THE INFLUENCE OF THE INEVITABLE UNCERTAINTIES ON THE INSPECTION PLANNING FOR STRUCTURES

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ABSTRACT

Life-time cost minimization is the optimal criterion for planning of inspection, repair and maintenance of fatigue deteriorating structures. However, most of the probabilities and cost items related to the cost analysis generally contain inevitable uncertainties in the actual structures. The appropriateness of inspection planning may be lost by several errors induced by such uncertainties. In this study, a sequential cost minimization method and its consistent formulation are applied for the estimation of the optimal inspection planning for fatigue deteriorating structures. The process of optimization is repeatedly carried out at every inspection. In this paper, the applicability of the method is examined for an actual member set consisting of structural elements with round fillet weld. The influence of inevitable uncertain parameters on inspection planning are discussed based on the Bayesian analysis and the sensitivity analysis.

Keywords: Inspection planning, Optimization, Uncertainties, Deteriorating structures

INTRODUCTION

The total cost minimization is the optimal criterion of decision making for design and maintenance of structures. In inspection planning problems, inspection intervals, inspection methods, repair qualities and so on are thought to be the optimized variables. In previous studies [1-3], the sequential cost minimization method was presented for the estimation of the optimal inspection planning of fatigue deteriorating structures without considering the effect of inevitable uncertainties. The optimized variables were only two parameters namely, the inspection method and the inspection interval to the next inspection for the structure. The optimization was repeatedly carried out at every inspection.

The fatigue deterioration property for any structure usually contains several uncertainties. In addition, most of the probabilities and cost items related to the cost analysis in the previous studies generally contain inevitable uncertainties. On the other hand, the sensitivity analysis of the uncertain parameters may provide a useful information for the improvement of fatigue design and inspection strategy. From these viewpoints, the influence of inevitable uncertain parameters on the inspection strategy is discussed in the present work in which the analysis is based on the sequential cost minimization method [2].

Usually, the inspections are limited, such as once a year or once every two years, for several operational and economical reasons. Also, repair quality is determined such that similar damage will never take place again in the member after repair, i.e.; Perfect Repair Model[4]. Under such conditions the optimization is mainly achieved by the selection of the most appropriate strategy from the possible combinations of inspection intervals and inspection methods allowed for the structure.

Hereinafter, the main tool used in the present analysis is the sequential cost minimization method. Therefore, a short review in the next section is presented for the method.

THE SEQUENTIAL COST MINIMIZATION METHOD (SHORT REVIEW)

The optimal inspection strategy can be achieved by the appropriate selection of inspection interval, inspection method, repair quality and so on. The sequential cost minimization method is used to estimate the optimal inspection strategy for deteriorating structures by using cost minimization approach and reliability theory. The sequential cost minimization method has three main steps to follow [2]:-

1. Estimation of the total expected operating cost for structure.
2. The selection of the optimal inspection method for a member set.
3. The selection of the appropriate inspection interval for a structure.

The expected operating cost for the whole structure can be estimated by applying the sequential cost minimization method, where this cost is the optimal one. The sequential cost minimization method has been applied for five inspection methods; No inspection (*NO*), Visual inspection (*VI*), Mechanical (Precise) inspection (*MI*), Visual and conditional mechanical inspection *(V&M)*, and Sampling mechanical inspection *(SM)*. Figure 1 shows how the structure is treated when applying sequential cost minimization method. All the structural members in each set were assumed to have the same strength property and were subjected to the same loading conditions. Each member has a possibility of failure due to the deterioration damage.

At a certain inspection during service, the total expected operating cost for the structure in the succeeding inspection interval is classified into two main groups:-

- Costs necessary in the present inspection, and
- Risks (expected costs) during the service period until the next inspection.

Figure 2 shows the total operating cost components in the succeeding inspection interval.

