NEW STABILITY APPROACH APPLIED TO LARGE-SCALE POWER SYSTEMS WITH GENERATOR FLUX DECAY

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ABSTRACT

In this work, the Lyapunov's direct method is used to carry out transient stability analysis of an N-machine power system considering a more sophisticated generator model. Each generator is represented by the so-called 2-axis model [1], in which the two voltage components E'_q and E'_d of the generator internal voltage E' are considered to be changing with time. The system loads are represented by constant shunt impedances, then the system nodes (except the generator internal nodes) are eliminated, and finally the system reduced admittance matrix of the order N, is computed. Applying the decomposition-aggregation method, the system is decomposed such that each subsystem includes three machines, instead of only one machine as considered so far [2], in addition to the comparison machine. Describing each generator by a fourth-order dynamic model, and considering non-uniform mechanical damping, the system mathematical model (the transfer conductances are included) is determined and decomposed into (N-1)/3 15th -order interconnected subsystems. Each of them is decomposed into free subsystem, containing six nonlinear functions,and interconnections. A vector Lyapunov function is constructed and used for the system aggregation. A square aggregation matrix of the order (N-1)/3 is obtained, whose stability implies asymptotic stability of the system equilibrium. As an illustrative example, the developed approach is applied to a 10-machine, 11-bus power system ,and an estimate for the system asymptotic stability domain is determined. The system transient stability computations are carried out considering a 3-phase short circuit fault, and the approach is used to determine directly the critical time for clearing the fault. It is found that the developed approach is suitable and can be easily used for practical and on-line stability studies of power systems (number of machines may be more than 10). The developed approach can also reduce the conservativeness of the decomposition-aggregation method.

Keywords: Power systems, Transient stability, Two-axis model, Lyapunov method, Large scale systems.

INTRODUCTION

The study of the transient stability of power systems began in about 1920. Since then, various methods have been developed to attack the problem. The numerical methods allow accurate and detailed representation of power system,but they are not suitable for on-line application due to large computation time. The Lyapunov's direct method, which has been used to carry out analysis of power systems since 1966, has the advantage of avoiding the need for a direct solution of the system nonlinear differential equations.

As the power systems increased in size and complexity so did the difficulties in applying the Lyapunov scalar method to stability analysis of these systems, and in particular when the problem of the stability domain estimation of the system is attacked [3-6].

Because of the high efficiency of the Lyapunov's direct method, it has important applications in power system design and operation. It can be used , for example, for estimating critical fault clearing time for on-line security assessment, and for emergency control [7]. However, the direct methods of stability analysis are acknowledged to provide satisfactory practical results, as far as the use of a simplified mathematical system description may be acceptable [8].

Attempt to overcome the Lyapunov scalar method drawback have led to the application of the general decomposition-aggregation method, which is based on Bellman's concept of vector Lyapunov functions [9]. The expected advantages of the decomposition-aggregation method are, however, numerous. It is obvious that the Lyapunov function of a low-order disconnected subsystem can handle more sophisticated generator and transmission models than a multimachine one. Exact estimates of the overall system stability domain may also be defined [10].

The decomposition-aggregation method has been used for stability analysis of an N-machine power system in a number of works [5, 6, 11-20]. In those works it is noted that the stability analysis was performed considering the generator classical model (hat is, the generator internal voltage E' is considered constant), which is equivalent to neglecting the effect of generators flux decays.

In References 7, 21-23 , the authors carried out power system stability analysis assuming each generator to be represented by the so-called one-axis model, in which the generator voltage component E'_q is changing with the time while the second component E'_d is assumed constant. Further, the authors used different forms of (scalar). Lyapunov functions which were constructed neglecting the transfer conductances G_{ij} . This essentially means that resistances of the system lines are neglected, in addition the system network cannot be simplified by eliminating the nodes at which system loads are connected.

In Reference 24, transient stability analysis of an N-machine power system was carried out considering the transfer conductances, and assuming the generator two internal voltage components E'_q and E'_d to be changing with time. The system was decomposed into (N-1), two-machine subsystems, and it is obtained (the uniform mechanical damping was considered)an aggregation matrix of the order (N - 1). A 3-machine, 4-bus , power system was used as an illustrative example.

In the present work, transient stability analysis of an N-machine power system is performed considering non-uniform mechanical damping, transfer conductances, and variations of the generator internal voltage E' in both the quadrature and direct axes. Applying the "four-machine" decomposition, the system is decomposed into only (N-1)/3, interconnected subsystems.

A scalar Lyapunov function of the form "quadratic form + sum of the integrals of six nonlinear functions", is accepted for each free subsystem. The free subsystem Lyapunov functions are used for constructing a vector Lyapunov function by which the system is aggregated. An aggrergation matrix of the order (N-1)/ 3, is obtained for the considered system.

POWER SYSTEM MODEL

Let us consider an N-machine power system with mechanical damping, and assume that the machine parameters M_i and P_{mi} of each machine are constant.

Now neglecting the stator resistance and assuming each machine to be represented by the two-axis model [1], in which the two voltage components E'_q and E'_d of the internal voltage E', are considered to be changing with the time. Then, the absolute motion of the i-th machine is described by the following three differential equations (see Notation),

$$M_i \ddot{\delta}_i + D_i \dot{\delta}_i = P_{mi} - P_{ei}$$
$$T'_{d0i} \dot{E}'_{qi} = E_{fdi} - E'_{qi} + (X_{di} - X'_{di}) I_{di}$$
$$T'_{q0i} \dot{E}'_{di} = -E'_{di} - (X_{qi} - X'_{qi}) I_{qi} \qquad (1)$$

where

$$P_{ei} = E'_{di} I_{di} + E'_{qi} I_{qi} - (X'_{qi} - X'_{di}) I_{di} I_{qi}$$
$$i = 1,2,\ldots\ldots,N \qquad (2)$$

Under the assumption $X'_{di} = X'_{qi}$ (machines with solid cylindrical rotors are considered), we get ,

$$P_{ei} = \sum_{j=1}^{N} Y_{ij}\{E'_{qi}\,[E'_{qj}\cos(\theta_{ij} - \delta_{ij}) - E'_{dj}\sin(\theta_{ij} - \delta_{ij})] + E'_{di}\,[E'_{dj}\cos(\theta_{ij} - \delta_{ij}) + E'_{qj}\sin(\theta_{ij} - \delta_{ij})]\},\ i = 1,2,\ldots\ldots,N \qquad (3)$$

where δ_i , is the rotor angle of the i-th machine, or position of the rotor q-axis from the common reference frame.

Note carefully that, in this work , the dynamics of the automatic voltage regulator (AVR) has been, for simplicity, neglected , and hence the voltage E_{fdi} in Equation 1 , is equal to its pretransient value $\hat{E}_{fdi}$.

Now, let us select the Nth machine as a comparison machine, and let the following (4N -1) state variables to be introduced,

$$\sigma_{iN} = \delta_{iN} - \delta^{\circ}_{iN}\ ,\ i \neq N$$

$$\omega_i = \dot{\delta}_i \qquad ,\ i = ,2,\ldots\ldots\ldots,N$$

$$E_{Qi} = E'_{qi} - \hat{E}'_{qi} \quad ; \quad E_{Di} = E'_{di} - \hat{E}'_{di}\ ,\ i = 1,2,\ldots\ldots,N \qquad (4)$$

where δ°_{iN} , $\hat{E}'_{di}$ and $\hat{E}'_{qi}$ are the steady-state (pretransient) values of δ_{iN}, E'_{di} and E'_{qi}, respectively.

Hence, the overall system motion is governed by the following state equations:

$$\dot{\sigma}_{iN} = \omega_i - \omega_N = \omega_{iN}$$

$$\dot{\omega}_i = \lambda_i\omega_i - (1/M_i)\{G_{ii}[E^2_{Qi} + 2E_{Qi}\hat{E}'_{qi} + E^2_{Di} + 2E_{Di}\hat{E}'_{di}] + \Sigma Y_{ij}\{A_{ij}\, f_{ij}(\sigma_{ij}) + \hat{A}_{ij}\, g_{ij}(\sigma_{ij})$$
$$+ [\hat{E}'_{iQj} + \hat{E}'_{di}E_{Dj}]\,\hat{f}_{ij}(\sigma_{ij}) - [\hat{E}'_{di}E_{Qj} - \hat{E}'_{qi}E_{Dj}]\,\hat{g}_{ij}(\sigma_{ij}) + [E_{Qi}(E_{Qj} + \hat{E}'_{qj}) +$$
$$+ E_{Di}(E_{Dj} + \hat{E}'_{dj})]\,\hat{f}_{ij}(\sigma_{ij}) - [E_{Di}(E_{Qj} + \hat{E}'_{qj}) - E_{Qi}(E_{Dj} + \hat{E}'_{dj})]\,\hat{g}_{ij}(\sigma_{ij})\}\}$$

$$\dot{E}_{Qi} = -\Gamma_i E_{Qi} + K_i\{G_{ii}E_{Di} + \Sigma Y_{ij}[\hat{E}'_{dj}\, f_{ij}(\sigma_{ij}) - \hat{E}'_{qj}\, g_{ij}(\sigma_{ij})$$
$$- E_{Qj}\,\hat{g}_{ij}(\sigma_{ij}) + E_{Dj}\,\hat{f}_{ij}(\sigma_{ij})]\}$$

$$\dot{E}_{Di} = -\mu_i E_{Di} - L_i\{G_{ii}E_{Qi} + \Sigma Y_{ij}[\hat{E}'_{qj}\, f_{ij}(\sigma_{ij}) + \hat{E}'_{dj}\, g_{ij}(\sigma_{ij})$$
$$+ E_{Qj}\,\hat{f}_{ij}(\sigma_{ij}) + E_{Dj}\,\hat{g}_{ij}(\sigma_{ij})]\}\ ,\ i = 1,2,\ldots,N \qquad (5)$$

where, Σ is defined as $\sum_{j\neq 1}^{N}$, and the following nonlinear functions are defined,

$$f_{ij}(\sigma_{ij}) = \cos(\sigma_{ij} + \delta^{\circ}_{ij} - \theta_{ij}) - \cos(\delta^{\circ}_{ij} - \theta_{ij})$$
$$g_{ij}(\sigma_{ij}) = \sin(\sigma_{ij} + \delta^{\circ}_{ij} - \theta_{ij}) - \sin(\delta^{\circ}_{ij} - \theta_{ij})$$
$$\hat{f}_{ij}(\sigma_{ij}) = \cos(\sigma_{ij} + \delta^{\circ}_{ij} - \theta_{ij})$$
$$\hat{g}_{ij}(\sigma_{ij}) = \sin(\sigma_{ij} + \delta^{\circ}_{ij} - \theta_{ij})$$
$$,\ i \neq j\ ,\ \ i, j = 1,2,\ldots, N \qquad (6)$$

It is to be noted that the two nonlinearities f_{ij} and g_{ij} of Equation 6, satisfy the following conditions

$$f_{ij}(0) = g_{ij}(0) = 0;\ 0 < f_{ij}(\sigma_{ij})/\sigma_{ij} \leq \xi_{ij}\ ;$$
$$0 < |g_{ij}(\sigma_{ij})/\sigma_{ij}| \leq \tilde{\xi}_{ij}\ ,\ \sigma_{ij} \neq 0\ ,$$
$$i \neq j\ ,\ i\ ,\ j = 1,2,\ldots, N \qquad (7)$$

where ξ_{ij} and $\tilde{\xi}_{ij}$ are positive numbers, and may be determined as,

$$\xi_{ij} = (\partial f_{ij}(\sigma_{ij})/\partial\sigma_{ij})\,|\sigma_{ij} = 0$$

and

$$\tilde{\xi}_{ij} = (\,|\,\partial g_{ij}(\sigma_{ij})/\partial\sigma_{ij}\,|)\,|\sigma_{ij} = 0 \qquad (8)$$

POWER SYSTEM DECOMPOSITION

In this paper, decomposition of the considered N-machine system is carried out as follows:

1. All the system loads are represented by constant impedances to ground (those impedances are obtained from the pretransient conditions of the system).
2. All the system nodes, except generators internal nodes, are eliminated . Hence, we obtain the system Nth-order reduced admittance matrix Y .
3. To reduce the conservativeness of the decomposition-aggregation method, it is found that each subsystem should contain a largest number of machines [16,17,19,20]. Accordingly the considered power system is decomposed such that each subsystem includes four machines, instead of only two machines as proposed so far [2].

Referring to the obtained Y-matrix, and using the "four-machine" decomposition [19] the system is decomposed into only (N-1)/ 3 subsystems, instead of (N-1) subsystems for the pair-wise decomposition.

Now, let us define the state vector X_I in the form

$$X_I = [\sigma_{iI,N},\ \sigma_{iI+1,N}, \sigma_{iI+2,N}, \omega_{iI}, \omega_{iI+1}, \omega_{iI+2}, \omega_N, E_{QiI},\ E_{QiI+1},\ E_{QiI+2}, E_{QN}, E_{DiI}, E_{DiI+1},\ E_{DiI+2}, E_{DN}]^T = [X_{I1}, X_{I2}, X_{I3}, \ldots\ldots, X_{I14}, X_{I15}]^T \quad (9)$$

Then, the mathematical model of the whole system (Equation 5) can be decomposed into S = (N-1)/3 15th-order interconnected subsystems, each of them can be written in the general form

$$\dot{X}_I = P_I X_I + B_I F_I(\sigma_I) + h_I(X),\ \sigma_I = C^T_I X_I, \quad I = 1,2,\ldots\ldots, S \quad (10)$$

where P_I , B_I and C_I are constant matrices with appropriate dimensions, and $F_I(\sigma_I)$ is a nonlinear vector function, whose elements are arbitrary chosen.

It is of importance to note that each subsystem of Equation 10, may be decomposed into the free (disconnected) subsystem

$$\dot{X}_I = P_I X_I + B_I F_I(\sigma_I), \quad \sigma_I = C^T_I X_I, \quad I = 1,2,\ldots\ldots, S \quad (11)$$

and the interconnections $h_I(X)$.

