equation in steady state in vector form is

$$\vec{\nabla}.\left(K\,\vec{\nabla}\,u\right) = 0$$

In scalar form using Cartesian coordinates we get

$$\frac{\partial}{\partial x}\left(K\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial u}{\partial y}\right) = 0 \qquad (21)$$

The boundary conditions are

$$\frac{\partial u(o,y)}{\partial x} = 0 \quad \text{on side AD, } 0 \le y \angle b \qquad (22)$$

$$u(x,o) = 0 \quad \text{on side AB, } 0 < x < a \qquad (23)$$

$$K\,\frac{\partial u(a,y)}{\partial x} = -hu\,(a,y)$$

$$\text{on side BC } 0 \le y < b \qquad (24)$$

$$u(x,b) = f(x) \quad \text{on side CD, } 0 < x < a \qquad (25)$$

$K = K(u) = K_o \exp(cu)$,

$$V = V(x,y) = \frac{K_o}{c}\left[\exp(cu(x,y)) - 1\right],$$

$$\frac{\partial V}{\partial x} = \frac{\partial V}{\partial u}.\frac{\partial u}{\partial x} = K\frac{\partial u}{\partial x},$$

$$\frac{\partial V}{\partial y} = \frac{\partial V}{\partial u}.\frac{\partial u}{\partial y} = K\frac{\partial u}{\partial y}.$$

The transformed conduction equation is (linear homogeneous partial differential equation)

$$\frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y}\right) = 0$$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0 \qquad (26)$$

The transformed boundary conditions are:

$$\frac{\partial V(o,y)}{\partial x} = K\frac{\partial u(o,y)}{\partial x} = 0 \qquad (27)$$

$$V(x,o) = \frac{K_o}{c}\left[\exp(cu(x,0)) - 1\right] = 0 \qquad (28)$$

$$\frac{\partial V(a,y)}{\partial x} = -\frac{h_o}{K_o}V(a,y) \qquad (29)$$

$$V(x,b) = \frac{K_o}{c}\left[\exp(cu(x,b)) - 1\right]$$

$$= \frac{K_o}{c}\left[\exp(cf(x)) - 1\right] \qquad (30)$$

The transformed boundary value problem is illustrated in Figure 2.

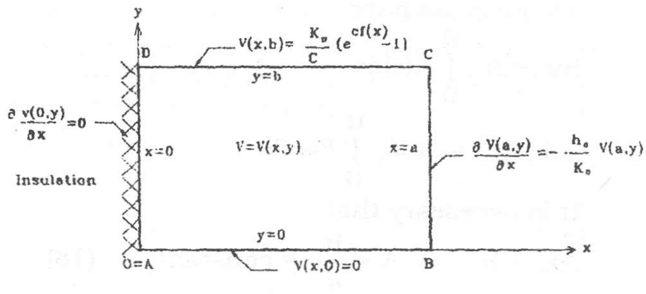

Figure 2 Transformed boundary conditions for a rectangular region considered in Problem 1

Now, we are going to apply the technique of "separation of variables" to solve this posed problem i.e. Equations 26 - 30. The details are given in Appendix A.

The formal solution is

$$V(x,y) = \sum_{n=1}^{\infty} A_n \sinh(\lambda_n y)\; X_n(x) \qquad (31)$$

where

$$\tan(\lambda_n a) = \frac{h_o}{K_o\lambda_n}$$

$$X_n(x) = \frac{\sqrt{2}\quad \cos(\lambda_n x)}{\sqrt{a + \dfrac{\left(\dfrac{h_o}{K_o}\right)}{\lambda_n^2 + \left(\dfrac{h_o}{K_o}\right)^2}}},$$

$$A_n = \frac{1}{\sinh \lambda_n b} \int_0^a \frac{K_o}{c}\left[\exp(cf(x)) - 1\right] X_n(x)\,dx$$

Then the solution of Equation 21 with the accompanying boundary conditions given by Equations 22 -25 is

$$u(x,y) = \frac{1}{c} \ln\left(1 + \frac{cV(x,y)}{K_o}\right) \qquad (32)$$

where V(x,y) is as given by Equation 31.

