ANALYSIS OF REINFORCED CONCRETE RIBBED SLABS

Mohie Eldin S. Shukry

Structural Engineering Department, Faculty of Engineering
Alexandria University, Alexandria, Egypt

ABSTRACT

This paper presents a numerical study using the Finite Element Method for the analysis of reinforced concrete square one way ribbed slabs subjected to either uniformly distributed loads or line loads acting along the ribs. The results of the analysis , in form of deformations, cracking patterns, strains and stresses in both concrete and steel and ultimate capacity, are presented. The results reveal that, for the selected ribbed slab dimensions, cracking of concrete and failure of the slabs initiated at flange near edges of the slabs.

Keywords: Reinforced concrete, Ribbed slabs, Joist floor systems

INTRODUCTION

Ribbed floors are economical for many buildings such as apartment houses, hotels and hospitals, where live loads are fairly small and the spans comparatively long. However, they are not suitable for heavy live loads such as in the case of warehouses, and heavy manufacturing buildings.

A one way ribbed slab (or joist floor system) consists of a series of small, closely spaced reinforced concrete T-beams framing into monolithically cast concrete girders which are in turn carried by the building columns. Regardless of the ratio of length to width of the slab panel, engineers used to consider these slabs as one way.

A type of one way ribbed slab system has evolved known as joist-band system in which the ribs are supported by broad girders having the same depth as the ribs. Separate beam forms are eliminated and the same deck forms the soffit (bottom) of both ribs and girders. The simplified formwork, faster construction, and level ceiling with no obstructing beams, all combine to achieve overall reduction in cost in most cases.

In practice, each rib is analyzed as a beam subjected to loads acting on the width tributary to one rib i.e. acting on a width equals to the distance between centerline of ribs, and then the rib is designed as a beam of flanged section.

Sometimes, line loads in form of partition loads act on ribbed slabs. When these loads act perpendicular to rib direction, the ribs are analyzed as beams subjected to concentrated loads. However, when a line load acts along a single rib, the distribution of such load on this rib and on the neighboring ribs is questionable.

This study presents a numerical nonlinear analysis using the Finite Element Method (FEM) to investigate the behaviour of a square ribbed slab in which ribs are arranged in one direction (i.e. one way) supported on main beams having the same depth as the ribs and the floor is subjected to either distributed loads or line loads acting along the ribs. For the second case of loading, the main variable studied was the position of the loaded rib.

THE FINITE ELEMENT ANALYSIS

A nonlinear Finite Element analysis was used in the present study. The element used is a specialization of the hexahedral solid element developed by Ahmad *et al.* [1] and is applied for the analysis of thick and thin plates. The element consists of eight nodes (corners and mid-sides) with three degrees of freedom at each node; a vertical translation (w), a rotation about x-axis (θ_X) and a rotation about y-axis ((θ_Y). The element was divided across its thickness into a number of layers with the steel reinforcement smeared into the concrete layers. Perfect bond was assumed between the layers. Five values of non-zero stresses (σ_X, σ_Y, τ_{XY}, τ_{YZ}, τ_{ZX}) and strains (ε_X, ε_Y, γ_{XY} , γ_{YZ}, γ_{ZX}) were considered. Modeling of concrete in compression, concrete in tension and the steel reinforcement and also the method of the analysis may be found in Reference 2.

DIMENSIONS AND REINFORCEMENT OF SLABS

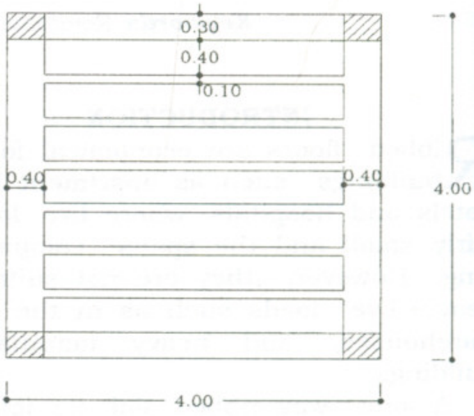
The slabs studied herein are square with dimensions 4 x 4 m as shown in Figure 1-a with the ribs spanned in one direction. The dimensions and reinforcement of the ribs were designed according to the Codes [3,4]. As shown in Figure 1-b, the overall thickness of the floor was 250 mm including 50 mm topping slab. The width of the ribs was taken as 100 mm and their spacing was 500 mm. Each rib was reinforced with 2 ϕ 13 mm bars while the flange was reinforced with ϕ 8 mm each 200 and 250 mm in rib direction and perpendicular to rib direction respectively. The two main beams, supported on four columns with effective span of 3.7 m, had 400 mm width, 250 mm height, and were reinforced with 5 ϕ 16 mm bars . The reinforcement used was mild steel bars with f_y = 240 N/mm^2. Concrete properties were as follows: cube strength f_{cu} = 25 N/mm^2, initial tangent modulus E_{co} = 21 kN/m^2 and tensile strength f_t = 3 N/mm^2.