Referring to Equations. 5 and 9, the matrix P_I is derived in the form (see Notation)

$$P_I = \left[\begin{array}{c|c|c|c} & I_3 & -a & O_{3\times 8} \\ \hline O_{15\times 3} & -P_{I1} & -2P_{I2} & -2P_{I3} \\ \hline & O_{8\times 4} & P_{I4} & P_{I5} \\ & & P_{I6} & -P_{I7} \end{array}\right] \quad (12)$$

where 0 and I , are zero and identity (square) matrices, respectively, of the indicated dimensions , and where

$$a = [\,1.0\ ,\ 1.0\ ,\ 1.0\,]^T$$
$$P_{I1} = \mathrm{diag}\,[\,\lambda_{iI}, \lambda_{iI+1}, \lambda_{iI+2}, \lambda_N]$$
$$P_{I2} = \mathrm{diag}\,[\hat{E}'_{qiI} G_{iI,iI}/M_{iI},\ \hat{E}'_{qiI+1} G_{iI+1,iI+1}/M_{iI+1},\ \hat{E}'_{qiI+2} G_{iI+2,iI+2}/M_{iI+2},\ \hat{E}'_{qN} G_{N,N}/M_N]$$
$$P_{I3} = \mathrm{diag}\,[\hat{E}'_{diI} G_{iI,iI}/M_{iI},\ \hat{E}'_{diI+1} G_{iI+1,iI+1}/M_{iI+1},\ \hat{E}'_{diI+2} G_{iI+2,iI+2}/M_{iI+2},\ \hat{E}'_{dN} G_{N,N}/M_N]$$
$$P_{I4} = \mathrm{diag}\,[\Gamma_{iI}, \Gamma_{iI+1}, \Gamma_{iI+2}, \Gamma_N]$$
$$P_{I5} = \mathrm{diag}\,[K_{iI} G_{iI,iI}, K_{iI+1} G_{iI+1,iI+1}, K_{iI+2} G_{iI+2,iI+2}, K_N G_{N,N}]$$
$$P_{I6} = \mathrm{diag}\,[L_{iI} G_{iI,iI}, L_{iI+1} G_{iI+1,iI+1}, L_{iI+2} G_{iI+2,iI+2}, L_N G_{N,N}]$$
$$P_{I7} = \mathrm{diag}\,[\mu_{iI}, \mu_{iI+1}, \mu_{Ii+2}, \mu_N] \quad (13)$$

Now, to define the matrices B_I , C_I and h_I, we must at first select elements of the nonlinear functions F_I. However, a largest asymptotic stability domain estimate can be obtained for a power system provided that each interconnected subsystem of Equation 10, is decomposed such that the free subsystem contains the largest number of nonlinearities [16,17,19,20].

Applying the proposed "four-machine" decomposition, it is found that each free subsystem can include at most 24 nonlinear functions, instead of only four nonlinearities for the two-machine decomposition [2]. However it is found, after expanding the free subsystem 24 nonlinearities, that there are only six nonlinear functions satisfy the Lurie sector condition on bounded intervals. These six functions are given as,

$$f_{I1}(\sigma_{I1}) = \sin(\sigma_{iI,N} + \delta^{\circ}_{iI,N}) - \sin(\delta^{\circ}_{iI,N})$$
$$f_{I2}(\sigma_{I2}) = \sin(\sigma_{iI+1,N} + \delta^{\circ}_{iI+1,N}) - \sin(\delta^{\circ}_{iI+1,N})$$
$$f_{I3}(\sigma_{I3}) = \sin(\sigma_{iI+2,N} + \delta^{\circ}_{iI+2,N}) - \sin(\delta^{\circ}_{iI+2,N})$$
$$f_{I4}(\sigma_{I4}) = \sin(\sigma_{iI,iI+1} + \delta^{\circ}_{iI,iI+1}) - \sin(\delta^{\circ}_{iI,iI+1})$$
$$f_{I5}(\sigma_{I5}) = \sin(\sigma_{iI,iI+2} + \delta^{\circ}_{iI,iI+2}) - \sin(\delta^{\circ}_{iI,iI+2})$$
$$f_{I6}(\sigma_{I6}) = \sin(\sigma_{iI+1,iI+2} + \delta^{\circ}_{iI+1,iI+2}) - \sin(\delta^{\circ}_{iI+1,iI+2}) \quad (14)$$

It is to be noticed that for the nonlinear functions of Equation 14, there exist positive constants $\varepsilon_{Ik} \in (0, \xi_{Ik})$, for which the following condition

$$\sigma_{Ik}\, f_{Ik}(\sigma_{Ik}) \geq \varepsilon_{Ik}\, \sigma^2_{Ik}, \quad k = 1,2,\ldots,6 \quad (15)$$

is satisfied on the compact interval of σ_{Ik},

$$U_{Ik} = [\underline{U}_{Ik}, \overline{U}_{Ik}], \quad k = 1,2,\ldots,6 \quad (16)$$

where $\underline{U}_{Ik}$, $\overline{U}_{Ik}$ are the negative and positive solutions, respectively, of the equation

$$f_{Ik}(\sigma_{Ik}) = \varepsilon_{Ik}\, \sigma_{Ik}, \quad k=1,2,\ldots,6 \quad (17)$$

Using the six functions of Equation 14, the nonlinear vector F_I is constructed in the form:

$$F_I(\sigma_I) = [f_{I1}(\sigma_{I1}), f_{I2}(\sigma_{I2}), f_{I3}(\sigma_{I3}), f_{I4}(\sigma_{I4}), f_{I5}(\sigma_{I5}), f_{I6}(\sigma_{I6})]^T \quad (18)$$

Hence, the corresponding C_I and B_I matrices are derived, referring to Equation. 6, as

$$C^T_I = \left[\begin{array}{ccc|c} & I_3 & & \\ \hline 1.0 & -1.0 & 0 & O_{6\times 12} \\ 1.0 & 0 & -1.0 & \\ 0 & 1.0 & -1.0 & \end{array}\right] \quad (19)$$

$$B_I = \begin{bmatrix} \multicolumn{2}{c}{0_{3\times 6}} \\ -B_{I1} & B_{I4} \\ a_{I1} & O_{1\times 3} \\ B_{I2} & B_{I5} \\ -a_{I2} & O_{1\times 3} \\ -B_{I3} & B_{I6} \\ a_{I3} & O_{1\times 3} \end{bmatrix} \quad (20)$$

where O is zero matrix with indicated dimension, and where (see Notation),

$$B_{I1} = \text{diag}\,[(H_{iI,N} + \hat{H}_{iI,N})/M_{iI},\ (H_{iI+1,N} + \hat{H}_{iI+1,N})/M_{iI+1},\ (H_{iI+2,N} + \hat{H}_{iI+2,N})/M_{iI+2}]$$
$$B_{I2} = \text{diag}\,[K_{iI}(\tilde{S}_{iI,N} - \overline{S}_{iI,N}),\ K_{iI+1}(\tilde{S}_{iI+1,N} - \overline{S}_{iI+1,N}),\ K_{iI+2}(\tilde{S}_{iI+2,N} - \overline{S}_{iI+2,N})]$$
$$B_{I3} = \text{diag}\,[L_{iI}(S_{iI,N} + \hat{S}_{iI,N}),\ L_{iI+1}(S_{iI+1,N} + \hat{S}_{iI+1,N}),\ L_{iI+2}(S_{iI+2,N} + \hat{S}_{iI+2,N})]$$
$$a_{I1} = (1/M_N)\,[(H_{iI,N} - \hat{H}_{iI,N}),\ (H_{iI+1,N} - \hat{H}_{iI+1,N}),\ (H_{iI+2,N} - \hat{H}_{iI+2,N})]$$
$$a_{I2} = K_N\,[(\tilde{S}_{N,iI} - \overline{S}_{N,iI}),\ (\tilde{S}_{N,iI+1} - \overline{S}_{N,iI+1}),\ (\tilde{S}_{N,iI+2} - \overline{S}_{N,iI+2})]$$
$$a_{I3} = L_N\,[(\hat{S}_{N,iI} + S_{N,iI}),\ (\hat{S}_{N,iI+1} + S_{N,iI+1}),\ (\hat{S}_{N,iI+2} + S_{N,iI+2})]$$

$$B_{I4}=\begin{bmatrix} -(H_{iI,iI+1}+\hat{H}_{iI,iI+1})/M_{iI} & -(H_{iI,iI+2}+\hat{H}_{iI,iI+2})/M_{iI} & 0 \\ (H_{iI,iI+1}-\hat{H}_{iI,iI+1})/M_{iI+1} & 0 & -(H_{iI+1,iI+2}+\hat{H}_{iI+1,iI+2})/M_{iI+1} \\ 0 & (H_{iI,iI+2}-\hat{H}_{iI,iI+2})/M_{iI+2} & (H_{iI+1,iI+2}-\hat{H}_{iI+1,iI+2})/M_{iI+2} \end{bmatrix}$$

$$B_{I5}=\begin{bmatrix} K_{iI}(\tilde{S}_{iI,iI+1}-\overline{S}_{iI,iI+1}) & K_{iI}(\tilde{S}_{iI,iI+2}-\overline{S}_{iI,iI+2}) & 0 \\ -K_{iI+1}(\tilde{S}_{iI+1,iI}-\overline{S}_{iI+1,iI}) & 0 & K_{iI+1}(\tilde{S}_{iI+1,iI+2}-\overline{S}_{iI+1,iI+2}) \\ 0 & -K_{iI+2}(\tilde{S}_{iI+2,iI}-\overline{S}_{iI+2,iI}) & -K_{iI+2}(\tilde{S}_{iI+2,iI+1}-\overline{S}_{iI+2,iI+1}) \end{bmatrix}$$

$$B_{I6}=\begin{bmatrix} -L_{iI}(\hat{S}_{iI,iI+1}+S_{iI,iI+1}) & -L_{iI}(\hat{S}_{iI,iI+2}+S_{iI,iI+2}) & 0 \\ -L_{iI+1}(\hat{S}_{iI+1,iI}+S_{iI+1,iI}) & 0 & -L_{iI+1}(\hat{S}_{iI+1,iI+2}+S_{iI+1,iI+2}) \\ 0 & L_{iI+2}(\hat{S}_{iI+2,iI}+S_{iI+2,iI}) & L_{iI+2}(\hat{S}_{iI+2,iI+1}+S_{iI+2,iI+1}) \end{bmatrix} \quad (21)$$

Now, using Equations 9,12,18-20, the free subsystem of Equation 11, is completely defined. Referring to Equation 5, the interconnections,that is, the matrix $h_I(X)$, is derived in the form,

$$h_I(X)=[\,0\,,0\,,0,\; h_{I4}(X)\,,h_{I5}(X),\ldots\ldots,\; h_{I14}(X)\,,h_{I15}(X)\,]^T \quad (22)$$

where,

$$\begin{aligned} h_{I4}(X)= -(1/M_{iI})\{\overline{\sum} & Y_{iI,j}[A_{iI,j}\,f_{iI,j}(\sigma_{iI,j})+\hat{A}_{iI,j}\,g_{iI,j}(\sigma_{iI,j})+(X_{I8}+\hat{E}_{qiI})[E_{Qj}\hat{f}_{iI,j}+E_{Dj}\hat{g}_{iI,j}] \\ & +(X_{I12}+\hat{E}'_{diI})[E_{Dj}\hat{f}_{iI,j}-E_{Qj}\hat{g}_{iI,j}]+X_{I8}\,\hat{E}'_{qj}\,\hat{f}_{iI,j}+\hat{E}'_{dj}\,\hat{g}_{iI,j}]+X_{I12}\,[\hat{E}'_{dj}\,\hat{f}_{iI,j}- \\ & \hat{E}_{qj}\,\hat{g}_{iI,j}]]+(\tilde{H}_{iI,N}-\overline{H}_{iI,N})\,R_{I1}(\sigma_{I1})+(\tilde{H}_{iI,iI+1}-\overline{H}_{iI,iI+1})\,R_{I4}(\sigma_{I4})+(\tilde{H}_{iI,iI+2}-\overline{H}_{iI,iI+2}) \\ & R_{I5}(\sigma_{I5})\} \end{aligned}$$

$$\begin{aligned} h_{I5}(X)= -(1/M_{iI+1})\{\overline{\sum} & Y_{iI+1,j}[A_{iI+1,j}\,f_{iI+1,j}(\sigma_{iI+1,j})+\hat{A}_{iI+1,j}\,g_{iI+1,j}(\sigma_{iI+1,j}) \\ & +(X_{I9}+\hat{E}'_{qiI+1})[E_{Qj}\,\hat{f}_{iI+1,j}+E_{Dj}\,\hat{g}_{iI+1,j}]+(X_{I13}+\hat{E}'_{diI+1})[E_{Dj}\,\hat{f}_{iI+1,j}-E_{Qj} \\ & \hat{g}_{iI+1,j}]+X_{I9}\,[\hat{E}'_{qj}\hat{f}_{iI+1,j}+E'_{dj}\,\hat{g}_{iI+1,j}]+X_{I13}\,[\hat{E}'_{dj}\,\hat{f}_{iI+1,j}-\hat{E}'_{qj}\,\hat{g}_{iI+1,j}]]+ \\ & (\tilde{H}_{iI+1,N}-\overline{H}_{iI+1,N})\,R_{I2}(\sigma_{I2})+(\tilde{H}_{iI+1,iI}+\overline{H}_{iI+1,iI})\,R_{I4}(\sigma_{I4})+(\tilde{H}_{iI+1,iI+2}- \\ & \overline{H}_{iI+1,iI+2})\,R_{I6}(\sigma_{I6})\} \end{aligned}$$

$$\begin{aligned} h_{I6}(X)= -(1/M_{iI+2})\{\overline{\sum} & Y_{iI+2,j}[A_{iI+2,j}\,f_{iI+2,j}(\sigma_{iI+2,j})+\hat{A}_{iI+2,j}\,g_{iI+2,j}(\sigma_{iI+2,j})+ \\ & (X_{I10}+\hat{E}'_{qiI+2})[E_{Qj}\,\hat{f}_{iI+2,j}+E_{Dj}\,\hat{g}_{iI+2,j}]+(X_{I14}+\hat{E}'_{diI+2})[E_{Dj}\,\hat{f}_{iI+2,j}-E_{Qj}\,\hat{g}_{iI+2,j}]+ \\ & X_{I10}\,[\hat{E}'_{qj}\,\hat{f}_{iI+2,j}+??'_{dj}\,\hat{g}_{iI+2,j}]+X_{I14}\,[\hat{E}'_{dj}\,\hat{f}_{iI+2,j}-\hat{E}'_{qj}\,??_{iI+2,j}]]+(\tilde{H}_{iI+2,N}-\overline{H}_{iI+2,N})\,R_{I3} \\ & (\sigma_{I3})+(\tilde{H}_{iI+2,iI}+\overline{H}_{iI+2,iI})\,R_{I5}(\sigma_{I5})+(\tilde{H}_{iI+2,iI+1}+\overline{H}_{iI+2,iI+1})\,R_{I6}(\sigma_{I6})\} \end{aligned}$$

$$h_{I7}(X) = -(1/M_N)\{\overline{\sum} Y_{N,j}[A_{N,j}\, f_{N,j}(\sigma_{N,j}) - \hat{A}_{N,j}\, g_{N,j}(\sigma_{N,j}) + (X_{I11} + \hat{E}'_{qN})[E_{Qj}\,\hat{f}_{N,j} + E_{Dj}\,\hat{g}_{N,j}] + (X_{I15} + \hat{E}'_{dN})[E_{Dj}\,\hat{f}_{N,j} - E_{Qj}\,\hat{g}_{N,j}] + X_{I11}[\hat{E}'_{qj}\,\hat{f}_{N,j} + \hat{E}'_{dj}\,\hat{g}_{N,j}] + X_{I15}[\hat{E}'_{dj}\,\hat{f}_{N,j} - \hat{E}'_{qj}\,\hat{g}_{N,j}]] + (\tilde{H}_{iI,N} + \overline{H}_{iI,N}) R_{I1}(\sigma_{I1}) + (\tilde{H}_{iI+1,N} + \overline{H}_{iI+1,N}) R_{I2}(\sigma_{I2}) + (\tilde{H}_{iI+2,N} + \overline{H}_{iI+2,N}) R_{I3}(\sigma_{I3})\}$$

