

# GENERALIZED ANALYSIS OF THE AC MACHINE WITH STATOR AND ROTOR PHASES CONNECTED IN SERIES

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## ABSTRACT

This paper is intended to be the first part in a series of a research work related to the alternating current machine when its stator and rotor phase windings are connected in series. Procedures have been established-based on the generalized machine theory-for handling a general machine with an  $m$ -phase stator and  $m$ -phase rotor under balanced or unbalanced excitation. An additional transformation matrix has been used to solve the linear voltage equations which contain time-varying coefficients. A better understanding of the energy conversion process in such machines has been achieved. The comparison between the theoretical and experimental results shows the validity of the presented analysis.

**Keywords:** Electrical Machines, Polyphase series connected ac machines, Reference Frame Transformation  
Generalized machine theory, Energy conversion.

## INTRODUCTION

The series connected  $m$ -to- $m$  winding machine is practically an ac machine with  $m$ -phases on the stator connected in series with  $m$ -phases on the rotor via sliding contacts. Each two series connected phases become one winding of periodically varying inductance during the steady state operation. This is the reason behind the name "parametric machine" usually found in the literature [1-4]. The connection of phases is carried out as follows: the stator phase number 1 is connected with the rotor phase number 1, the stator phase number 2 is connected with the rotor phase number  $m$ , the stator phase number 3 is connected with the rotor phase number  $m-1$ , and so on until finally we find that the stator  $m$ -th phase is to be connected with the rotor phase number 2. By this sequence of connection when this machine is fed from balanced supply, two rotating fields are established in the air gap:

- a forward rotating field owing to the stator normal connection and,

- a backward rotating field due to the sequence of rotor connection.

Each one of the two fields rotates with an angular speed equal to the supply angular frequency. The two fields will be in synchronism if, and only if, the rotor angular speed is equal to twice the supply angular frequency in the forward direction. This case of operation is compatible with the condition of energy conversion [5]. Because the rotor neither slips behind the rotating field nor be in synchronism with it, such a machine is neither induction machine nor synchronous one. This machine can be operated as a generator when a suitable capacitor bank is connected across its terminals and the rotor is driven by means of a prime mover [6]. In this mode of operation, the frequency of the output voltage is equal to half the rotor electrical angular frequency.

It is clear that the nature of electric connection in such machines draws our attention towards the symmetrical component transformation. The analysis presented in this paper involves:



- the mathematical model in terms of a set of variables before and after the symmetrical component transformation,
- the variable coefficient transformation without losing the essential information.
- the effect of transformation on the electromagnetic torque and,
- the agreement between theoretical and experimental results.

Before generalizing the treatment, the 3 phase series connected ac machine will be discussed. Saturation is not included.

**THE 3 TO 3 SERIES CONNECTED PHASE AC MACHINE**

Figure 1 shows the proper connection of phases in our case which can be made in a 3 phase wound rotor induction motor. The voltage equations for the stator and rotor windings can be written in the following matrix form :

$$V^s = \{R^s + L^{ss} p\} I^s + P \{L^{sr} I^r\} \quad (1)$$

$$V^r = \{R^r + L^{rr} p\} I^r + P \{L^{rs} I^s\} \quad (2)$$

where the stator voltage column, rotor voltage column, stator current column and rotor current column matrices are:

$$V^s = [V_A \ V_B \ V_C]_t$$

$$V^r = [V_a \ V_b \ V_c]_t$$

$$I^s = [i_A \ i_B \ i_C]_t$$

$$I^r = [i_a \ i_b \ i_c]_t$$

and the suffix t means the transpose of the matrix.

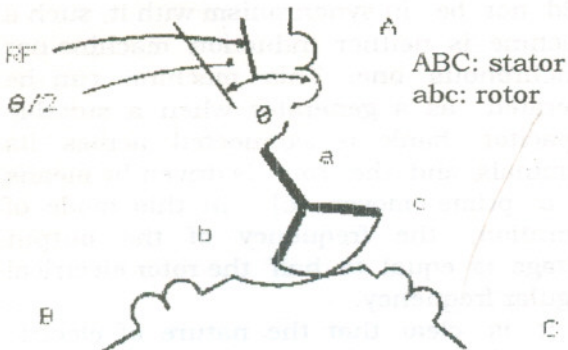


Figure 1 Connection of phases in a 3 to 3 series connected phase ac machine

Since the machine is symmetric the resistance per phase is the same for the three phases, then  $R_A = R_B = R_C = R_s$  where  $R_s$  is the stator resistance per phase. Similarly for the rotor  $R_a = R_b = R_c = R_r$  where  $R_r$  is the rotor resistance per phase. That is to say that  $R^s$  and  $R^r$  can be written as :  $R^s = R_s U_3$  and  $R^r = R_r U_3$  where  $U_3$  is 3 by 3 unit matrix.