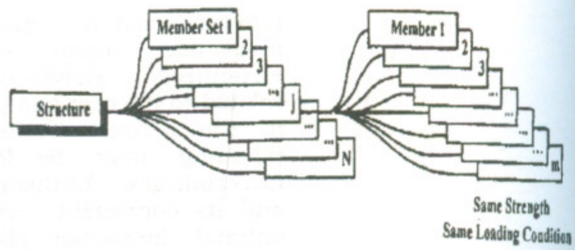

Figure 1 Representation of structure, member sets and mebers

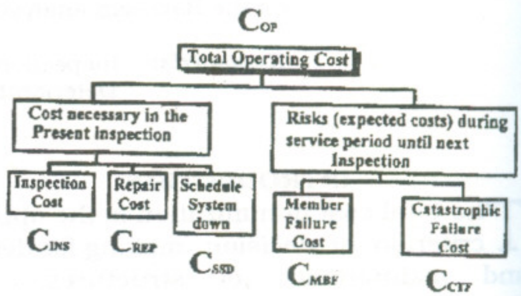

Figure 2 Cost necessary for structure in an inspection interval

When the sequential cost minimization method is applied for the selection of the appropriate inspection interval among two or more inspection intervals, cost comparison should be performed. Figure 3 gives an idea about the technical procedure when applying the proposed method in case that three inspection intervals of once a year, once every two years and once every four years are allowed for the structure. The sequential cost minimization method has been discussed in detail in Reference 2.

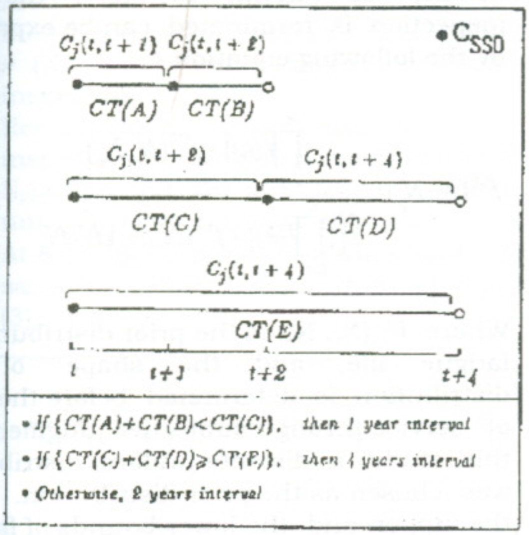

Figure 3 Selection of suitable inspection interval

Total Expected Operating Cost

The estimation of the total expected operating cost corresponding to each member set for the above mentioned five inspection methods, and the cost evaluation equations were developed by the same author [2]. The Markov chain model [5] used for the estimation of the probabilities appearing in the cost evaluation equations are also evaluated in the same Reference 2. The total expected operating cost is given by the following :-

$$C_{op} = \sum_{j=0}^{life-time} C_j(t, t+S)$$

where, S = 1 or 2 or 4 years (1)

where;

C_{op} The total expected operating cost

C_j The expected operating cost for member set j in one inspection interval.

$$C_j = C_{INS}+C_{REP}+C_{MBF}+C_{CTF} \quad (2)$$

C_{INS} Inspection cost of each member set.

C_{REP} Expected repair cost of the detected damages during inspection.

C_{MBF} Expected loss due to a member failure.

C_{CTF} Risk against catastrophic failure.

S The inspection interval.

T The time at which the inspection is carried out.

BAYESIAN ANALYSIS

Method of analysis

Most of the probabilities and the cost items required in the analysis contain several uncertainties in the actual structure. In this section, the influence of uncertainty of fatigue life for the members on inspection planning is discussed based on the Bayesian analysis [6]. The formulation of the Bayesian analysis is presented for a structural member set with deterioration, in which the fatigue crack initiation and propagation lives are the uncertain parameters [7]. At the time of the *n-th* inspection, one event among the following three events can be obtained.

