TORSION ON HOMOGENEOUS ANISOTROPIC CYLINDRICAL PRISMATIC BAR

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ABSTRACT

In modern industry, there are many kinds of anisotropic materials that are natural like wood or synthetic like polymers, Please cancel, or reinforced plastics. These non-metallic materials are superior to metallic materials in different practical applications, because of insulation to electricity or heat, stain-resistance, lightness, etc. This work introduces a numerical iterative solution for pure torsion equation of a continuously homogeneous prismatic bar, which is cut longitudinally from a solid cylinder, having a material with rectilinear anisotropy. The problem is constructed using a small physical parameter, which characterizes the material anisotropy. A mathematical model is derived using Fourier series analysis, and the stress components are obtained as conversion expansions. A simple technique is used to avoid singular points. A computer program is designed, and illustrative examples for orthotropic and non-orthotropic materials are introduced to show the effectiveness of the suggested solution. Acceptable numerical results are obtained, and convergence is discussed.

Keywords: Pure torsion, Cylindrical prismatic bar, Rectilinear Anisotropy, Stress components.

INTRODUCTION

The theory of generalized torsion was first worked out by Voigt, and the rigorous theory of pure torsion was developed by Saint-Venant. There are several works on the theory of pure torsion, and among these; a large monograph mentioned by Lekhnitskii [1], Mamrilla [2] and Sarkisyan[3], who were interested in torsion on homogeneous and non-homogenous anisotropic bodies. The Problem of torsion on homogeneous prismatic bar which is cut from hollow cylinder having rectilinear anisotropy is stated and mathematical model is suggested to solve the problem [4]. In Reference 5, the solution is rearranged in a numerical form and computer program is designed to study convergence and stability of the suggested solution which fails in solving the problem at the singular points where the prism's angle equal $\pi/2$ or $3\pi/2$ also when the prism is cut from solid cylinder. This work is an improvement to the solution in Reference 4 for a prism cut from solid cylinder also singular points were taken in consideration.

STATEMENT OF THE PROBLEM

Consider the continuously homogeneous prismatic bar, which is cut from a solid cylinder having radius b, the prism's cross section is a circular sector having angle α. The origin of coordinates is at the center of the edge cross section which lies at XY plane, and Z axis coincides with the axis of the solid cylinder. The cylinder's material has rectilinear anisotropy and the planes of the cross sections are planes of elastic symmetry. The forces being distributed over the ends are reduced at either of them to a twisting moment Mt. Four stress components out of six are zeros : $\sigma_x = \sigma_y = \sigma_z = \tau_{xy} = 0$, and the others are related to the stress function $\psi(x,y)$ as :

$$\tau_{xz}(x,y) = \frac{\partial\psi(x,y)}{\partial y}, \text{and } \tau_{yz}(x,y) = -\frac{\partial\psi(x,y)}{\partial x} \quad (1)$$

where, σ_x, σ_y and σ_z are the normal stress components and τ_{xy}, τ_{xy} and τ_{yz} are the tangent stress components. The stress function $\psi(x, y)$ satisfies the second-order partial differential Equation

$$a_{44}\frac{\partial^2\psi}{\partial x^2} - 2a_{45}\frac{\partial^2\psi}{\partial x\partial y} + a_{55}\frac{\partial^2\psi}{\partial y^2} = -2\vartheta \quad (2)$$

Where a_{44} and a_{55} are coefficients of elasticity and ϑ is the angle of twist per unit length [1]. The torsion rigidity C_t is given by:

$$C_t = \frac{2}{\vartheta}\oint\psi(x,y).dx.dy \quad (3)$$

The stress function $\psi(x,y)$ vanishes on the contour of the cross section

$$\psi(x,y)\big|_{contour} = 0 \quad (4)$$

A SUGGESTED SOLUTION TECHNIQUE

Equation 2 is transformed from Cartesian coordinates into polar coordinates as follows:

$$\psi(x,y) = \phi(r,\theta)$$
$$T[\phi(r,\theta)] + \delta.S[\phi(r,\theta 0\} = -A_o \quad (5)$$

where T[ ] and S[ ] are two differential operators,

$$T[\ ] = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}, \text{ and}$$

$$S[\ ] = a_1(\theta)(\frac{\partial^2}{\partial r^2} - \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}) - 2a_2(\theta)\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial\theta}),$$