The inductance matrices  $L^{ss}$  and  $L^{rr}$  which will specify the self and mutual inductances of the stator and rotor windings are of the forms:

$$L^{ss} = \begin{bmatrix} L_s + L_{ms} & -0.5L_{ms} & -0.5L_{ms} \\ -0.5L_{ms} & L_s + L_{ms} & -0.5L_{ms} \\ -0.5L_{ms} & -0.5L_{ms} & L_s + L_{ms} \end{bmatrix} \quad (3)$$

$$L^{rr} = \begin{bmatrix} L_r + L_{mr} & -0.5L_{mr} & -0.5L_{mr} \\ -0.5L_{mr} & L_r + L_{mr} & -0.5L_{mr} \\ -0.5L_{mr} & -0.5L_{mr} & L_r + L_{mr} \end{bmatrix} \quad (4)$$

where  $L_s, L_r$  are the stator and rotor leakage inductances per phase and  $L_{ms}, L_{mr}$  are the stator and rotor magnetizing inductances per phase. The mutual inductance between the stator and rotor will take the form :

$$L^{sr} = M \begin{bmatrix} \cos\theta & \cos(\theta + 2\pi/3) & \cos(\theta + 4\pi/3) \\ (\cos\theta - 2\pi/3) & \cos\theta & \cos(\theta + 2\pi/3) \\ \cos(\theta - 4\pi/3) & \cos(\theta - 2\pi/3) & \cos\theta \end{bmatrix} \quad (5)$$

where M is the maximum value of the inductance between a stator phase and a rotor phase when  $\theta$  between them is zero. The matrix  $L^{rs}$  is the transpose of  $L^{sr}$ .

Now the machine under consideration has six-phase voltage equations given by Equations 1 and 2. However, it can be described by three voltage equations only because of the series connection. The three voltage equations are:  $V_{Aa} = V_A + V_a, V_{Bc} = V_B + V_c, V_{Cb} = V_C + V_b$

When the machine is fed from balanced sinusoidal supply then, these voltages can be written in the form of :



## Generalized Analysis of the ac Machine With Stator and Rotor Phases Connected in Series

$$\begin{aligned} V_{Aa} &= \sqrt{2} V \cos(\omega t + \phi_v) \\ V_{Bc} &= \sqrt{2} V \cos(\omega t + \phi_v - 2\pi/3) \\ V_{Cb} &= \sqrt{2} V \cos(\omega t + \phi_v + 2\pi/3) \end{aligned} \quad (6)$$

Also the currents can be written in the form:

$$\begin{aligned} i_A &= \sqrt{2} I \cos(\omega t + \phi_i) \\ i_B &= \sqrt{2} I \cos(\omega t + \phi_i - 2\pi/3) \\ i_C &= \sqrt{2} I \cos(\omega t + \phi_i + 2\pi/3) \end{aligned} \quad (7)$$

Where the phase angles  $\phi_v$  and  $\phi_i$  are measured from an arbitrary reference. The electrical equation of motion will be:

$$V = ZI \quad (8-a)$$

and the mechanical equation of motion will be:

$$T_e - T_L = J\dot{\theta} \quad (8-b)$$

where  $V$  represents:  $[V_{Aa} \ V_{Bc} \ V_{Cb}]_t$  and  $I$  represents:

$[i_A \ i_B \ i_C]$ ,  $J$  is the inertia,  $T_L$  is the load torque and  $T_e$  is the developed torque. The impedance matrix  $Z$  will be:

$$Z = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix}$$

in which:

$$Z_{11} = (R_s + R_r) + (L_s + L_r) P + (L_{ms} + L_{mr}) P + 2MP \cos \theta$$

$$Z_{12} = Z_{21} = 2MP \cos(\theta - 2\pi/3) - 0.5(L_{mr} + L_{mr}) P$$

$$Z_{13} = Z_{31} = 2MP \cos(\theta + 2\pi/3) - 0.5(L_{ms} + L_{mr}) P$$

$$Z_{22} = (R_s + R_r) + (L_s + L_r) P + (L_{ms} + L_{mr}) P + 2MP \cos(\theta + 2\pi/3)$$

$$Z_{23} = Z_{32} = 2MP \cos(\theta) - 0.5(L_{ms} + L_{mr}) P$$

$$Z_{33} = (R_s + R_r) + (L_s + L_r) P + (L_{ms} + L_{mr}) P + 2MP \cos(\theta - 2\pi/3) \quad (9)$$