Problem 2

A plate is bounded by two concentric sectors of circles of inner radius a and outer radius b and has a central angle α. The boundary surfaces at $r = a$, $r = b$, and $\theta = \alpha$ are all kept at zero temperature, the boundary surface at $\theta = 0$ is kept insulated as illustrated in Figure 3.

The heat is generated in the plate at a constant rate per unit volume. It is required to obtain an expression for the steady-state temprature distribution $u(r, \theta)$ everywhere in that plate.

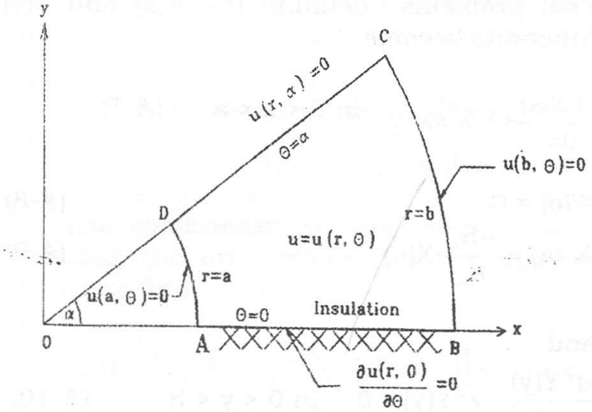

Figure 3 Boundary conditions for an annular region considered in Problem 2

Solution : The mathematical formulation of the problem is:

The conduction equation in steady state in vector form is

$$\vec{\nabla}.\left(K\,\vec{\nabla}\,u\right) + g = 0$$

In scalar form using polar coordinates we have

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\,K\frac{\partial u}{\partial r}\right) + \frac{\partial}{\partial \theta}\left(\frac{K}{r}\frac{\partial u}{\partial \theta}\right)\right] + g = 0 \qquad (33)$$

The boundary conditions are:

$$\frac{\partial u(r,o)}{\partial \theta} = 0 \qquad \text{on AB, } a < r < b \qquad (34)$$

$$u(a,\theta) = 0 \qquad \text{on AD, } 0 < \theta \le \alpha \qquad (35)$$

$$u(r,\alpha) = 0 \qquad \text{on CD, } a < r < b \qquad (36)$$

$$u(b,\theta) = 0 \qquad \text{on BC, } 0 \le \theta \le \alpha \qquad (37)$$

$K = Ku) = K_o \exp(cu)$,

$$V = V(r,\theta) = \frac{K_o}{c}\left[\exp(cu(r,\theta)) - 1\right],$$

$$\frac{\partial V}{\partial r} = \frac{\partial V}{\partial u}\frac{\partial u}{\partial r} = K\frac{\partial u}{\partial r}.$$

$$\frac{\partial V}{\partial \theta} = \frac{\partial V}{\partial u}.\frac{\partial u}{\partial \theta} = K\frac{\partial u}{\partial \theta}$$

The transformed conduction equation is (linear non-homogeneous partial differential equation)

$$\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{\partial}{\partial \theta}\left(\frac{1}{r}\frac{\partial V}{\partial \theta}\right)\right] + g = 0$$

$$\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + g = 0 \qquad (38)$$

The transformed boundary conditions are:

$$\frac{\partial V(r,o)}{\partial \theta} = K\frac{\partial u(r,o)}{\partial \theta} = 0 \qquad (39)$$

$$V(a,\theta) = \frac{K_o}{c}\left[\exp(cu(a,\theta)) - 1\right] = 0 \qquad (40)$$

$$V(r,\alpha) = \frac{K_o}{c}\left[\exp(cu(r,\alpha)) - 1\right] = 0 \qquad (41)$$

$$V(b,\theta) = \frac{K_o}{c}\left[\exp(cu(b,\theta)) - 1\right] = 0 \qquad (42)$$

The transformed boundary value problem is shown in Figure 4.