$a_1(\theta) = \cos(2\theta) - K_1\sin(2\theta)$, and
$a_2(\theta) = \sin(2\theta) + K_1\cos(2\theta)$
where δ is a small physical parameter that is always less than unity [3,4]

$$\delta = \frac{a_{44} - a_{55}}{a_{44} + a_{55}},\ 0 \le \delta < 1,\ K_1 = \frac{2a_{45}}{a_{44} - a_{55}}$$

and $A_0 = \dfrac{4\vartheta}{a_{44} + a_{55}}$

and the stress components, and torsion rigidity are given as:

$$\tau_{xz}(r,\theta) = \sin(\theta)\frac{\partial\phi(r,\theta)}{\partial r} + \frac{\cos(\theta)}{r}\frac{\partial\phi}{\partial\theta}, \quad (6\text{-a})$$

$$\tau_{yz}(r,\theta) = -\cos(\theta)\frac{\partial\phi(r,\theta)}{\partial r} + \frac{\sin(\theta)}{r}\frac{\partial\phi(r,\theta)}{\partial r},$$

and (6-b)

$$C_t = \frac{2}{\vartheta}\iint_{domain}\phi(r,\theta).rdr.d\theta \quad (6\text{-c})$$

Solution of Equation 5 is assumed in the form [2,3]

$$\phi(r,\theta) = \eta(\phi_0(r,\theta) + \delta.\phi_1(r,\theta) + \delta^2.$$
$$\phi_2(r,\theta) + \dots\dots\dots) = \eta\sum_{j=0,1,2,3,\dots}^{\infty}\delta^j.\phi_j(r,\theta) \quad (7)$$

Substituting from Equation 7 into Equation 5 and equating coefficients of δ^j, (j=0,1,2,...) we obtain the differential equations

$$T[\phi_0] = \frac{-A_0}{\eta} = f_0, \quad (8)$$
$$T[\phi_1] = -S[\phi_0] = f_1(r,\theta), \quad (9)$$
$$T[\phi_2] = -S[\phi_1] = f_2(r,\theta) \quad (10)$$

in general $\Gamma[\phi_j] = -S[\phi_{j-1}] = f_i(r,\theta)$ and $f_o = -A_o/\eta$.
The boundary conditions in Equation 4 will be:

$$\phi_j(b,\theta) = 0 \quad \text{and} \quad \phi_j(r,0) = \phi_j(r,\alpha) = 0$$

Substituting from Equation 7. into Equation 6, the stress components and torsion rigidity will be

$$\tau_{xz}(r,\theta) = \eta(\tau_{0xz}(r,\theta) + \delta.\tau_{1xz}(r,\theta) + \delta^2.$$
$$\tau_{2xz}(r,\theta) + \dots\dots\dots) = \eta\sum_{j=0,1,2,3,\dots}^{\infty}\delta^j.\tau_{jxz}(r,\theta) \quad (11)$$

$$\tau_{yz}(r,\theta) = \eta(\tau_{oyz}(r,\theta) + \delta.\tau_{1yz}(r,\theta) + \delta^2.$$
$$\tau_{2yz}(r,\theta) + \dots\dots\dots) = \eta\sum_{j=0,1,2,3,\dots}^{\infty}\delta^j.\tau_{jyz}(r,\theta) \quad (12)$$

and

$$C_t = \eta(C_{0t} + \delta.C_{1t} + \delta^2.C_{2t} + \dots\dots)$$
$$= \eta\sum_{j=0,1,2,3,\dots}^{\infty}\delta^j.C_{jt} \quad (13)$$

where the first approximation in Equations 11, 12, and 13 are

$$\tau_{0yz}(r,\theta) = -\cos(\theta)\frac{\partial\phi_0(r,\theta)}{\partial r} + \frac{\sin(\theta)}{r}\frac{\partial\phi_0(r,\theta)}{\partial\theta}, \quad (14\text{-a})$$

$$\tau_{0xz}(r,\theta) = \sin(\theta)\frac{\partial\phi_0(r,\theta)}{\partial r} + \frac{\cos(\theta)}{r}\frac{\partial\phi_0(r,\theta)}{\partial\theta}, \quad (14\text{-b})$$

and

$$C_{0t} = \frac{2}{\vartheta}\int_0^{\alpha}\int_a^{b}\phi_0(r,\theta).rdrd\theta \quad (14\text{-c})$$