This means that:

$$\begin{aligned} V_{Aa} &= Z_{11} i_A + Z_{12} i_B + Z_{13} i_C \\ V_{Bc} &= Z_{21} i_A + Z_{22} i_B + Z_{23} i_C \\ V_{Cb} &= Z_{31} i_A + Z_{32} i_B + Z_{33} i_C \end{aligned} \quad (10)$$

By substituting the values of impedances (Equation 9), currents (Equation 7) and voltages (Equation 6) in any one of the three

relations given by Equation 10 we come to the following voltage equation:

$$V \cos(\omega t + \phi_v) = RI \cos(\omega t + \phi_i) + j\omega L I \cos(\omega t + \phi_i) + \{j(3/2)\omega L_m I \cos(\omega t + \phi_i) + j3\omega M I \cos(\theta - \omega t + \phi_i)\} \quad (11)$$

In which the first two terms of the right hand side represent the resistance and leakage reactance voltage drops while the last two terms between brackets play the role of the back EMF (-E).

It is also possible to write this equation in the following phasor form where  $\theta = 2\omega t$  in case of steady operation

$$\begin{aligned} V / \phi_v &= RI / \phi_i + j\omega L I / \phi_i + (-E / \phi_e) \\ \text{and } (-E / \phi_e) &= \{j(3/2)\omega L_m I / \phi_i + j\omega (3M) I / \phi_i\} \end{aligned} \quad (12)$$

where:

$$R = R_s + R_r, \quad L = L_s + L_r, \quad \text{and } L_m = L_{ms} + L_{mr}$$

The phasor diagram shown in Figure 2 for motor mode; where the input phase current  $I$  is taken as reference axis i.e. Equation 12 is rotated by  $\phi_i$  in the clockwise direction. The angle between  $(-E)$  and  $V$  phasors is the torque angle  $\delta$ .

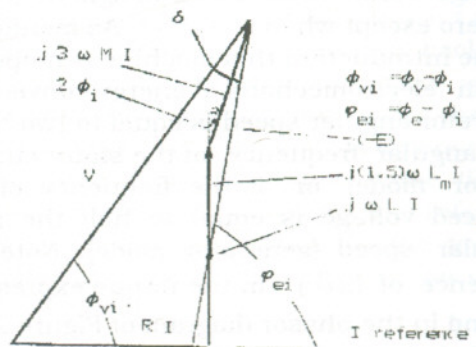


Figure 2 Phasor diagram

The equivalent circuit is shown in Figure 3

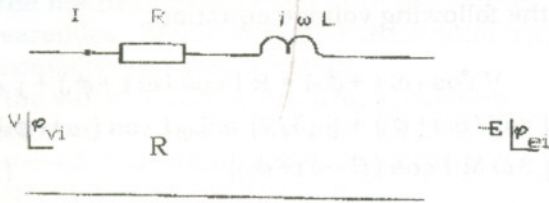


Figure 3 The equivalent circuit.

**Torque**

The electromagnetic torque can be obtained from the following equation:

$$T_e = 0.5 [i_A i_B i_C] \begin{bmatrix} 0 & \partial L^{sr} / \partial \theta \\ \dots & \dots \\ \partial L^{rs} / \partial \theta & 0 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \\ i_A \\ i_B \\ i_C \end{bmatrix} \quad (13)$$

Substituting the currents from Equation 7, mutual inductances from Equation 5 and simplification then the torque equation is reduced to:

$$T_e = -4.5 M I^2 \sin \{ (\theta - 2 \omega t) - 2 \phi_i \} \quad (14)$$

It is clear that for any value of  $\omega$  the average value of the electromagnetic torque is zero except when  $\theta = 2 \omega t$ . As mentioned in the introduction this machine can operate as an electromechanical energy converter if the rotor angular speed is equal to two times the angular frequency of the stator current (motor mode) or if the frequency of the induced voltage is equal to half the rotor angular speed (generator mode). Note the presence of  $(2 \phi_i)$  in the torque expression 14 and in the phasor diagram of Figure 2.

Owing to the space dependent factor contained in  $L^{sr}$  and  $L^{rs}$  the mathematical model of Equation 8 contains nonlinear differential equations with time varying coefficients. In most cases the solutions to these differential equations cannot be affected directly. Fortunately techniques have been developed [7,8,9] which do permit

the matrices to be diagonalized. Thereby the equations can be reduced to a set which can be solved independently. An appropriate transformation is used to diagonalize the parameter matrices using the techniques given in [7-9]. Owing to the nature of the machine under consideration the symmetrical component transformation will be used (suffix p will be used for the positive sequence, n for the negative sequence and z for the zero sequence).