$$h_{I8}(X) = K_{iI}\{\overline{\sum} Y_{iI,j}[\hat{E}'_{dj}\, f_{iI,j}(\sigma_{iI,j}) - \hat{E}'_{qj}\, g_{iI,j}(\sigma_{iI,j}) - E_{Qj}\,\hat{g}_{iI,j} + E_{Dj}\,\hat{f}_{iI,j}] + (S_{iI,N} + \hat{S}_{iI,N}) R_{I1}(\sigma_{I1}) + (\hat{S}_{iI,iI+1} + S_{iI,iI+1}) R_{I4}(\sigma_{I4}) + (\hat{S}_{iI,iI+2} + S_{iI,iI+2}) R_{I5}(\sigma_{I5})\}$$

$$h_{I9}(X) = K_{iI+1}\{\overline{\sum} Y_{iI+1,j}[\hat{E}'_{dj}\, f_{iI+1,j}(\sigma_{iI+1,j}) - \hat{E}'_{qj}\, g_{iI+1,j}(\sigma_{iI+1,j}) - E_{Qj}\,\hat{g}_{iI+1,j} + E_{Dj}\,\hat{f}_{iI+1,j}] + (S_{iI+1,N} + \hat{S}_{iI+1,N}) R_{I2}(\sigma_{I2}) + (\hat{S}_{iI+1,iI} + S_{iI+1,iI}) R_{I4}(\sigma_{I4}) + (\hat{S}_{iI+1,iI+2} + S_{iI+1,iI+2}) R_{I6}(\sigma_{I6})\}$$

$$h_{I10}(X) = K_{iI+2}\{\overline{\sum} Y_{iI+2,j}[\hat{E}'_{dj}\, f_{iI+2,j}(\sigma_{iI+2,j}) - \hat{E}'_{qj}\, g_{iI+2,j}(\sigma_{iI+2,j}) - E_{Qj}\,\hat{g}_{iI+2,j} + E_{Dj}\,\hat{f}_{iI+2,j}] + (S_{iI+2,N} + \hat{S}_{iI+2,N}) R_{I3}(\sigma_{I3}) + (\hat{S}_{iI+2,iI} + S_{iI+2,iI}) R_{I5}(\sigma_{I5}) + (\hat{S}_{iI+2,iI+1} + S_{iI+2,iI+1}) R_{I6}(\sigma_{I6})\}$$

$$h_{I11}(X) = K_N\{\overline{\sum} Y_{N,j}[\hat{E}'_{dj}\, f_{N,j}(\sigma_{N,j}) - \hat{E}'_{qj}\, g_{N,j}(\sigma_{N,j}) - E_{Qj}\,\hat{g}_{N,j} + E_{Dj}\,\hat{f}_{N,j}] + (S_{N,iI} + \hat{S}_{N,iI}) R_{I1}(\sigma_{I1}) + (\hat{S}_{N,iI+1} + S_{N,iI+1}) R_{I2}(\sigma_{I2}) + (\hat{S}_{N,iI+2} + S_{N,iI+2}) R_{I3}(\sigma_{I3})\}$$

$$h_{I12}(X) = -L_{iI}\{\overline{\sum} Y_{iI,j}[\hat{E}'_{qj}\, f_{iI,j}(\sigma_{iI,j}) + \hat{E}'_{dj}\, g_{iI,j}(\sigma_{iI,j}) + E_{Qj}\,\hat{f}_{iI,j} + E_{Dj}\,\hat{g}_{iI,j}] + (\overline{S}_{iI,N} - \tilde{S}_{iI,N}) R_{I1}(\sigma_{I1}) + (\overline{S}_{iI,iI+1} - \tilde{S}_{iI,iI+1}) R_{I4}(\sigma_{I4}) + (\overline{S}_{iI,iI+2} - \tilde{S}_{iI,iI+2}) R_{I5}(\sigma_{I5})\}$$

$$h_{I13}(X) = -L_{iI+1}\{\overline{\sum} Y_{iI+1,j}[\hat{E}'_{qj}\, f_{iI+1,j}(\sigma_{iI+1,j}) + \hat{E}'_{dj}\, g_{iI+1,j}(\sigma_{iI+1,j}) + E_{Qj}\,\hat{f}_{iI+1,j} + E_{Dj}\,\hat{g}_{iI+1,j}] + (\overline{S}_{iI+1,N} - \tilde{S}_{iI+1,N}) R_{I2}(\sigma_{I2}) + (\overline{S}_{iI+1,iI} - \tilde{S}_{iI+1,iI}) R_{I4}(\sigma_{I4}) + (\overline{S}_{iI+1,iI+2} - \tilde{S}_{iI+1,iI+2}) R_{I6}(\sigma_{I6})\}$$

$$h_{I14}(X) = -L_{iI+2}\{\overline{\sum} Y_{iI+2,j}[\hat{E}'_{qj}\, f_{iI+2,j}(\sigma_{iI+2,j}) + \hat{E}'_{dj}\, g_{iI+2,j}(\sigma_{iI+2,j}) + E_{Qj}\,\hat{f}_{iI+2,j} + E_{Dj}\,\hat{g}_{iI+2,j}] + (\overline{S}_{iI+2,N} - \tilde{S}_{iI+2,N}) R_{I3}(\sigma_{I3}) + (\overline{S}_{iI+2,iI} - \tilde{S}_{iI+2,iI}) R_{I5}(\sigma_{I5}) + (\overline{S}_{iI+2,iI+1} - \tilde{S}_{iI+2,iI+1}) R_{I6}(\sigma_{I6})\}$$

$$h_{I15}(X) = -L_N\{\overline{\sum} Y_{N,j}[\hat{E}'_{qj}\, f_{N,j}(\sigma_{N,j}) + \hat{E}'_{dj}\, g_{N,j}(\sigma_{N,j}) + E_{Qj}\,\hat{f}_{N,j} - E_{Dj}\,\hat{g}_{N,j}] + (\overline{S}_{N,iI} - \tilde{S}_{N,iI}) R_{I1}(\sigma_{I1}) + (\overline{S}_{N,iI+1} - \tilde{S}_{N,iI+1}) R_{I2}(\sigma_{I2}) + (\overline{S}_{N,iI+2} - \tilde{S}_{N,iI+2}) R_{I3}(\sigma_{I3})\} \qquad (23)$$

In Equation 23 , $\overline{\Sigma}$ is defined as, $\sum_{j\notin JIN}$ and the nonlinear functions $\hat{f}_{k,j}$ and $\hat{g}_{k,j}$, $k\in J_{IN}$, are given by Equation 6. Also in Equation 23, the following six nonlinearities are defined,

$$R_{I1}(\sigma_{I1}) = \cos(\sigma_{iI,N} + \delta^{\circ}_{iI,N}) - \cos(\delta^{\circ}_{iI,N})$$
$$R_{I2}(\sigma_{I2}) = \cos(\sigma_{iI+1,N} + \delta^{\circ}_{iI+1,N}) - \cos(\delta^{\circ}_{iI+1,N})$$
$$R_{I3}(\sigma_{I3}) = \cos(\sigma_{iI+2,N} + \delta^{\circ}_{iI+2,N}) - \cos(\delta^{\circ}_{iI+2,N})$$
$$R_{I4}(\sigma_{I4}) = \cos(\sigma_{iI,iI+1} + \delta^{\circ}_{iI,iI+1}) - \cos(\delta^{\circ}_{iI,iI+1})$$
$$R_{I5}(\sigma_{I5}) = \cos(\sigma_{iI,iI+2} + \delta^{\circ}_{iI,iI+2}) - \cos(\delta^{\circ}_{iI,iI+2})$$
$$R_{I6}(\sigma_{I6}) = \cos(\sigma_{iI+1,iI+2} + \delta^{\circ}_{iI+1,iI+2}) - \cos(\delta^{\circ}_{iI+1,iI+2}) \quad (24)$$

POWER SYSTEM AGGREGATION

An aggregation matrix, $A = [\alpha_{IJ}]$, is constructed, whose elements (real numbers) obey the inequality [24]

$$\dot{V}_I(X_I) \le \sum_{j=1}^{S} \alpha_{IJ}\, U_I(X_I)\, U_J(X_J), \quad I = 1,2,\ldots,S \quad (25)$$

where U_I and U_J are comparison functions, and they are chosen in the form [13]

$$U_k(X_k) = \| X_k \| = (X^T_k X_k)^{1/2} \quad \text{for } k = 1,2,\ldots,S \quad (26)$$

In Equation 25 , $V_I(X_I)$ is a Lyapunov function for the Ith free subsystem, and it is selected, in this work, in the form [6,12, 13, 16-18, 20] ,

$$V_I(X_I) = X_I^T H_I X_I + \sum_{m=1}^{6} \gamma_{Im} \int_0^{\sigma I} f_{Im}(\sigma_{Im})\, d\sigma_{Im}, \quad I = 1,2,\ldots,S \quad (27)$$

where H_I ,is an 15th-order symmetric positive definite matrix , the functions f_{Im} are given by Equation 14, and γ_{Im} are arbitrary positive numbers, and they are chosen in the form (see Notation),

$$\gamma_{I1} = 2\, h^I_{44}\,(\hat{H}_{iI,N} + H_{iI,N})/M_{iI},$$
$$\gamma_{I2} = 2\, h^I_{55}\,(\hat{H}_{iI+1,N} + H_{iI+1,N})/M_{iI+1}$$
$$\gamma_{I3} = 2\, h^I_{66}\,(\hat{H}_{iI+2,N} + H_{iI+2,N})/M_{iI+2}$$
$$\gamma_{I4} = 2\, h^I_{44}\,(\hat{H}_{iI,iI+1} + H_{iI,iI+1})/M_{iI}$$
$$\gamma_{I5} = 2h^I_{44}(\hat{H}_{iI,iI+2} + H_{iI,iI+2})/M_{iI},$$
$$\gamma_{I6} = 2h^I_{55}(\hat{H}_{iI+1,iI+2} + H_{iI+1,iI+2})/M_{iI+1} \quad (28)$$

where h^I_{44} , h^I_{55} and h^I_{66} are arbitrary (positive) diagonal elements of the matrix H_I.

It is to be noted that, the left-hand side of Equation 25 can be written in the form

$$\dot{V}_I(X_I) = \dot{V}_I(X_I)_f + [\text{grad } V_I(X_I)]^T h_I(X), \quad I=1,2,3\ldots,S \quad (29)$$

where ??$_I$(X_I)$_f$ is the total time derivative of V_I (X_I) along the motion of the Ith free subsystem of Equation 11.

Stability Criterion

According to theorem 1 of Reference 24 , stability of the aggregation matrix , $A=[\alpha_{ik}]$, or, equivalently, if it is satisfied the Hick`s conditions

$$(-1)^k \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1k} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2k} \\ & \vdots & & \\ \alpha_{k1} & \alpha_{k2} & \cdots & \alpha_{kk} \end{bmatrix} > 0, \quad k=1,2,\ldots,S \quad (30)$$

implies asymptotic stability of the system equilibrium.

Aggregation Matrix

To determine an aggregation matrix for the considered power system, the two terms in the right-hand side of Equation 29 , are computed .Then they are completely majorized using the following majorizations,

$$|R_{Ik}(\sigma_{Ik})| \le \hat{\xi}_{Ik}|\sigma_{Ik}|,\ \hat{\xi}_{Ik} = |\partial R_{Ik}(\sigma_{Ik})/\partial\sigma_{Ik}|\sigma_{Ik}=0 \ ,$$
$$k=1,2,\ldots,6$$

$$|f_{ij}(\sigma_{ij})| \le \xi_{ij}(|X_i| + |X_j|), \xi_{ij} = |\sin(\theta_{ij} - \delta^\circ_{ij})|$$

$$|g_{ij}(\sigma_{ij})| \le \hat{\xi}_{ij}(|X_j| + |X_j|) \ ,$$

$$\hat{\xi}_{ij} = |\cos(\theta_{ij} - \delta^\circ_{ij})| \quad i \ne j\ , i \in J_{IN}\ , j \in J_K$$

$$a\sin(\theta-\delta) + b\cos(\theta-\delta) \le \sqrt{(a^2+b^2)} \qquad (31)$$

where the constants a and b can have positive, negative or even zero values.

Finally, the system aggregation matrix A= $[\alpha_{Ik}]$ of order (N-1)/3 is obtained, and its elements are defined, referring to Equation 25, as

$$\alpha_{Ik} = \begin{cases} -\lambda^*_I & ,K=I \\ 2Z_{Ik} & ,K \ne I \quad K, I=1,2,\ldots, S=(N-1)/3 \end{cases} \qquad (32)$$

In Equation 32, λ^* is the minimal (positive) eigenvalue of the 18th-order symmetric matrix R , whose elements are given by Equation (A-1) , and Z_{IK} is defined by Equation (A-2) (see Appendix).

It is of importance to note that , the stability of the matrix A, can be easily satisfied when largest values are obtained for λ^* and/or when values of Z_{IK} are the smallest. However,values of Z_{Ik} can be appreciably decreased by decomposing a power system such that strong interconnections among machines be included in the subsystems instead of exposing them as interconnections among subsystems.

NUMERICAL EXAMPLE

In this example, the approach developed is applied to the 10-machine, 11-bus system shown in Figure 1, (the lines admittances are given in p.u.). For an application of the approach to practical stability studies of the considered system, it is assumed that a 3-phase short circuit fault is occurred near bus 7, at 11 % length of the tie-line between buses 7 and 11.The fault is cleared by opening the circuit breaker C.B$_1$, located at bus 7. The system stability computations are carried out as follows:

1. Using the Newton-Raphson method, buses voltages and their angles are obtained for the post-fault (the fault is cleared) system. Inserting the reactances X'_d and X_q of the generators at the respective system buses , it is computed for each generator the internal angle δ , the voltages E'_q and E'_d .The results are given in Table 1.

Table 1 Post-fault equilibrium state results.

Bus No.	δ° (deg)	$\hat{E}'_q$	$\hat{E}'_d$	$\hat{E}_{fd}$
1	13.13	1.05157	- 0.01496	1.05484
2	10.67	1.14624	- 0.00946	1.15341
3	7.64	1.13267	- 0.00554	1.13669
4	4.28	1.06455	- 0.00987	1.06710
5	2.34	1.07645	- 0.00350	1.07926
6	5.22	1.08650	- 0.01441	1.09363
7	13.37	1.07131	- 0.01113	1.07418
8	7.02	1.06913	- 0.00649	1.07313
9	8.40	1.05273	- 0.00821	1.05744
10	0.18	1.10040	- 0.00304	1.10316

2. The system loads are represented by equivalent shunt admittances, which are computed using the pre-fault (normal operation) condition. Inserting the reactance X'_d of each generator , the post-fault system admittance matrix is constructed , and it is reduced to the 10th-order (symmetric) admittance matrix **Y** , whose elements are given in Table 2.