Now Equation 8 is solved for the first approximation $\phi_0(r,\theta)$ using Fourier series (sine half-range expansion) for both f_0 and $\phi_0(r,\theta)$ as:

$$f_0 = \sum_{k=1,3,5,....}^{\infty} b_k.\sin(\lambda_k\theta)$$

$$\phi_0(r,\theta) = \sum_{k=1,3,5,....}^{\infty} R_k(r).\sin(\lambda_k\theta)$$

where $b_k = \frac{1}{k}$, $\eta = \frac{-169}{\pi(a_{44}+a_{55})}$, and

$$\lambda_k = \frac{\pi k}{\alpha} \quad (15)$$

Substituting from Equations 15 into Equation 8 and obtaining the ordinary differential Equation

$$\frac{d^2R_k(r)}{dr^2} + \frac{1}{r}\frac{dR_k(r)}{dr} - \frac{\lambda_k^2}{r^2}R_k(r) = b_k \quad (16)$$

which has a complete solution in the form

$$R_k(r) = C_k.r^{\lambda_k} + D_k.r^{-\lambda_k} + V_k(r) \quad (17)$$

and $\phi_0(r,\theta)$ in Equation 15 will be in the form

$$\phi_0(r,\theta) = \sum_{k=1,3,5,....}^{\infty} (C_k.r^{\lambda_k} + D_k.r^{-\lambda_k} + V_k(r)).\sin(\lambda_k\theta)$$

where

$$C_k = \frac{-V_k(b)}{b^{\lambda_k}} \; D_k=0, \; V_k(r) = V_K.r^2,$$

and $V_k = \frac{1}{k(4-\lambda_k^2)}$, $\lambda_k \neq \pm 2$ or

$$\phi_0(r,\theta) = B_0 \sum_{k=1,3,5,...} B_k(\rho^2 - \rho^{\lambda_k}).\sin(\lambda_k\theta),$$

where

$$\rho = \frac{r}{b}, \quad B_0 = \frac{-b^2\pi}{\alpha}$$

$$B_k = \frac{1}{\lambda_k(\lambda_k^2-4)} \quad (18\text{-a})$$

Substituting from equation 18-a into Equation14, obtaining

$$\tau_{0yz}(r,\theta) = \frac{B_0}{2}\sum_{k=1,3,5,...}^{\infty} B_k\{-\sin((\lambda_k-1)\theta).L_1 + \sin((\lambda_k+1)\theta).L_2\} \quad (18\text{-b})$$

$$\tau_{0xz}(r,\theta) = \frac{B_0}{2}\sum_{k=1,3,5,....}^{\infty} B_k\{.\cos((\lambda_k-1)\theta).L_1 + \cos((\lambda_k+1)\theta).L_2\} \quad (18\text{-c})$$

where

$$L_1 = \frac{r(\lambda_k+2)}{b^2} - \frac{2\lambda_k r^{\lambda_k - 1}}{b^{\lambda_k}}, \text{and} \quad L_2 = \frac{r(\lambda_k-2)}{b^2}$$

$$C_{0t} = B_0 b^2 \sum_{k=1,3,5,....}^{\infty} \frac{1}{\lambda_k^2(\lambda_k+2)^2} \quad (18\text{-d})$$

$\phi_0(r,\theta)$ in Equation 18-a is substituted in the partial differential Equation 9 which is solved by the same technique for $\phi_0(r,\theta)$, then obtaining the second approximation in Equations 11, 12, and 13 as

$$\phi_1(r,\theta) = \sum_{n=2,4,6,...}^{\infty}(C_n.r^{\lambda_n} + V_n(r)).\sin(\lambda_n\theta) \quad (19\text{-a})$$

where $C_n = \frac{-V_n(b)}{b^{\lambda_n}}$

$$V_n(r) = \sum_{k=1,3,5,...}^{\infty} U_{n,k}.r^{\lambda_k} + Q_{n,k}.r^2,$$

$$\lambda_n = \frac{\pi n}{\alpha} \quad U_{n,k} = E(n,k)\frac{4C_k.\lambda_k(\lambda_k-1)}{\lambda_n^2 - \lambda_k^2}$$

$$\lambda_n \neq \lambda_k$$

$$Q_{n,k} = \frac{V_k.\lambda_k}{4-\lambda_n^2}\begin{bmatrix} 2(F(n,k)-E(n,k)) \\ -\lambda_k(F(n,k)+E(n,k)) \end{bmatrix}$$