**SYMMETRICAL COMPONENT TRANSFORMATION**

If  $V = C V'$ , and  $I = C I'$ , then,  $V' = C^{-1} V$  and  $I' = C^{-1} I$  where  $V$  and  $I$  are defined in Equation 8.  $V'$  is the column  $[V_p V_n V_z]_t$  and  $I'$  is the column  $[i_p i_n i_z]_t$  i.e the positive, negative and zero sequence voltages and currents. The matrix  $C$  takes the form:

$$C = 1 / \sqrt{3} \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix} \quad (15)$$

where  $a = \exp (j 2 \pi / 3)$

For this transformation matrix  $C^{-1} = C_t^*$  where \* denotes the conjugate. It is now desired to effect the transformation of the Equation 8-a as follows:

$$\begin{aligned} V' &= C^{-1} V = C^{-1} Z I \\ V' &= C^{-1} Z C I' \\ V' &= Z' I' \end{aligned} \quad (16)$$

where  $Z' = C^{-1} Z C$  is the transformed matrix which takes the following form after multiplication and simplification :

$$Z' = \begin{bmatrix} Z'_{sr} & Z'_{12} & Z'_{13} \\ Z'_{21} & Z'_{sr} & Z'_{23} \\ Z'_{31} & Z'_{32} & Z'_{33} \end{bmatrix} \quad (17)$$

where:

$$\begin{aligned} Z'_{sr} &= (R_s + R_r) + (L_s + L_r) P + 1.5 (L_{ms} + L_{mr}) P; \\ Z'_{33} &= (R_s + R_r) + (L_s + L_r) P; \\ Z'_{12} &= 2 M P \exp (j \theta); \\ Z'_{21} &= 3 M P \exp (-j \theta); \text{ and,} \\ Z'_{13} &= Z'_{23} = Z'_{31} = Z'_{32} = 0 \end{aligned} \quad (18)$$



this means that:

$$\begin{aligned} V_p &= Z'_{sr} i_p + Z'_{12} i_n; \\ V_n &= Z'_{21} i_p + Z'_{sr} i_n \text{ and} \\ V_z &= Z'_{33} i_z \end{aligned} \quad (19)$$

or, in matrix form :

$$[V] = [Z'] [i] \quad (19-a)$$

It is noted that, contrary to both synchronous and asynchronous machines, each of the positive and negative sequence quantities is dependent on each other. It is noted also that due to the lack of coupling between stator and rotor zero sequence systems, they do not share in the process of energy conversion. Moreover, the zero sequence current is unique in the machine and circulates through a closed circuit composed of both the stator and rotor windings. A rather interesting result is the unusual way in which the angle  $\theta$  enters into  $V_p$  and  $V_n$ . It is noticed that the (positive/negative) sequence current induces a voltage of the form  $P \{3 M \exp (+j\theta)\}$  in the equation of the (negative/positive) sequence voltage. This means that each kind of sequences acts with the help of the other and not lonely (as a member of a team) in energy conversion.

### EFFECT OF SYMMETRICAL COMPONENT TRANS-FORMATION ON TORQUE

The effect of symmetrical component transformation on the electromagnetic torque is obtained using the torque equation:

$$T_e = 0.5 [I_t^* (\partial L / \partial \theta) I], \quad I = C I', \quad I' = C^{-1} I, \quad I_t^* = I_t'^* C_t'$$

which means that:

$$T_e = 0.5 \{ I_t'^* C_t'^* (\partial L / \partial \theta) C I' \}$$

After substitution with the appropriate values and simplification this torque expression becomes:

$$T_e = j(1.5)M \{ i_p^* i_n \exp (j\theta) - i_p i_n^* \exp (-j\theta) \} \quad (20)$$

The previous equation is a real quantity. The following instantaneous symmetrical components  $i_p$  and  $i_n$  :

$$\begin{aligned} i_p &= (\sqrt{3}/\sqrt{2}) I [\cos (\omega t + \phi i) + j \sin (\omega t + \phi i)] \\ i_n &= (\sqrt{3}/\sqrt{2}) I [\cos (\omega t + \phi i) - j \sin (\omega t + \phi i)] \end{aligned}$$

in Equation 20 where  $i_p^* = i_n$  and  $i_n^* = i_p$ , torque expression becomes:

$$T_e = j (1.5) M \{ i_n^2 \exp (j\theta) - i_p^2 \exp (-j\theta) \} \quad (21)$$

Equation 20 can be reduced to equation 14 by considering a symmetric machine fed from balanced supply.