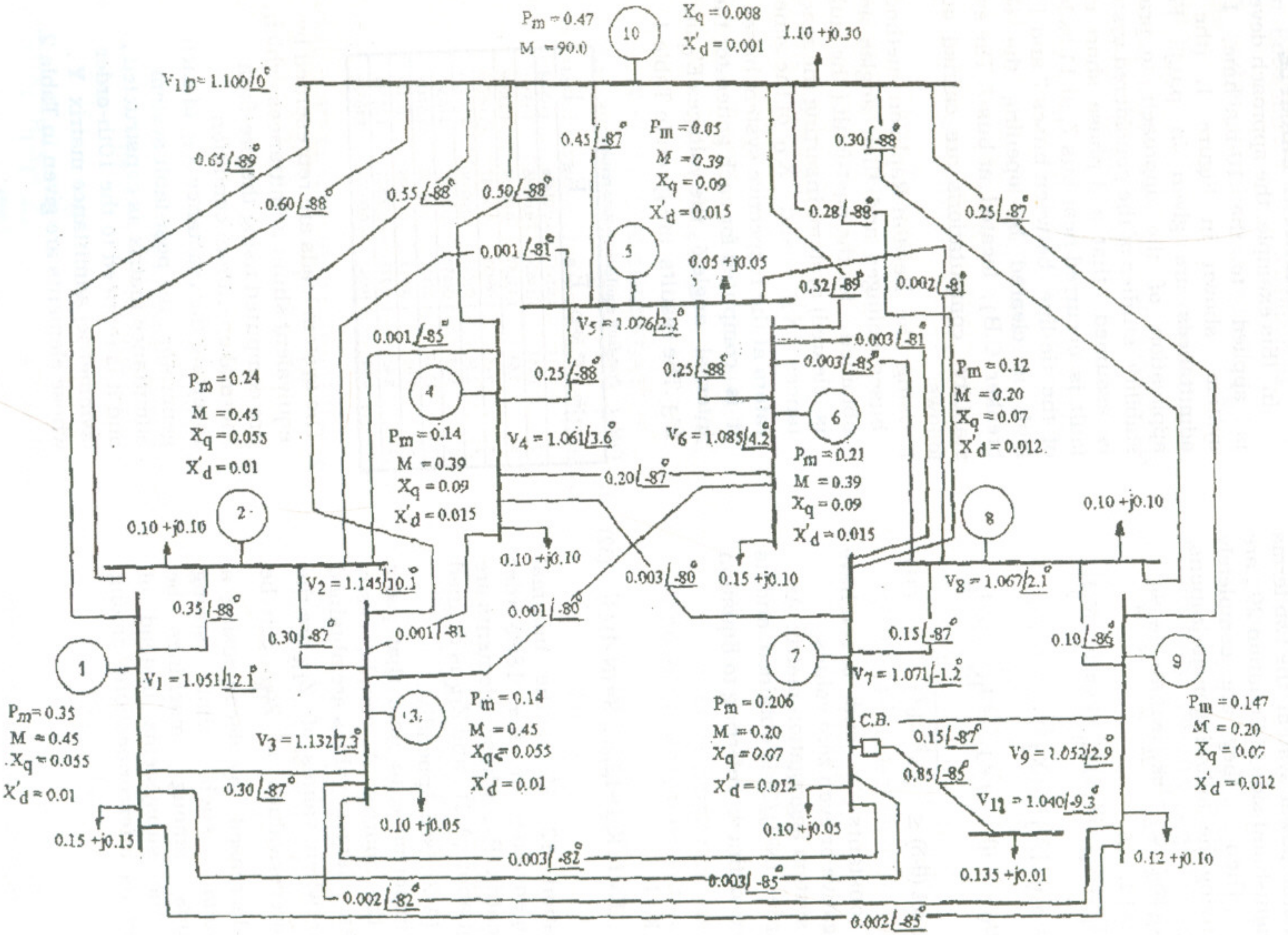


Figure 1 10-machine system all values in p.u

Table 2. Reduced admittance matrix for post-fault system (Moduli in (p.u.) and Arguments in (deg.)).

1	1.42766 ∠-83.15	0.34177 ∠91.84	0.29362 ∠92.83	0.00032 ∠92.90
	0.00029 ∠93.88	0.00033 ∠91.85	0.00313 ∠94.73	0.00020 ∠93.00
	0.00213 ∠94.78	0.64168 ∠90.86		
2	1.31336 ∠-84.67	0.29392 ∠92.86	0.00127 ∠94.76	0.00124 ∠98.13
	0.00031 ∠93.00	0.00018 ∠94.31	0.00018 ∠93.82	0.00016 ∠95.10
	0.59343 ∠91.88			
3	1.18560 ∠-83.88	0.00125 ∠97.88	0.00025 ∠95.08	0.00126 ∠98.22
	0.00311 ∠97.67	0.00017 ∠94.14	0.00211 ∠97.66	0.54486 ∠91.88
4	1.03273 ∠-83.20	0.24347 ∠91.83	0.19499 ∠92.77	0.00307 ∠99.57
	0.00017 ∠94.56	0.00013 ∠95.09		
	0.49336 ∠91.85			
5	0.98215 ∠-85.21	0.24342 ∠91.81	0.00015 ∠95.91	0.00210 ∠98.57
	0.00011 ∠95.93	0.44520 ∠92.87		
6	1.05429 ∠-81.74	0.00308 ∠98.52	0.00310 ∠94.72	0.00014 ∠94.18
	0.51276 ∠90.83			
7	0.63962 ∠-79.86	0.14798 ∠92.85	0.14805 ∠92.83	0.27774 ∠91.88
8	0.64671 ∠-80.03	0.09888 ∠93.83	0.29725 ∠91.87	
9	0.60439 ∠-77.11	0.24804 ∠92.86		
10	4.42601 ∠-76.50			

3. **Referring to the system reduced matrix Y, computed in step 2, and selecting machine 10 as the reference machine, the system is decomposed into three " four-machine " interconnected subsystems. Then, the following parameters are selected,**

$\lambda_i = 4.0\,,\ i = ,2,3,.......,9\,,\ \lambda_{10} = 25\,; \quad T'_{d0i} = 3.0\,,\ T'_{q0i} = 1.0\ ,\ i = 1, 2, 3\,,....., 10$

$h^k_{14} = h^k_{25} = h^k_{36} = 1.0 \quad ,\ k = 1\,,2\,,3$

$h^1_{44} = h^1_{55} = h^1_{66} = 0.74\,,\ h^1_{77} = 6.0\,,\ h^1_{88} = h^1_{99} = h^1_{10,10} = 500,\ h^1_{11,11} = 50.0$

$h^1_{12,12} = h^1_{13,13} = h^1_{14,14} = 100\,,h^1_{15,15} = 12.5\ ;\ \varepsilon_{11} = 0.92\,,\varepsilon_{12} = 0.94,\ \varepsilon_{13} = 0.95$

$h^2_{44} = h^2_{55} = h^2_{66} = 0.68\,,\ h^2_{77} = 5.0\ ,\ h^2_{88} = h^2_{99} = h^2_{10,10} = 300,\ h^2_{11,11} = 50.0$

$h^2_{12,12} = h^2_{13,13} = h^2_{14,14} = 50.0\,,\ h^2_{15,15} = 11.5\ ;\ \varepsilon_{21} = 0.93,\ \varepsilon_{22} = \varepsilon_{23} = 0.94$

$h^3_{44} = h^3_{55} = h^3_{66} = 0.70,\ h^3_{77} = 6.0\,,\ h^3_{88} = h^3_{99} = h^3_{10,10} = 500\ ,\ h^3_{11,11} = 35.0$

$h^3_{12,12} = h^3_{13,13} = h^3_{14,14} = 130\ ,\ h^3_{15,15} = 11.5\ ;\varepsilon_{31} = 0.84, \varepsilon_{32} = \varepsilon_{33} = 0.89$

Finally, using expression (32), we compute the aggregation matrix

$$A = \begin{bmatrix} -1.31656 & 0.632208 & 0.398732 \\ 0.684626 & -1.048120 & 0.398555 \\ 0.755088 & 0.645856 & -1.099290 \end{bmatrix}$$

which satisfies conditions (30), and hence it is a stable matrix . This implies asymptotic stability of the system equilibrium.

Now, to determine a stability domain estimate for the whole system , the matrix , $[A^T B + B^T A]$, with the matrix B is in the form , B = diag [2.0 , 1.55 0.8] , is computed and found to be negative definite. Then, according to theorem 4 of Reference , we conclude that

$$\Re_1 = \{\ \mathbf{X} :\ (2.0\ V_1\ (X_1) + 1.55\ V_2\ (X_2) + 0.80\ V_3\ (X_3)) \le \gamma_1\} \tag{33}$$

where, $\gamma_1 = \min\,(2.0\hat{V}_1,\ 1.55\hat{V}_2,\ \ 0.80\hat{V}_3)$, is an estimate of the system asymptotic stability domain. Using the following equation (see Appendix of Reference 17) ,

$$\hat{V}_I = \min_{m=1,2,3,\ldots,6} \; \min_{X_I^m \in \{X_I^{m-}, X_I^{m+}\}} \{(X_I^m)^T H_I X_I^m \sum_{k=1}^{6} \gamma_{Ik} \int_0^{\sigma_{Ik}^m} f_{Ik}(\sigma_{Ik})\, d\sigma_{Ik}\}$$

we compute $\hat{V}_1 = 0.5240$, $\hat{V}_2 = 0.9980$ and $\hat{V}_3 = 1.5772$, and hence we determine $\gamma_1 = 1.0480$. In terms of the original physical variables of the system (see Equation 5) the estimate $\Re_1$ is written as

$$\Re_1 = \{(\delta, \omega, E_Q, E_D) : (2.0V_1(\delta, \omega, E_Q, E_D) + 1.55V_2(\delta, \omega, E_Q, E_D) + V_3(\delta, \omega, E_Q, E_D)) \leq 1.0480\} \quad (34)$$

4. To determine the critical time for clearing the faulted line, the system equations (see Equation 5) are solved using the step- by -step method , and the obtained results are used for computing the three subsystems Lyapunov functions of Equation 27. It is found, referring to Equation 34 , that the critical clearing time t_{cc} for the fault equals 0.031 sec.

It is of importance to note that, using the standard step-by-step integration method, the system equations need to be solved several times to determine the critical clearing time. Hence, the developed stability (direct) approach is faster and can save stability computing time. However, the standard step-by-step method is known as an off-line method.

Figures 2a , b, c and d , show variations (note that the time is computed just after disconnecting the faulted line) of the third subsystem (contains machines 7, 8, 9 and 10) states. It is of importance to note that the states X_7, X_{11} and X_{15} of the third subsystem, in addition to the whole states of the first and second subsystems has negligible variations during the fault duration time.

It is clear , referring to Figures 2a ,b ,c and d, that the system will regain its prefault (steady-state) condition if the fault is cleared before, or when, $t_c = 0.031$ sec.

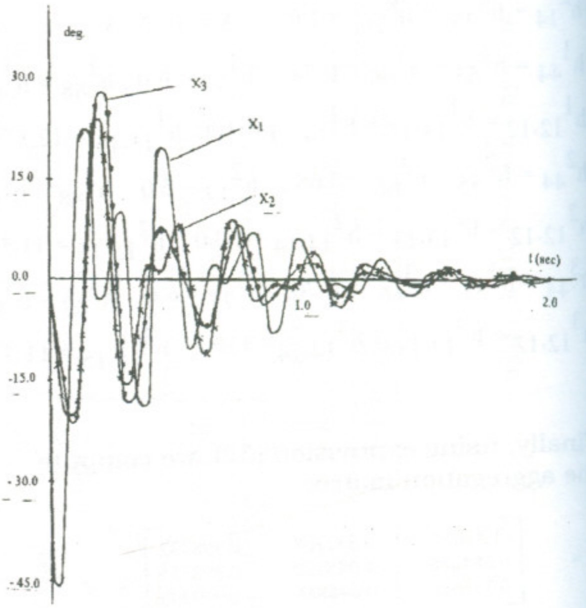


Figure 2-a Variations of the states X1, X2 and X3 of the third subsystem after clearing the fault.

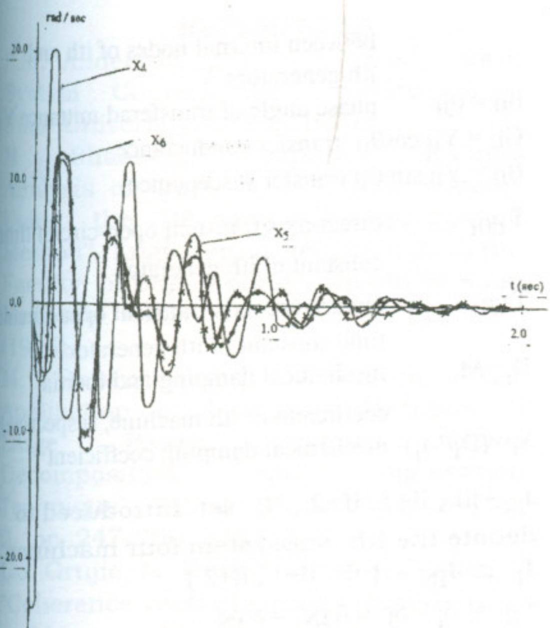
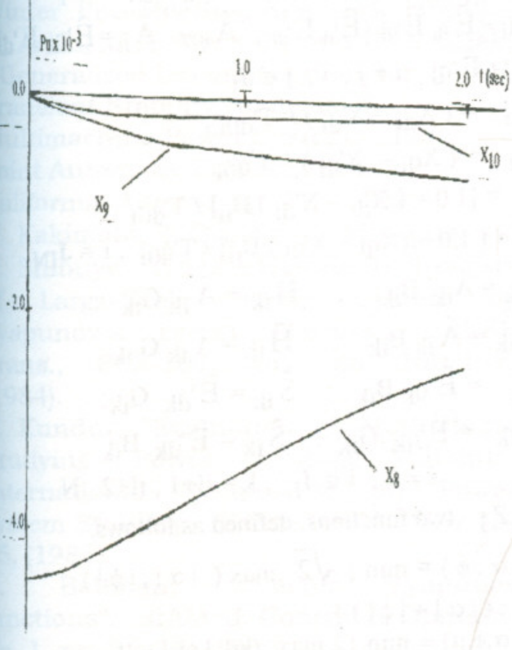


Figure 2-b Variation of the states X4, X5 and X6 of the third subsystem after clearing the fault.



igure 2-c Variation of the states X8, X9 and X 10 of the third substem after clearing the fault

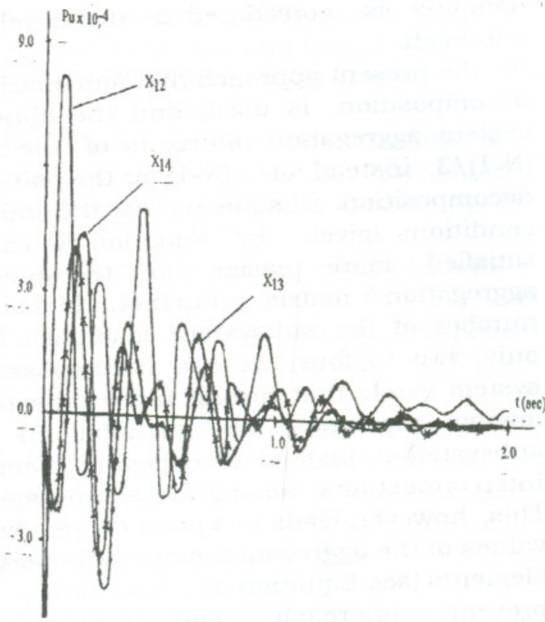


Figure 2-d Variation of the states X12, X13 and X14 of the third subsystem after clearing the fault.