- ***Event A*** : event that the member is found to have failed at the *n-th* inspection, or at an equivalent event that failure of a member occurred during the period between the last inspection and the present inspection.
- ***Event B1*** : event that a member is found not to have failed at the *n-th* inspection and a fatigue crack is detected in the event.
- ***Event B2*** : event that a member is found not to have failed at the *n-th* inspection and no crack is detected. This event consists of the following two mutually exclusive events : the first event is that no crack exists in the member and the second is that crack exists in the member but not detected at the n-th inspection.

Figure 4 shows the all event at the n-th inspection. The probabilities of the above mentioned events, if visual inspection is applied at the present inspection, can be expressed by the following equations (see Figure 4).

$$P_r[Event.A] = \frac{P_{F2}(n-1,n)}{P_{F2}(n-1,n)+P_{SV}(n)} \quad (3)$$

$$P_r[Event.B1] = \frac{P_{DV} \times P_{SV}(n)}{P_{F2}(n-1,n)+P_{SV}(n)} \quad (4)$$

$$P_r[\text{Event. B2}] = \frac{(1-P_{DV}) \times P_{SV}(n)}{P_{F2}(n-1,n)+P_{SV}} \quad (5)$$

Where;

PF2 : Probability of occurrence of member failure in the succeeding inspection interval applying visual inspection method at the present inspection.

PSV : Probability that a member has not experienced repair and failure until the present inspection.

PDV : Probability of detecting a defect by visual inspection method.

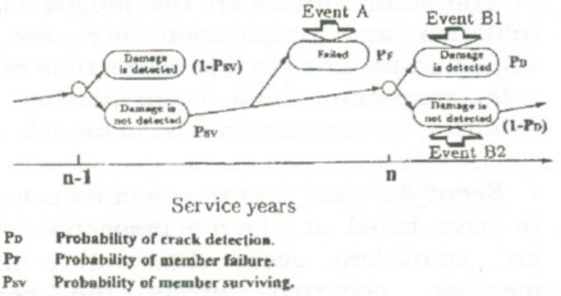

Figure 4 The event tree at the n-th inspection

The sequential cost minimization method is applied for five inspection methods, for the other inspection methods, similar equations can be used after changing the respective probabilities *PF2* and *PDV* with the corresponding one. The likelihood function *L(n)* of the entire event for the whole member set as a result of the *n-th* inspection is calculated as :-

$$L(n) = \{P_r[\text{Event.A}]\}^{m1} \times \{P_r[\text{Event.B1}]\}^{m2} \times \{P_r[\text{Event.B2}]\}^{(G-m1-m2)} \quad (6)$$

Where *"G"* is the total number of members in the set which survived until the *(n th-1)* inspection without experience of both failure and repair events. *"m1"* and *"m2"* are the numbers of members for which the results of the *n-th* inspection are the event A and the event B1, respectively. *"G-m1-m2"* is the number of members for which results of the n-th inspection is event B2. Employing the Bayes theorem, the posterior density of fatigue life after the *n-th* inspection is terminated can be expressed by the following equation:-

$$f^{(n)}(N_c,N_p) = \frac{\{\prod_{k=1}^{n} L(k)\} \times f^{(0)}(N_c,N_p)}{\iint \{\prod_{k=1}^{n} L(k)\} \times f^{(0)}(N_c,N_p)\, dN_c dN_p} \quad (7)$$

Where $f^{(0)}(N_c, N_p)$ is the prior distribution of fatigue life, and the shape of this distribution is determined before the start of service through subjective judgment. In this study, a discrete uniform distribution was chosen as the prior distribution. First, the upper and the lower bounds of fatigue life were assumed so as to include the true value of fatigue life in the range. Then, the range was divided into *k*-lots with uniform intervals. Also the same probability of $1/K$ was given for every lot.

$$f^{(0)}(N_c(i), N_p(i)) = \frac{1}{K} \quad (i=1,2,\ldots\ldots,k) \quad (8)$$

Applying the Bayes equation (7) repeatedly from the first inspection, the uncertainties of the fatigue life can be reduced successively. When the posterior density $f^{(0)}(N_c, N_p)$ of fatigue life is given, the expected total cost for the member set until the next inspection, *Cj(t,t+S),* can be calculated by the following integration :-

$$C_j(t, t+S) = \iint C_j(t, t+SN_c, N_p) * f^{(n)}(N_c, N_p) dN_c dN_p \quad (9)$$

Figure 5 shows the procedures of the inspection planning in which sequential cost minimization method and Bayesian analysis are jointly applied.