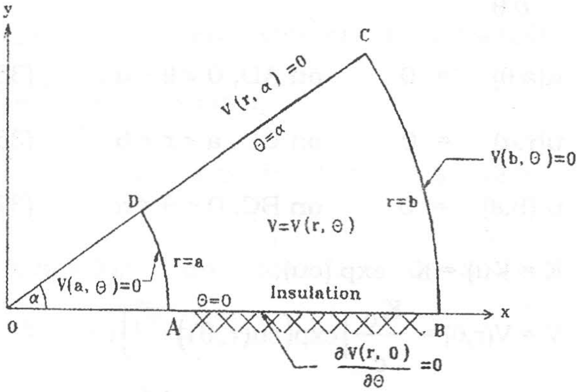

Figure 4 Transformed boundary conditions for an annular region considered in Problem 2

The finite Fourier transform [8], is used to solve Equations 38-42 for the non-homogeneous problem. The details are given in Appendix B. The formal solution is

$$V(r,\theta) = \sum_{n=1}^{\infty} \sqrt{\frac{2}{\alpha}}.\cos(\lambda_n\theta).\frac{g}{4-\lambda_n^2}.\frac{2\sqrt{2\alpha}}{(2n-1)\pi}(-1)^{n+1}$$
$$\left\{\frac{a^2\left(\frac{r}{b}\right)^{\lambda_n} - b^2\left(\frac{r}{a}\right)^{\lambda_n}}{\left(\frac{a}{b}\right)^{\lambda_n} - \left(\frac{b}{a}\right)^{\lambda_n}} + \frac{b^2\left(\frac{a}{r}\right)^{\lambda_n} - a^2\left(\frac{b}{r}\right)^{\lambda_n}}{\left(\frac{a}{b}\right)^{\lambda_n} - \left(\frac{b}{a}\right)^{\lambda_n}} - r^2\right\} \qquad (43)$$

where

$$\lambda_n = \frac{(2n-1)\pi}{2\alpha} \qquad n = 1,2,3,4, ...$$

Then the solution of Equation 33 together with the associated boundary conditions given by Equations 34-37 is

$$u(r,\theta) = \frac{1}{c}\ln\left(1+\frac{cV(r,\theta)}{K_o}\right) \qquad (44)$$

where $V(r,\theta)$ is as given by Equation 43.

APPENDIX A

$$\frac{\partial^2 V(x,y)}{\partial x^2} + \frac{\partial^2 V(x,y)}{\partial y^2} = 0$$

$$0 < x < a,\ 0 < y < b \qquad (A\text{-}1)$$

$$\frac{\partial V(0,y)}{\partial x} = 0$$

$$V(x,0) = 0 \qquad (A\text{-}3)$$

$$\frac{\partial V(a,y)}{\partial x} = \frac{-h_o}{K_o}V(a,y) \qquad (A\text{-}4)$$

$$V(x,b) = \frac{K_o}{c}\left\{\exp(cf(x)) - 1\right\} \qquad (A\text{-}5)$$

Solution: Assuming a solution of the form

$$V(x,y) = X(x)\,Y(y) \qquad (A\text{-}6)$$

The problems defining the X(x) and Y(y) functions become

$$\frac{d^2X(x)}{dx^2} + \lambda^2 X(x) = 0 \quad \text{in } 0 < x < a \qquad (A\text{-}7)$$

$$X'(o) = 0 \qquad (A\text{-}8)$$

$$X'(a) = \frac{-h_o}{K_o}X(a) \qquad (A\text{-}9)$$

and

$$\frac{d^2Y(y)}{dy^2} - \lambda^2 Y(y) = 0 \quad \text{in } 0 < y < b \qquad (A\text{-}10)$$

$$Y(o) = 0 \qquad (A\text{-}11)$$

The solution of the eigenvalue problem (A-7), (A-8) and (A-9) [Sturm-Liouville System], is obtained in the normalized eigenfunction form

$$X_n(x) = X(\lambda n, x) = \frac{\sqrt{2}\quad \cos(\lambda_n x)}{\sqrt{a + \dfrac{\left(\frac{h_o}{K_o}\right)}{\lambda_n^2 + \left(\frac{h_o}{K_o}\right)^2}}}$$

and the eigenvalues λ_n are the positive roots of $\lambda_n \tan(\lambda_n a) = \frac{h_o}{K_o}$