CONCLUSIONS

It is developed, in the present work, a new Lyapunov stability approach applied to an N-machine power system. The approach developed is applied, in a numerical example, to a 10-machine, 11-bus power system. It is determined an estimate for the system asymptotic stability domain. Then a 3-phase short circuit fault is assumed near one of the system buses, and the approach is used to determine directly the critical time for clearing the fault. The following salient conclusions are drawn:

1. The approach developed is suitable for application to real power systems. Note that the transfer conductances G_{ij} ,are taken into consideration in the present approach. This essentially means that resistances of the system lines are considered. In addition, system loads may be represented by shunt admittances, and hence the system network can be greatly simplified by eliminating the system nodes (except the generators internal nodes).
2. The present approach is more suitable for application to real power systems than that developed so far. It is to be noted that non-uniform, instead of uniform, mechanical

damping is considered in the developed approach.

3. In the present approach the "four-machine" decomposition is used, and the obtained system aggregation matrix is of the order (N-1)/3, instead of (N-1) for the pair-wise decomposition. This means that the stability conditions (given by Equation 30) can be satisfied more easier for the present aggregation matrix. Further, increasing number of the subsystem machines, from only two to four, we can decompose the system such that strong interconnections among machines be included in the subsystems instead of exposing them as interconnections among those subsystems. This, however, leads to a great decrement in values of the aggregation matrix off-diagonal elements (see Equation 32). Accordingly, the present approach can reduce the conservativeness of the decomposition-aggregation method.
4. The present approach is simple and it can be easily used for practical and on-line stability studies of multi-machine power systems (number of machines may be more than 10).

NOMENCLATURE

P_{mi} mechanical power delivered to ith machine

P_{ei} electrical power delivered by ith machine

δ_i rotor angle, or position of the rotor q-axis from the reference

X_{di}, X_{qi} direct-axis, quadrature-axis synchronous reactances

X'_{di}, X'_{qi} d-axis, q-axis transient reactances

E_{fd} exciter voltage referred to the armature circuit

E'_i voltage behind d-axis transient reactance

E'_{di}, E'_{qi} d-axis, q-axis components of the voltage E'_i

E_q armature emf corresponding to the field current

$\hat{E}_{fdi}, \hat{E}'_{qi}$, $\hat{E}'_{di}$ post-fault (final) values of the voltages E_{fdi}, E'_{qi} and E'_{di}, respectively

ω_i rotor speed with respect to the synchronous speed

$Y_{ij} = Y_{ji}$ modulus of transfer admittance between internal nodes of ith and jth generators

$\theta_{ij} = \theta_{ji}$ phase angle of transferad mittance Y_{ij}

$G_{ij} = Y_{ij} \cos\theta_{ij}$ transfer conductance

$B_{ij} = Y_{ij} \sin\theta_{ij}$ transfer susceptance

T'_{d0i} direct-axis transient open-circuit time constant of ith generator

T'_{q0i} quadrature-axis transient open-circuit time constant of ith generator

D_i, M_i mechanical damping and inertia coefficient of ith machine, respectively

$\lambda_i = (D_i/M_i)$ mechanical damping coefficient

$J_{IN} = \{iI, iI+1, iI+2, N\}$ set introduced to denote the Ith subsystem four machines

$J_I \subset J_{IN} = \{iI, iI+1, iI+2\}$

$\delta_{ij} = \delta_i - \delta_j = \delta_{iN} - \delta_{jN}$

$\sigma_{ij} = \delta_{ij} - \delta^o_{ij} = \sigma_{iN} - \sigma_{jN}$,

$\sigma_{kN} = \delta_{kN} - \delta^o_{kN}$, $k \neq N$, $k = i, j$

$A_{kN} = A_{Nk} = \hat{E}'_{qk}\hat{E}'_{qN} + \hat{E}'_{dk}\hat{E}'_{dN}$;

$\hat{A}_{kN} = -\hat{A}_{kN} = \hat{E}'_{qN}\hat{E}'_{dk} - \hat{E}'_{qk}\hat{E}'_{dN}$, $k \in J_I$

$A_{ij} = \hat{E}'_{qi}\hat{E}'_{qj} + \hat{E}'_{di}\hat{E}'_{dj}$; $\hat{A}_{ij} = -\hat{A}_{ji} = \hat{E}'_{qi}\hat{E}'_{dj} - \hat{E}'_{di}\hat{E}'_{qj}$, $i \neq j$, $i, j \in J_I$

$K_i = (X_{di} - X'_{di}) / T'_{d0i}$,

$L_i = (X_{qi} - X'_{qi}) / T'_{q0i}$

$\Gamma_i = [1.0 - (X_{di} - X'_{di}) B_{ii}] / T'_{d0i}$;

$\mu_i = [1.0 - (X_{qi} - X'_{qi}) B_{ii}] / T'_{q0i}$, $i \in J_{IN}$

$H_{ik} = A_{ik} B_{ik}$; $\hat{H}_{ik} = \hat{A}_{ik} G_{ik}$;

$\overline{H}_{ik} = \hat{A}_{ik} B_{ik}$; $\tilde{H}_{ik} = A_{ik} G_{ik}$

$S_{ik} = \hat{E}'_{qk} B_{ik}$; $\hat{S}_{ik} = \hat{E}'_{dk} G_{ik}$;

$\overline{S}_{ik} = \hat{E}'_{qk} G_{ik}$; $\tilde{S}_{ik} = \hat{E}'_{dk} B_{ik}$

, $i \neq k$, $i \in J_I$, $k = iI+1, iI+2, N$

Z_2, Z_3 two functions, defined as follows:

$Z_2(\alpha, \phi) = \min\{\sqrt{2}\max(|\alpha|, |\phi|); (|\alpha| + |\phi|)\}$

$Z_3(\alpha, \phi, \mu) = \min\{2\max(|\alpha|, |\phi|, |\mu|); (|\alpha| + |\phi| + |\mu|); Z_2[Z_2(\alpha, \phi), \mu]; Z_2[Z_2(\phi, \mu), \alpha]; Z_2[Z_2(\mu, \alpha), \phi]\}$

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APPENDIX

Definition of the elements of the matrix R_I:

Elements of the 18th-order symmetric matrix R, (see Equation 32) are given as,

$$r^I_{11} = (2h^I_{14}/M_{iI})\{(H_{iI,N} + \hat{H}_{iI,N})\varepsilon_{I1} - |\overline{H}_{iI,N} - \tilde{H}_{iI,N}|\hat{\xi}_{I1} - m_{iI,iI+1}\hat{\xi}_{I4} - m_{iI,iI+2}\hat{\xi}_{I5} - \Sigma\max(U_{iI,j};\hat{U}_{iI,j})\}$$

$$r^I_{22} = (2h^I_{25}/M_{iI+1})\{(H_{iI+1,N} + \hat{H}_{iI+1,N})\varepsilon_{I2} - |\overline{H}_{iI+1,N} - \tilde{H}_{iI+1,N}|\hat{\xi}_{I2} - m_{iI+1,iI}\hat{\xi}_{I4} - m_{iI+1,iI+2}\hat{\xi}_{I6} - \Sigma\max(U_{iI+1,j};\hat{U}_{iI+1,j})\}$$

$$r^I_{33} = (2h^I_{36}/M_{iI+2})\{(H_{iI+2,N} + \hat{H}_{iI+2,N})\varepsilon_{I3} - |\overline{H}_{iI+2,N} - \tilde{H}_{iI+2,N}|\hat{\xi}_{I3} - m_{iI+2,iI}\hat{\xi}_{I5} - m_{iI+2,iI+1}\hat{\xi}_{I6} - \Sigma\max(U_{iI+2,j};\hat{U}_{iI+2,j})\}$$

$$r^I_{44} = 2(h^I_{14} - \lambda_{iI}h^I_{44}), \qquad r^I_{55} = 2(h^I_{25} - \lambda_{iI+1}h^I_{55})$$

$$r^I_{66} = 2(h^I_{36} - \lambda_{iI+2}h^I_{66}), \quad r^I_{77} = 2\lambda_N h^I_{77}, \quad r^I_{88} = 2\Gamma_{iI}h^I_{88}, \quad r^I_{99} = 2\Gamma_{iI+1}h^I_{99}$$

$$r^I_{10,10} = 2\Gamma_{iI+2}h^I_{10,10}, \quad r^I_{11,11} = 2\Gamma_N h^I_{11,11}, \quad r^I_{12,12} = 2\mu_{iI}h^I_{12,12}$$

$$r^I_{13,13} = 2\mu_{iI+1}h^I_{13,13}, \quad r^I_{14,14} = 2\mu_{iI+2}h^I_{14,14}, \quad r^I_{15,15} = 2\mu_N h^I_{15,15}$$

$$r^I_{12} = -\Big[\max[(\hat{E}'_{qiI}h^I_{14}\tilde{m}_{iI,iI+1});(\hat{E}'_{qiI+1}h^I_{25}\tilde{m}_{iI+1,iI}/M_{iI+1})] + \max[(|\hat{E}'_{diI}|h^I_{14}\hat{m}_{iI,iI+1}/M_{iI});(|\hat{E}'_{diI+1}|h^I_{25}\hat{m}_{iI+1,iI}/M_{iI+1})\Big]\hat{\xi}_{I4}$$

$$r^I_{13} = -\Big[\max[(\hat{E}'_{qiI}h^I_{14}\tilde{m}_{iI,iI+2}/M_{iI});(\hat{E}'_{qiI+2}h^I_{36}\tilde{m}_{iI+2,iI}/M_{iI+2})] + \max[(|\hat{E}'_{diI}|h^I_{14}\hat{m}_{iI,iI+2}/M_{iI});(|\hat{E}'_{diI+2}|h^I_{36}\hat{m}_{iI+2,iI}/M_{iI+2})\Big]\hat{\xi}_{I5}$$

$$r^I_{14} = -(h^I_{44}/M_{iI})\Big\{|\overline{H}_{iI,N} - \tilde{H}_{iI,N}|\hat{\xi}_{I1} + m_{iI,iI+1}\hat{\xi}_{I4} + m_{iI,iI+2}\hat{\xi}_{I5} + \Sigma[\max(U_{iI,j};\hat{U}_{iI,j}) + \tilde{U}_{iI,j}]\Big\}$$

$$r^I_{15} = -m_{iI+1,\, iI}\, h^I_{55}\, \hat{\xi}_{I4} / M_{iI+1} \quad , \quad r^I_{16} = -m_{iI+2,\, iI}\, h^I_{66}\, \hat{\xi}_{I5} / M_{iI+2}$$

$$r^I_{17} = -\{\, | H_{iI,N} + \hat{H}_{iI,N} |\, h^I_{44}\, \xi_{I1} / M_{iI} \} - \overline{D}_{iI,N}\, h^I_{77} - \lambda_{iI}\, h^I_{14}$$

$$r^I_{18} = -(h^I_{14}/M_{iI})[Y_{iI,N}\hat{E}'_N + V_{iI,iI} + |D_{iI,iI+1} + D_{iI,iI+2}| + \hat{m}_{iI,iI+1} + \hat{m}_{iI,iI+2}]$$
$$- K_I\, [Y_{iI,N}(\hat{E}'_{qN}\hat{\xi}_{I1} + |\hat{E}'_{dN}|\, \xi_{I1}) + \hat{m}_{iI,iI+1}\hat{\xi}_{I4} + \hat{m}_{iI,iI+2}\hat{\xi}_{I5}] -$$
$$- [(h^I_{14}/M_{iI}) + K_I]\, \Sigma\, C_{iI,j}$$

$$r^I_{19} = -[\hat{E}'_{iI}\, Y_{iI,iI+1}\, h^I_{14} / M_{iI}] - \grave{K}_I\, \hat{m}_{iI+1,iI}\, \hat{\xi}_{I4}$$

$$r^I_{1,10} = -[\hat{E}'_{iI}\, Y_{iI,iI+2}\, h^I_{14}/M_{iI}] - \tilde{K}_I\, \hat{m}_{iI+2,iI}\, \hat{\xi}_{I5}$$

$$r^I_{1,11} = -[\hat{E}'_{iI}\, Y_{iI,N}\, h^I_{14}/M_{iI}] - \overline{K}_I\, \overline{C}_{iI,N}$$

$$r^I_{1,12} = -(h^I_{14}/M_{iI})[Y_{iI,N}\hat{E}'_N + \hat{V}_{iI,iI} + |\hat{D}_{iI,iI+1} + \hat{D}_{iI,iI+2}| + \tilde{m}_{iI,iI+1} + \tilde{m}_{iI,iI+2}] -$$
$$- L_I\, [Y_{iI,N}(\hat{E}'_{qN}\xi_{I1} + |\hat{E}'_{dN}|\, \hat{\xi}_{I1}) + \tilde{m}_{iI,iI+1}\hat{\xi}_{I4} + \tilde{m}_{iI,iI+2}\hat{\xi}_{I5}] -$$
$$- [(h^I_{14}/M_{iI}) + L_I]\, \Sigma\, \tilde{C}_{iI,j}$$

$$r^I_{1,13} = -[\hat{E}'_{iI}\, Y_{iI,iI+1} h^I_{14}/M_{iI}] - \hat{L}_I\, \tilde{m}_{iI+1,iI}\, \hat{\xi}_{I4}$$

$$r^I_{1,14} = -[\hat{E}'_{iI}\, Y_{iI,iI+2}\, h^I_{14} / M_{iI}] - \tilde{L}_I\, \tilde{m}_{iI+2,iI}\, \hat{\xi}_{I5}$$

$$r^I_{1,15} = -[\hat{E}'_{iI}\, Y_{iI,N}\, h^I_{14} / M_{iI}] - \overline{L}_I\, \hat{C}_{iI,N}$$