1. At the n *th* inspection time, first select the inspection quality and the time of (n*th*+1) inspection using the Equation 9. At this time, $f^{(n-1)}(N_c, N_p)$ obtained at the (n*th*-1) inspection is used as the posterior distribution. Then carry out the inspection following the decided method.

After the completion of the n *th* inspection, calculate posterior density $f^{(n-1)}$ (N_c, N_p) by using the result of the n *th* inspection.

3. Recalculate the time of (n*th*+1) inspection by using the obtained $f^{(n-1)}$ (N_c, N_p). Then put the structure into service until the (n*th*+1) inspection time.
4. At the (n*th*+1) inspection time, repeat the same procedure as shown in (1), (2) and (3).

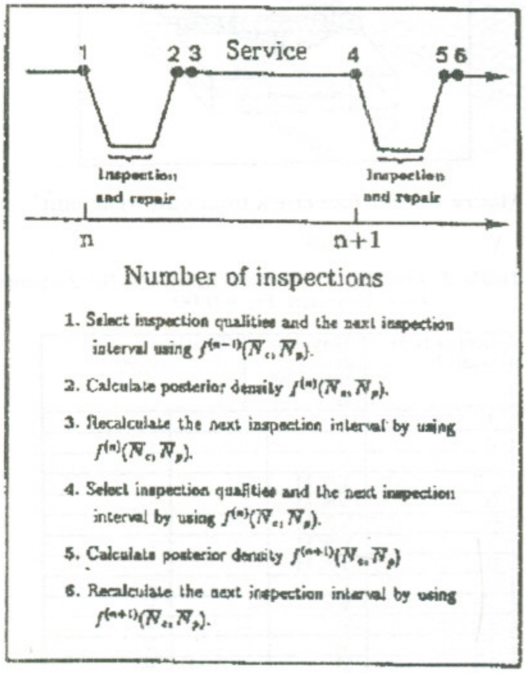

Figure 5 Bayesian analysis

In the above procedure, it is made as a rule to calculate the next inspection interval twice, considering that the posterior density of fatigue life may be changed in the period between before and after inspection. This is feature of the calculation.

NUMERICAL EXAMPLE

Assume a single member set consisting of 200 members of welded joints similar to that shown in Figure 6. The welded joint has a possibility of failure by fatigue damage initiated in the weld toe. The surface crack initiated from the weld toe of a plate with a 20 mm thickness is treated as the deterioration damage. The mean lives of crack initiation and propagation are N_c=25 years and N_p=15 years, respectively. The crack growth curve is described by Paris's equation with the stress intensity facto ΔK calculated by linear elastic fracture mechanics. N_c and N_p follow two parameter Weibull distributions with shape parameters of 3.0 and 4.0, respectively.

Then, inspections are periodically carried out once a year, and the inspection quality is selected from the following four methods: *NO, VI, MI* and *V&M* methods. The probability of crack detection *POD* curves are assumed as a function of crack depth d, as follows[2]:-

$$POD(VI) = 1.0 - \exp\{-0.2 \times (d - 3.0)\} \text{ ,and}$$
$$POD(MI) = 1.0 - \exp\{-0.4 \times (d - 1.0)\} \quad (10)$$