### VARIABLE COEFFICIENT TRANSFORMATION

The set of voltage equations given by Equation 19 still involves variable coefficients owing to the factor of the form  $p \{3 M \exp (\pm j\theta)\}$ . In the simple case of rotation at constant speed  $\theta = 2 \omega t$ , these equations are linear, with time - varying coefficients. since a general solution of such equations is unknown, some additional progress towards a solution by means of introducing a new transformation is needed. The additional transformation matrix is denoted by  $D$  and is of course a function of  $\theta$ . the transformed quantities will be the forward and backward components in the form of :  $i_f = i_p \exp (-j\theta/2)$  and  $i_b = i_n \exp (j\theta/2)$ . Note that the forward and backward currents have the same amplitude as the sequence components from which they are derived, only the frequency is different. For these quantities a reference frame {RF} is assumed as shown in Figure 1 The reference frame {RF} rotates with  $\omega$  in the forward direction in space. The matrix  $D$  ( for which  $D^{-1} = D_t^*$ ) will be :

$$D = \begin{bmatrix} \exp(j\theta/2) & 0 & 0 \\ 0 & \exp(j\theta/2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (22)$$



Applying this transformation on Equation 19-a then:

$$[V''] = [Z''] [I''] \quad (23)$$

where:

$$V'' = D^{-1} V' = [V_f V_b V_z]_t, I'' = D^{-1} I' = [i_f i_b i_z]_t$$

$$\text{and } Z'' = \{D^{-1} Z' D\}$$

which becomes:

$$Z'' = \begin{bmatrix} Z'_{sr} & 3MP & 0 \\ 3MP & Z'_{sr} & 0 \\ 0 & 0 & Z'_{33} \end{bmatrix} \quad (24)$$

The forward, backward and zero voltage equations can be written as follows:

$$V_f = Z'_{sr} i_f + 3 M P i_b \quad (25)$$

$$V_b = 3 M P i_f + Z'_{sr} i_b \quad (26)$$

$$V_z = Z'_{33} i_z \quad (27)$$

Observe that, the additional transformation has removed the angular dependence  $\theta$  while the zero sequence system does not change. Remembering that, the rotor circuit is fed directly from the supply voltage like the stator, the corresponding forms of Equations 25-27 under steady state sinusoidal excitation result by replacing P by  $j\omega$  as follows:

$$V_f = (R + j\omega L + j 1.5 \omega L_m) i_f + j 3 \omega M i_b \quad (28)$$

$$V_b = j 3 \omega M i_f + (R + j\omega L + j 1.5 \omega L_m) i_b \quad (29)$$

$$V_z = (R + j\omega L) i_z \quad (30)$$

The set of voltage equations (28-30) may be represented by a set of equivalent circuits as shown in Figure 4.

### TORQUE IN TERMS OF FORWARD AND BACKWARD CURRENTS .

The torque  $T_e$  in terms of forward and backward components will be of the form :

$$T_e = 0.5 \{i_t^{**} [D_t^* E D] I''\} \quad (31)$$

where  $E = C_t^* (\partial L / \partial \theta) C$

which leads us to the following very simple form:

$$T_e = 0.5 I_t^{**} G I'' \quad (32)$$

where G is the following matrix:

$$G = D_t^* E D \quad (33)$$

After substituting the relevant values and simplification the torque expression becomes:

$$T_e = j 1.5 M \{i_f^* i_b - i_b^* i_f\} \quad (34)$$

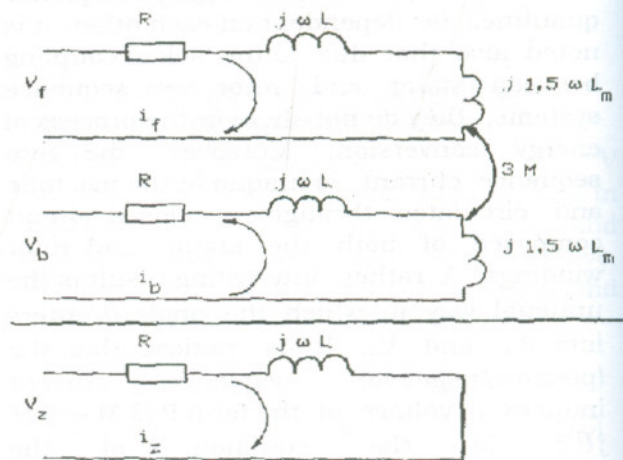


Figure 4 The proposed equivalent circuit of the ac machine with stator and rotor 3 phases connected in series.