$$r^I_{23} = -\{\max[(\hat{E}'_{qiI+1}\, h^I_{25}\, \tilde{m}_{iI+1,iI+2}/M_{iI+1}); (\hat{E}'_{qiI+2}\, \tilde{m}_{iI+2,iI+1}\, h^I_{36} / M_{iI+2})] +$$
$$+ \max[(|\hat{E}'_{diI+1}|h^I_{25}\, \hat{m}_{iI+1,iI+2}/M_{iI+1}); (\hat{E}'_{diI+2}|\hat{m}_{iI+2\, iI+1} h^I_{36}/M_{iI+2})]\}\, \hat{\xi}_{I6}$$

$$r^I_{24} = -h^I_{44}\, m_{iI,iI+1}\, \hat{\xi}_{I4} / M_{iI}$$

$$r^I_{25} = -(h^I_{55} / M_{iI+1})\{|\overline{H}_{iI+1,N} - \tilde{H}_{iI-1,N}|\, \hat{\xi}_{I2} + m_{iI+1,iI}\, \hat{\xi}_{I4} + m_{iI+1,iI+2}\, \hat{\xi}_{I6} +$$
$$+ \Sigma\, [\max(U_{iI+1,j}\, ;\, \hat{U}_{iI+1,j}) + \tilde{U}_{iI+1,j}]\}$$

$$r^I_{26} = -h^I_{66}\, m_{iI+2,iI+1}\, \hat{\xi}_{I6} / M_{iI+2}$$

$$r^I_{27} = -\{\, |H_{iI+1,N} + \hat{H}_{iI+1,N}|\, h^I_{55}\, \xi_{I2} / M_{iI+1}\} - \overline{D}_{iI+1,N}\, h^I_{77} - \lambda_{iI+1}\, h^I_{25}$$

$$r^I_{28} = -[\hat{E}'_{iI+1}\, Y_{iI,iI+1}\, h^I_{25} / M_{iI+1}] - K_I\, \hat{m}_{iI,iI+1}\, \hat{\xi}_{I4}$$

$$r^I_{29} = -(h^I_{25}/M_{iI+1})\{Y_{iI+1,N}\hat{E}'_N + V_{iI+1,iI+1} + |D_{iI+1,iI} + D_{iI+1,iI+2}| + \hat{m}_{iI+1,iI} +$$
$$+ \hat{m}_{iI+1,iI+2}\} - \hat{K}_I\{Y_{iI+1,N}(\hat{E}'_{qN}\hat{\xi}_{I2} + |\hat{E}'_{dN}|\, \xi_{I2}) + \hat{m}_{iI+1,iI}\, \hat{\xi}_{I4} +$$
$$+ \hat{m}_{iI+1,iI+2}\, \hat{\xi}_{I6}\} - [(h^I_{25} / M_{iI+1}) + \hat{K}_I]\, \Sigma\, C_{iI+1,j}$$

$$r^I_{2,10} = -[\hat{E}'_{iI+1}\, Y_{iI+1,iI+2}\, h^I_{25} / M_{iI+1}] - \tilde{K}_I\, \hat{m}_{iI+2,iI+1}\, \hat{\xi}_{I6}$$

$r^I_{2,11} = -[\hat{E}'_{iI+1} Y_{iI+1,N} h^I_{25} / M_{iI+1}] - \overline{K}_I \overline{C}_{iI+1,N}$

$r^I_{2,12} = -[\hat{E}'_{iI+1} Y_{iI+1,iI} h^I_{25} / M_{iI+1}] - L_I \tilde{m}_{iI,iI+1} \hat{\xi}_{I4}$

$r^I_{2,13} = -(h^I_{25}/M_{iI+1}) \{Y_{iI+1,N} \hat{E}'_N + \hat{V}_{iI+1,iI+1} + |\hat{D}_{iI+1,iI} + \hat{D}_{iI+1,iI+2}| + \tilde{m}_{iI+1,iI}$
$+ \tilde{m}_{iI+1,iI+2}\} - \hat{L}_I\{Y_{iI+1,N} (\hat{E}'_{qN} \xi_{I2} + |\hat{E}'_{dN}| \hat{\xi}_{I2}) + \tilde{m}_{iI+1,iI} \hat{\xi}_{I4} +$
$+ \tilde{m}_{iI+1,iI+2} \hat{\xi}_{I6}\} - [(h^I_{25}/M_{iI+1}) + \hat{L}_I] \Sigma \tilde{C}_{iI+1,j}$

$r^I_{2,14} = -[\hat{E}'_{iI+1} Y_{iI+1,iI+2} h^I_{25} / M_{iI+1}] - \tilde{L}_I \tilde{m}_{iI+2,iI+1} \hat{\xi}_{I6}$

$r^I_{2,15} = -[\hat{E}'_{iI+1} Y_{iI+1,N} h^I_{25} / M_{iI+1}] - \overline{L}_I \hat{C}_{iI+1,N}$

$r^I_{2,16} = -|[(h^I_{25}/M_{iI+1}) - (h^I_{14}/M_{iI})] H_{iI,iI+1} - [(h^I_{25}/M_{iI+1}) + (h^I_{14}/M_{iI})] \hat{H}_{iI,iI+1}|$

$r^I_{34} = -h^I_{44} m_{iI,iI+2} \hat{\xi}_{I5} / M_{iI}$, $r^I_{35} = -h^I_{55} m_{iI+1,iI+2} \hat{\xi}_{I6} / M_{iI+1}$

$r^I_{36} = -(h^I_{66} / M_{iI+2}) \{ |\overline{H}_{iI+2,N} - \tilde{H}_{iI+2,N}| \hat{\xi}_{I3} + m_{iI+2,iI} \hat{\xi}_{I5} + m_{iI+2,iI+1} \hat{\xi}_{I6} +$
$+ \Sigma[\max(U_{iI+2,j} ; \hat{U}_{iI+2,j}) + \tilde{U}_{iI+2,j}]\}$

$r^I_{37} = -\{|H_{iI+2,N} + \hat{H}_{iI+2,N}| h^I_{66} \xi_{I3} / M_{iI+2}\} - \overline{D}_{iI+2,N} h^I_{77} - \lambda_{iI+2} h^I_{36}$

$r^I_{38} = -[\hat{E}'_{iI+2} Y_{iI,iI+2} h^I_{36} / M_{iI+2}] - K_I \hat{m}_{iI,iI+2} \hat{\xi}_{I5}$

$r^I_{39} = -[\hat{E}'_{iI+2} Y_{iI+1,iI+2} h^I_{36} / M_{iI+2}] - \hat{K}_I \hat{m}_{iI+1,iI+2} \hat{\xi}_{I6}$

$r^I_{3,10} = -(h^I_{36}/M_{iI+2}) \{Y_{iI+2,N} \hat{E}'_N + V_{iI+2,iI+2} + |D_{iI+2,iI} + D_{iI+2,iI+1}| + \hat{m}_{iI+2,iI} +$
$+ \hat{m}_{iI+2,iI+1}\} - \tilde{K}_I\{Y_{iI+2,N} (\hat{E}'_{qN} \hat{\xi}_{I3} + |E'_{dN}| \xi_{I3}) + \hat{m}_{iI+2,iI} \hat{\xi}_{I5} +$
$+ \hat{m}_{iI+2,iI+1} \hat{\xi}_{I6}\} - [(h^I_{36}/M_{iI+2}) + \tilde{K}_I] \Sigma C_{iI+2,j}$

$r^I_{3,11} = -[\hat{E}'_{iI+2} Y_{iI+2,N} h^I_{36} / M_{iI+2}] - \overline{K}_I \overline{C}_{iI+2,N}$

$r^I_{3,12} = -[\hat{E}'_{iI+2} Y_{iI,iI+2} h^I_{36} / M_{iI+2}] - L_I \tilde{m}_{iI,iI+2} \hat{\xi}_{I5}$

$r^I_{3,13} = -[\hat{E}'_{iI+2} Y_{iI+1,iI+2} h^I_{36} / M_{iI+2}] - \hat{L}_I \tilde{m}_{iI+1,iI+2} \hat{\xi}_{I6}$

$r^I_{3,14} = -(h^I_{36} / M_{iI+2}) \{Y_{iI+2,N} \hat{E}'_N + \hat{V}_{iI+2,iI+2} + |\hat{D}_{iI+2,iI} + \hat{D}_{iI+2,iI+1}| + \tilde{m}_{iI+2,iI}$
$+ \tilde{m}_{iI+2,iI+1}\} - \tilde{L}_I \{Y_{iI+2,N} (\hat{E}'_{qN} \xi_{I3} + |\hat{E}'_{dN}| \hat{\xi}_{I3}) + \tilde{m}_{iI+2,iI} \hat{\xi}_{I5}$
$+ \tilde{m}_{iI+2,iI+1} \hat{\xi}_{I6}\} - [(h^I_{36}/M_{iI+2}) + \tilde{L}_I] \Sigma \tilde{C}_{iI+2,j}$

$r^I_{3,15} = -[\hat{E}'_{iI+2} Y_{iI+2,N} h^I_{36} / M_{iI+2}] - \overline{L}_I \hat{C}_{iI+2,N}$

$r^I_{3,17} = -|[(h^I_{36}/M_{iI+2}) - (h^I_{14}/M_{iI})] H_{iI,iI+2} - [(h^I_{36}/M_{iI+2}) + (h^I_{14}/M_{iI})]$
$\hat{H}_{iI,iI+2}|$

$r^I_{3,18} = -[(h^I_{36}/M_{iI+2}) - (h^I_{25}/M_{iI+1})]H_{iI+1,iI+2} - [(h^I_{36}/M_{iI+2}) + (h^I_{25}/M_{iI+1})]$
$\hat{H}_{iI+1,iI+2}|$, $r^I_{47} = -h^I_{14}$

$r^I_{48} = -d_I\{\hat{E}'_N Y_{iI,N} + V_{iI,iI} + |D_{iI,iI+1} + D_{iI,iI+2}| + \hat{m}_{iI,iI+1} + \hat{m}_{iI,iI+2} + \Sigma C_{iI,j}\}$

$r^I_{49} = -d_I \hat{E}'_{iI} Y_{iI,iI+1}$, $r^I_{4,10} = -d_I \hat{E}'_{iI} Y_{iI,iI+2}$, $r^I_{4,11} = -d_I \hat{E}'_{iI} Y_{iI,N}$

$r^I_{4,12} = -d_I\{\hat{E}'_N Y_{iI,N} + \hat{V}_{iI,iI} + |\hat{D}_{iI,iI+2} + \hat{D}_{iI,iI+2}| + \tilde{m}_{iI,iI+1} + \tilde{m}_{iI,iI+2} + \Sigma \tilde{C}_{iI,j}\}$

$r^I_{4,13} = r^I_{49}$, $r^I_{4,14} = r^I_{4,10}$, $r^I_{4,15} = r^I_{4,11}$, $r^I_{57} = -r^I_{25}$, $r^I_{58} = -\hat{d}_I \hat{E}'_{iI+1} Y_{iI,iI+1}$

$r^I_{59} = -\hat{d}_I\{\hat{E}'_N Y_{iI+1,N} + V_{iI+1,iI+1} + |D_{iI+1,iI} + D_{iI+1,iI+2}| + \hat{m}_{iI+1,iI} + \hat{m}_{iI+1,iI+2}$
$+ \Sigma C_{iI+1,j}\}$

$r^I_{5,10} = -\hat{d}_I \hat{E}'_{iI+1} Y_{iI+1,iI+2}$, $r^I_{5,11} = -\hat{d}_I \hat{E}'_{iI+1} Y_{iI+1,N}$, $r^I_{5,12} = r^I_{58}$

$r^I_{5,13} = -\hat{d}_I\{\hat{E}'_N Y_{iI+1,N} + \hat{V}_{iI+1,iI+1} + |\hat{D}_{iI+1,iI} + \hat{D}_{iI+1,iI+2}| + \tilde{m}_{iI+1,iI} + \tilde{m}_{iI+1,iI+2}$
$+ \Sigma \tilde{C}_{iI+1,j}\}$, $r^I_{5,14} = r^I_{5,10}$, $r^I_{5,15} = r^I_{5,11}$

$r^I_{5,16} = -|(\hat{d}_I - d_I) H_{iI,iI+1} - (\hat{d}_I + d_I)\hat{H}_{iI,iI+1}|$

$r^I_{67} = -h^I_{36}$, $r^I_{68} = -\tilde{d}_I \hat{E}'_{iI+2} Y_{iI,iI+2}$, $r^I_{69} = -\tilde{d}_I \hat{E}'_{iI+2} Y_{iI+1,iI+2}$

$r^I_{6,10} = -\tilde{d}_I\{\hat{E}'_N Y_{iI+2,N} + V_{iI+2,iI+2} + |D_{iI+2,iI} + D_{iI+2,iI+1}| + \hat{m}_{iI+2,iI} + \hat{m}_{iI+2,iI+1}$
$+ \Sigma C_{iI+2,j}\}$

$r^I_{6,11} = -\tilde{d}_I \hat{E}'_{iI+2} Y_{iI+2,N}$, $r^I_{6,12} = r^I_{68}$, $r^I_{6,13} = r^I_{69}$

$r^I_{6,14} = -\tilde{d}_I\{\hat{E}'_N Y_{iI+2,N} + \hat{V}_{iI+2,iI+2} + |\hat{D}_{iI+2,iI} + \hat{D}_{iI+2,iI+1}| + \tilde{m}_{iI+2,iI} + \tilde{m}_{iI+2,iI+1}$
$+ \Sigma \tilde{C}_{iI+2,j}\}$, $r^I_{6,15} = r^I_{6,11}$

$r^I_{6,17} = -|(\tilde{d}_I - d_I) H_{iI,iI+2} - (\tilde{d}_I + d_I)\hat{H}_{iI,iI+2}|$

$r^I_{6,18} = -|(\tilde{d}_I - \hat{d}_I) H_{iI+1,iI+2} - (\tilde{d}_I + \hat{d}_I)\hat{H}_{iI+1,iI+2}|$

$r^I_{78} = -\bar{d}_I \hat{E}'_N Y_{iI,N}$, $r^I_{79} = -\bar{d}_I \hat{E}'_N Y_{iI+1,N}$, $r^I_{7,10} = -\bar{d}_I \hat{E}'_N Y_{iI+2,N}$

$r^I_{7,11} = -\bar{d}_I\{V_{N,N} + \hat{E}'_{iI} Y_{iI,N} + \hat{E}'_{iI+1} Y_{iI+1,N} + \hat{E}'_{iI+2} Y_{iI+2,N} + \Sigma C_{N,j}\}$

$r^I_{7,12} = r^I_{78}$, $r^I_{7,13} = r^I_{79}$, $r^I_{7,14} = r^I_{7,10}$

$r^I_{7,15} = -\bar{d}_I\{\hat{V}_{N,N} + \hat{E}'_{iI} Y_{iI,N} + \hat{E}'_{iI+1} Y_{iI+1,N} + \hat{E}'_{iI+2} Y_{iI+2,N} + \Sigma \tilde{C}_{N,j}\}$

$r^I_{89} = -Y_{iI,iI+1}\sqrt{\{K_I^2 + \hat{K}_I^2 - 2K_I\hat{K}_I\beta_{Ia}\}}$