Figure 2-a shows the Finite Element mesh used in the analysis together with the size of the problem. Due to symmetry in dimensions and loading, only one half of the slab was modeled. Figure 2-b shows the layered element for both ribs and flange.

Four slabs were analyzed under different loads. Slab A was subjected to a uniformly distributed load while slabs B, C, and D were subjected to line loads along the ribs at different positions, as shown in Figure 3 and given in Table 1.

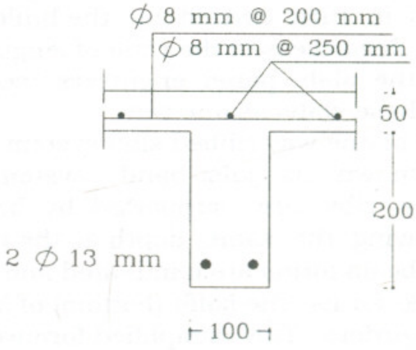
1. at top surface of flange near edge parallel to ribs
2. at bottom surface of loaded rib
3. at bottom surface of main beam

To check the validity of the proposed analysis, the four slabs were analyzed using program SAP90 [5]. The maximum values of deflection obtained at low load level (i.e. before concrete cracking) are compared to those obtained by the proposed analysis in Table 2. The table indicates that a difference of 13 % to 22 % was recorded from both analyses. It should be noted that in program SAP90, a four -node shell element was used for flange, ribs and solid parts.



PLAN

(a)



Cross Section

(b)

Figure 1 (a) RC ribbed slab analyzed;
(b) Dimensions and reinforcement of ribs

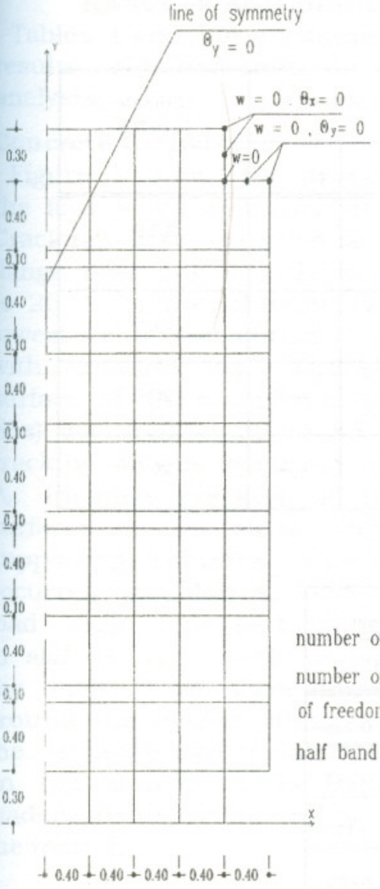


Figure 2-a Finite Element mesh

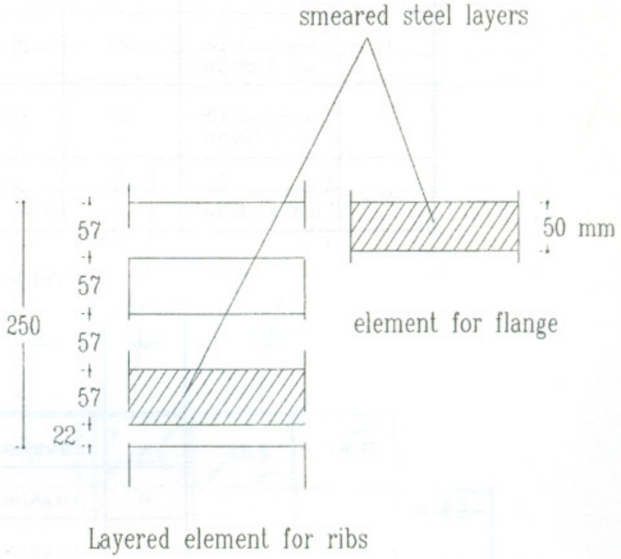
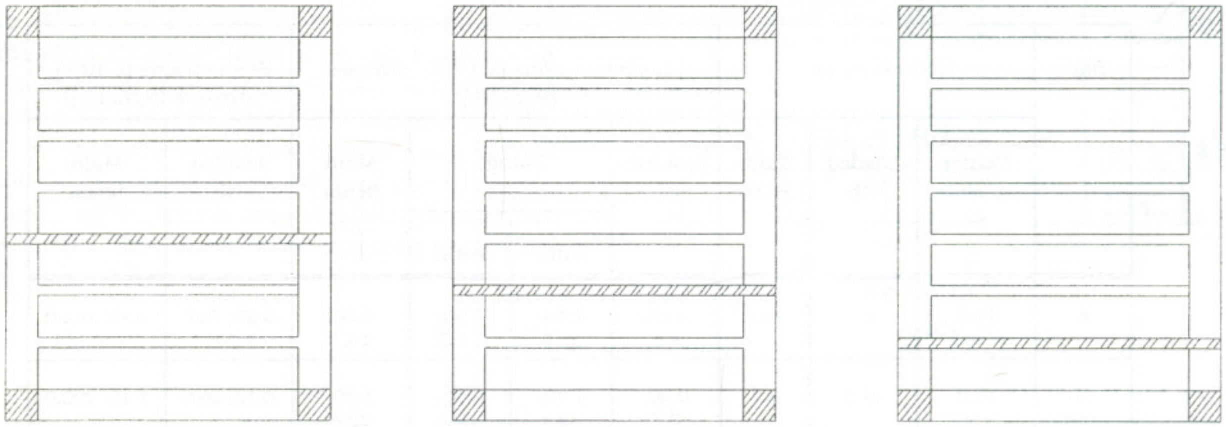