The solution of Equations A-10, A-11 is given in the form

$Y(y) = A \sinh(\lambda_n y)$, where A = constant

The complete solution for V(x,y) is constructed as

$$V(x,y) = \sum_{n=1}^{\infty} A_n \sinh(\lambda_n y) X_n(x),$$

where A_n are Constants

which satisfies the transformed heat conduction Equation (A-1) and its three homogeneous boundary conditions (A-2), (A-3) and (A-4).

The coefficients A_n should be determined so that it also satisfies the nonhomogeneous boundary condition (A-5). The application of the boundary condition at y = b yields

$$V(x,b) = \sum_{n=1}^{\infty} A_n \mathrm{Sinh}(\lambda_n b) X_n(x) = \frac{K_o}{c}\left[\exp(cu) - 1\right]$$

in 0 < x < a

The coefficients A_n are determined by utilizing the orthogonality of the funsction $X_n(x)$, we find

$$A_n \sinh(\lambda_n b) = \int_0^a \frac{K_o}{c}\left\{e^{cf(x)} - 1\right\} X_n(x)\,dx$$

APPENDIX B

$$\frac{\partial^2 V(r,\theta)}{\partial r^2} + \frac{1}{r}\frac{\partial V(r,\theta)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V(r,\theta)}{\partial \theta^2} + g = 0$$

$a < r < b,\ 0 < \theta < \alpha$ (B-1)

$\frac{\partial V(r,0)}{\partial \theta} = 0$ $\quad a < r < b$ (B-2)

$V(a, \theta) = 0$ $\quad 0 < \theta < \alpha$ (B-3)

$V(r, \alpha) = 0$ $\quad a < r < b$ (B-4)

$V(b, \theta) = 0$ $\quad 0 < \theta < \alpha$ (B-5)

Solution: The corresponding linear homogeneous problem is

$$\frac{\partial^2 V(r,\theta)}{\partial r^2} + \frac{1}{r}\frac{\partial V(r,\theta)}{\partial r} + \frac{1}{r^2}\frac{\partial^2 V(r,\theta)}{\partial \theta^2} = 0 \quad \text{(B-6)}$$

$\frac{\partial V(r,0)}{\partial \theta} = 0$ (B-7)

$V(a, \theta) = 0$ (B-8)

$V(r, \alpha) = 0$ (B-9)

$V(b, \theta) = 0$ (B-10)

If we assume a solution in the form R(r) H(θ) then substituting in Equation B-6, we get

$$\frac{r^2 R''}{R} + r\frac{R'}{R} = -\frac{H''}{H} = \lambda^2$$

When the boundary conditions (B-7) and (B-9) are imposed on the separated function, we obtain a Sturm-Liouville systen in H(θ) given by :

H" + ?? H = 0 (B-11)

$H'(o) = 0$ (B-12)

$H(\alpha) = 0$ (B-13)

The eigenvalues are $\lambda_n = \frac{(2n-1)\pi}{2\alpha}$,

(n =1,2,3,...), with the orthonormal eigenfunctions $H_n(\theta) = \sqrt{\frac{2}{\alpha}}\quad \cos(\lambda_n \theta)$

If we apply the finite Fourier transform associated with this system to (B-1), we get

$$\int_{\theta=0}^{\alpha}\left(\frac{\partial^2 V(r,\theta)}{\partial r^2}+\frac{1}{r}\frac{\partial V(r,\theta)}{\partial r}+\frac{1}{r^2}\frac{\partial^2 V(r,\theta)}{\partial \theta^2}+g\right)H_n(\theta)d\theta = 0$$

$$\frac{\partial^2}{\partial r^2}\int_{0}^{\alpha} V(r,\theta)H_n(\theta)d\theta + \frac{1}{r}\frac{\partial}{\partial r}\int_{0}^{\alpha} V(r,\theta)H_n(\theta)d\theta$$

$$+\frac{1}{r^2}\int_{0}^{\alpha}\frac{\partial^2 V(r,\theta)}{\partial^2}H_n(\theta)d\theta + g\int_{0}^{\alpha}H_n(\theta)d\theta = 0$$