$r^I_{8,10} = -Y_{iI,iI+2}\sqrt{\{K_I^2 + \tilde{K}_I^2 - 2K_I\tilde{K}_I\beta_{Ib}\}}$

$r^I_{8,11} = -Y_{iI,N}\sqrt{\{K_I^2 + \bar{K}_I^2 - 2K_I\bar{K}_I\beta_{Na}\}}$

$r^I_{8,12} = -|L_I - K_I|\, G_{iI,iI}$

$$r^{I}_{8,13} = -Y_{iI,iI+1}\sqrt{\{K_I^2 + \hat{L}_I^2 - 2\,K_I\,\hat{L}_I\,\beta_{Ia}\}}$$

$$r^{I}_{8,14} = -Y_{iI,iI+2}\sqrt{\{K_I^2 + \tilde{L}_I^2 - 2\,K_I\,\tilde{L}_I\,\beta_{Ib}\}}$$

$$r^{I}_{8,15} = -Y_{iI,N}\sqrt{\{K_I^2 + \overline{L}_I^2 - 2\,K_I\,\overline{L}_I\,\beta_{Na}\}}$$

$$r^{I}_{8,16} = -K_I\,\tilde{m}_{\,iI,iI+1} \quad , \quad r^{I}_{8,17} = -K_I\,\tilde{m}_{\,iI,iI+2}$$

$$r^{I}_{9,10} = -Y_{iI+1,iI+2}\sqrt{\{\hat{K}_I^2 + \tilde{K}_I^2 - 2\,\hat{K}_I\,\tilde{K}_I\,\beta_{Ic}\}}$$

$$r^{I}_{9,11} = -Y_{iI+1,N}\sqrt{\{\hat{K}_I^2 + \overline{K}_I^2 - 2\,\hat{K}_I\,\overline{K}_I\,\beta_{Nb}\}}$$

$$r^{I}_{9,12} = -Y_{iI,iI+1}\sqrt{\{\hat{K}_I^2 + L_I^2 - 2\,\hat{K}_I\,L_I\,\beta_{Ia}\}}$$

$$r^{I}_{9,13} = -|\hat{L}_I - \hat{K}_I|\,G_{iI+1,iI+1}$$

$$r^{I}_{9,14} = -Y_{iI+1,iI+2}\sqrt{\{\hat{K}_I^2 + \tilde{L}_I^2 - 2\,\hat{K}_I\,\tilde{L}_I\,\beta_{Ic}\}}$$

$$r^{I}_{9,15} = -Y_{iI+1,N}\sqrt{\{\hat{K}_I^2 + \overline{L}_I^2 - 2\,\hat{K}_I\overline{L}_I\,\beta_{Nb}\}}$$

$$r^{I}_{9,16} = -\hat{K}_I\,\tilde{m}_{iI+1,iI} \quad , \quad r^{I}_{9,18} = -\hat{K}_I\,\tilde{m}_{\,iI+1,iI+2}$$

$$r^{I}_{10,11} = -Y_{iI+2,N}\sqrt{\{\tilde{K}_I^2 + \overline{K}_I^2 - 2\tilde{K}_I\,\overline{K}_I\,\beta_{Nc}\}}$$

$$r^{I}_{10,12} = -Y_{iI,iI+2}\sqrt{\{\tilde{K}_I^2 + L_I^2 - 2\,\tilde{K}_I\,L_I\,\beta_{Ib}\}}$$

$$r^{I}_{10,13} = -Y_{iI+1,iI+2}\sqrt{\{\tilde{K}_I^2 + \hat{L}_I^2 - 2\,\tilde{K}_I\,\hat{L}_I\,\beta_{Ic}\}}$$

$$r^{I}_{10,14} = -|\,\tilde{L}_I - \tilde{K}_I\,|\,G_{\,iI+2,iI+2}$$

$$r^{I}_{10,15} = -Y_{iI+2,N}\sqrt{\{\tilde{K}_I^2 + \overline{L}_I^2 - 2\,\tilde{K}_I\,\overline{L}_I\,\beta_{Nc}\}}$$

$$r^{I}_{10,17} = -\tilde{K}_I\,\tilde{m}_{\,iI+2,iI} \quad , \quad r^{I}_{10,18} = -\tilde{K}_I\,\tilde{m}_{\,iI+2,iI+1}$$

$$r^{I}_{11,12} = -Y_{iI,N}\sqrt{\{\overline{K}_I^2 + L_I^2 - 2\,\overline{K}_I\,L_I\,\beta_{Na}\}}$$

$$r^{I}_{11,13} = -Y_{iI+1,N}\sqrt{\{\overline{K}_I^2 + \hat{L}_I^2 - 2\,\overline{K}_I\,\hat{L}_I\,\beta_{Nb}\}}$$

$$r^{I}_{11,14} = -Y_{iI+2,N}\sqrt{\{\overline{K}_I^2 + \tilde{L}_I^2 - 2\,\overline{K}_I\,\tilde{L}_I\,\beta_{Nc}\}}$$

$$r^{I}_{11,15} = -|\,\overline{L}_I - \overline{K}_I\,|\,G_{\,N,N}$$

$$r^{I}_{12,13} = -Y_{iI,iI+1}\sqrt{\{L_I^2 + \hat{L}_I^2 - 2\,L_I\,\hat{L}_I\,\beta_{Ia}\}}$$

$$r^{I}_{12,14} = -Y_{iI,iI+2}\sqrt{\{L_I^2 + \tilde{L}_I^2 - 2\,L_I\,\tilde{L}_I\,\beta_{Ib}\}}$$

$$r^{I}_{12,15} = -Y_{iI,N}\sqrt{\{L_I^2 + \overline{L}_I^2 - 2\,L_I\,\overline{L}_I\,\beta_{Na}\}}$$

$$r^{I}_{12,16} = -L_I\,\hat{m}_{\,iI,iI+1} \quad , \quad r^{I}_{12,17} = -L_I\,\hat{m}_{\,iI,iI+2}$$

$$r^{I}_{13,14} = -Y_{iI+1,iI+2}\sqrt{\{\hat{L}_I^2 + \tilde{L}_I^2 - 2\,\hat{L}_I\,\tilde{L}_I\,\beta_{Ic}\}}$$

$$r^{I}_{13,15} = -Y_{iI+1,N}\sqrt{\{\hat{L}_I^2 + \overline{L}_I^2 - 2\,\hat{L}_I\,\overline{L}_I\,\beta_{Nb}\}}$$

$$r^{I}_{13,16} = -\hat{L}_I\,\hat{m}_{\,iI+1,iI} \quad , \quad r^{I}_{13,18} = -\hat{L}_I\,\hat{m}_{\,iI+1,iI+2}$$

$$r^I_{14,15} = -Y_{iI+2,N} \sqrt{\{\tilde{L}_I^{\,2} + \overline{L}_I^{\,2} - 2\,\tilde{L}_I\,\overline{L}_I\,\beta_{Nc}\}}$$

$$r^I_{14,17} = -\tilde{L}_I\,\hat{m}_{\,iI+2,iI} \quad , \qquad r^I_{14,18} = -\tilde{L}_I\,\hat{m}_{\,iI+2,iI+1}$$

$$r^I_{16,16} = 2\,h^I_{14}\,(A_{iI,iI+1}\,B_{iI,iI+1} + \hat{A}_{iI,iI+1}\,G_{iI,iI+1})\,/\,M_{iI}\,\xi_{I4}$$

$$r^I_{17,17} = 2\,h^I_{14}\,(A_{iI,iI+2}\,B_{iI,iI+2} + \hat{A}_{iI,iI+2}\,G_{iI,iI+2})\,/\,M_{iI}\,\xi_{I4}$$

$$r^I_{18,18} = 2\,h^I_{25}\,(A_{iI+1,iI+2}\,B_{iI+1,iI+2} + \hat{A}_{iI+1,iI+2}\,G_{iI+1,iI+2})\,/\,M_{iI+1}\,\xi_{I6} \qquad \text{(A-1)}$$

while the other elements of this matrix are equal zero. In Equation 35 , recall that Σ is given as $\sum_{j\notin JIN}$, and it is equivalent to $\sum_{K\neq I}^{S}\sum_{j\in JK}$ Further, the following constants are defined,

$$m_{k,j} = [\hat{E}'_{qk}\,\hat{E}'_{qj}\,G_{k,j} - \hat{E}'_{dj}\,B_{k,j}\,| + \hat{E}'_{dk}\,||\hat{E}'_{dj}\,G_{k,j} + \hat{E}'_{qj}\,B_{kj}\,|\,] \quad , k \neq j\,,\; k, j \in J_I$$

$$\hat{m}_{k,j} = |\hat{E}'_{qj}\,B_{kj} + \hat{E}'_{dj}\,G_{kj}\,| \quad , \quad \tilde{m}_{k,j} = |\hat{E}'_{dj}\,B_{kj} - \hat{E}'_{qj}\,G_{kj}\,| \quad , k \neq j\,,\; k, j \in J_{IN}$$

$$D_{i,j} = (\hat{E}'_{qj}\,G_{ij} - \hat{E}'_{dj}\,B_{ij})\cos\delta_{ij}\,, \quad \hat{D}_{i,j} = (\hat{E}'_{qj}\,B_{ij} + \hat{E}'_{dj}\,G_{ij})\cos\delta_{ij}\,, \; i \neq j\,,\; i, j \in J_I$$

$$C_{i,j} = (|\hat{E}'_{dj}\,|\xi_{ij} + \hat{E}'_{qj}\hat{\xi}_{ij})\,Y_{ij} \quad , \quad \tilde{C}_{i,j} = (|\hat{E}'_{dj}\,|\hat{\xi}_{ij} + \hat{E}'_{qj}\xi_{ij})\,Y_{ij} \;, i \neq j\,,\; i \in J_{IN}$$

$$U_{k,j} = \hat{E}'_{qk}\,Y_{kj}\,|\,\hat{E}'_{dj}\,|\hat{\xi}_{kj} \quad , \hat{U}_{k,j} = |\,\hat{E}'_{dk}\,|\;Y_{kj}\;\hat{E}'_{qj}\,\hat{\xi}_{kj} \quad , \quad \tilde{U}_{k,j} = A_{kj}\,Y_{kj}\,\xi_{kj}$$

$$\overline{C}_{k,N} = (\hat{E}'_{qk}\,\hat{\xi}_{Nk} + |\,\hat{E}'_{dk}\,|\;\xi_{Nk})\,Y_{kN} \quad , \quad \hat{C}_{k,N} = (\hat{E}'_{qk}\,\xi_{Nk} + |\,\hat{E}'_{dk}\,|\;\hat{\xi}_{Nk})\,Y_{kN}\;\overline{D}_{k,N} = (A_{kN}\,\xi_{N,j} + |\,\hat{A}_{kN}\;|\,\hat{\xi}_{Nk})\,Y_{kN}\,/\,M_N \quad , k \in J_I$$

$$V_{k,k} = 2.0\,\hat{E}'_{qk}\,G_{k,k} \quad , \quad \hat{V}_{k,k} = 2.0\,|\hat{E}'_{dk}\,|\,G_{k,k} \quad , k \in J_{IN}$$

$$d_I = h^I_{44}\,/\,M_{iI} \quad , \hat{d}_I = h^I_{55}\,/\,M_{iI+1} \quad , \tilde{d}_I = h^I_{66}\,/\,M_{iI+2} \;, \quad \overline{d}_I = h^I_{77}\,/\,M_N$$

$$K_I = K_{iI}\,h^I_{88} \;, \quad \hat{K}_I = K_{\,iI+1}\,h^I_{99} \;, \quad \tilde{K}_I = K_{iI+2}\,h^I_{10,10} \;, \quad \overline{K}_I = K_N\;h^I_{11,11}$$

$$L_I = L_{iI}\,h^I_{12,12} \;, \quad \hat{L}_I = L_{iI+1}\;h^I_{13,13} \;, \quad \tilde{L}_I = L_{\,iI+2}\,h^I_{14,14} \;, \quad \overline{L}_I = L_N\;h^I_{15,15}$$

Definition of the elements Z_{IK}:

The off-diagonal elements Z_{IK}, of the aggregation matrix A, are defined as follows (see Notation)

$$Z_{IK} = Z_3\,(Z_{IKa}\,;\,Z_{IKb}\;;\,Z_{IKc})$$

where,

$$Z_{IKa} = Z_3\;(Z_{Ia}\;;\;\hat{Z}_{Ia}\;;\;\tilde{Z}_{Ia})$$

$$Z_{IKb} = Z_3\;(Z_{Ib}\;;\;\hat{Z}_{Ib}\;;\;\tilde{Z}_{Ib})$$

$$Z_{IKc} = Z_3\;(Z_{Ic}\;;\;\hat{Z}_{Ic}\;;\;\tilde{Z}_{Ic})$$

and where,

$$Z_{Ia} = Z_2\,[Z_3\,(Z_{iI,iK}\;;\;Z_{iI+1,iK}\;;\;Z_{iI+2,iK})\;;Z_{N,iK}\;]$$

$$Z_{iI,iK} = Z_3\;[e_I\;\{\tilde{U}_{iI,iK} + \max\,[d_{\,iI,iK}\;;\;\tilde{d}_{iI,iK}]\}\;\;;K_I\,C_{\,iI,iK}\;;\;L_I\,\tilde{C}_{iI,iK}\,]$$

$$Z_{iI+1,iK} = Z_3\;[\hat{e}_I\;\{\tilde{U}_{iI+1,iK} + \max\,[d_{\,iI+1,iK}\;;\;\tilde{d}_{iI+1,iK}]\}\;;\;\hat{K}_I\,C_{\,iI+1,iK}\;;\;\hat{L}_I\,\tilde{C}_{iI+1,iK}\,]$$

$$Z_{iI+2,iK} = Z_3 [\tilde{e}_I\{\tilde{U}_{iI+2,iK} + \max [d_{iI+2,iK} ; \tilde{d}_{iI+2,iK}]\} ; \tilde{K}_I C_{iI+2,iK} ; \tilde{L}_I \tilde{C}_{iI+2,iK}]$$

$$Z_{N,iK} = Z_I [\bar{e}_I \{\tilde{U}_{N,iK} + \max [d_{N,iK} ; \tilde{d}_{N,iK}]\} ; \bar{K}_I C_{N,iK} ; \bar{L}_I \tilde{C}_{N,iK}]$$

$$\hat{Z}_{Ia} = Z_2 [Z_3 (Z_{iI,iK+1} ; Z_{iI+1,iK+1} ; Z_{iI+2,iK+1}) ; Z_{N,iK+1}]$$

$$Z_{iI,iK+1} = Z_3 [e_I \{\tilde{U}_{iI,iK+1} + \max [d_{iI,iK+1} ; \tilde{d}_{iI,iK+1}]\} ; K_I C_{iI,iK+1} ; L_I \tilde{C}_{iI,iK+1}]$$