Figure 2-b Elements for ribs and flange



CASE B

Loaded rib : No. 3

CASE C

Loaded rib : No. 2

CASE D

Loaded rib : No. 1

Figure 3 Cases of Loading

Table 1 Results of the analysis

Slab	Loading	Cracking load			Load at first steel yield			Ultimate capacity
		(1)	(2)	(3)	(1)	(2)	(3)	
A	uniformly distributed, kN/m^2	2.5	12.5	7.5	11.25	----	----	15.0
B	line load, rib no. 3, kN/m	5.0	21.0	14.0	19.0	---	---	21.0
C	line load, rib no. 2, kN/m	5.0	17.0	16.0	18.0	20	----	20.0
D	line load, rib no. 1, kN/m	6.0	20.0	22.0	26.0	----	----	26.0

Table 2 Values of deflection (mm)

Slab	load	center or loaded rib		main beam	
		SAP90	FE	SAP90	FE
A	2.5 kN/m^2	0.495	0.608	0.247	0.296
B	4.0 kN/m	0.442	0.555	0.211	0.244
C	4.0 kN/m	0.337	0.417	0.151	0.179
D	4.0 kN/m	0.196	0.233	0.056	0.072

Table 3 Deflections, strains and stresses at ultimate load

Slab	Deflection, mm			Concrete strains (x 10^{-3}) , stresses (N/mm^2)^a				Steel strains (x 10^{-3}), stresses (N/mm^2)^b	
	Center of slab	Loaded rib	Main beam	Loaded rib	Flange		Main beam	Loaded rib	Main beam
					y-dir.	x-dir.			
A	16.4	---	10.4	0.46, 8.6	1.40, 28.0	0.50, 17.5	1.00, 18.1	0.35, 70.0	0.81, 162.0
B	11.2	11.2	8.7	0.20, 3.7	1.70, 32.4	0.60, 15.2	1.30, 22.4	0.13, 26.0	1.10, 220.0
C	5.4	10.8	2.9	1.50. 13.5	1.40, 31.1	0.90, 34.0	0.60, 13.38	1.20, 240.0	0.50, 100.0
D	1.5	4.2	1.1	0.30, 7.0	1.20, 25.2	0.35, 13.0	1.10, 20.0	0.25, 50.0	0.86, 172.6

a = compressive strains and stresses, b = tensile strains and stresses

RESULTS AND DISCUSSIONS

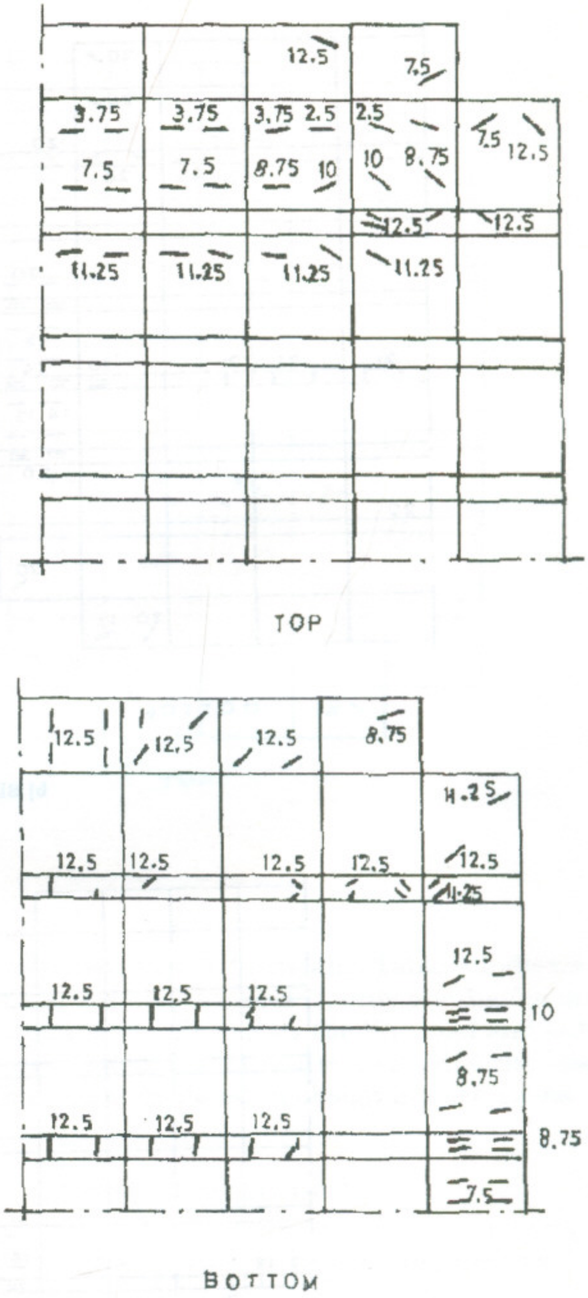
Tables 1 and 3 give a summary of the main results obtained from the Finite Element analysis.