It is also noted that, the backward field induces voltage by mutual induction in the circuit of the forward voltage and the forward field induces the same voltage in the circuit of the backward voltage. The coupled circuit representation is used in Figure 4 because each circuit cannot act without the other.

### THE D-Q MODEL

The d-q model can be derived directly from the forward and backward components as follows :

$$V_{dq} = H^{-1} V_{fb} = \{H^{-1} Z_{fb} H\} H^{-1} i_{fb}$$

where H is the transformation matrix:

$$(1/\sqrt{2}) \begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \quad (35)$$



then  $Z_{dq} = H^{-1} Z_{fb} H$  will be

$$\begin{bmatrix} Z'_{sr} + 3MP & 0 \\ 0 & Z'_{sr} + 3MP \end{bmatrix} \quad (36)$$

This is why other researchers handled such machine as if it were a synchronous machine of hypothetical saliency.

**TORQUE IN TERMS OF D-Q TERMS:**

From Equation 32 it is possible to write torque equation as follows:

$$T = 0.5 I_t^{**} [H_t^* GH] I'''$$

while the result of multiplication between brackets is :

$$\begin{bmatrix} 0 & 3M \\ 3M & 0 \end{bmatrix} \quad (37)$$

and the torque becomes:

$$T = 3 M i_d i_q \quad (38)$$

which gives the same value obtained from Equation's 14, 20 and 34. This is obtained by substituting:

$$\begin{aligned} i_d &= (1/\sqrt{2}) (i_r + i_b) \text{ and,} \\ i_q &= (1/\sqrt{2}) (-j i_r + j i_b) \end{aligned}$$

in Equation 38.

**THE GENERALIZED MACHINE**

The generalized machine consists of a stator (likewise the rotor) which has m identical coils per pair of poles located around the air gap periphery which are spaced  $\alpha = 2\pi/m$  electrical degrees apart ( $m \geq 3$ ). Also the machine is assumed to be of non-salient pole type with a uniform air gap. The resistance parameter matrices may be written directly as  $R_s U_m$  for the stator and  $R_r U_m$  for the rotor where  $U_m$  is  $m \times m$  unit matrix. The inductance parameters matrices possess cylindrical symmetry [11]. The mutual inductance between the stator i-th coil and the rotor k-th coil can be defined by the element:

$$L^{s'ik} = M \cos \{ \theta + (k-i) \alpha \} \quad (39)$$

The matrix C Equation 15 will be  $m \times m$  matrix in which the first row is:

$$(1/\sqrt{m}) [a^1_m a^2_m a^3_m \dots a^m_m]$$

the second row is :

$$(1/\sqrt{m}) [a^1_{m-1} a^2_{m-1} a^3_{m-1} \dots a^m_{m-1}]$$

the third row is :

$$(1/\sqrt{m}) [a^1_{m-2} a^2_{m-2} a^3_{m-2} \dots a^m_{m-2}]$$

and so on until the m-th row which is :

$$(1/\sqrt{m}) [a^1_1 a^2_1 a^3_1 \dots a^m_1]$$

where  $a = \exp (j 2 \pi / m)$ .

In this case we will find m sequence system. The first sequence and the sequence number (m-1) only share in energy conversion. The zero sequence here is the m-th sequence. The symmetrical component transformation reveals an impedance matrix Z' Equation 17 of the elements  $Z'_{11} = Z'_{(m-1)(m-1)} = (R_s + R_r) + (L_s + L_r) P + (m/2)(L_{ms} + L_{mr})P$ ,  $Z'_{1(m-1)} = m MP \exp (j\theta)$ ,  $Z'_{(m-1)1} = m MP \exp (-j\theta)$  and  $Z_{mm} = (R_s + R_r) + (L_s + L_r) P$  only while other elements are zero. this means that only the first, the (m-1) and the m-th sequence systems are found. This needs an additional matrix D of m rows and m columns. In this matrix the diagonal contains only the elements  $D_{11}$  and  $D_{(m-1)(m-1)}$  are the same as in Equation 22 while the element  $D_{mm}$  is one.

The forward, backward and zero voltage equations will be:

$$V_f = Z'_{11} i_f + m MP i_b \quad (40)$$

$$V_b = m MP i_f + Z'_{22} i_b \quad (41)$$

$$V_z = Z'_{mm} i_m \quad (42)$$

These equations can be represented by the equivalent circuit shown in Figure 5. Torque Equations 14, 20 and 34 and 38 will be :

$$T = (m^2/2) I^2 M \sin \{ (\theta - 2 \omega t) - 2 \phi_i \} \quad (43)$$

$$T = (m/2) M [i^*_{m-2} i_2 \exp(j\theta) - i_1 i^*_{m-2} \exp(-j\theta)] \quad (44)$$

$$T = m M i_f i_b \quad (45)$$

$$T = m M i_d i_q \quad (46)$$

respectively, where m is still the original number of phases.