The assumed cost items are : *CVI*= visual inspection cost for a member = 10$, *CMI*= mechanical inspection cost for a member =100$, *CSSD* = schedule system down cost =2x10^5^$, C_{ASD}= accidental system down cost= 10^6^$, C_{RD} = repair cost of a damaged member = from 10^3 $to 10^6 $ in which it was treated as time dependent, C_{RF}= repair cost of failed member = from $10^5$$ to 10^6 $ in which it was also treated as time dependent, C_{CF}= risk of catastrophic failure =$2\text{x}10^8$$. The transition probability to catastrophic failure was assumed to be small as P_{FC}=0.005. In the analysis, the uncertainty exists only in the values of N_c and N_p , and all the other parameters are statistically determined or deterministic. The range of an assumed prior distribution is (5 years ~*Nc*~ 50 years) and (5 years ~*Np* ~ 30 years). The entire fatigue and inspection processes of the 200 members were simulated by the Monte Carlo simulation.

Table 1 shows the results of the Bayesian analysis, in which the inspection qualities and the numbers of detected cracks at every inspection are shown. The "Truth" in the table expresses the inspection strategy obtained for the condition that the true fatigue property of member set is known in advance. In the table, the *V&M* method is selected until the 8-th inspection, and the *VI* method is selected after that. Figure 3 shows the change of the posterior density of the fatigue life as well as the prior density. The peak is not seen in the posterior distribution until the 10th inspection. However, the possibilities of the first two combinations of fatigue lives, (*Nc=5, Np=5*) and (*Nc=10, Np=7*) are removed from the posterior distribution at the 10th inspection. One crack is first detected at the 14th inspection. As the result, the peak of the posterior distribution becomes clearly apparent at the point of true fatigue life. The above results indicate that the *V&M* method becomes profitable when the degree of uncertainty is large in the fatigue life of members.

Through in similar analysis , it was found that the *SM* method also becomes profitable when uncertainty exists in the fatigue life of members.

SENSITIVITY ANALYSIS

In this section, a sensitivity analysis was carried out giving a large change for a parameter *PFC* (The transition probability to catastrophic failure). The initial uncertainty of *PFC* is usually maintained during the whole service life, because a catastrophic failure may be a rare event. A wide range of uncertainty of *PFC* in the inspection planning must be prepared. The analysis was carried out changing the *PFC* gradually from 0.0 to 0.5 for the welded joint shown in the Figure 6. All of the analytical conditions are the same as the previous numerical example, except *Nc, Np* and *CSSD*. In this analysis, it was assumed that *Nc* is 50 years, *Np* is 20 years, and *CSSD* is 2×10^4 \$. When *PFC* =0, it means that the member failure will never be developed into a catastrophic failure, and when *PFC*=0.5, it means that the member failure will be developed into a catastrophic failure with a probability of 50%.

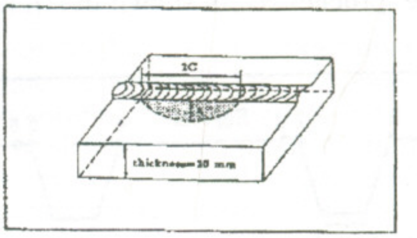

Figure 6 Surface crack from butt weld joint

Table 1 Results of Bayesian analysis N_c=25 years, N_p = 15 years, P_{ID} = 0.00

Inspection year 1	Bayesian approach	Truth	
1	--	-	--
2	--	--	--
3	V	--	--
	V& M	--	--
5	V& M	--	--
6	V& M	--	--
7	V& M	--	--
8	V& M	--	--
9	V	--	--
10	V	--	V
11	V	--	V
12	V	--	V
13	V	--	V
14	V	1	V
15	V	--	V
16	V	2	V
17	V	1	M
18	V	2	V
19	V	6	V
20	M	16	M

N_C Mean life of crack initiation., N_p Mean life of crack propagation.