Concrete Cracking

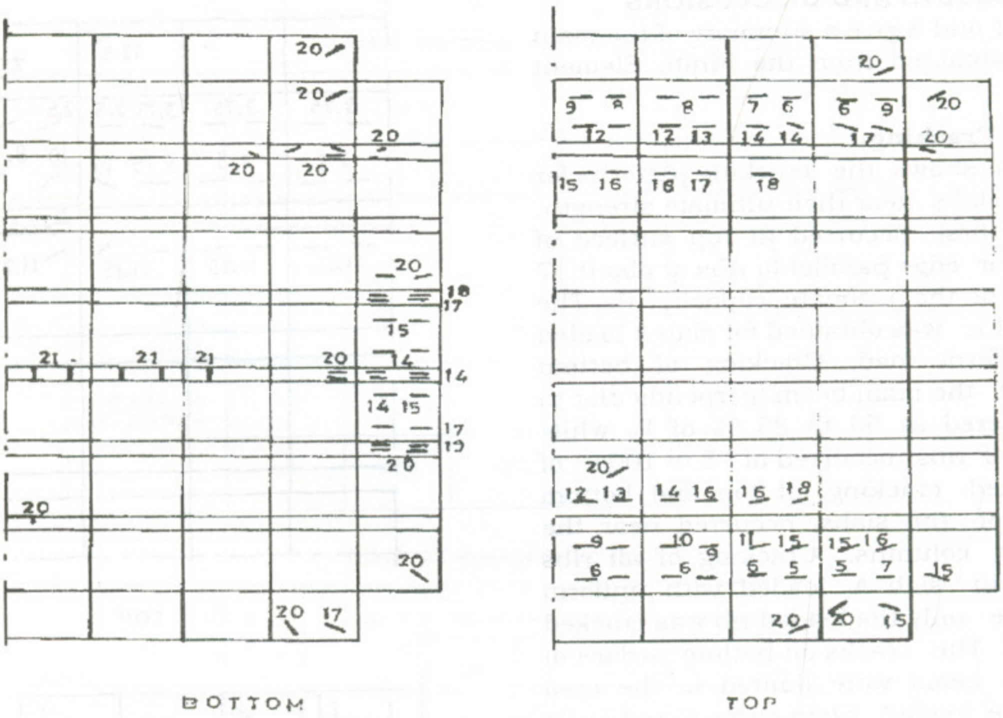
Figure 4 shows the cracking patterns for the four slabs near their ultimate strength. Cracking first occurred at top surface of flange near edge parallel to ribs at about 17 to 25 % of the ultimate capacity P_u. The lowest value was obtained for slab A loaded with uniform load. Cracking of bottom surface of the main beam (perpendicular to ribs) occurred at 50 to 85 % of P_u while cracking of ribs occurred at 75 to 100 % of P_u. Inclined cracking, at top and bottom surfaces of the slabs, occurred near the supporting columns. Cracking of all ribs occurred in slab A, loaded with uniform load, while only the loaded rib was cracked in slab B. The cracks on bottom surface of the main beam were limited to the area around the loaded rib in cases C and D. At the ultimate load of slab D, cracks occurred on top surface of the flange around the loaded rib especially at its connection with the main beam.