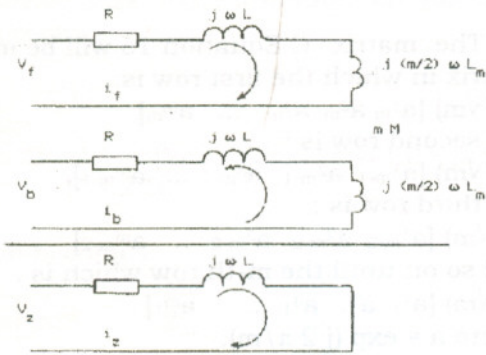


Figure 5 The equivalent circuit of the generalized machine.

**EXPERIMENTAL INVESTIGATION**

In order to prove the validity of the presented analysis, the theoretical results were compared with previously made experimental results in Alexandria University [4]. In these experimental study a three phase slip ring induction motor (its design data are given in the Appendix) was used as a test motor after connecting its stator and rotor phases in series according to the sequence drawn in Figure 1. Input power, input current, applied voltage, output torque and speed were measured for different loads. The developed torque can be calculated from any torque expression given in this paper. By subtracting the mechanical losses the output torque can be computed.

For certain load the power factor is known from the measured input power, current and voltage. According to this power factor and the applied voltage the equivalent circuit of Figure 3 can be solved to get the current and the input power. The torque angle  $\delta$  can be obtained theoretically from

the phasor diagram of Figure 2, Figures 6 (a, b, c and d) show both computed and test results of the input current I (Ampere), power factor angle  $\phi$  (degrees), the output torque T (N.m) and the input power  $P_i$  (watt) respectively versus the torque angle  $\delta$  (degrees).

**CONCLUSIONS**

The general machine theory was applied to the ac machine when the stator phases are connected in series with the rotor phases in a certain sequence. Symmetrical component transformation was performed to show that:

contrary to the conventional poly phase ac machines the positive sequence system depends on the negative sequence system and vice versa. The two systems look like two coupled circuits where each circuit current induces voltage by mutual induction in the other circuit. The zero sequence system is unique in the machine and is not torque producer. Torque is produced due to the interaction between the (positive/negative) current and the flux of the (negative/positive) current. The problem of variable coefficient was solved by means of the forward and backward transformation. Both forward and backward components depend on each other. The very simple d-q model shows why other researchers surmise that it looks like a synchronous machine of hypothetical saliency. This machine exhibits the characteristic of synchronous machine : by increasing load the torque angle increases keeping speed constant. The presented analysis agreed with the previously published experimental study.