Interchanging the orders of integration and differentiation for the first and second terms of this equation, and integrating by parts the third term, together with the fact that $H_n(\alpha) = 0$, $H_n'(0) = 0$ we finally obtain

$$r^2\frac{d^2\bar{V}(r,\lambda n)}{dr^2}+r\frac{d\bar{V}(r,\lambda n)}{dr}-\lambda_n^2\bar{V}(r,\lambda n) = -gr^2\frac{2\sqrt{2\alpha}}{(2n-1)\pi}(-1)^{n+1} \quad \text{(B-14)}$$

This is an ordinary differential equation (Cauchy-Euler equation type) for $\bar{V}(r,\lambda n)$.

When we take finite Fourier transform of boundary conditions (B-8),(B-10) we obtain boundary conditions for Equation (B-14)

$$\bar{V}(a,\lambda n) = 0 \quad \text{(B-15)}$$

$$\bar{V}(b,\lambda n) = 0 \quad \text{(B-16)}$$

The solution of Equations (B-14), (B-15), and (B-16) is

$$\bar{V}(r,\lambda_n) = \frac{g}{4-\lambda_n^2}\frac{2\sqrt{2\alpha}}{(2n-1)\pi}(-1)^{n+1}\left\{\frac{a^2\left(\frac{r}{b}\right)^{\lambda n}-b^2\left(\frac{r}{a}\right)^{\lambda n}}{\left(\frac{a}{b}\right)^{\lambda n}-\left(\frac{b}{a}\right)^{\lambda n}} + \frac{b^2\left(\frac{a}{r}\right)^{\lambda n}-a^2\left(\frac{b}{a}\right)^{\lambda n}}{\left(\frac{a}{b}\right)^{\lambda n}-\left(\frac{b}{a}\right)^{\lambda n}} - r^2\right\}$$

Inverting this equation, the solution becomes

$$V(r,\theta) = \sum_{n=1}^{\infty}\bar{V}(r,\lambda_n)H_n(\theta)$$

NOMENCLATURE

u	temperature in Kelvin, K
V	transformed temperature , K
K_o	thermal conductivity at 0 K
K	thermal conductivity $\left(\frac{W}{m\,K}\right)$
$F(u) = \frac{K}{K_o}$	auxiliary function, dimensionless
c	conductivity coefficient $\left(\frac{1}{K}\right)$
u_i	general orthogonal coordinates
$\vec{q}$	heat flux vector (W/m^2)
h_i	scale factor
$\vec{e}_i$	unit tangent vector
h_o	film coefficient at 0 K
h	film coefficient $\left(\frac{W}{m^2\,K}\right)$
$L(u) = \frac{h}{h_o}$	auxiliary function, dimensionless
g	internal heat generation (W/m^3)
$\vec{\nabla}$	vector differential operator del
∇^2	Laplacian operator
x,y	cartesian coordinates
r,θ	polar coordinates
A, A_n	constants

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Received May 12,1998
Accepted. October 1, 1998

الانتقال الحرارى المستقر في الاجسام الصلبة في وجود توصيل حرارى أسي

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قسم الرياضيات والفيزياء الهندسية - جامعة الإسكندرية

ملخص البحث

يتناول البحث طريقة رياضة تحليلية لإيجاد الحلول للمعادلة التفاضلية الجزئية اللاخطية للتوصيل الحرارى في الحالة التي يعتمد عليها معامل التوصيل علي درجة الحرارة في صورة أسية . وتبنى الطريقة على تطبيق تحويل " كيرشوف" الذى يغير المعادلة الأصلية اللاخطية إلى صورة خطية. وتم توضيح الطريقة على مسألتين أحداهما متجانسة وأخرى غير متجانسة.