Ultimate Strength

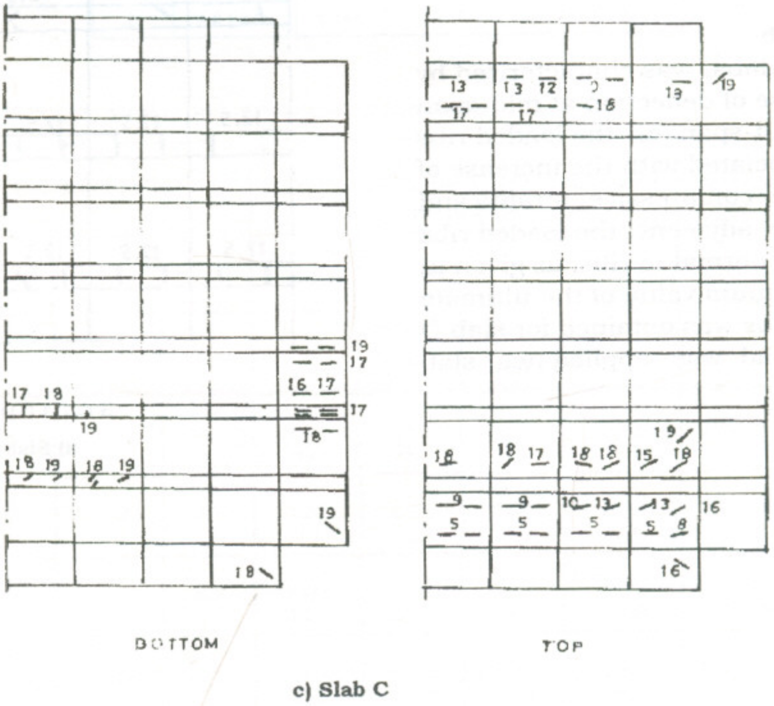
Failure of the slabs was characterized by the large increase of deflection at mid-span (slab A) or at mid-span of the loaded ribs (slabs B to D) associated with the increase of the values of the compressive strains and stresses in flange adjacent the loaded ribs and in direction normal to ribs, as given in Table 3. The maximum value of the ultimate capacity of the slabs was obtained for slab D where the line load was applied near slab edge.



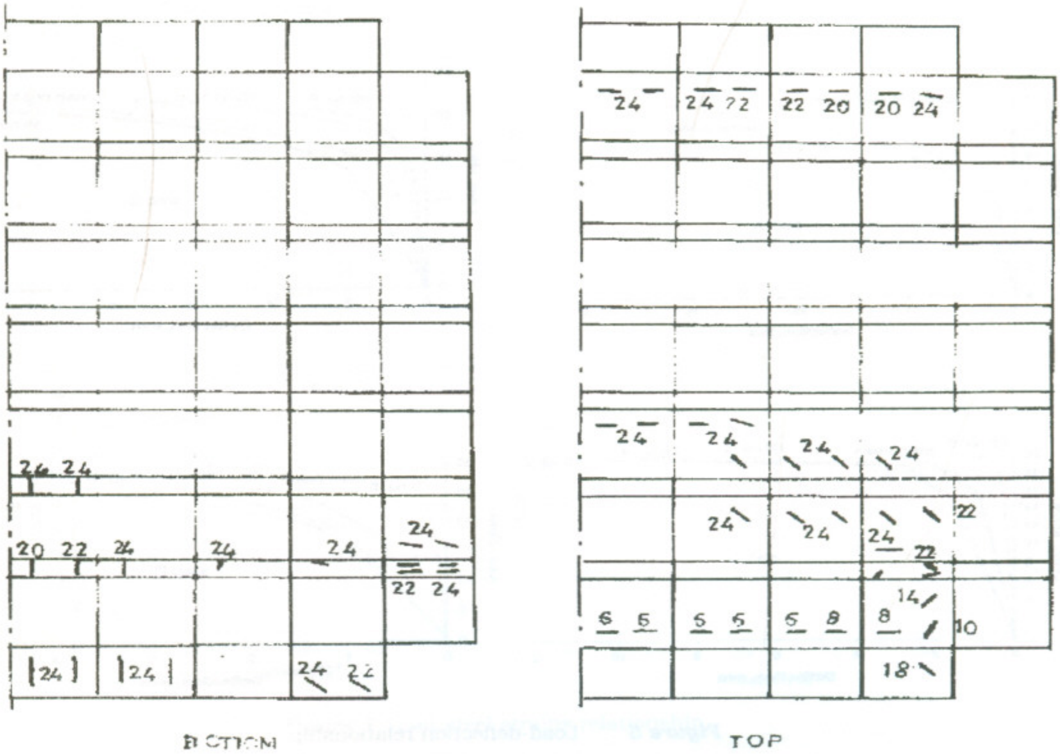
a) Slab A



b) Slab B



c) Slab C



(d) Slab D

Figure 4 Cracking patterns for slabs close to ultimate loads

Deflection of slabs

The load-deflection relationships for all the slabs are shown in Figure 5. Slab A deformed under uniform load into a dished surface rather than a cylindrical one. The values of deflection, at ultimate load, decreased as the line load moved away from the slab center.

As shown in Figure 6, the deflection pattern for slab B was similar to that for slab A, while for slabs C and D, the high values of deflection were limited for only one-half of the slab length.

The ratios of the maximum value of deflection of the loaded rib at mid-span to that of other ribs are given in Table 4.