$$F(n,k) = \frac{1}{2}\left(\frac{-\sin(\alpha_1)-K_1.\cos(\alpha_1)}{\alpha_1} + \frac{\sin(\alpha_2)-K_1.\cos(\alpha_2)}{\alpha_2} + \frac{2K_1.\pi n}{\alpha_1.\alpha_2}\right)$$

$$E(n,k) = \frac{1}{2}\left(\frac{\sin(\alpha_3)+K_1.\cos(\alpha_3)}{\alpha_3} + \frac{-\sin(\alpha_4)+K_1.\cos(\alpha_4)}{\alpha_4} - \frac{2K_1.\pi n}{\alpha_3.\alpha_4}\right)$$

$\alpha_1 = \pi(n + K) + 2\alpha$, $\quad \alpha_2 = \pi(n - K) - 2\alpha$,

$\alpha_3 = \pi(n - K) + 2\alpha$, $\quad \alpha_4 = \pi(n + K) - 2\alpha$,

$$\tau_{1yz} = \frac{1}{2}\sum_{n=2,4,6..}^{\infty} -\sin((\lambda_n-1)\theta)M_1 + \sin((\lambda_n+1)\theta)M_2 \quad (19\text{-b})$$

$$\tau_{1yz} = \frac{1}{2}\sum_{n=2,4,6..}^{\infty} -\cos((\lambda_n-1)\theta)M_1 + \cos((\lambda_n+1)\theta)M_2 \quad (19\text{-c})$$

$$M_1=2\lambda_n C_n\, r^{\lambda_n-1} + \sum_{k=1,3,5..}^{\infty} (\lambda_n+\lambda_k)U_{n,k}\, r^{\lambda_k-1} +$$

$$(\lambda_n+2)Q_{n,k}\, r$$

$$M_2 = \sum_{k=1,3,5..}^{\infty} (\lambda_n-\lambda_k)U_{n,k}\, r^{\lambda_k-1} + (\lambda_n-2)Q_{n,k}\, r, \text{ and}$$

$$C_{lt} = 0 \qquad \text{(19-d)}$$

$\phi_1(r,\theta)$ in Equation 19-a is substituted in the partial differential Equation 10 which is solved by the same technique for $\phi_2(r,\theta)$, then obtaining the third approximation in Equations 11, 12, and 13 as

$$\phi_2(r,\theta) = \sum_m^{\infty}(C_m.r^{\lambda_m} + V_m(r)).\sin(\lambda_m\theta) \qquad \text{(20-a)}$$

$$C_m = \frac{-V_m(b)}{b^{\lambda_m}},$$

$$V_m(r) = \sum_{n=2,4,6,....}^{\infty} B_1.r^{\lambda_n} + \sum_{k=1,3,5,....}^{\infty} B_2.r^{\lambda_k} + B_3.r^2,$$

$$\lambda_m = \frac{\pi m}{\alpha}$$

$$B_1 = \frac{-4C_n.\lambda_n.(\lambda_n-1).E(m,n)}{\lambda_n^2-\lambda_m^2}, \quad \lambda_n \neq \lambda_m$$

$$B_2 = \frac{-2U_{n,k}.(\lambda_n+\lambda_k)(\lambda_k-1)E(m,n)}{\lambda_k^2-\lambda_m^2} + \frac{2U_{n,k}.(\lambda_n+\lambda_k)(\lambda_k-1)F(m,n)}{\lambda_k^2-\lambda_m^2} + \frac{-4C_k.\lambda_k(\lambda_k-1)H_{m,k}}{\lambda_k^2-\lambda_m^2}$$

$$B_3 = \frac{-2Q_{n,k}.(\lambda_n+2)E(m,n)}{4-\lambda_m^2} + \frac{2Q_{n,k}.(\lambda_n-2)F(m,n)}{4-\lambda_m^2}$$

$$+\frac{V_k.\lambda_k(4-\lambda_k^2)G_{m,k}}{(4-\lambda_m^2)(2+\lambda_k)} + \frac{-V_k.\lambda_k(4-\lambda_k^2)H_{m,k}}{(2-\lambda_k)(4-\lambda_m^2)}$$