$$Z_{iI+1,iK+1} = Z_3 [\hat{e}_I \{\tilde{U}_{iI+1,iK+1} + \max [d_{iI+1,iK+1} ; \tilde{d}_{iI+1,iK+1}]\} ; \hat{K}_I C_{iI+1,iK+1} ; \hat{L}_I \tilde{C}_{iI+1,iK+1}]$$

$$Z_{iI+2,iK+1} = Z_3 [\tilde{e}_I\{\tilde{U}_{iI+2,iK+1} + \max [d_{iI+2,iK+1} ; \tilde{d}_{iI+2,iK+1}]\} ; \tilde{K}_I C_{iI+2,iK+1} ; \tilde{L}_I \tilde{C}_{iI+2,iK+1}]$$

$$Z_{N,iK+1} = Z_3 [\bar{e}_I\{\tilde{U}_{N,iK+1} + \max [d_{N,iK+1} ; \tilde{d}_{N,iK+1}]\} ; \bar{K}_I C_{N,iK+1} ; \bar{L}_I \tilde{C}_{N,iK+1}]$$

$$\tilde{Z}_{Ia} = Z_2 [Z_3 (Z_{iI,iK+2} ; Z_{iI+1,iK+2} ; Z_{iI+2,iK+2}) ; Z_{N,iK+2}]$$

$$Z_{iI,iK+2} = Z_3 [e_I \{\tilde{U}_{iI,iK+2} + \max [d_{iI,iK+2} ; \tilde{d}_{iI,iK+2}]\} ; K_I C_{iI,iK+2} ; L_I \tilde{C}_{iI,iK+2}]$$

$$Z_{iI+1,iK+2} = Z_3 [\hat{e}_I \{\tilde{U}_{iI+1,iK+2} + \max [d_{iI+1,iK+2} ; \tilde{d}_{iI+1,iK+2}]\} ; \hat{K}_I C_{iI+1,iK+2} ; \hat{L}_I \tilde{C}_{iI+1,iK+2}]$$

$$Z_{iI+2,iK+2} = Z_3 [\tilde{e}_I\{\tilde{U}_{iI+2,iK+2} + \max [d_{iI+2,iK+2} ; \tilde{d}_{iI+2,iK+2}]\} ; \tilde{K}_I C_{iI+2,iK+2} ; \tilde{L}_I \tilde{C}_{iI+2,iK+2}]$$

$$Z_{N,iK+2} = Z_3 [\bar{e}_I\{\tilde{U}_{N,iK+2} + \max [d_{N,iK+2} ; \tilde{d}_{N,iK+2}]\} ; \bar{K}_I C_{N,iK+2} ; \bar{L}_I \tilde{C}_{N,iK+2}]$$

$$Z_{Ib} = Z_2 [Z_3 (\bar{Z}_{iI,iK} ; \bar{Z}_{iI+1,iK} ; \bar{Z}_{iI+2,iK}) ; \bar{Z}_{N,iK}]$$

$$\bar{Z}_{iI,iK} = Z_3 [e_I \hat{E}'_{iI} ; K_I \xi_{iI,iK} ; L_I \hat{\xi}_{iI,iK}] Y_{iI,iK}$$

$$\bar{Z}_{iI+1,iK} = Z_3 [\hat{e}_I \hat{E}'_{iI+1} ; \hat{K}_I \xi_{iI+1,iK} ; \hat{L}_I \hat{\xi}_{iI+1,iK}] Y_{iI+1,iK}$$

$$\bar{Z}_{iI+2,iK} = Z_3 [\tilde{e}_I \hat{E}'_{iI+2} ; \tilde{K}_I \xi_{iI+2,iK} ; \tilde{L}_I \hat{\xi}_{iI+2,iK}] Y_{iI+2,iK}$$

$$\bar{Z}_{N,iK} = Z_3 [\bar{e}_I \hat{E}'_N ; \bar{K}_I \xi_{N,iK} ; \bar{L}_I \hat{\xi}_{N,iK}] Y_{N,iK}$$

$$\hat{Z}_{Ib} = Z_2 [Z_3 (\bar{Z}_{iI,iK+1} ; \bar{Z}_{iI+1,iK+1} ; \bar{Z}_{iI+2,iK+1}) ; \bar{Z}_{N,iK+1}]$$

$$\bar{Z}_{iI,iK+1} = Z_3 [e_I \hat{E}'_{iI} ; K_I \xi_{iI,iK+1} ; L_I \hat{\xi}_{iI,iK+1}] Y_{iI,iK+1}$$

$$\bar{Z}_{iI+1,iK+1} = Z_3 [\hat{e}_I \hat{E}'_{iI+1} ; \hat{K}_I \xi_{iI+1,iK+1} ; \hat{L}_I \hat{\xi}_{iI+1,iK+1}] Y_{iI+1,iK+1}$$

$$\bar{Z}_{iI+2,iK+1} = Z_3 [\tilde{e}_I \hat{E}'_{iI+2} ; \tilde{K}_I \xi_{iI+2,iK+1} ; \tilde{L}_I \hat{\xi}_{iI+2,iK+1}] Y_{iI+2,iK+1}$$

$$\bar{Z}_{N,iK+1} = Z_3 [\bar{e}_I \hat{E}'_N ; \bar{K}_I \xi_{N,iK+1} ; \bar{L}_I \hat{\xi}_{N,iK+1}] Y_{N,iK+1}$$

$$\tilde{Z}_{Ib} = = Z_2 [Z_3 (\bar{Z}_{iI,iK+2} ; \bar{Z}_{iI+1,iK+2} ; \bar{Z}_{iI+2,iK+2}) ; \bar{Z}_{N,iK+2}]$$

$$\bar{Z}_{iI,iK+2} = Z_3 [e_I \hat{E}'_{iI} ; K_I \xi_{iI,iK+2} ; L_I \hat{\xi}_{iI,iK+2}] Y_{iI,iK+2}$$

$$\bar{Z}_{iI+1,iK+2} = Z_3 [\hat{e}_I \hat{E}'_{iI+1} ; \hat{K}_I \xi_{iI+1,iK+2} ; \hat{L}_I \hat{\xi}_{iI+1,iK+2}] Y_{iI+1,iK+2}$$

$$\bar{Z}_{iI+2,iK+2} = Z_3 [\tilde{e}_I \hat{E}'_{iI+2} ; \tilde{K}_I \xi_{iI+2,iK+2} ; \tilde{L}_I \hat{\xi}_{iI+2,iK+2}] Y_{iI+2,iK+2}$$

$$\bar{Z}_{N,iK+2} = Z_3 [\bar{e}_I \hat{E}'_N ; \bar{K}_I \xi_{N,iK+2} ; \bar{L}_I \hat{\xi}_{N,iK+2}] Y_{N,iK+2}$$

$$Z_{Ic} = Z_2 [Z_3 (\hat{Z}_{iI,iK} ; \hat{Z}_{iI+1,iK} ; \hat{Z}_{iI+2,iK}) ; \hat{Z}_{N,iK}]$$

$$\hat{Z}_{iI,iK} = Z_3 [e_I \hat{E}'_{iI} ; K_I \hat{\xi}_{iI,iK} ; L_I \xi_{iI,iK}] Y_{iI,iK}$$

$$\hat{Z}_{iI+1,iK} = Z_3 [\hat{e}_I \hat{E}'_{iI+1} ; \hat{K}_I \hat{\xi}_{iI+1,iK} ; \hat{L}_I \xi_{iI+1,iK}] Y_{iI+1,iK}$$

$$\hat{Z}_{iI+2,iK} = Z_3 [\tilde{e}_I \hat{E}'_{iI+2} ; \tilde{K}_I \hat{\xi}_{iI+2,iK} ; \tilde{L}_I \xi_{iI+2,iK}] Y_{iI+2,iK}$$

$$\hat{Z}_{N,iK} = Z_3 [\bar{e}_I \hat{E}'_N ; \bar{K}_I \hat{\xi}_{N,iK} ; \bar{L}_I \xi_{N,iK}] Y_{N,iK}$$

$$\hat{Z}_{Ic} = Z_2 [Z_3 (\hat{Z}_{iI,iK+1} ; \hat{Z}_{iI+1,iK+1} ; \hat{Z}_{iI+2,iK+1}) ; \hat{Z}_{N,iK+1}]$$

$$\hat{Z}_{iI,iK+1} = Z_3 [e_I \hat{E}'_{iI} ; K_I \hat{\xi}_{iI,iK+1} ; L_I \xi_{iI,iK+1}] Y_{iI,iK+1}$$

$$\hat{Z}_{iI+1,iK+1} = Z_3 [\hat{e}_I \hat{E}'_{iI+1} ; \hat{K}_I \hat{\xi}_{iI+1,iK+1} ; \hat{L}_I \xi_{iI+1,iK+1}] Y_{iI+1,iK+1}$$

$$\hat{Z}_{iI+2,iK+1} = Z_3 [\tilde{e}_I \hat{E}'_{iI+2} ; \tilde{K}_I \hat{\xi}_{iI+2,iK+1} ; \tilde{L}_I \xi_{iI+2,iK+1}] Y_{iI+2,iK+1}$$

$$\hat{Z}_{N,iK+1} = Z_3 [\bar{e}_I \hat{E}'_N ; \bar{K}_I \hat{\xi}_{N,iK+1} ; \bar{L}_I \xi_{N,iK+1}] Y_{N,iK+1}$$

$$\tilde{Z}_{Ic} = Z_2 [Z_3 (\hat{Z}_{iI,iK+2} ; \hat{Z}_{iI+1,iK+2} ; \hat{Z}_{iI+2,iK+2}) ; \hat{Z}_{N,iK+2}]$$

$$\hat{Z}_{iI,iK+2} = Z_3 [e_I \hat{E}'_{iI} ; K_I \hat{\xi}_{iI,iK+2} ; L_I \xi_{iI,iK+2}] Y_{iI,iK+2}$$

$$\hat{Z}_{iI+1,iK+2} = Z_3 [\hat{e}_I \hat{E}'_{iI+1} ; \hat{K}_I \hat{\xi}_{iI+1,iK+2} ; \hat{L}_I \xi_{iI+1,iK+2}] Y_{iI+1,iK+2}$$

$$\hat{Z}_{iI+2,iK+2} = Z_3 [\tilde{e}_I \hat{E}'_{iI+2} ; \tilde{K}_I \hat{\xi}_{iI+2,iK+2} ; \tilde{L}_I \xi_{iI+2,iK+2}] Y_{iI+2,iK+2}$$

$$\hat{Z}_{N,iK+2} = Z_3 [\bar{e}_I \hat{E}'_N ; \bar{K}_I \hat{\xi}_{N,iK+2} ; \bar{L}_I \xi_{N,iK+2}] Y_{N,iK+2} \qquad \text{(A-2)}$$

In Equation 36 , we define the following constants,

$$e_I = Z_2 (h^I_{14} ; h^I_{44}) / M_{iI} \quad , \quad \hat{e}_I = Z_2 (h^I_{25} ; h^I_{55}) / M_{iI+1}$$

$$\tilde{e}_I = Z_2 (h^I_{36} ; h^I_{66}) / M_{iI+2} \quad , \quad \bar{e}_I = h^I_{77} / M_N$$

$$d_{i,j} = \hat{E}_{qi} | E'_{dj} | Y_{ij} \hat{\xi}_{ij} \quad , \quad \tilde{d}_{i,j} = |\hat{E}'_{di} | E'_{qj} Y_{ij} \hat{\xi}_{ij}$$

$$C_{i,j} = (|\hat{E}'_{dj} | \xi_{ij} + \hat{E}'_{qj} \hat{\xi}_{ij}) Y_{ij}$$

$$\tilde{C}_{i,j} = (|\hat{E}'_{dj} | \hat{\xi}_{ij} + E'_{qj} \xi_{ij}) Y_{ij} \quad , \quad i \neq j \quad , i \in J_{IN} \quad , j \in J_K$$

$$\beta_{Ia} = \cos (2 \theta_{iI,jI+1}) \quad , \quad \beta_{Ib} = \cos (2 \theta_{iI,jI+2}) \quad , \quad \beta_{Ic} = \cos (2 \theta_{iI+1,jI+2})$$

$$\beta_{Na} = \cos (2 \theta_{iI,N}) \quad , \quad \beta_{Nb} = \cos (2 \theta_{iI+1,N}) \quad , \quad \beta_{Nc} = \cos (2 \theta_{iI+2,N})$$

معيار أتزان جديد يطبق على أنظمة القدرة كبيرة المقياس مع تضاؤل مجال المولد

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ملخص البحث

فى هذا البحث استخدمت طريقه ليابونوف المباشره لأنجاز تحليل الاتزان الأنتقالى لنظام قدره يشتمل على عدد "ن" أله مع الأخذ فى الاعتبار نموذج للمولد اكثر تعقيدا. تم تمثيل كل مولد بما يسمى نموذج المحورين، والذى فيه نعتبر أن مركبتى الجهد الموازيه والمتعامده للجهد الداخلى للمولد متغيره مع الزمن.

- أحمال المولد يتم تمثيلها بمعاوقات توازى ثابته، وعندئذ يتم أزاله كل عقد النظام (فيما عدا العقد الداخله للمولدات) وفى النهايه نحصل على مصفوفه النظام المخفضه من الدرجه ن.
- بتطبيق طريقه الفك والتركيب، يفك النظام بحيث أن كل تحت نظام يشتمل على ثلاثه مولدات اضافه الى الأله المقارنه.
- تم وصف كل مولد بنموذج ديناميكى من الدرجه الرابعه مع الأخذ فى الاعتبار حاله الأخماد الميكانيكى الغير متماثل تم الحصول على النموذج الرياضى للنظام. تم تقسيم هذا النموذج الى عدد $\frac{N-1}{3}$ تحت أنظمه مرتبطه كل منها من الدرجه الخامسه عشر، ثم تم فك كل تحت نظام حر (يشتمل على سته دوال غير خطيه)، وارتباطات.
- تم تكوين داله ليابونوف متجهه وأستخدمت لأجراء التراكب للنظام، تم الحصول على مصفوفه تراكب من الدرجه $\frac{N-1}{3}$ للنظام، اتزان هذه المصفوفه يتضمن اتزان النظام.
- كمثال توضيحى طبق معيار الاتزان المقدم على نظام قدره يشتمل على عشره مولدات، أحدى عشر قضيب. تم الحصول على تقدير لحيز اتزان النظام.

  تم اجراء حسابات الاتزان الانتقالى للنظام مع الأخذ فى الاعتبار حدوث قصر ثلاثى الأوجه بجوار أحد قضبان النظام، ثم استخدم معيار الاتزان لإيجاد الزمن الحرج بطرقه مباشره لعزل منطقه القصر.
- وجد أن معيار الاتزان المقدم مناسبا ويمكن استخدامه بسهوله للدراسات العمليه والمباشره لأنظمه القدره كبيرة المقياس والتى تشتمل على أكثر من عشره مولدات.
- وجد ايضا أن المعيار المقدم للاتزان يمكنه انقاص القصور فى طريقه الفك والتركيب.