As given in Table 4, as the line load moved away from the slab center towards the edge, the values of deflection of other ribs decreased drastically. Table 4 indicates that, for slabs C and D, only one-half of the slabs was affected by the line load and only the loaded rib with the two adjacent ribs may be considered in design in such cases

Table 4 Deflection results

Slab	Ratio of maximum deflection of loaded rib to that of rib no. :					
	1	2	3	4	5	6
A	2.1	1.3	1	1	1.3	2.1
B	2.2	1.3	1	1.1	1.6	3.6
C	1.1	1	1.6	2.6	4.7	12
D	1	1.3	2.1	3.4	5.8	15.5

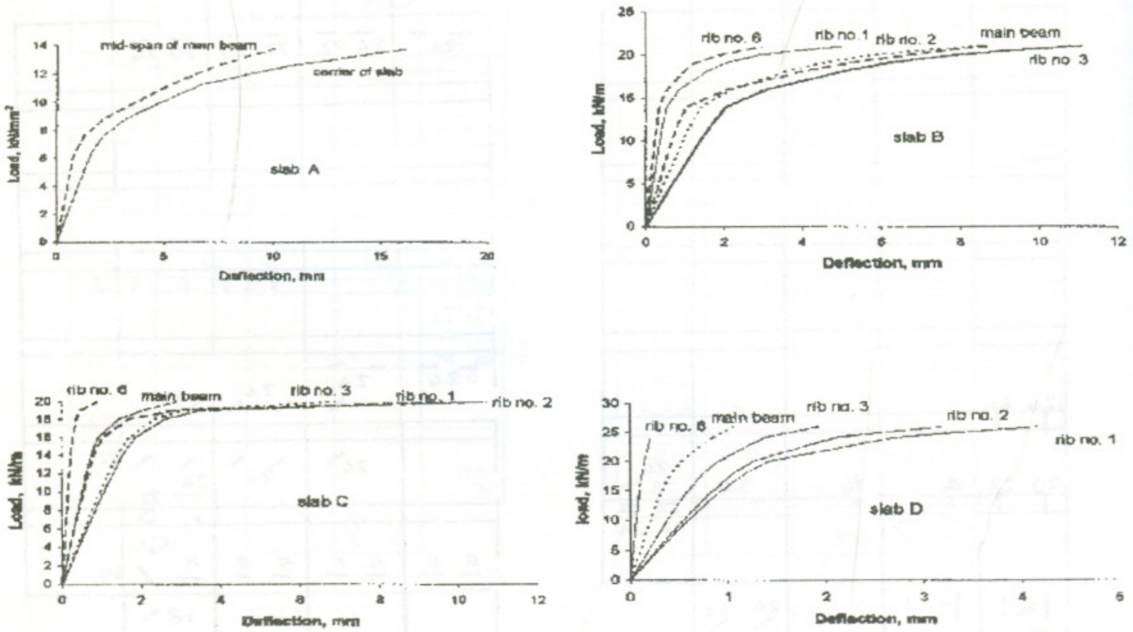


Figure 5 Load-deflection relationship

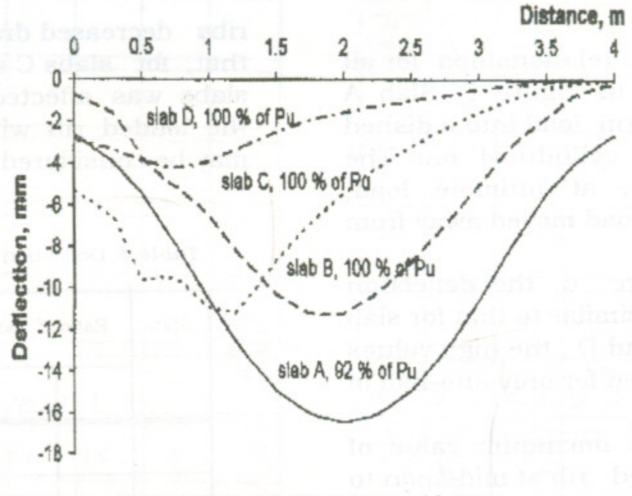


Figure 6 Variation of defletion along mid-span

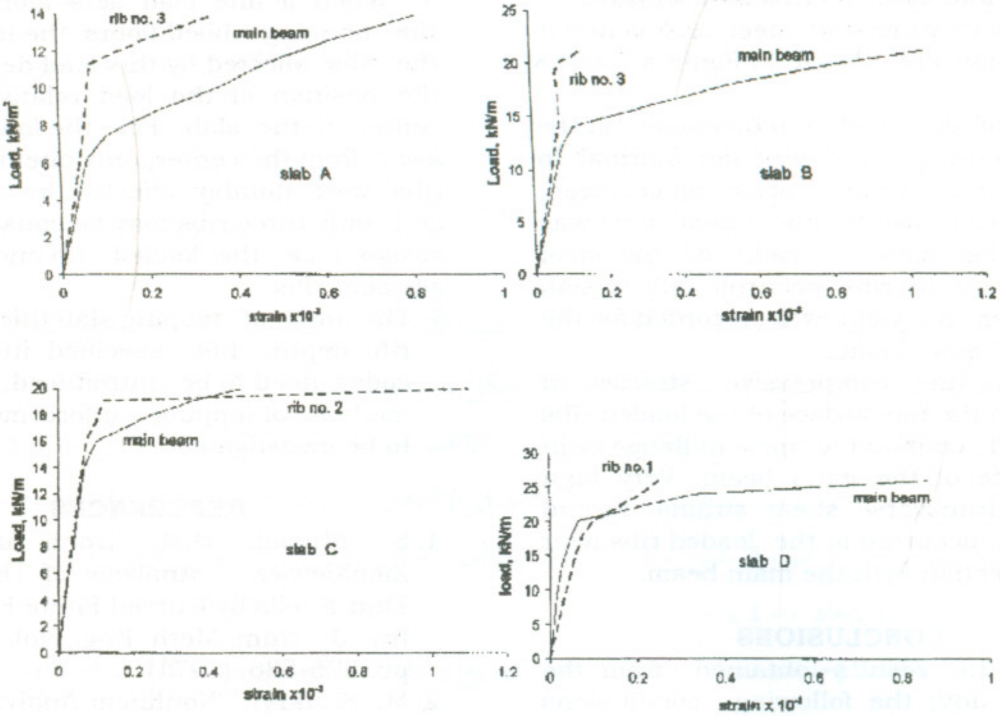


Figure 7 Load-steel strains relationship

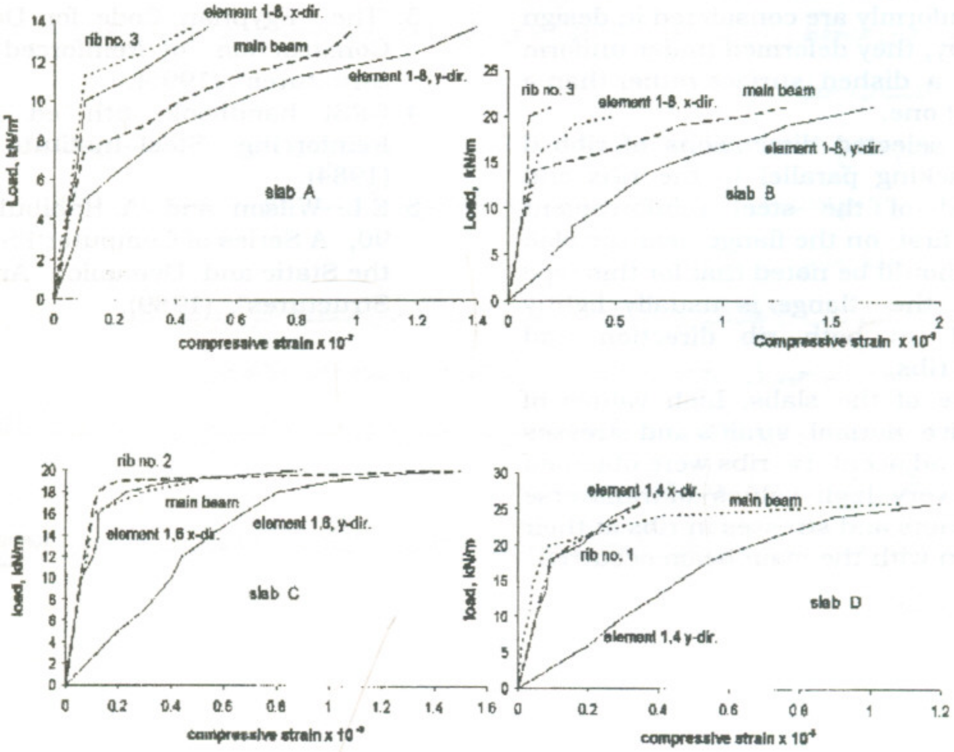


Figure 8 Load-concrete strains relationship

Concrete and steel strains and stresses

The load-strains in steel and concrete relationships are shown in Figures 7 and 8 respectively.

Yield of the steel reinforcement in the flange occurred in y-direction (normal to ribs) near edge, where cracks first occurred, at 73 to 100 % of P_u, the lowest value was recorded for slab A. Yield of the steel reinforcement in ribs occurred only in slab C. However, no yield was recorded for the steel in the main beam.

Generally, the compressive stresses in concrete in the top surface of the loaded ribs were small cmpared to those in flange or in top surface of the main beam. Very high values of transverse shear strains e_{yz} and stresses t_{yz} occurred at the loaded ribs near their connection with the main beam.

CONCLUSIONS

From the results obtained from the present study, the following conclusions could be drawn :

1- Although the ribbed slabs supported on beams having same depth of ribs and loaded uniformly are considered in design as one-way, they deformed under uniform load into a dished surface rather than a cylindrical one.