$$\lambda_k \neq \lambda_m \quad ,\lambda_m \neq \pm 2$$

$$G_{m,k} = \frac{1-K_1^2}{4}\left(\frac{-\sin(\gamma_1)}{\gamma_1} + \frac{\sin(\gamma_2)}{\gamma_2}\right)$$

$$-\frac{K_1}{2}\left(\frac{\cos(\gamma_1)}{\gamma_1} + \frac{\cos(\gamma_2)}{\gamma_2} - \frac{2\pi m}{\gamma_1.\gamma_2}\right)$$

$$H_{m,k} = \frac{1-K_1^2}{4}\left(\frac{\sin(\gamma_3)}{\gamma_3} - \frac{\sin(\gamma_4)}{\gamma_4}\right)$$

$$+\frac{K_1}{2}\left(\frac{\cos(\gamma_3)}{\gamma_3} + \frac{\cos(\gamma_4)}{\gamma_4} - \frac{2\pi m}{\gamma_3.\gamma_4}\right)$$

$\gamma_1=\pi(m+k)+4\alpha$, $\gamma_2=\pi(m+k)+4\alpha$

$\gamma_3=\pi(m-k)+4\alpha$, $\gamma_4=\pi(m+k)-4\alpha$

$$\tau_{2yz} = \frac{1}{2}\sum_{m=1,2,3,....}^{\infty} -\sin((\lambda_m-1)\theta).S_1 + \sin((\lambda_m+1)\theta).S_2 \qquad \text{(20-b)}$$

$$\tau_{2xz} = \frac{1}{2}\sum_{m=1,2,3,....}^{\infty} \cos((\lambda_m-1)\theta).S_1 + \cos((\lambda_m+1)\theta).S_2 \qquad \text{(20-c)}$$

$$S_1 = 2\lambda_m.C_m.r^{\lambda_m-1} + \sum_{n=2,4,6,....}^{\infty} B_1(\lambda_m+\lambda_n)r^{\lambda_n-1} ++$$

$$\sum_{k=1,3,5,....}^{\infty} B_2(\lambda_m+\lambda_k)r^{\lambda_k-1} + B_3(\lambda_m+2)r$$

$$S_2 = \sum_{n=2,4,6,....}^{\infty} B_1(\lambda_m-\lambda_n)r^{\lambda_n-1} + \sum_{k=1,3,5,....}^{\infty}$$

$$B_2(\lambda_m-\lambda_k)r^{\lambda_k-1} + B_3(\lambda_m-2)r$$

$$C_{2t} = 4b^2 \sum_{m=1,3,5,....}^{\infty} \frac{1}{\lambda_m}\left(\frac{C_m.b^{\lambda_m}}{2+\lambda_m} + \sum_{n=2,4,6,....}^{\infty} \frac{B_1.b^{\lambda_n}}{2+\lambda_n}\right.$$

$$\left. + \sum_{k=1,3,5,....}^{\infty} \frac{B_2.b^{\lambda_k}}{2+\lambda_k} + \frac{B_3.b^2}{4}\right) \qquad \text{(20-d)}$$

and so on to calculate the fourth approximation, etc,..... .

AVOIDING SINGULIRITES

The preceding solution has singularities where sector angle αequal $\pi/2$ or $3\pi/2$ which deals to $\lambda_k=2$. To avoid this singularities, the solution is divided into two parts. The first one is the summation of the series's terms without the term at which $\lambda_k=2$, and the second part is the term at which $\lambda_k=2$, and Lo Hospital theorem is applied to obtain its value.

For sector angle α equal $\pi/2$ or $3\pi/2$ the stress function$\phi_0(r, \theta)$ in equation 18-a will be in the form

$$\phi_0(r,\theta) = \phi_0(r,\theta)_{|\lambda_k\neq 2} + \phi_0(r,\theta)_{|\lambda_k=2} \qquad (21)$$

where

$$\phi_0(r,\theta)_{|\lambda_k\neq 2} = B_0 \sum_{k=1,3,5,..} B_k(\rho^2-\rho^{\lambda_k}).\sin(\lambda_k\theta), \text{ and}$$

$$\phi_0(r,\theta)_{|\lambda_k=2} = B_0 \lim_{\lambda_k=2}\{B_k(\rho^2-\rho^{\lambda_k})\sin(\lambda_k\theta)\}$$

$$= \frac{\pi b^2}{8\alpha}(\rho^2 \ln\rho).\sin(2\theta)$$

Therefore $\phi_0(r,\theta)$ in Equation 21 is substituted in the partial differential Equation 9 which is solved for $\phi_1(r,\theta)$, and the solution is completed.