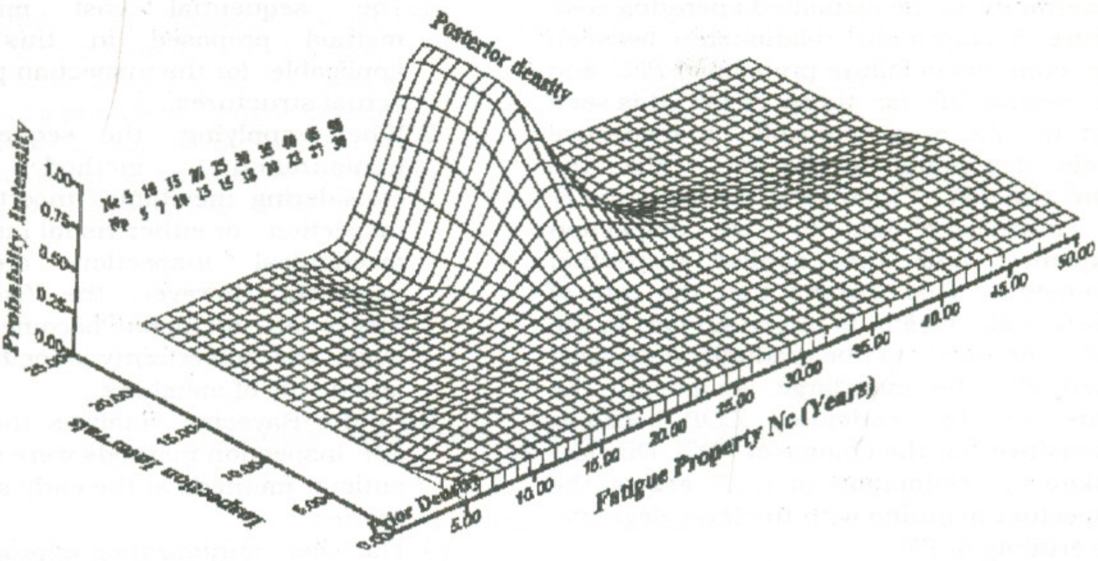

Figure 7 Posterior density of fatigue life

Table 2 Influence of a large change of P_F on inspection strategy

P_{fc}	C_F	Inspection years and qualities										Cumulative operation cost US $	Without inspection
	1.1 X 10^6	4	8	12	16	20	24	--	--	--	--	2.11 x 10^5	1.8 X 10^6
0.00	2.0 X 10^6	V	M	M	V	V	M	--	--	--	--		
	3.1 X 10^6	4	8	12	16	20	24	--	--	--	--	2.66 X 10^6	5.1 X 10^6
0.01	4.0 X 10^6	V	M	M	M	M	M	--	--	--	--		
	11.0 X 10^6	4	8	10	12	14	16	18	20	22	--	3.42 x 10^5	5.1 x 10^5
0.05	12.0 X 10^6	M	M	M	V	V	V	V	M	V	--	3.91 x 10^5	1.8 x 10^5
	21.0 X 10^6	4	8	10	12	14	16	18	20	22	-		
0.10	22.0 X 10^6	M	M	M	V	V	M	V	M	M	--	4.06 x 10^5	3.4 x 10^5
	41.0 X 10^6	4	6	8	10	12	14	`6	18	20	22	4.82 X 10^5	6.8 x 10^5
0.20	42.0 X 10^6	M	M	M	M	V	M	V	M	M	M		
	100.6 X 10^6	4	6	8	10	12	14	16	18	20	22	4.82 X10^6	1.7 X 10^6
0.50	11.1 X 10^6	M	M	M	M	M	V	M	M	M	M		