2- For the selected dimensions of ribbed slabs, cracking parallel to the ribs and also yield of the steel reinforcement occurred first on the flange near the slab edge. It should be noted that for this type of slabs, the flange is usually lightly reinforced in both rib direction and normal to ribs.

3- At failure of the slabs, high values of compressive normal strains and stresses in flange adjacent to ribs were obtained and also very high values of transverse shear strains and stresses in ribs at their connection with the main beam occurred.

4- When a line load acts along a rib of the one-way ribbed floors, the number of the ribs affected by this load depends on the position of the load relative to the center of the slab. For the loads acting away from the center, only one half of the ribs were notably affected by this load and only three ribs may be considered in design ; i.e. the loaded rib and the two adjacent ribs.

5- The ratio of topping slab thickness to rib depth, not specified in present codes, need to be introduced, and method of topping reinforcement need to be investigated.

REFERENCES

1. S. Ahmad, B.M. Irons and O.C. Zienkiewicz, " Analysis of Thick and Thin Shells by Curved Finite Element", Int. J. Num. Meth. Eng., Vol. 3, No. 4, pp. 575-586, (1971).
2. M. Shukry, "Nonlinear Analysis of RC Slabs with Openings", Alexandria Engineering Journal, Vol. 32, No. 1, pp. C41-49, (1993).
3. "The Egyptian Code for Design and Construction of Reinforced Concrete Structures", (1995).
4. CRSI handbook, 6th ed. ,Concrete Reinforcing Steel Institute, Chicago, (1984).
5. E.L. Wilson and A. Habibulloh, " SAP 90, A Series of Computer Programs for the Static and Dynamic Analysis of Structures", (1989).

Received October 2, 1997
Accepted May 6, 1998

تحليل البلاطات الخرسانية المسلحة ذات الأعصاب

محيى الدين صلاح شكرى

هندسة أنشائية - جامعة الاسكندرية

ملخص البحث

يتناول هذا البحث دراسة عددية باستخدام طريقة العناصر المحددة لتحليل البلاطات الخرسانية المسلحة ذات الأعصاب فى الاتجاه الواحد والبلاطات التى تم تحليلها فى هذا البحث مربعة الشكل ذات أبعاد ٤,٠٠ x ٤,٠٠ مترا ذات أعصاب فى اتجاه واحد بارتفاع كلى ٢٥٠ مم وعرض ١٠٠ مم والمسافه بين محاور الأعصاب ٥٠٠ مم والتسليح السفلى لهم ٢ ϕ ١٣مم بينما كان سمك شفه الضغط ٥٠ مم وبتسليح ϕ ٨ كل ٢٠٠مم فى الاتجاهين. وترتكز الأعصاب على كمرتين رئيسئتين ذات بحر فعال ٣,٧مترا وبعرض ٤٠٠ مم ولها نفس ارتفاع الأعصاب (أى ٢٥٠ مم) وبتسليح سفلى ٥ ϕ ١٦مم. وترتكز الكمرتين الرئيسيتين على أربعة أعمدة بأبعاد ٤٠٠ x ٣٠٠ مم.

وقد تمت دراسة أربعة بلاطات إحداهم معرضة لأحمال منتظمة التوزيع والثلاث الأخريات معرضة لأحمال خطية موازية للأعصاب وفى أماكن مختلفة بالنسبة لمنتصف البلاطات.

وقد تم عمل برنامج لاخطى على الحاسب الآلى لتحليل هذه البلاطات وللتحقق من صحة التحليل المقترح تمت مقارنة بعض النتائج عند مستوى أحمال منخفض (أى قبل حدوث التشرخ فى الخرسانة) مع النتائج التى تم الحصول عليها من برنامج SAP90 وهو برنامج لتحليل المنشآت خطيا.

وتم تقديم ومناقشة بعض نتائج التحليل وهى الترخيم والتشكلات، شكل التشريخ فى الخرسانة للسطحين السفلى والعلوى للبلاطات، الانفعالات والاجهادات فى كل من الخرسانة وحديد التسليح، وكذلك المقاومة القصوى للبلاطة.

وقد أوضحت نتائج التحليل مايلى:

١- بالرغم من أن البلاطات ذات الأعصاب فى الاتجاه الواحد المعرضه لأحمال منتظمة التوزيع يتم تصميمها ككمرات بقطاع T فى اتجاه واحد إلا أن شكل الترخيم فى البلاطات كان أشبه بالبلاطات المصمته ذات الاتجاهين.

٢- حدوث شروخ فى الخرسانة موازية للأعصاب وكذلك حدوث الخضوع فى صلب التسليح فى شفة الضغط بالقرب من طرف البلاطات وبدء الانهيار للبلاطات فى هذه الأماكن.

٣- عند تعرض البلاطات لحمل خطى على طول العصب فان عدد الأعصاب المتأثرة بهذا الحمل يتوقف على مكان الحمل الخطى بالنسبة لمنتصف البلاطة.