ALGORITHM DEVELOPMENT

1- INPUT

i) Coefficients of elasticity (a_{44}, a_{55}, a_{45}) ,and angle of twist per unit length (ϑ).

ii) Sector's dimensions (b,α).

iii) Coordinates (r,θ) of the point at which stress components and torsion rigidity are calculated.

2- Calculate(δ,K_1) by using Equation 5, and(η,λ_K) using Equation 15.

3- Calculate V_k, V_k, (r),C_K) then $\phi_0(r, \theta)$ by Equation 18-a.

4- Calculate (L_1,L_2), then τ_{0xz} using Equation 18-b, τ_{0xz} using Equation 18-c, and C_{ot} using Equation 18-d.

5- Calculate (α_1, α_2, α_3, α_4), (F(n,k), E(n,k)), ($U_{n,k}$, $Q_{n,K}$, V_n(r) , (λn,cn) and $\phi_0(r, \theta)$ by using equation 19-a.

6- Calculate (M_1,M_2), τ_{lyz} using Equation 19-b, and τ_{lxz} using Equation 19-c.

7- Calculate (γ_1, γ_2, γ_3, γ_4),($G_{m,k}$, $H_{m,k}$) (F(m,n), E(m,n), (B_1,B_2,B_3),$V_m(r)$, (λ_m, C_m) and $\phi_2(r,\theta)$ using Equation 20-a.

8- Calculate (S_1=,S_2), τ_{2yz} using Equation 20-b, τ_{2xz} using Equation 20-c, and C_{2t} using Equation 20-d.

9- OUTPUT:

i) The stress function $\phi(r,\theta)$ using Equation 7.

ii) The stress components, $\tau_{xz}(r,\theta)$ using Equation 11, and $\tau_{yz}(r,\theta)$ using Equation 12.

iii) Torsion rigidity C_t using Equation 13.

A computer program is designed in FORTRAN language, and runs on personal computer. Double precision leads to faster convergence and stability than single precision. The following tables are chosen as samples of the obtained results.

CASE STUDIES

In the following, all numerical values are calculated in double precision form, all written results are performed to the fixed significant figures in the last two successive approximations , and at the point where $(r,\theta) = (\frac{b}{2},\frac{\alpha}{2})$.

EXAMPLE :

This example is introduced to show solution's convergence and stability, for orthotropic and nonorthotropic materials respectively.

a- Consider an orthotropic prism (a_{45}=0) having sector dimensions as; b= unit length,$\alpha=\pi/4$.

Tables 1 and 2 give double precision numerical values for the first and third approximations for stress function, stress components and torsion rigidity against truncation number, as in Equations 18, and 20 respectively.

Table 1 Numerical values for the first approximation for stress function, stress components and torsion rigidity against truncation number for orthotropic prism.

k	$10x\phi_0$	k	$10x\tau_{0xz}$	k	$10x\tau_{0yz}$	k	$10xC_{ot}$
5	-0.151	7	-0.151	3	0.36	7	-0.711
11	-0.1512	13	-0.1518	7	0.366	13	-0.7116
43	-0.15124	53	-0.15184	23	0.3665	29	-0.71169
97	-0.151249	75	-0.151840	39	0.36657	49	-0.711699

In Equations 19-a, to 19-d d, for k=100, and n=48 the results $\phi_1 \approx 0.0$, $\tau_{1xz} = 0.01020$, $\tau_{1yz} = 0.00422$, and C_{1t}=0.0 are obtained. No more accuracy is gained from n=48 to n=100.

Table 2 The first approximation is always independent of the coefficients of elasticity for orthotropic or non-orthotropic materials. k=100, n=100

M	$10^4x\phi_2$	M	$100x\tau_{2xz}$	M	$100x\tau_{2yz}$	M	10^3xC_{2t}
5	-0.1	14	-0.8	6	-0.5	27	-0.20
25	-0.15	66	-0.88	19	-0.58	39	-0.201
27	-0.159	88	-0.889	88	-0.583	79	-0.2017
49	-0.1593	100	-0.8893	100	-0.5831	95	-0.20177

b- Consider non-orthotropic prism $a_{45}\neq$ 0.0)having sector dimensions as; b=unit length, $\alpha=\pi/3$, and K_1=0.1.Tables 3-5 give double precision numerical values for the first, second and third approximations for stress function, stress components and torsional rigidity against truncation number, as in Equations 18, 19 and 20 respectively.