V: Visual inspection M: Mechanical inspection

Table 2 shows the results of inspection planning for respective *PFC*'s. The more the increase in *PFC* , the more frequently and precisely the inspections are being carried out. The table compares two operating costs (*COP*s). The first *COP* was obtained for the condition that the member set was put into operation following the predicted inspection schedule. The second one was for the condition that the member set was put into operation without inspection in the entire service life. If no inspections are carried out, the values of *COP*s are influenced dramatically by the change of *PFC*. However, if inspections are carried out following the predicted schedule, the change of *PFC* is condensed about two times. This means that the inspection planning based on the cost minimization approach itself has an effect to decrease the uncertainty of the estimated operating cost. Figure 8 shows the relationship between the cumulative failure probability *PFC* and the service life for these cases. It is seen that the *PF* 's are controlled to respective levels depending on the *PFC*. Also, the more the *PFC* increases, the lower the P_F is controlled, due to the inspection frequency and the applied inspection qualities. This means that the rate of failure risk, which is uncertain part in the *COP* , becomes to be gradually decreased when *PFC* becomes large. As a result, the value of the estimated *COP* becomes insensitive for the change of *PFC* . This fact makes the estimation of *COP* easy at the inspection planning with the large degree of uncertainty of *PFC*.

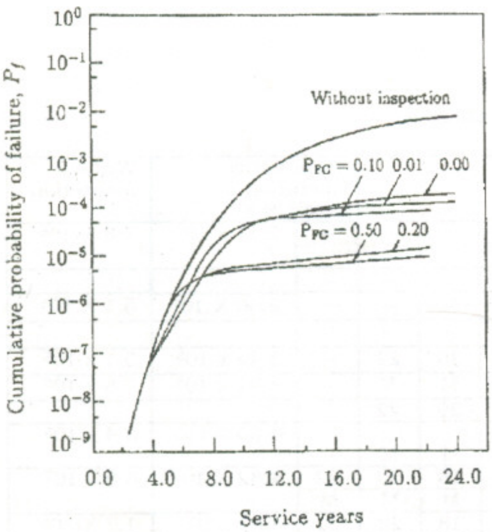

Figure 8 P_F-S_Y curves

CONCLUSIONS

In this paper, a sequential cost minimization method was used for the estimation of the inspection planning of fatigue deteriorating structures. The influence of inevitable uncertain parameters on inspection planning is discussed based on the Bayesian analysis and the sensitivity analysis.

1. The sequential cost minimization method proposed in this paper is applicable for the inspection planning of actual structures.
2. When applying the sequential cost minimization method without considering inevitable uncertainties, no inspection, or either visual inspection or mechanical inspection are usually expected. However, the *V&M* and *SM* inspection methods become profitable when the uncertainty is included in the fatigue life of members.
3. In the Bayesian analysis, the *V&M* and *SM* inspection methods were selected as optimal methods at the early stage of the service.
4. The cost minimization approach has an effect to reduce the uncertainty of the estimated operating cost when a large uncertainty exists in the failure risk of members.

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تأثير ملازمة عدم التأكد على خطة الفحص للمنشآت

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ملخص البحث

بناء على مبدأ تقليل التكلفة طوال فترة خدمة المنشأ التي تعتبر أكفاء الطرق لتحديد عمليات الفحص والصيانة والإصلاح للمنشآت المتهالكة نتيجة الكلل. على الرغم من انه فى المنشآت الحقيقية فان معظم الاحتمالات ومفردات التكلفة المتعلقة بعمليات التحليل هذه تحتوى على ملازمة عدم تأكد مما يؤدى الى حدوث أخطاء فى استنتاج خطة الفحص والصيانة للمنشأ. فى هذا البحث تم استخدام طريقة إتباع اقل التكلفة ومعادلاتها لاستنتاج أكفاء خطة الفحص والصيانة للمنشآت المتهالكة نتيجة الكلل. عمليات الاستنتاج بنيت على تكرار الطريقة المقترحة عند كل زمن فحص طوال فترة خدمة المنشأ . ثم تم تطبيق الطريقة المقترحة على منشأ مكون من عدد من الأعضاء ملحومة بلحام مستدير واخذ فى الاعتبار تأثير ملازمة عدم التأكد وكذلك حساسية التحليل على خطة الفحص لاستنتاج أكفاء خطة الفحص والصيانة للمنشآت المتهالكة نتيجة الكلل.