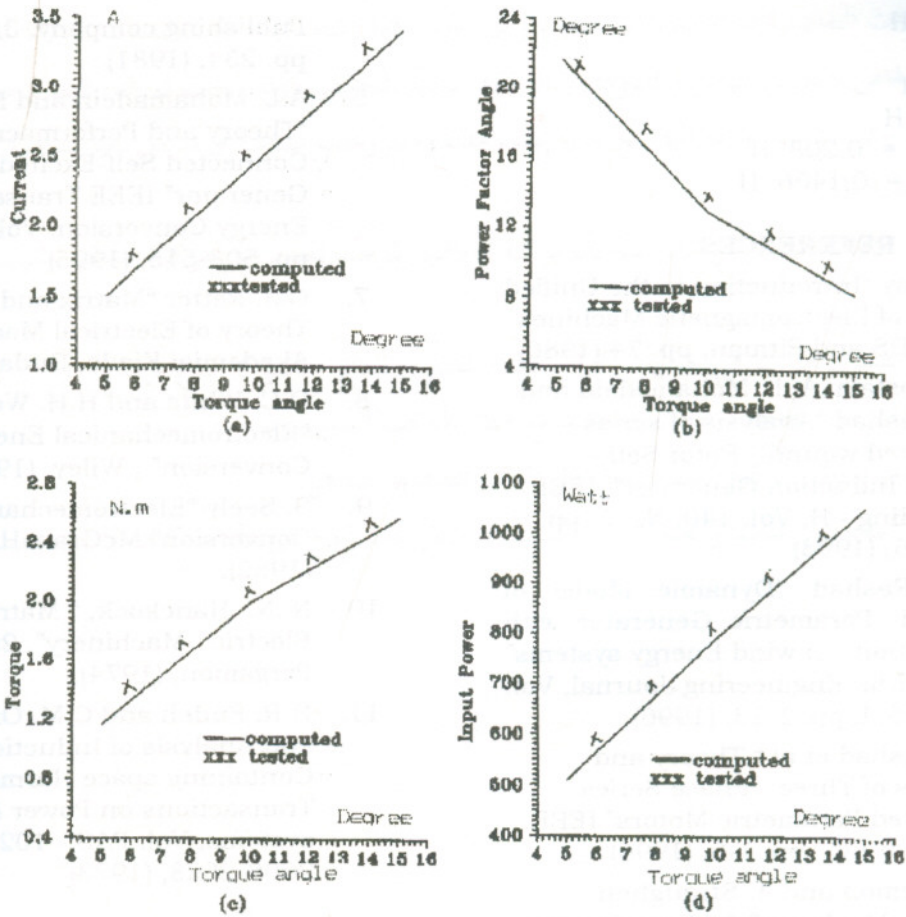


Figure 6 Theoretical and experimental results

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**NOMENCLATURE**

- V :voltage, volt
- I or I :current, ampere
- R :resistance, ohm
- L :inductance, henery
- p :operator
- M :maximum value of mutual inductance, henery
- Z :impedance, ohm
- $\Phi$  :phase angle, degree
- $\theta$  :space angle, degree
- $\delta$  :torque angle, degree
- $\omega$  :angular frequency, rad/sec

*Suffixes:*

- s:stator
- r :rotor
- ss :stator self inductance
- rr :rotor self inductance
- m :magnetizing inductance

**APPENDIX**

The test slip ring induction motor is of the following data:  
 Power : 2.2 KW , frequency : 50 Hz , four poles.  
 stator : 380 V , Y-connected , 3.6 A ,  $R_s = 2.1 \Omega/\text{phase}$  ,  
 Rotor : 328 V , Y-connected , 4.2 A ,  $R_r = 1.96 \Omega/\text{phase}$  ,  
 $M = 0.169 \text{ H}$   
 stator to rotor turns ration = 1.16.



$$L_{ms} = 0.196 \text{ H}$$

$$L_s = 0.012 \text{ H}$$

$$L_{mr} = 0.146 \text{ H}$$

$$L_r = 6 \times 10^{-4} \text{ H}$$

$$L_{ss} = L_s + L_{ms} = 0.208 \text{ H}$$

$$L_{rr} = L_r + L_{mr} = 0.1466 \text{ H}$$

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## التحليل العام لآلات التيار المتغير عند توصيل لفات العضو الثابت بالتوالي مع لفات العضو الدائر

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### ملخص البحث

يقدم هذا البحث الذي يعتبر بداية بحثية لأعمال قادمة في نفس المجال تحليلاً مبنياً على النظرية العامة لآلات الكهربية خاصة بالآلات التيار المتغير عند توصيل لفات العضو الثابت مع لفات العضو الدائر بالتوالي بينما تكون تلك اللغات متعددة ومتماثلة ومتساوية كما أن الآلة تكون عرضة للتغذية من منبع متزن أو غير متزن ويواجه التحليل المقدم باستعمال المركبات



المتماثلة مشكلة المعادلات التفاضلية ذات المعاملات المتغيرة والتي يقوم بحلها باستعمال التحويل للمركبات الأمامية والخلفية ثم يشتق منها النموذج الخاص بالمحورين المباشر والعمودي وبالوصول لنهاية التحليل يكون البحث قد قدم فهما لتحويل الطاقة في هذا النوع من الآلات والتي نشأت الحاجة إليها مع بداية إنشاء مدن ومشروعات صناعية في مناطق نائية تستعمل فيها طاقة الرياح وقد خلص البحث إلى .

- أنه على العكس من الآلات التزامنية واللاتزامنية فإن مركبة التعاقب الموجب تعتمد على مركبة التعاقب السالب والعكس صحيح .

- أن مركبة التعاقب الصفري للثابت هي نفسها مركبة التعاقب الصفري للدائر ولكنها لا تنتج عزمًا .

- أن العزم ينتج من تفاعل تيار التعاقب الموجب/السالب مع فيض التعاقب السالب/الموجب

- أن المركبة الأمامية تعتمد على المركبة الخلفية بينما تظل المركبة الصفرية كما هي .

- أن النموذج المشتق للمحورين المباشر والعمودي بين السبب في النظر لهذا النوع من الآلات كأنها آلات تزامنية ذات بروز وهمي .