Table 3 the numerical values for the first approximation for stress function, stress components and torsion rigidity against truncation number for non-orthotropic prism.

k	$10\phi_0$	k	$10x\tau_{0xz}$	k	$10x\tau_{0xz}$	k	$100xC_{0t}$
5	-0.24	7	-0.232	5	0.40	9	-0.137
15	-0.2409	37	-0.23250	11	0.402	19	-0.13708
39	-0.24095	83	-0.232504	37	0.4027	77	-0.137088

Table 4 the numerical values for the second approximation for stress function, stress components and torsion rigidity against truncation number for non-orthotropic prism.

N	$10x\tau_{1xz}$	N	$10x\tau_{1yz}$
12	o.23	8	0.13
36	0.237	34	0.137
86	0.2376	98	0.1372

Table 5 the numerical values for the third approximation for stress function, stress components and torsion rigidity against truncation number for non-orthotropic prism.

M	$10x\phi_2$	M	τ_{2xz}	M	τ_{2yz}	M	C_{2t}
7	-0.31	6	0.170	3	-0.36	11	0.1514
23	-0.317	32	0.1702	6	-0.368	15	0.15144
91	-0.31789	78	0.1728	37	-0.3687	53	0.151444

CONCLUSIONS

A numerical method is introduced for solving the problem of pure torsion equation of a continuously homogeneous prismatic bar, cut longitudinally from solid cylinder, having a material with rectilinear anisotropy and a circular sector cross section. A mathematical model is derived using Fourier series analysis, and the stress components are obtained as conversion expansions dependent on a small physical parameter, which characterizes the material anisotropy. The solution is suitable for orthotropic and non-orthotropic materials with acceptable convergence and stability. Numerical results show that:

1- Whenever the physical parameter is smaller, the truncation error is smaller, and the number of approximating terms for acceptable results is fewer. Therefore, when the physical parameter is known, the accuracy of the solution can be roughly defined.
2- First approximation is always independent of coefficients of elasticity for orthotropic or non-orthotropic materials.
3- The series's summations for orthotropic materials are dependent on the prism's dimensions and independent of material's coefficients of elasticity.
4- Whenever dimensions are smaller the series converge faster and better, therefore a suitable length's unit is chosen for small errors.
5- Personal computer needs few seconds to calculate the above results in double precision.

NOMENCLATURE

M_t	Twisting moment
ψ,ϕ	Stress functions
ϑ	Angle of twist per unit length
τ_{xz}, τ_{yz}	Tangent stress components
C_t	Torsion rigidity
δ	Physical parameter

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لى على منشور أسطوانى متجانس له خواص متباينه اتجاهيا

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ملخص البحث

فى الصناعات الحديثه كثيراً ما تُستخدم المواد غير المعدنية ذات الخواص المتباينة أتجاهياً منها ما هو طبيعى مثل الأخشاب أو مُصنع مثل البوليمرات أو الأألياف الزجاجيه أو البلاستيك المقوى. تتميز هذه المواد عن المواد المعدنيـــة بخواص مفيدة مثل عزل الكهرباء و عزل الحرارة و مقاومة الصدأ و خفة الوزن. يقدم هذا البحث طريقة عددية تكرارية لحساب الأجهادات الناتجة من تأثير عزم التواء على منشور مقتطع طولياً من أسطوانة مصمتة مصنوعـــة مـــن مـــادة متجانسة و متباينة الخواص خطياً . و يعتمد البناء الرياضى للحل على معامل فيزيائى يعتمد على الخواص الإتجاهيه لمادة المنشور. و يفرض الحل فى صورة متسلسله تقاربيه تستنتج حدودها بطريقه تكراريه ، كما تستنتج مركبات الإجهاد فى صورة متسلسلات فوريير التقاربيه ، و أمكن معالجة الحل عند النقط المنفرده. تم تصميم برنامج للحاسب و بواســـطته أمكن حل المسأله على منشور مقتطع ممن ماده لها خواص متشابهه و أخرى لها خواص متباينه لتوضيح الحل المقـــترح و توضيح التقارب ، كما أمكن الحصول على نتائج عدديه مقبوله و مناقشتها.